# Topological Theory of Hopf Bundle and Mass

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**Abstract**

Why a particle has the specific rest mass it does is an open question. An alternative theory of mass is put forward. Mass is due to the intersection of a Hopf bundle and 3-space. The masses of six lighter hyperons and electron are derived as functions of the proton and neutron masses. Nine free parameters are thereby reduced to two. The most significant outcome is the derivation of the electron mass.

**Keywords:** hyperon, electron, hypersphere, Hopf, Higgs, mass splitting

In the Standard Model the Higgs field imparts mass to fundamental particles. In the crowd analogy the field acts like a throng impeding the progress of a celebrity across a room.[1] The slower the progress, the stronger the interaction, the heavier the particle. If we dig a little bit deeper, particles that exhibit internal Lie group symmetry at higher energy states gain mass when spontaneous symmetry breaking couples with the Higgs field.[2, 3] The caveat is the Higgs field interacts with quarks, leptons and some bosons, but not photons; while the bulk of Hadron mass is due to quark confinement and not the Higgs field. Unable to predict why a particle has the precise mass that it does, the Standard Model leaves particle mass an open question. An alternative theory of mass is put forward that rethinks why a particle notices a force. Symmetry preservation (not symmetry breaking) is the cause of mass. The intersection of the particle and field is also responsible for the entirety of a particle's mass. This simplifying premise enables the calculation of six light hyperons and electron as functions of the proton and neutron masses.

The topological theory considers a particle to be a Hopf bundle. The geometry is well understood.[4, 5, 6, 7] A Hopf bundle maps a 3-sphere to a 2-sphere. The 3-sphere is the set of four dimensional points $S^3$. The 2-sphere is a two dimensional surface described by the set of three dimensional points $S^2$. A Hopf fibration continuously maps $S^3$ to $S^2$. This is done with Hopf maps. A Hopf map ($h : S^3 \to S^2$) is a surjective function that maps a subset of $S^3$ elements to a point in $S^2$. An individual Hopf map describes a circle (Hopf circle). Continuous mapping entails an infinite number of maps for each point in $S^3$; this requires an infinite bundle of circles that in total connect each S2 point to every point in S3. The total space is transitive.

To this potted account of a Hopf bundle we bring a physical interpretation. A 'Hopf particle', as we shall call it, is also a 3-sphere that interacts with an ambient three dimensional space (3-space). Like the Higgs field, the 3-space is a field with a ground state. An external force is a vector in 3-space. On the surface of the Hopf particle the force applies to a point that also belongs to a bundle of Hopf circles. This raises the question of the differing topologies of circle and point. Continuous retraction is impossible. Only by *cutting* may the circle retract to a point. If a circle does not break, the force must *jump* topologies. On this view, a particle with topology that deform retracts to a point offers no resistance and is massless, whilst topological discontinuity is interpreted as physical resistance to change in location and speed. With the bundle of Hopf circles at point of contact related to every point in $S^3$, and a total space that is transitive, the size of the 3-sphere is the measure of resistance to an external force.

Five equations characterise Hopf particle mass. The first tells us mass is determined by the size of the 3-sphere. For example, if the mass of the proton is 938.272 MeV then $r \approx 3.622$ MeV. I.E.

$$M = 2\pi^2 r^3. \tag{1}$$

The volume of a 2-sphere is the space the Hopf particle occupies in the ambient 3-space. This is the volume of an ordinary ball.

$$V = \frac{2M}{3\pi} = \frac{4\pi}{3} r^3. \tag{2}$$

At Eq. (2), $r$ is the radius derived at Eq. (1). In the case of the proton $V \approx 199.108$ MeV. The 3-sphere's extra fourth dimension does not contribute to the 3-space volume; it is dark in the sense it is not a direction the ball can be forced to move within the limitations of 3-space.

$$\rho = \frac{M}{V} = \frac{3\pi}{2}. \tag{3}$$

Eq. (3) means the ball is hyper-dense. The excess mass, we call 'hypermass', is evidence of the extra dimension. Hypermass (H) is the difference between mass and volume.

$$H = M - V. \tag{4}$$

Hopf particle mass has the Hopf/hypermass signature (H-signature):

$$M = (H)(\frac{\rho}{\rho - 1}). \tag{5}$$

H-signatures found in the mass data suggest lighter hyperons are Hopf particles. For what follows the 2018 CODATA recommended values are used for the proton and neutron masses (ignoring the standard deviation).[8]

$$\begin{aligned} M_p &= 938.272\ 088\ 16\ \ \pm 0.000\ 000\ 29\ \mathrm{MeV}/c^2. \\ M_n &= 939.565\ 420\ 52\ \ \pm 0.000\ 000\ 54\ \mathrm{MeV}/c^2. \end{aligned} \tag{6}$$

All other masses derived in this paper are a function of $M_p$ and $M_n$. For instance, the light $\Sigma$ (Sigma) masses are the following functions.

$$M_{\Sigma^+} = (2M_p - M_n)(\frac{\rho}{\rho - 1}) \approx 1189.3712. \tag{7}$$

$$M_{\Sigma^0} = (M_n)(\frac{\rho}{\rho - 1}) \approx 1192.6546. \tag{8}$$

$$M_{\Sigma^-} = (4M_n - 3M_p)(\frac{\rho}{\rho - 1}) \approx 1197.5797. \tag{9}$$

All three derived values are close to the observed masses. The Particle Data Group (PDG) fit for $M_{\Sigma^+}$ is 1189.37 $\pm$0.07.[9] While the PDG fit for $M_{\Sigma^0}$ is 1192.642 $\pm$0.024, Eq. (8) is particularly close to Wang 1192.65 $\pm$0.020.[10] Eq. (9), however, is over four standard deviations shy of the PDG value (1197.449 $\pm$0.030). The present PDG fit for $M_{\Sigma^-}$ draws on three results. Schmidt (1197.43) and Gurev (1197.417) are too low to be the value derived here, though Eq. (9) is within one standard deviation of Gall (1197.532 $\pm$0.057).[11, 12, 13] The H-signatures for the $\Xi$ (Xi) pair introduce a complication that provides a way to check whether Eqs. (8, 9) are correct.

$$M_{\Xi^0} = (M_{\Sigma^0})(\frac{\rho}{\rho - 1}) - V_p \approx 1314.8104. \tag{10}$$

$$(M_{\Sigma^-})(\frac{\rho}{\rho - 1}) - V_p \approx 1321.0622. \tag{11}$$

Eq. (10) is within one standard deviation of the PDG fit and is close to Fanti (1314.82 $\pm$0.06)[14], but a problem looms. When the basic pattern of Eq. (10) is repeated at Eq. (11) the result (1321.0622) is over nine standard deviations adrift of the PDG fit for $M_{\Xi^-}$. The present PDG recommended value (1321.71 Mev) relies on a 2006 study of a large 1992-1995 data sample.[15] Realistically, the 2006 result makes a future nine standard deviation downward adjustment unlikely. Accepting Eq. (11) will not do, we are about to see why [15] is accurate.

If $M_{\Sigma^-}$ is close to 1321.71 a fudge $\approx$ 0.51 is needed to adjust the Eq. (11) value. The electron mass $\approx$ 0.511 MeV is an obvious candidate. For the moment we call the additional weighting value 'W'. I.E.

$$M_{\Xi^-} = (M_{\Sigma^-} + W)(\frac{\rho}{\rho - 1}) - V_p. \tag{12}$$

At face value $W$ appears ad hoc, but there is a firm reason for thinking otherwise. There are a few more equations to walk through before we can see why. First, we give the formula for the $\Omega^-$ (Omega) mass.

$$M_{\Omega^-} = \left(\frac{3M_{\Xi^0} + 2M_{\Xi^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right). \tag{13}$$

Given Eqs. (8, 9, 10, 12, 13), and using Eq. (2) and Eq. (13) to also find $V_{\Omega^-}$, we use the following equivalences to determine $W$.

$$\left(\frac{(M_{\Sigma^0})(M_{\Xi^-}) - (M_{\Sigma^0})(M_{\Xi^0})}{M_{\Sigma^-} - M_{\Sigma^0}} - M_{\Xi^0} - V_{\Omega^-}\right)\left(\frac{\rho - 1}{\rho}\right) = 1. \tag{14}$$

$$\left(\frac{(M_{\Sigma^-})(M_{\Xi^-}) - (M_{\Sigma^-})(M_{\Xi^0})}{M_{\Sigma^-} - M_{\Sigma^0}} - M_{\Xi^-} - V_{\Omega^-}\right)\left(\frac{\rho - 1}{\rho}\right) = 1. \tag{15}$$

Eqs. (14, 15) insist their mix of mass and volume are equivalent to the pure number $\frac{\rho}{\rho-1}$. If the equivalence holds true this is evidence for the topological theory of mass. The proximity of $W$ to $M_e$ gives support to this idea. When Eqs. (14, 15) = 1, $W \approx 0.510\ 998\ 961\ 080$. This compares to 2018 CODATA value 0.510 998 9500 $\pm$0.000 000 0015.[8] An adjustment within one standard deviation to either $M_p$ or $M_n$ at Eq. (6) allows $W$ to come within one standard deviation of the CODATA $M_e$ value. From this we conclude it is highly likely $W = M_e$ (MeV), and future adjustments to $M_p$ and $M_n$ will see $W$ and $M_e$ converge. If so, the mass value at Eqs. (8, 9, 10) are accurate to four decimal places, while the values for $M_{\Xi^-}$ and $M_{\Omega^-}$ are within one standard deviation of the PDG recommendation. I.E.

$$M_{\Xi^-} = (M_{\Sigma^-} + M_e)(\frac{\rho}{\rho - 1}) \approx 1321.7109. \tag{16}$$

$$M_{\Omega^-} \approx 1672.4824 \text{ (Eq. 13)}. \tag{17}$$

Before concluding, there is a question to clear up concerning which system of units is the right system to describe electron mass. In eV, Eqs. (14, 15) = $\frac{M_e\ eV}{M_e\ MeV}$ = 1,000,000; or in Kg, $\frac{M_e\ Kg}{M_e\ MeV} = 1.78 \cdot 10^{-30}$. It seems the formulae only resolve to 1 when the numerator is in MeV. It is difficult to believe nature privileges increments of one million electron volts. We find the answer lies in an obsolete cgs unit of magnetomotive force, the Gilbert (Gb).[16] As the unit of current in an electric circuit is the Volt, the Gilbert is a unit of magnetic flux in a magnetic circuit. The SI units for magnetomotive force are Ampere (A) and turn (tr). Turns are the winding number of an electromagnetic coil. The winding number is the number of times the coil wraps around a point. In SI units a Gilbert is equal to:

$$1\ Gb = \frac{10}{4\pi}\ A \cdot tr. \tag{18}$$

The magnetic permeability $\mu_0$ (mu zero) is proportional to the energy stored in a magnetic field.

$$\mu_0 \approx (4\pi)(10^{-7})\ N \cdot A^{-2}. \tag{19}$$

The revaluation of SI units in 2019 means $\mu$ is no longer an exact value. However, it is sufficiently close to the number $4\pi \times 10^{-7}$ for the difference to be negligible. Magnetic permeability is related to electric permittivity $\varepsilon_0$ (epsilon nought) by the following equivalence.

$$\varepsilon_0 = \frac{1}{\mu_0 c^2}. \tag{20}$$

$\varepsilon_0$ is proportional to the energy stored in an electric field. We divide a Gilbert by $\varepsilon_0$ and use Eq. 20 to simplify and parse dimensions.

$$\frac{1\ Gb}{\varepsilon_0} \approx (10^{-6})(c^2)\ N \cdot A^{-1} \cdot tr. \tag{21}$$

The arrangement of units of Eq. (21) converts mass denominated in eV/$c^2$ into rest energy described in Newton-Volt-turns, where $n$ is the number of electron volts and one turn is the winding number of a Hopf circle. I.E.

$$\left(\frac{n\ eV}{c^2}\right)\left(\frac{1\ Gb}{\varepsilon_0}\right) \approx n \times 10^{-6}\ N \cdot V \cdot tr. \tag{22}$$

The final value given in NVtr is numerically indistinguishable from MeV.

The discrepant topologies of point and circle offer an economical theory of mass, but not one that plays well with the Standard Model. The smattering of results presented here are a long way from a thorough-going theory, while the many questions left open make it easy to discount a challenge to the Standard Model. Nonetheless, the $\Sigma$, $\Xi$, $\Omega$ and electron masses are derived as functions of the proton and neutron. It is the first time this has been done.

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