Topological Theory of Hopf Bundle and Mass

Garry Goodwin
garry_goodwin@hotmail.co.uk

August 7, 2024(Revised)

Abstract

Why a particle has the specific rest mass it does is an open question. An alternative theory of mass is put forward. Rest mass is due to discrepant topologies found at the intersection of a Hopf bundle and 3-space. The rest masses of six lighter hyperons and electron are derived as functions of the proton and neutron, reducing nine free parameters to two. The most significant outcome is the derivation of the electron mass due to baryon mass splitting.

Keywords: Hopf fibration, Higgs field, topology, mass splitting

In the Standard Model the Higgs field imparts mass to fundamental particles. In the crowd analogy the field acts like a throng impeding a celebrity as they attempt to cross a room.[1] The slower the progress, the stronger the interaction and the heavier the particle. If we dig a little bit deeper, particles that exhibit internal Lie group symmetry at higher energy states gain mass when spontaneous symmetry breaking couples with the Higgs field.[2, 3] The caveat is the Higgs field interacts with quarks, leptons and some bosons, but not photons; while the bulk of Hadron mass is due to quark confinement and not the Higgs field. Unable to predict why a particle has the precise mass that it does, the Standard Model leaves particle rest mass an open question. An alternative theory of mass begins to address this problem by rethinking how a particle resists a force. Mass is due to homotopic non-equivalence between force and particle. The intersection of the particle and field is also responsible for the entirety of a particle's mass. This simplifying premise enables the calculation of values for six light hyperons and the electron as functions of the proton and neutron masses.

The topological theory put forward considers a particle to be an S^3 Hopf bundle. The geometry is well understood.[4, 5, 6, 7] A Hopf bundle maps a 3-sphere to a 2-sphere. The 3-sphere is the set of four dimensional points S^3. The 2-sphere is a two dimensional surface described by the set of three dimensional points S^2. A Hopf fibration continuously maps S^3 to S^2. This is done with Hopf maps. A Hopf map ($h : S^3 \to S^2$) is a surjective function that maps a subset of S^3 elements to a point in S^2. An individual Hopf map describes a circle (Hopf circle). Continuous mapping entails an infinite number of maps for each point in S^3; this requires an infinite bundle of circles that in total connect each S2 point to every point in S3. The total space is transitive.

A physical interpretation is added to this potted account of a Hopf bundle. A 'Hopf particle', as we shall call it, is a 3-sphere that intersects an ambient three dimensional manifold (3-space). In 3-space the Hopf particle appears as a 2-sphere. A force external to the Hopf particle is a 3-space vector. The location an external force contacts the Hopf particle has the topology of a point in 3-space; this contrasts the bundle of Hopf circles that form the surface of the 3-sphere at the same location. The topological discrepancy raises the question of the homotopic non-equivalence of circle and point. The problem may be pictured as a cone mapping. If we imagine the point is the apex of the cone, unless its topology is punctured, the point force is unable to pass to the base circle. In reverse, only by *cutting* may the circle deform retract to a point. On this view, a particle with topology that deform retracts to a point offers no resistance and is massless. In the case of Hopf particle and external 3-pace force, resistance to change in location and speed is interpreted as a manifestation of the force having to *jump* topologies. In other words, particle mass results from a Hopf particle not breaking symmetry when acted on by a 3-space force. With every location on the 2-sphere a bundle of Hopf circles related to every point on the 3-sphere, and a total space that is transitive, the totality of the 3-sphere is the measure of resistance to an external force regardless of the magnitude of the 3-space force.

Five equations characterise Hopf particle rest mass. The first tells us mass is determined by the size of the 3-sphere. For example, if the mass of the proton is 938.272 MeV then $r \approx 3.622$ MeV. I.E.

$$M = 2\pi^2 r^3. \tag{1}$$

The volume of a 2-sphere is the space the Hopf particle occupies in the ambient 3-space. This is the volume of an ordinary ball.

$$V = \frac{2M}{3\pi} = \frac{4\pi}{3} r^3. \tag{2}$$

At Eq. (2), r is the radius derived at Eq. (1). In the case of the proton $V_p \approx 199.108$ MeV; this is the volume measured in 3-space.

$$\rho = \frac{M}{V} = \frac{3\pi}{2}. \tag{3}$$

Eq. (3) means a Hopf particle is a hyper-dense ball in 3-space. The excess mass, here called 'hypermass', is indirect evidence of an extra dimension. Hypermass (H) is the difference between mass and volume.

$$H = M - V. \tag{4}$$

Hopf particle mass has the Hopf/hypermass signature (H-signature):

$$M = (H)(\frac{\rho}{\rho - 1}). \tag{5}$$

H-signatures found in the mass data suggest lighter hyperons are Hopf particles. For the initial three input values the 2024 CODATA recommended values are used while ignoring the standard deviation.[8]

$$\begin{aligned} M_p &= 938.272\ 089\ 43\ \text{MeV}/c^2 \\ M_n &= 939.565\ 421\ 94\ \text{MeV}/c^2 \\ M_e &= 0.5109\ 998\ 959\ 069\ \text{MeV}/c^2 \end{aligned} \tag{6}$$

For instance, the Σ rest masses each have an H-signature that is a function of the M_p and M_n values.

$$M_{\Sigma^+} = (2M_p - M_n)(\frac{\rho}{\rho - 1}) \approx 1189.3712. \tag{7}$$

$$M_{\Sigma^0} = (M_n)(\frac{\rho}{\rho - 1}) \approx 1192.6546. \tag{8}$$

$$M_{\Sigma^-} = (4M_n - 3M_p)(\frac{\rho}{\rho - 1}) \approx 1197.5797. \tag{9}$$

All three derived values are close to the observed masses. The Particle Data Group (PDG) fit for M_{Σ^+} is 1189.37 $\pm$0.07.[9] While the PDG fit for M_{Σ^0} is 1192.642 $\pm$0.024, Eq. (8) is particularly close to Wang 1192.65 $\pm$0.020.[10] Eq. (9), however, is over four standard deviations shy of the PDG value (1197.449 $\pm$0.030). The present PDG fit for M_{Σ^-} draws on three results. Schmidt (1197.43) and Gurev (1197.417) are too low to be the value derived here, though Eq. (9) is within one standard deviation of Gall (1197.532 $\pm$0.057).[11, 12, 13]

The H-signatures for the Ξ pair introduce a complication that provides a way to check whether Eqs. (8, 9) are reliable.

$$M_{\Xi^0} = (M_{\Sigma^0})(\frac{\rho}{\rho - 1}) - V_p \approx 1314.8104. \tag{10}$$

$$(M_{\Sigma^-})(\frac{\rho}{\rho - 1}) - V_p \approx 1321.0622. \tag{11}$$

Eq. (10) is within one standard deviation of the PDG fit and is close to Fanti (1314.82 $\pm$0.06)[14], but a problem looms. When the basic pattern of Eq. (10) is repeated at Eq. (11) the result 1321.0622 is over nine standard deviations adrift of the PDG fit for M_{Ξ^-}. The present PDG recommended value 1321.71 Mev draws on a large 1992-1995 data sample [15] making a future nine standard deviation downward adjustment unlikely. If M_{Σ^-} is indeed close to 1321.71, a fudge ≈ 0.511 is needed. I.E.

$$M_{\Xi^-} = (M_{\Sigma^-} + M_e)(\frac{\rho}{\rho - 1}) - V_p \approx 1321.7109. \quad (12)$$

At face value the extra Me weighting appears ad hoc, but there is a firm reason for thinking otherwise. First, we give the formula for the Ω^- (Omega) mass.

$$M_{\Omega^-} = \left(\frac{3M_{\Xi^0} + 2M_{\Xi^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right) \approx 1672.4824. \quad (13)$$

From Eq (13), $V_\Omega = 354.9118$ using Eqs. (1, 2). The next formula is evidence the M_e weighting is not ad hoc.

$$\left[(M_{\Sigma^0})\left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V_{\Omega^-}\right]\left(\frac{\rho - 1}{\rho}\right)(M_e) \approx 0.511. \quad (14)$$

Eq. (14) gives a value ≈ 0.511 regardless of the unit of mass used. When the three input values are $M_e = 1, M_p = 1836.152\ 673\ 426, M_n = 1838.683\ 662$ (CODATA 2024) it is particularly obvious 0.511 is not an artefact. Nonetheless, while the final number may be denominated in any system of units, the resemblance to M_e in MeV is puzzling. It is certainly wrong to suggest nature privileges an arbitrary system of units. However, given it takes only a small adjustment within one standard deviation to M_n for the Eq. (14) value to converge on the CODATA 2024 recommended value for M_e in MeV, it is unlikely the resemblance is a coincidence. Importantly, while the final value ≈ 0.511 depends on the input value used for M_e, Eq (14) allows the numerical value for M_e to be calibrated without referencing the CODATA value. On this condition M_e is no longer a free parameter. For instance, using the values for M_p and M_n introduced at Eq. (6), $M_e =$ Eq. (14) when $M_e \approx 0.510\ \ 999\ \ 021$. Thus, given it is accepted Eq. (14) $\propto M_e$ in MeV, we can say the number of input values is reduced from three to two, with M_e a complicated function of Eq. (14).

It is also worth noting that if the CODATA and PDG 2024 rest mass values in MeV are used as inputs at Eq. (14) the result is ≈ 0.55. Topological theory predicts future adjustments to the observed rest masses will see Eq. (14) converge on a number closer to 0.511.

Admittedly, the pattern of mass splitting seen in Eqs. (7, 8, 9, 10, 11, 12, 13) lacks a detailed theory to explain why we might expect 0.511 at Eq. (14). Looking at this from another direction, Eq. (14) also hints the mass of the electron is due to the relationship between baryonic matter. These questions are left open for the time being. Nonetheless, the equations this paper set forth allow mass values for $\Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-, \Omega^-$ and the electron to be derived as functions of p and n. Nine free parameters are thereby reduced to two.

References

[1] David Miller. Politics, solid state and the Higgs. *Physics World*, 6(9):27, 1993.

[2] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys. Rev. Lett.*, 13:321–323, Aug 1964.

[3] Peter W. Higgs. Broken Symmetries and the Masses of Gauge Bosons. *Phys. Rev. Lett.*, 13:508–509, Oct 1964.

[4] Heinz Hopf. Über die abbildungen der dreidimensionalen sphäre auf die kugelfläche. *Mathematische Annalen*, 104(1):637–665, 1931.

[5] N Steenrod. *The Topology of Fibre Bundles*. Princeton University Press, Princeton, 1951.

[6] Herman Gluck, Frank Warner, and Wolfgang Ziller. The geometry of the Hopf fibrations. *Enseign. Math.(2)*, 32(3-4):173–198, 1986.

[7] David W Lyons. An elementary introduction to the hopf fibration. *Mathematics magazine*, 76(2):87–98, 2003.

[8] Eite Tiesinga, Peter J. Mohr, David B. Newell, and Barry N. Taylor. Codata recommended values of the fundamental physical constants: 2022. 2024.

[9] R. L. Workman et al. Review of particle physics. *PTEP*, 2022:083C01, 2022.

[10] M. H. L. S. Wang et al. A precise measurement of the Σ^0 mass. *Phys. Rev. D*, 56:2544–2547, 1997.

[11] P. Schmidt. Precise hyperon masses. *Phys. Rev.*, 140:B1328, 1965.

[12] M. P. Gurev et al. First measurement of the x-ray emission of Σ^- atoms by means of a crystal diffraction spectrometer. *JETP Lett.*, 57:400–405, 1993.

[13] K. P. Gall et al. Precision measurements of the K^- and Σ^- masses. *Phys. Rev. Lett.*, 60:186–189, 1988.

[14] V. Fanti et al. Precision measurement of the Ξ^0 mass and the branching ratios of the decays $\Xi^0 \to \Lambda$ gamma and $\Xi^0 \to \Sigma^0$ gamma. *Eur. Phys. J. C*, 12:69–76, 2000.

[15] J. Abdallah et al. Masses, lifetimes and production rates of Ξ^- and anti-Ξ^+ at lep 1. *Phys. Lett. B*, 639:179–191, 2006.