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Evidential aggregation-based DEMATEL functions and its application in expert decision system for criminal cases

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Abstract In real criminal cases, the decision outcome is often influenced by many complex factors, such as the importance of initial evidence and the prioritization of evidence. How to model these information in an integrated manner to provide technical tools for case detection so as to find the real suspect is of great importance for social security and stability. To address the above issues, this paper proposes a novel soft likelihood function based on the Decision Making Trial and Evaluation Laboratory (DEMATEL) method. Firstly, the proposed method well preserves the preference of decision-maker (DM) in the soft likelihood function proposed by Yager et al. Secondly, the method takes into account the modeling of associated information. In addition, it also extends the soft likelihood function to reflect the preferences of DMs through the importance of evidence. Finally, based on these designed algorithms, a decision processing model for criminal cases is constructed, which systematically provides a guiding process for case detection. Numerical examples and applications show the practicality and effectiveness of the proposed method.

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1 Introduction

1.1 Literature review

Decision-making is closely related to human activities, which is based on human judgment and cognition. Intelligent systems are essentially simulations of human thinking, and are an effective mechanism for automatically or semi-automatically transforming information from different sources into providing decision-making results for humans. With the development of society, decision making plays an increasingly important role in actual life [1–3]. Up to now, there are many researches and applications of decision-based derivatives, such as group decision making [4–6], multi-attribute decision making [7,8], ethical decision making [9–11], strategic decision making in business [12,13], etc [14,15].

However, real world information is often abstract and uncertain. Therefore, how to model uncertain information to obtain valuable information for people to make decisions and judgments is a crucial hot topic. In order to make full use of the potential value of information, many scholars have carried out extensive research and created many studies on uncertain information modeling, such as fuzzy sets [16–18], Dempster-Shafer evidence theory [19–22], aggregation-based operator [23–26], Z numbers [27,28], D numbers [29,30], evidence reasoning [31,32], etc. Like these methods, likelihood function is also one of the important information processing and modeling tools. However, the initial likelihood function can produce conflicting results, when fusing compatible evidence from multiple sources. One extreme example is, when there is an event with probability 0 in the evidence group, the likelihood function will also be equal to 0. That is, in this case, a small probability event vetoes the role of other evidence, which is obviously unreasonable. To solve this problem, Yager et al. [33] first created a more moderate likelihood function in 2017, which is called the soft likelihood function. The proposed soft likelihood function introduces weights of ordered weighted averaging (OWA) operators based on decision makers' preferences, which makes the original likelihood function more flexible in combining compatible evidence. A practical utility is that, it allows for the optimal allocation of available resources. In criminal cases, an interesting phenomenon has been demonstrated. That is, for evidence collected from different suspects, the higher the value of the fused soft likelihood function, the stronger the willingness of the officer to investigate the person. Due to the advantages of soft likelihood function in dealing with uncertain information, many scholars have extended it to other fields.

For instance, Jiang et al. [34] studied the belief structure of Dempster-Shafer evidence theory based on soft likelihood function. Ma et al. [35] constructed the method of transforming triangular fuzzy number into basic belief assignment based on soft likelihood function in evidence theory. Fei et al. designed intuitionistic fuzzy decision-making [36] and interval-valued fuzzy decision-making [37] using soft likelihood function, etc [38,39]. In addition, combined with the soft likelihood function based on the OWA operator, some recent practical applications, namely expert decision-making [40] and multi-sensor data fusion [41] and healthy waste management [42] have also been paid attention to by the author. The above practical applications demonstrate the potential of soft likelihood functions to move successfully from theory to practice.

Now, let us return to the development of the soft likelihood function itself. At first, Song and Deng [43] proposed a new soft likelihood function based on power ordered weighted average (POWA) operator, which considered the degree of support between evidence probabilities. Subsequently, through theoretical analysis and experimental research, The author in [44] found two major flaws in the soft likelihood function proposed by Song and Deng (On the one hand, they method does not reflect the weights of OWA operators. This means that their approach only reflects the "P" (i.e., power) in POWA, but ignores the "OWA", so it is not strictly a form of POWA. On the other hand, it does not reflect the preferences of decision makers well. That is, the fusion results of the likelihood function and the decision maker's preference level do not show a correlation feature, but this property is quite important in the soft likelihood function, as originally discussed by Yager in literature [33]). To overcome the above problems, a modified soft likelihood function is proposed by the author in [44]. However, to further enrich the theoretical connotation of the soft likelihood function, there are many interesting questions worthy of extensive exploration.

1.2 Challenge and motivation

By reviewing and mining the existing soft likelihood function itself, we find that, in all soft likelihood functions, both of the following assumptions are considered to be prerequisites. **1). The sources of evidence are independent. 2). The modeling of uncertainty evidence information is based on the probability priority.** However, according to the logic of reasoning, in the real world, there is often a correlation between things. As you might expect, these two conditions may be the drawbacks that limit its development, to some extent. Below, we will explain in detail the reasons why they are not met, which can be seen as the motivation for the creation of this paper.

Why independence is a special constraint?

Causal inference is a fundamental and important feature in the detection of a criminal case. The basis for establishing this feature is the correlation between evidence, or the value of influence transmission, as it is called. In fact, even in the usual case, our opinions about something are rarely independent. When evaluating the difference between an objective thing, for example, we say that A is better than B, and C is much better than B. In the above sentence, the "better" and the "much better" are relative and overlap. This means that "much better" is at least 100% of "better", if not more. Thus, in this case, independence is only a special case (which we can fully achieve by adjusting the size of the quantified indicator), and may even be a defect.

Is the priority order of the evidence itself more needed than its probability information?

For decision makers, the criteria for information modeling do not depend solely on the probability of things happening. Let's explain in the same practical scenario as above. Suppose that in a criminal case, investigators obtained three pieces of evidence at the crime scene, namely fingerprints, handwriting, and evidence of footprints. Through forensic expert identification, the three probabilities of identifying the suspect x_i are 0.4, 0.6, and 0.5, respectively. According to the existing research of soft likelihood function, when the decision maker adopts more positive preferences, the higher the priority given by the handwriting to the suspect x_i. That is, the decision maker thinks the suspect x_i with a high probability the designation of is based on handwriting. However, in the process of decision-making, modeling based on the importance of evidence is crucial. In this instance, the fingerprint information is evidence that reflects the only physiological characteristics of the suspect x_i. This means that it is more important than other evidence. Our position here is that the more important the evidence, the more priority should be given. In this situation, the more positive the decision maker's preference, the fingerprint evidence will have the highest degree of identification of the suspect, although its identification probability is not very high (Under these two different viewpoints, the order in which the evidence is given importance is shown in Fig. 1). Therefore, this way provides a distinctive way of identifying the ultimate offender.

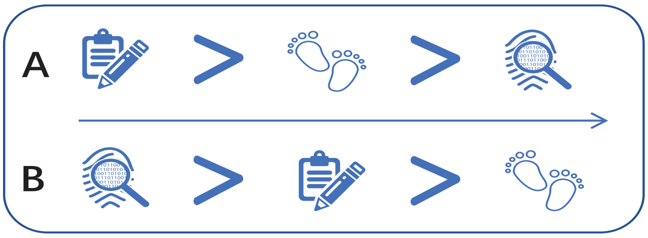

Fig. 1: Part A represents the result of the existing soft likelihood function assigning priority order to the evidence set. Obviously, we can see that this is their respective probability-based representation. Part B is the order in which we consider the importance of the evidence (i.e., its intrinsic properties) to be greater than its probabilistic information.

More further, in response to the above two questions, the motivation for the study of soft likelihood functions in this paper can be summarized as the following two points, respectively: 1). how to expose potential correlations between evidence; and 2). how to build a new compatible evidence fusion system based on the inherent properties of evidence.

1.3 Our solution

Inspired by the DEMATEL method and the two important issues found above, in this paper, we propose a novel soft likelihood function based on DEMATEL. First of all, by using DEMATEL to comprehensively consider the correlation information between the evidences. And then, a evidence correlation matrix is formed to obtain the importance ranking of the evidence. Next, based on the OWA operator, the preference characteristics of decision makers are integrated. Finally, the information fusion result is obtained through the distribution of weights. The idea of the proposed method can be summarized as: the modeling of uncertain information is based on the importance of evidence. That is, in the decision-making process, when decision makers adopt positive preferences, the more important information has the higher priority. Conversely, when policymakers adopt pessimistic preferences, less important information has higher priority. All evidence is equally important if policymakers adopt neutral preferences. On the basis of the above efficacy, a criminal case analysis system has been specially developed. In addition, the effectiveness of the proposed method is illustrated by its application in criminal forensic cases. Moreover, the superiority of the proposed method is illustrated by comparison and analysis.

1.4 Outline

The paper is organized as follows. Section 2 introduces the background knowledge required for this paper. Section 3 proposes a novel soft likelihood function based on the DEMATEL method. And, based on this, a system model for criminal cases is constructed. Section 4 applies the proposed method to criminal forensic cases in the real world, to show the actual effect. Section 5 compares and analyzes the rationality and superiority of the proposed method, and demonstrates the reliability of the method through sensitivity tests. Finally, the work done in this paper is summarized in Section 6.

2 Preliminaries

In this section, we review some basic concepts, such as ordered weighted averaging (OWA) operators, soft likelihood function, and Decision Making Trial and Evaluation Laboratory (DEMATEL) method.

2.1 Ordered weighted averaging operators

Definition 1. An OWA aggregator operator of dimension n is a mapping domain, i.e., OWA: $\mathbb{R}^n \rightarrow \mathbb{R}$. It is related to a vector of n dimensions. That is

$$\omega = \begin{bmatrix} w_1 \cdots w_n \end{bmatrix}^T \tag{1}$$

In the above equation, ω indicates the OWA weight vector, and its component w_j is called the weight of OWA. The elements in w_i always satisfy the following two constraints, for all j

$$s.t. \begin{cases} 0 \leq w_i \leq 1 \\ \sum_i w_i = 1 \end{cases} \tag{2}$$

Based on the above, eventually, this OWA aggregation operator is formally represented as follows

$$OWA(a_1, \cdots, a_n) = \sum_{i=1}^{n} w_i b_i \tag{3}$$

with b_i is the jth largest of the a_j.

If we let λ be an index function and λ_i be the index of the ith largest parameter value, then the OWA aggregation operator can be redefined as below

$$OWA(a_1, ..., a_n) = \sum_{i=1}^{n} w_i a_{\lambda_i} \tag{4}$$

The OWA operator provides many categories for aggregating different weight vectors. Some special examples are given as follows

- ω^*: If $w_1 = 1$ and $w_j = 0$ for $j \neq 1$, this operator can be written as $OWA(a_1, ..., a_n) = a_{\lambda(1)} = Max_i(a_i)$.
- ω_*: If $w_n = 1$ and $w_j = 0$ for $j \neq n$, in this case, we can get that $OWA(a_1, ..., a_n) = a_{\lambda(n)} = Min_i(a_i)$.
- ω_n: When $w_j = (1/n)$ for $j = 1$ to n, this operator can be described as $OWA(a_1, ..., a_n) = (1/n)\sum_{i=1}^{n} a_i$.
- $\omega_{[K]}$: When $w_K = 1$ and $w_j = 0$ for $j \neq K$, in this case, we can obtain that $OWA(a_1, ..., a_n) = a_{\lambda(K)}$.

2.2 OWA weights based on attitudinal preferences

For obtaining the weight values of OWA, many approaches have been investigated. One of the popular ways is a function-based representation. Suppose the monotonic function is a mapping F: $[0, 1] \to [0, 1]$, which satisfies the following three constraint conditions

$$s.t. \begin{cases} F(x) \geq f(y) \ if \ x > y \\ F(0) = 0 \\ F(1) = 1 \end{cases} \tag{5}$$

in which F is called a BUM function. Based on this function, the w_j for $j = 1$ to n such that

$$w_j = f(\frac{j}{n}) - f(\frac{j-1}{n}) \tag{6}$$

where w_j has all the properties of OWA weights [45,46].

Yager [45] shows that, for a given BUM function F, a representative measure of optimism can be obtained, which is described in mathematical notation as

$$Opt(F) = \int_0^1 F(x)dx \tag{7}$$

For a significant function F, a useful form is $F(x) = x^{\wp}$ for $\wp \geq 0$. Based on this, the degree of optimism α, is determined as

$$\alpha = \int_0^1 x^{\wp} dx = \frac{x^{\wp+1}}{\wp+1}\Big|_0^1 = \frac{1}{\wp+1} \tag{8}$$

It is easy to deduce that $0 \leq \alpha \leq 1$. Using this function form, the OWA weights for $j = 1$ to n is defined as

$$w_j = F(\frac{j}{n}) - F(\frac{j-1}{n}) = (\frac{j}{n})^{\wp} - (\frac{j-1}{n})^{\wp} \tag{9}$$

Finally, from the perspective of given preference α, we can obtain

$$w_j = (\frac{j}{n})^{\frac{1-\alpha}{\alpha}} - (\frac{j-1}{n})^{\frac{1-\alpha}{\alpha}} \tag{10}$$

2.3 Soft likelihood function

Yager et al. [33] originally proposed the application of the likelihood function to criminal investigation cases to determine the probability of a suspect committing a crime.

Definition 2. For m independent sources of evidence supporting the guilt of suspect x_i, let p_{ij} represent the jth body of evidence that is used as proof that x_i is the offender. One way to find the offender is defined as

$$L_i = \prod_{j=1}^{m} p_{ij} \tag{11}$$

Definition 3. Let λ_i be the index function and the likelihood function be denoted as

$$Prod_i(j) = \prod_{k=1}^{j} p_{i\lambda_i(k)} \tag{12}$$

where $p_{i\lambda_i(k)}$ is the kth maximum compatible probability associated with suspicious x_i. It can be seen that $Prod_i(j)$ is monotonically decreasing as a function of j, i.e., if $j_1 < j_2$, then $Prod_i(j_1) \geq Prod_i(j_2)$. In addition, since $0 \leq p_{i\lambda_i(k)} \leq 1$, $0 \leq Prod_i(j) \leq 1$ can be also easily observed.

By the above equation, it is easy to observe that the result of aggregation of this compatible body of evidence is a product of m p_{ij} with respect to x_i.

However, when there exists a very small probability, such as $p_{ij} = 0$, for any $j = 1$ to m, the result thus obtained is $L_i = 0$. Obviously, this is against common human perception, since the body of conflicting evidence completely eliminates the possibility of other events occurring.

Considering that the representation of the likelihood function is too strict, Yager et al. considered using the OWA aggregation operator to flexibly assign weights to obtain a soft likelihood function.

Definition 4. For the OWA aggregation of the weight vector ω, and $Prod_i(j)$, a softer likelihood function for each element x_i, denoted as $L_{i,\omega}$. That is

$$L_{i,\omega} = \sum_{j=1}^{m} w_j Prod_i(j) \tag{13}$$

in which ω is the OWA weighting vector of dimension m. It satisfies all the previously given constraints.

Furthermore, taking into account the special form of the weighted vector ω, the soft likelihood function can be expressed in the following form. That is

1) ω^{ς}: For $j = 2$ to m, $w_j = 0$; and $w_1 = 1$. In this case, we have $L_{i,W^{\varsigma}} = Prod_i(1) = p_{i\lambda_i(1)}$.
2) ω_{ς}: For $j \neq 1$ to $m-1$, $w_j = 0$; and $w_m = 1$. In this case, we have $L_{i,W_{\varsigma}} = Prod_i(m) = \prod_{j=1}^{m} p_{ij}$.
3) ω_n: For $j = 1$ to m, $w_j = (1/m)$. In this case, we have $L_{i,\omega_n} = (1/m) \sum_{j=1}^{m} Prod_i(j) = (1/m) \sum_{j=1}^{m} (\prod_{k=1}^{j} p_{i\lambda_i(k)})$.

For the given function $f(x) = x^{\wp}$ ($\wp = (1-\alpha)/\alpha$), the soft likelihood function is denoted as follows

$$L_{i,\alpha}^{\omega} = \sum_{j=1}^{m} \left((\frac{j}{m})^{\frac{1-\alpha}{\alpha}} - (\frac{j-1}{m})^{\frac{1-\alpha}{\alpha}} \right) \prod_{k=1}^{j} p_{i\lambda_i(k)} \tag{14}$$

where $0 \leq \alpha \leq 1$. Normally, α takes 0.1, 0.2, ..., 1. Different values of α indicate different preferences of decision makers. The larger the α, the more optimistic it is.

2.4 Decision Making Trial and Evaluation Laboratory method

The DEMATEL method, was proposed by scholars A. Gabus and E. Fontela of the Battelle Laboratory in the United States in 1974, to understand complex and difficult problems in the real world [47]. It is a systematic analysis method using graph theory and matrix tools. Since its inception, this method has been widely concerned by many scholars, because it can make full use of the knowledge and experience of experts to deal with complex social problems with uncertain system elements, especially for those systems with uncertain element relationships. Up to now, it has been widely used in many fields, such as social economy [48,49], management science [50], information fusion [51,52], decision making, etc.

Here, this DEMATEL implementation step is briefly introduced as follows.

- Step 1: Starting from the research purpose, determine the research indicators or elements. Quantify the interrelationship between the elements to get a direct impact matrix.
- Step 2: By normalizing the original relation matrix, and obtaining the norm directly affecting the matrix.
- Step 3: From the normalization direct impact matrix, a comprehensive impact matrix is calculated.
- Step 4: According to the comprehensive impact matrix, four factors are calculated.
- Step 5: Plot and explain the degree of centrality and causation derived from the calculation. Further treatment is carried out according to the actual situation, such as removal of non-core elements, which is used in conjunction with systematic methods such as interpretation of structural model.

3 Proposed technology

3.1 Detailed motivations and ideas

In the real world, decision-making conditions for DMs are often complex. Under certain conditions, the results of experts' decisions are limited by objective or subjective factors such as external conditions, physical conditions, professionalism, etc. Therefore, DMs do not always make adequately correct decisions under any conditions. In this subsection, a novel soft likelihood function based on DEMATEL method is proposed. Based on this, a novel expert decision system for criminal cases is constructed.

In the proposed method, the decision preference parameter α surmounts the above-mentioned problems very well. When DMs adopt positive attitudinal characteristics, it indicates that the DM is more confident in his (or her) professional level or is in good physical condition. That is to say, it is often possible to make relatively correct decisions in this transition. On the contrary, when pessimistic attitude characteristics are adopted, it indicates that the DM may not be in good physical condition or do not know much about the object of the final decision.

The mechanisms by which the methods in this paper form the effects mentioned above are as follows. In the design of the soft likelihood function, the evidence is first ranked according to the importance of the evidence through the relevance of the evidence. The larger α indicates that the more important evidence is given priority consideration by DMs. In this case, the results made by DMs are often correct. Conversely, the smaller α is, the less important evidence is considered by DMs. At this time, the DMs may make a decision error or misjudge. When DMs adopt neutral decision preferences, all evidence is equally important.

3.2 The theoretical model

The method is clearly illustrated through following steps.

Step 1: Determine the evidence set $E = \{e_i \mid i = 1, 2, \cdots, n\}$.

Step 2: Construct the evidence impact matrix (EAM). The interrelationship between the collected evidence is determined by the case analysis expert. Then, the case analysis expert determines the magnitude of this impact through scoring. In the scoring process, because events are vague, it doesn't make sense to use accurate measurements. Of course, there are many fuzzy set-based methods to characterize expert multi-granularity fuzzy uncertainty information. Here, we adopt a straightforward rank-based term set as an example. The commonly used method has a 5-level scale, that is, a method of taking $0-4$ to measure, such as: $\{none, small, normal, large, very\ large\}$ or $\{none, veryweak, normal, strong, very\ strong\}$, etc. Assuming a total of n evidences are collected, one EAM is given as

$$EAM = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & 0 \end{bmatrix} \tag{15}$$

where the evidence itself does not need to be compared, that is, the values on the diagonal of the matrix are usually denoted by 0. In this matrix, element a_{ij} denotes the direct influence of evidence a_i on evidence a_j.

Step 3: Normalize the evidence association matrix. In the following formula, the right-hand side is used to calculate the sum of each row in EAM. Next, the maximum value is taken from these values using the $max(\cdot)$ function.

$$Maxval = max(\sum_{j=1}^{n} a_{ij}) \tag{16}$$

Then, this normalized EAM is defined as

$$N = [\frac{a_{ij}}{Maxval}]_{n\times n} \tag{17}$$

in which $1 \leq i \leq n$ and $1 \leq j \leq n$.

Step 4: Obtain the comprehensive evidence association matrix ($CEAM$). Based on this normalized EAM, i.e., the N, the $CEAM = [t_{ij}]_{n\times n}$ is defined as

$$CEAM = (N + N^2 + N^3 + \cdots + N^k) = \sum_{k=1}^{\infty} N^k \tag{18}$$

$$\Updownarrow$$

$$CEAM = N(I - N)^{-1} \tag{19}$$

Where $N \times N$ means the indirect relationship of increase, which includes both the amount of increase between values that are not zero in the EAM, as well as the value of zero that becomes non-zero through the transfer of influence between elements. The I denotes as the identity matrix, and t_{ij} is denoted as the comprehensive degree in the $CEAM$ to which the evidence i affects the evidence j.

Step 5: Calculate the degree of influence D_i and the degree of being influenced C_i. The D_i is the sum of the values of each line of the $CEAM$, which represents the comprehensive influence value of the corresponding evidence of each line on all other evidences. The set consisting of all D is denoted as D.

$$D = \{D_1, D_2, \cdots, D_n\} \tag{20}$$

in which

$$D_i = \sum_{j=1}^{n} t_{ij},\ i = 1, 2, \cdots, n \tag{21}$$

The C_i is the sum of the values of each column of the $CEAM$, which indicates that the corresponding evidence of each column is affected by the comprehensive influence of all other evidences. And the set consisting of all D is denoted as C.

$$C = \{C_1, C_2, \cdots, C_n\} \tag{22}$$

in which

$$C_i = \sum_{j=1}^{n} t_{ij},\ j = 1, 2, \cdots, n \tag{23}$$

Step 6: Compute the degree of centrality and the degree of causation. The degree of centrality of evidence i obtained by adding the influence degree and the affected degree, which is denoted as M_i. The degree of centrality indicates how important the evidence is.

$$M_i = D_i + C_i,\ i = 1, 2, \cdots, n \tag{24}$$

The D_i of the evidence i is subtracted from the C_i to obtain the degree of causation of the evidence, which is denoted as R_i.

$$R_i = D_i - C_i,\ i = 1, 2, \cdots, n \tag{25}$$

If the degree of causation is greater than 0, it indicates that the evidence has a great influence on other factors, which is called the causal factor. On the contrary, it is called the result factor.

Step 7: Plot the *Cartesian* coordinate of the degree of centrality - the degree of causation. In the *Cartesian* coordinate, from left to right, it shows that the evidence is more and more important; from bottom to top, it show that the evidence is more relevant to other evidence. In this paper, ranking is based on the importance of the evidence, that is, the order from left to right in the *Cartesian* coordinate. We believe that the more important the evidence is in the decision-making process, the more this evidence should be given priority. The evidences is sorted in descending order using the index function δ.

Step 8: Calculate the soft likelihood function. Before that, we need to get the value of the likelihood function of the compatible evidence. Based on the ranking result of index function δ in **Step 7**, the following formula is used to calculate the likelihood function value of evidences.

$$Prod_i(j) = \prod_{k=1}^{j} p_{i\delta_i(k)} \tag{26}$$

where $p_{i\delta_i(k)}$ is the kth maximum compatible probability and $Prod_i(j)$ is the product of the j largest probabilities.

Combining the weights of OWA operators, the soft likelihood function can be expressed as

$$\tilde{S}^{\omega}_{i,n} = \sum_{j=1}^{n} w_j Prod_i(j) \tag{27}$$

Finally, according to Eq. (10), a soft likelihood function based the DEMATEL method is designed as

$$\tilde{S}^{\alpha}_{i,n} = \sum_{j=1}^{n} \left(\pounds^{\alpha}_{j,n} - \pounds^{\alpha}_{j-1,n}\right) \prod_{k=1}^{j} p_{i\delta_i(k)} \tag{28}$$

in which

$$s.t. \begin{cases} \pounds^{\alpha}_{j,n} = (\frac{j}{n})^{\frac{1-\alpha}{\alpha}} \\ \pounds^{\alpha}_{j-1,n} = (\frac{j-1}{n})^{\frac{1-\alpha}{\alpha}} \\ \pounds^{\alpha}_{j,n} - \pounds^{\alpha}_{j-1,n} = w_j \end{cases} \tag{29}$$

where, $\alpha \in [0, 1]$, it usually takes the value of 0.1, 0.2, ..., 1. A larger α indicates that the decision maker in a criminal case is more optimistic about the suspect x_i.

3.3 Building expert decision-making systems

A decision model diagram of the proposed method is shown in Fig. 2. First, some evidence at the scene of the crime is collected by professionals. Then, through identification and recognition, forensic experts give the degree of support each piece of evidence has for the suspect. Next, a comprehensive evidence correlation matrix is obtained by the case analysis expert by analyzing and evaluating the facts of the case. Through computing some necessary calculations, a result based on the centrality of the evidence and the degree of cause is obtained. Finally, by integrating the attitudinal characteristics, i.e., preferences, of the decision makers, the final decision result of the case for a particular suspect is available.

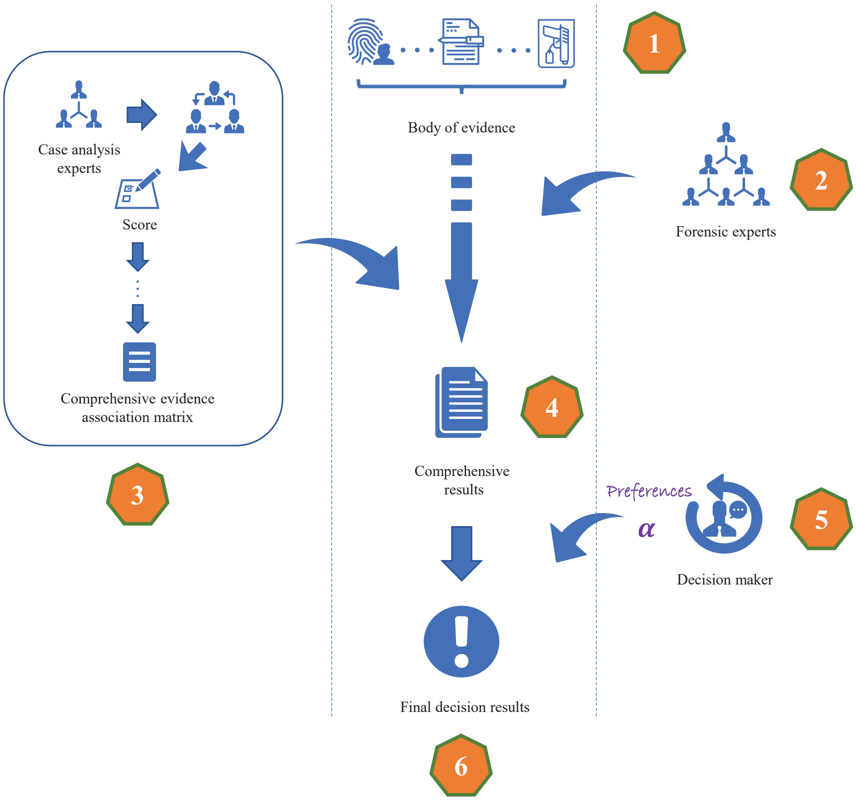

Fig. 2: An expert decision-making system for criminal cases based on the proposed method

4 Application in criminal forensic medicine cases

In this section, the superiority of this method is further illustrated by its application in criminal forensic cases. Some correlative terms and fundamental data used in this section are taken from [33].

4.1 Problem description

At the crime scene, forensic inspectors work with investigators to investigate the place where the case occurred, and obtain five trace evidences left by the criminal through technical means, namely evidence A, B, C, D and E. Through the analysis of medical experts, the probability of each kind of evidence identifying the criminal suspect x_i is respectively

$$E = \{p_{i1} = 0.5, p_{i2} = 1, p_{i3} = 0.3, p_{i4} = 0.8, p_{i5} = 0.7\}$$

It is important to note that case analysis experts point out that the five types of evidence collected are not completely independent of each other. There is a certain correlation between them. The correlation between the evidence is expressed as Table 1.

Table 1: The assessment results of case analysis experts

Body of evidence	A	B	C	D	E
A	-	$*$	-	$**$	$****$
B	-	-	$***$	-	-
C	-	-	-	-	$*$
D	-	-	$*$	-	$***$
E	-	-	-	-	-

[1] The experts used a 5-level assessment guideline, i.e., a scale of 0-4. Where "-" indicates that there is no impact between bodies of evidence. The count of "$*$" indicates the level of influence. That is, the higher the number of "$*$", the greater the interaction effect between the bodies of evidence.

4.2 Implementation of the proposed solution

According to the background of the case, the identification of the criminal suspect x_i has become complicated due to the correlation between the evidence. It can be seen that modeling based on the correlation or impact of evidence to extract key evidence information becomes even more important. The steps of the proposed method are shown below.

Step 1: Determine the evidence set. Through the analysis of the case, the evidence set we obtained is as follows: $E = \{A, B, C, D, E\}$.

Step 2: Construct the evidence association matrix (EAM). According to the description of the case analysis experts, we use a 5-level scale, that is, a $0-4$ method to measure the correlation between the evidence, where $\{none = 0, small = 1, normal = 2, large = 3, very\ large = 4\}$. Based on semantics and scale, the EAM constructed is

$$EAM = \begin{bmatrix} 0 & 1 & 0 & 2 & 4 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 3: Normalize the evidence association matrix. Using Eq. (16), we can get $Maxval = max(7, 3, 1, 4, 0) = 7$. Then, through Eq. (17), the normalized EAM is

$$N = \begin{bmatrix} 0 & 0.1429 & 0 & 0.2857 & 0.5714 \\ 0 & 0 & 0.4286 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1429 \\ 0 & 0 & 0.1429 & 0 & 0.4286 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 4: Obtain the comprehensive evidence association matrix ($CEAM$). By Eq. (19), the $CEAM$ obtained is denoted as

$$CEAM = \begin{bmatrix} 0 & 0.1429 & 0.1021 & 0.2857 & 0.7084 \\ 0 & 0 & 0.4286 & 0 & 0.0612 \\ 0 & 0 & 0 & 0 & 0.1429 \\ 0 & 0 & 0.1429 & 0 & 0.4490 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 5: Calculate the degree of influence D_i and the degree of being influenced C_i. Using Eq. (21) and Eq. (23), the calculation results of the D_i and C_i of the evidences are shown in Table 2.

Table 2: The degree of influence and the degree of being influenced

Item	Body of evidence				
	A	B	C	D	E
The degree of influence (D_i)	1.2391	0.4898	0.1429	0.5919	0
The degree of being influenced (C_i)	0	0.1429	0.6736	0.2857	1.3616

Step 6: Compute the degree of centrality and the degree of causation. By Eqs. (24)-(25), the calculation results of the degree of centrality and the degree of causation of the evidences are shown in Table 3.

Table 3: The degree of centrality and the degree of causation

Item	Body of evidence				
	A	B	C	D	E
The degree of centrality (M_i)	1.2391	0.6327	0.8165	0.8776	1.3616
The degree of causation (R_i)	1.2391	0.3469	-0.5307	0.3062	-1.3616

Step 7: Plot the *Cartesian* coordinate of the degree of centrality - the degree of causation. According to M_i and R_i in Table 3 in **Step 6**, the *Cartesian* coordinate are shown in Fig. 3.
Recalling the previous discussion, we know that, in the *Cartesian* coordinate, from left to right, this evidence is more important than other evidence. Therefore, using index function δ, we can get that the sort result is: $\delta_i(1) = 5, \delta_i(2) = 1, \delta_i(3) = 4, \delta_i(4) = 3, \delta_i(5) = 2$. Further, the ranking results of the

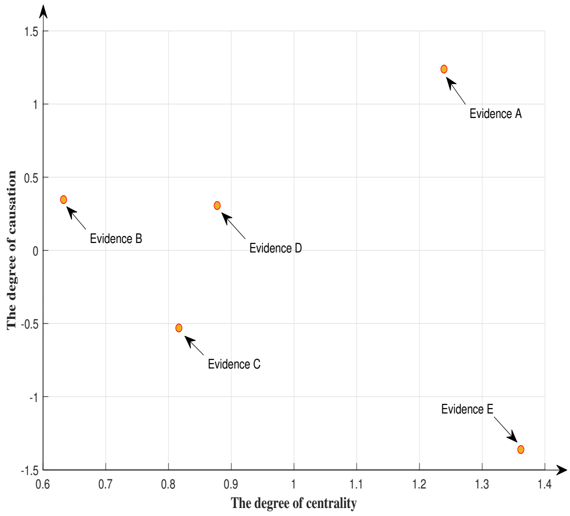

Fig. 3: The Cartesian coordinate of the degrees of centrality and causation

probability of the body of evidence can be obtained as $p_{i\delta_i(1)} = p_{i5} = 0.7$, $p_{i\delta_i(2)} = p_{i1} = 0.5$, $p_{i\delta_i(3)} = p_{i4} = 0.8$, $p_{i\delta_i(4)} = p_{i3} = 0.3$, $p_{i\delta_i(5)} = p_{i2} = 1$.

Step 8: Compute the soft likelihood function. Using Eq. (26), the calculation results of the soft likelihood function are derived. Next, with Eqs. (27)-(28), we calculate the soft likelihood function under three specific preferences, i.e., $\alpha = 0.1$ (the negative attitude), $\alpha = 0.5$ (the neutral attitude), $\alpha = 0.9$ (the positive attitude). The obtained results are 0.0860, 0.2996, 0.6251, respectively. In addition, for other preferences, i.e., $\alpha = 0.2$, $\alpha = 0.3$, $\alpha = 0.4$, $\alpha = 0.6$, $\alpha = 0.7$, $\alpha = 0.8$, the calculation results of the soft likelihood function are shown in Table 4.

Table 4: The value of the soft likelihood function with $\alpha = 0.2, 0.3, 0.4, 0.6, 0.7, 0.8$

	$\alpha = 0.2$					$\alpha = 0.3$				
j	$\mathcal{L}^{\alpha}_{j,n}$	$\mathcal{L}^{\alpha}_{j-1,n}$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$	$\mathcal{L}^{\alpha}_{j,n}$	$\mathcal{L}^{\alpha}_{j-1,n}$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$
1	0.0016	0.0000	0.0016	0.7000	0.0011	0.0234	0.0000	0.0234	0.7000	0.0164
2	0.0256	0.0016	0.0240	0.3500	0.0084	0.1179	0.0234	0.0945	0.3500	0.0331
3	0.1296	0.0256	0.1040	0.2800	0.0291	0.3036	0.1179	0.1857	0.2800	0.0520
4	0.4096	0.1296	0.2800	0.0840	0.0235	0.5941	0.3036	0.2905	0.0840	0.0244
5	1.0000	0.4096	0.5904	0.0840	0.0496	1.0000	0.5941	0.4059	0.0840	0.0341
			$\sum_{j=1}^{5} w_j = 1$		$\sum_{j=1}^{5} w_j Prod_i(j) = \mathbf{0.1118}$			$\sum_{j=1}^{5} w_j = 1$		$\sum_{j=1}^{5} w_j Prod_i(j) = \mathbf{0.1600}$
	$\alpha = 0.4$					$\alpha = 0.6$				
j	$\mathcal{L}^{\alpha}_{j,n}$	$\mathcal{L}^{\alpha}_{j-1,n}$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$	$\mathcal{L}^{\alpha}_{j,n}$	$\mathcal{L}^{\alpha}_{j-1,n}$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$
1	0.0894	0.0000	0.0894	0.7000	0.0626	0.3420	0.0000	0.3420	0.7000	0.2394
2	0.2530	0.0894	0.1635	0.3500	0.0572	0.5429	0.3420	0.2009	0.3500	0.0703
3	0.4648	0.2530	0.2118	0.2800	0.0593	0.7114	0.5429	0.1685	0.2800	0.0472
4	0.7155	0.4648	0.2508	0.0840	0.0211	0.8618	0.7114	0.1504	0.0840	0.0126
5	1.0000	0.7155	0.2845	0.0840	0.0239	1.0000	0.8618	0.1382	0.0840	0.0116
			$\sum_{j=1}^{5} w_j = 1$		$\sum_{j=1}^{5} w_j Prod_i(j) = \mathbf{0.2241}$			$\sum_{j=1}^{5} w_j = 1$		$\sum_{j=1}^{5} w_j Prod_i(j) = \mathbf{0.3811}$
	$\alpha = 0.7$					$\alpha = 0.8$				
j	$\mathcal{L}^{\alpha}_{j,n}$	$\mathcal{L}^{\alpha}_{j-1,n}$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$	$\mathcal{L}^{\alpha}_{j,n}$	$\mathcal{L}^{\alpha}_{j-1,n}$	w_j	$Prod_i(j)$	$w_j Prod_i(j)$
1	0.5017	0.0000	0.5017	0.7000	0.3512	0.6687	0.0000	0.6687	0.7000	0.4681
2	0.6752	0.5017	0.1735	0.3500	0.0607	0.7953	0.6687	0.1265	0.3500	0.0443
3	0.8034	0.6752	0.1281	0.2800	0.0359	0.8801	0.7953	0.0848	0.2800	0.0238
4	0.9088	0.8034	0.1054	0.0840	0.0089	0.9457	0.8801	0.0656	0.0840	0.0055
5	1.0000	0.9088	0.0912	0.0840	0.0077	1.0000	0.9457	0.0543	0.0840	0.0046
			$\sum_{j=1}^{5} w_j = 1$		$\sum_{j=1}^{5} w_j Prod_i(j) = \mathbf{0.4643}$			$\sum_{j=1}^{5} w_j = 1$		$\sum_{j=1}^{5} w_j Prod_i(j) = \mathbf{0.5462}$

4.3 Analysis and discussion

Obviously, from the results in the above tables, we can see that when DMs adopt different preferences for the same set of evidence obtained, the soft likelihood function calculated using the proposed method shows significant changes. More specifically, when the DM adopts the most positive preference ($\alpha = 0.9$), he (or she) is more inclined to the identification result of the evidence E on the suspect x_i. When the DM adopts the most negative preference ($\alpha = 0.1$), he (or she) is more inclined to the identification result of the suspect x_i by evidence B. When the DM adopts neutral preferences ($\alpha = 0.5$), that is, all evidence is equally important, then the soft likelihood function takes the middle value between positive and negative preferences are adopted.

In general, when the DM's decision preferences are more positive, it indicates that the DM can make the right decision. In this case, the higher the probability that the suspect x_i is considered to be a criminal, that is, the higher the value of the soft likelihood function. When the DM's decision preference is more negative, it indicates that the DM is disturbed by other factors, and the decision result may be wrong. In this case, it is likely to cause misjudgment. Therefore, we should reduce the probability that this DM refers to the suspect x_i, that is, the lower the value of the soft likelihood function.

5 Comparison and analysis

This section will compare and discuss with the existing methods to explain the rationality of the method. Through sensitivity analysis, the reliability of the proposed method is shown. In addition, the superiority of the method is further emphasized.

5.1 Rationality analysis

Reviewing the literature, the existing research on soft likelihood function can be divided into two categories. The first category is to extend the soft likelihood function proposed by Yager et al. to other application areas. Such scholars include Jiang et al. [34], Fei et al. [37,36,53,39], Li et al. [54], and some of the author's work [42,41,40]. The second category is inspired by Yager et al.'s soft likelihood function based on OWA operators [33], and develops a new soft likelihood function. In this category, Song and Deng [43] proposed a new soft likelihood function based on the POWA operators. Through theoretical analysis and experimental research, the author in [44] found defects in the soft likelihood function proposed by Song and Deng, and proposed an improved soft likelihood function. Therefore, in this subsection, the rationality analysis of the proposed method is mainly compared with the soft likelihood function proposed by Yager et al. [33], Song and Deng [43] and the author in [44].

As introduced in subsection 2.3, the soft likelihood function proposed by Yager et al. [33] is mainly to overcome the defect of conflicts when using the likelihood function to fuse compatible evidence. From Eq. (12), the probabilities are sorted by the index function $\lambda_i(k)$, where $p_{i\lambda_i(k)}$ represents the product of the $k-th$ largest compatible probability. In other words, the idea of soft likelihood function proposed by Yager et al. can be summarized as: when the decision preference is more positive, the more probable the evidence is. At this time, this evidence is more important, so it has higher priority than other evidence; on the contrary, the more pessimistic the decision preference is, the more probable the evidence is. At this time, this evidence is just as important, so it has higher priority than other evidence; in particular, when decision makers adopt a neutral preference, all evidence is equally important at this time. The soft likelihood function proposed by Song and Deng [43] is based on the POWA operators [23]. Compared with the soft likelihood function proposed by Yager et al. [33], The main advantages of the Song and Deng's method are recapitulated as follows. First, the POWA operators is better than the OWA operators takes into account the probabilistic information of the evidence. The weight of POWA in this method is based on probability, which can be seen from the support function. Second, the soft likelihood function is more gentle than the soft likelihood function of Yager et al. This is because the likelihood function takes the form of an arithmetic square root. However, through theoretical analysis and experimental research, the author [44] found shortcomings of the soft likelihood function proposed by Song and Deng. On the one hand, although the POWA operators considers the probability information, it does not reflect the weight of the OWA operators. On the other hand, the proposed method does not reflect the preferences of decision makers well. To overcome the above problems, an improved soft likelihood function is proposed. Through comprehensive analysis, we can know that although the weights assigned to the likelihood function are different, the method of Yager et al. [33] uses the weight of the OWA operators, and the method of Song and Deng [43] and the author [44] use the weight of the POWA operators. However, in essence, the idea of the soft likelihood function by Song and Deng is consistent with the soft likelihood function proposed by Yager et al. This is because the ranking of the likelihood function is based on the maximum compatible probability.

It should be emphasized that, in this paper, we consider the fact that the evidence is not always independent of each other in practical applications, expanding the scope of application of soft likelihood function. The DEMATEL method, is used to model the uncertain information to obtain more important evidence information. And the index function is used to sort according to the importance of the evidence. That is, the ranking of the probability of evidence in the proposed method is based on the importance of the evidence. The idea of the proposed method can be summarized as: the more positive the decision preferences, the more important the evidence is. Moreover, the more important evidence has higher priority. On the contrary, the more pessimistic the decision preference, the more inclined to the less important evidence. At this time, the less important evidence has higher priority. In particular, when the decision maker adopts a neutral preference, all evidence is equally important at this time.

5.2 Sensitivity analysis

In this subsection, a sensitivity test is performed to demonstrate the reliability of the proposed method.

Assume at the scene of the crime, seven trace evidences were collected and seven forensic experts were invited to evaluate the evidence. Case analysis experts have shown that there is some correlation between these evidences, but it is impossible to know which specific evidence will have a crucial impact on the outcome of the case. The information provided by all the above experts is needed to obtain the probability of the suspect's crime. The decision results of the seven experts on each trace of evidence are as follows:

$$E^{'} = \{p_{i1} = 0.2, p_{i2} = 1, p_{i3} = 0.9, p_{i4} = 0.7, p_{i5} = 0.6, p_{i6} = 0.5, p_{i7} = 0.8\}$$

where p_{ij} represents the probability that the jth forensic expert supports the suspect $x_i$crime.

And then, we use the proposed method to model the evidence information and obtain the ranking of the importance of the evidence. The total number of possible evidence ranking results is $7 \times 6 \times 5! = 5040$. In this test, we randomly obtain 6 possible evidence information ranking results from these 5040 evidence ranking results. The details are shown below.

- Case one: $p_{i6} \succ p_{i3} \succ p_{i2} \succ p_{i1} \succ p_{i7} \succ p_{i5} \succ p_{i4}$;
- Case two: $p_{i2} \succ p_{i7} \succ p_{i1} \succ p_{i6} \succ p_{i4} \succ p_{i5} \succ p_{i3}$;
- Case three: $p_{i4} \succ p_{i6} \succ p_{i2} \succ p_{i5} \succ p_{i7} \succ p_{i3} \succ p_{i1}$;
- Case four: $p_{i5} \succ p_{i7} \succ p_{i6} \succ p_{i1} \succ p_{i3} \succ p_{i4} \succ p_{i2}$;
- Case five: $p_{i7} \succ p_{i1} \succ p_{i5} \succ p_{i4} \succ p_{i3} \succ p_{i2} > p_{i6}$;
- Case six: $p_{i3} \succ p_{i4} \succ p_{i6} \succ p_{i1} \succ p_{i5} \succ p_{i7} > p_{i2}$;

in which, $p_{ip} \succ p_{iq}$ indicates that evidence p is more important than evidence q.

Next, we choose six decision preferences, which are $\alpha = 0.1$, $\alpha = 0.3$, $\alpha = 0.4$, $\alpha = 0.6$, $\alpha = 0.7$ and $\alpha = 0.9$, respectively. Finally, in each case, the calculation results of the soft likelihood function with the six different preferences are shown in Fig. 4.

Obviously, the changes of the soft likelihood function are different under the six different decision preferences, which is caused by the different importance of the evidence in each case. On the whole, when the decision maker's decision preferences are more positive, the degree of support for the suspect's x_i crime is higher. According to the analysis in [44], we know that this is in line with reality. Clearly, it can be seen from the above tests that the proposed method can reasonably represent the decision results of decision makers under different preferences in any evidence sequence of any set of evidence sets. Therefore, the proposed method has strong robustness and reliability.

5.3 Superiority of the proposed method

When the correlation between the obtained evidence is not considered, the fusion result of compatible evidence is completely based on the size of the probability. However, in real-world applications, the evidence is not completely independent. At this point, how to model uncertain information is critical. In this paper, a novel soft likelihood function based on DEMATEL is proposed, which provides a new idea for the modeling of associated evidence information.

In the application of criminal forensic cases, the evidence collected is relevant. If multiple evidence information is extracted according to the method proposed by Yager et al. [33], Song and Deng [43] and the author in [44], this will obviously lose some important information, that is, the correlation characteristics between the

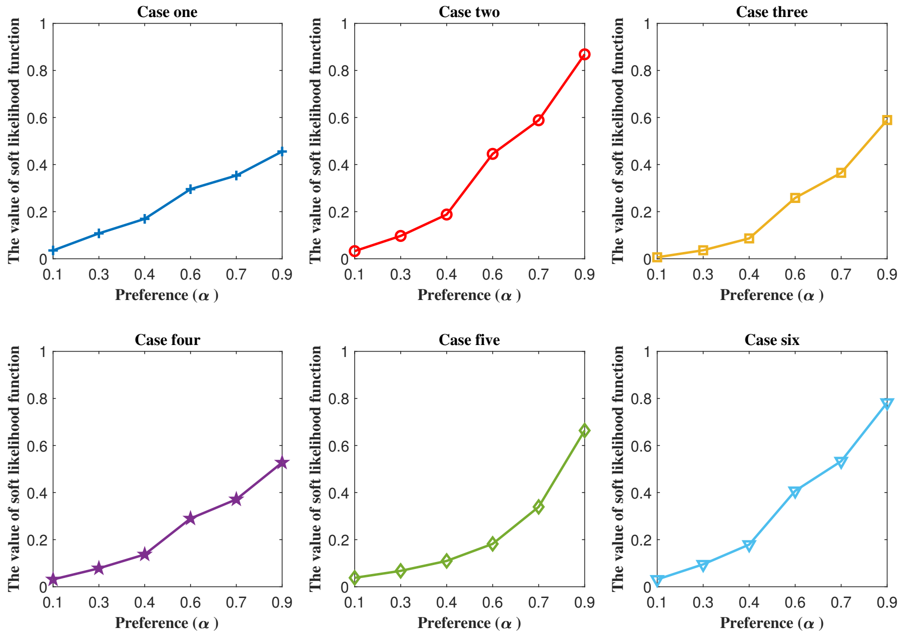

Fig. 4: Soft likelihood function under different preferences in cases one to six

evidence. At this time, how to comprehensively consider the opinions of forensic experts and case analysis experts and deal with uncertain information based on the general opinions is very important. As a result, in the context of criminal cases, the qualitative analysis of the pros and cons of different models is shown in Table 5.

Table 5: Comparison of the functions of different models

Method	The relevance between evidence	The importance of evidence
Yager et al. [33]	✗	✗
Song and Deng [43]	✗	✗
The author in [44]	✗	✗
Proposed method in this paper	✓	✓

[1] In the table, "✓" indicates that the method has the function and "✗" indicates that it does not.

In the proposed method, DEMATEL method is used to comprehensively consider the opinions of all experts to obtain the ranking of important information. In the end, the fusion of evidence adjusts the priority of evidence according to the preferences of decision makers. This provides an innovative solution for decision makers' judgment of criminal suspects. Subsequently, we consider an application example to explain the effectiveness of the proposed method. In this application, modeling is based on the potential correlations between evidence pointed out by case analysis experts. However, more broadly, in real-world applications, this association can be given more meaning, such as influence, judgment, recognition, and so on. There are many practical problems that can be solved by the proposed methods, such as the assessment of job satisfaction, the results of funny lesson scheduling programs, etc. Finally, in a nutshell, the proposed method is superior in solving this type of problem.

6 Conclusion

Likelihood function are one of the important tools for processing certain information. However, since the original soft likelihood function was too strict, Yager et al. developed a soft likelihood function. Because soft likelihood function is more flexible in dealing with uncertain information, it has been successfully applied in many fields.

However, by reviewing existing research, we find that the fusion results of soft likelihood function are all in the case where the evidence is independent of each other. However, in the real world, it is more common for evidence to be related. Therefore, the modeling of related evidence information is very important. In this paper, inspired by the DEMATEL method, a novel soft likelihood function is proposed. First, the comprehensive information correlation matrix is considered by DEMATEL to form a comprehensive evidence correlation matrix, so as to obtain the importance ranking of evidence. Then, based on the OWA operators, the preference characteristics of decision makers are considered. Finally, the information fusion result is obtained through the distribution of weights. The idea of the proposed method can be summarized as: the modeling of uncertain information is based on the importance of evidence. That is, in the decision-making process, when a decision maker adopts a positive preference, the more important information has the higher priority. Conversely, when policymakers adopt pessimistic preferences, less important information has higher priority. All evidence is equally important if policymakers adopt neutral preferences. The application in criminal forensic cases illustrates the effectiveness of the proposed method. The comparison and analysis show the superiority of the proposed method.

In further work, we intend to further study the properties of the proposed soft likelihood function. In addition, we hope to extend the proposed method to suit more applications.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

1. Max H Bazerman and Don A Moore. *Judgment in managerial decision making*. Wiley New York, 1994.
2. Feifei Jin, Ying Zhang, Harish Garg, Jinpei Liu, and Jia Chen. Evaluation of small and medium-sized enterprises' sustainable development with hesitant fuzzy linguistic group decision-making method. *Applied Intelligence*, pages 1–21, 2021.
3. Xinyang Deng, Wen Jiang, and Zhen Wang. An information source selection model based on evolutionary game theory. *Applied Mathematics and Computation*, 385:125362, 2020.
4. R Scott Tindale and Jeremy R Winget. Group decision-making. In *Oxford Research Encyclopedia of Psychology*. 2019.
5. Zaoli Yang, Harish Garg, Jinqiu Li, Gautam Srivastava, and Zehong Cao. Investigation of multiple heterogeneous relationships using a q-rung orthopair fuzzy multi-criteria decision algorithm. *Neural Computing and Applications*, 33(17):10771–10786, 2021.
6. Ming Tang, Huchang Liao, and Hamido Fujita. Delegation mechanism-based large-scale group decision making with heterogeneous experts and overlapping communities. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2021.
7. Zeshui Xu. *Uncertain multi-attribute decision making: Methods and applications*. Springer, 2015.
8. Liguo Fei and Yuqiang Feng. A dynamic framework of multi-attribute decision making under pythagorean fuzzy environment by using dempster–shafer theory. *Engineering Applications of Artificial Intelligence*, 101:104213, 2021.
9. Dennis P Wittmer. Ethical decision-making. In *Handbook of administrative ethics*, pages 507–534. Routledge, 2019.
10. Shuping Wan. Power average operators of trapezoidal intuitionistic fuzzy numbers and application to multi-attribute group decision making. *Applied mathematical modelling*, 37(6):4112–4126, 2013.
11. Shuping Wan and Zhihong Yi. Power average of trapezoidal intuitionistic fuzzy numbers using strict t-norms and t-conorms. *IEEE Transactions on Fuzzy Systems*, 24(5):1035–1047, 2015.
12. Bill Wooldridge and Birton Cowden. Strategic decision-making in business. In *Oxford Research Encyclopedia of Business and Management*. 2020.
13. Zhinan Hao, Zeshui Xu, Hua Zhao, and Zhan Su. Optimized data manipulation methods for intensive hesitant fuzzy set with applications to decision making. *Information Sciences*, 2021.
14. Ronald R Yager. Generalized regret based decision making. *Engineering Applications of Artificial Intelligence*, 65:400–405, 2017.
15. Ronald R Yager and Naif Alajlan. Importance-based multicriteria decision making with interval valued criteria satisfactions. *International Journal of Intelligent Systems*, 34(12):3336–3344, 2019.
16. Lotfi A Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.
17. Krassimir T Atanassov. Intuitionistic fuzzy sets. In *Intuitionistic fuzzy sets*, pages 1–137. Springer, 1999.
18. Yige Xue, Yong Deng, and Harish Garg. Uncertain database retrieval with measure–based belief function attribute values under intuitionistic fuzzy set. *Information Sciences*, 546:436–447, 2021.
19. Arthur P. Dempster. Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, 38(2):325–339, 1967.
20. Fuyuan Xiao. Ced: A distance for complex mass functions. *IEEE Transactions on Neural Networks and Learning Systems*, page Doi: 10.1109/TNNLS.2020.2984918, 2020.
21. Yong Deng. Information volume of mass function. *International Journal of Computers, Communications & Control*, 15(6):3983, 2020.
22. Fuyuan Xiao. Ceqd: A complex mass function to predict interference effects. *IEEE Transactions on Cybernetics*, page Doi: 10.1109/TCYB.2020.3040770, 2020.
23. Ronald R Yager. The power average operator. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 31(6):724–731, 2001.
24. Moxian Song, Wen Jiang, Chunhe Xie, and Deyun Zhou. A new interval numbers power average operator in multiple attribute decision making. *International Journal of Intelligent Systems*, 32(6):631–644, 2017.
25. Shanshan Meng, Nan Liu, and Yingdong He. Gifihia operator and its application to the selection of cold chain logistics enterprises. *Granular Computing*, 2(3):187–197, 2017.

26. Chengqing Wang, Xinggan Fu, Shanshan Meng, and Yingdong He. Multi-attribute decision-making based on the spifgia operators. *Granular Computing*, 2(4):321–331, 2017.
27. Lotfi A Zadeh. A note on z-numbers. *Information Sciences*, 181(14):2923–2932, 2011.
28. Bingyi Kang, Pengdan Zhang, Zhenyu Gao, Gyan Chhipi-Shrestha, Kasun Hewage, and Rehan Sadiq. Environmental assessment under uncertainty using dempster–shafer theory and z-numbers. *Journal of Ambient Intelligence and Humanized Computing*, pages 1–20, 2019.
29. Xinyang Deng, Yong Hu, Yong Deng, and Sankaran Mahadevan. Supplier selection using ahp methodology extended by d numbers. *Expert Systems with Applications*, 41(1):156–167, 2014.
30. Fuyuan Xiao. A multiple-criteria decision-making method based on d numbers and belief entropy. *International Journal of Fuzzy Systems*, 21(4):1144–1153, 2019.
31. Jianbo Yang and Dongling Xu. Evidential reasoning rule for evidence combination. *Artificial Intelligence*, 205:1–29, 2013.
32. Xiaoge Zhang, Sankaran Mahadevan, and Xinyang Deng. Reliability analysis with linguistic data: An evidential network approach. *Reliability Engineering & System Safety*, 162:111–121, 2017.
33. Ronald R Yager, Paul Elmore, and Fred Petry. Soft likelihood functions in combining evidence. *Information Fusion*, 36:185–190, 2017.
34. Wen Jiang and Weiwei Hu. An improved soft likelihood function for dempster–shafer belief structures. *International Journal of Intelligent Systems*, 33(6):1264–1282, 2018.
35. Tianshuo Ma and Fuyuan Xiao. An improved method to transform triangular fuzzy number into basic belief assignment in evidence theory. *IEEE Access*, 7:25308–25322, 2019.
36. Liguo Fei, Yuqiang Feng, Luning Liu, and Weicheng Mao. On intuitionistic fuzzy decision-making using soft likelihood functions. *International Journal of Intelligent Systems*, 34(9):2225–2242, 2019.
37. Liguo Fei. On interval-valued fuzzy decision-making using soft likelihood functions. *International Journal of Intelligent Systems*, 34(7):1631–1652, 2019.
38. Yangxue Li and Yong Deng. Tdbf: Two-dimensional belief function. *International Journal of Intelligent Systems*, 2019.
39. Liguo Fei, Yuqiang Feng, and Luning Liu. On pythagorean fuzzy decision making using soft likelihood functions. *International Journal of Intelligent Systems*, 34(12):3317–3335, 2019.
40. Ye Tian, Lili Liu, Xiangjun Mi, and Bingyi Kang. Zslf: A new soft likelihood function based on z-numbers and its application in expert decision system. *IEEE Transactions on Fuzzy Systems*, 2020.
41. Xiangjun Mi, Tongxuan Lv, Ye Tian, and Bingyi Kang. Multi-sensor data fusion based on soft likelihood functions and owa aggregation and its application in target recognition system. *ISA transactions*, 112:137–149, 2021.
42. Xiangjun Mi, Ye Tian, and Bingyi Kang. A hybrid multi-criteria decision making approach for assessing health-care waste management technologies based on soft likelihood function and d-numbers. *Applied Intelligence*, pages 1–20, 2021.
43. Yutong Song and Yong Deng. A new soft likelihood function based on power ordered weighted average operator. *International Journal of Intelligent Systems*, 34(11):2988–2999, 2019.
44. Xiangjun Mi, Ye Tian, and Bingyi Kang. A modified soft-likelihood function based on powa operator. *International Journal of Intelligent Systems*, 35(5):869–890, 2020.
45. Ronald R Yager. Quantifier guided aggregation using owa operators. *International Journal of Intelligent Systems*, 11(1):49–73, 1996.
46. Ronald R Yager. Families of owa operators. *Fuzzy sets and systems*, 59(2):125–148, 1993.
47. E Fontela and A Gabus. Dematel, innovative methods. 1974.
48. Minglang Tseng, Kuojui Wu, Li Ma, Tsai Chi Kuo, and Fuyume Sai. A hierarchical framework for assessing corporate sustainability performance using a hybrid fuzzy synthetic method-dematel. *Technological Forecasting and Social Change*, 144:524–533, 2019.
49. KEK Vimal, P Sasikumar, Kaliyan Mathiyazhagan, M Nishal, and K Sivakumar. 10 analyzing the drivers of sustainable procurement using the dematel approach. *Sustainable Procurement in Supply Chain Operations*, page 10, 2019.
50. Naveen Kumar and K Mathiyazhagan. Modeling the interrelationship of critical success factors adoption of sustainable lean manufacturing using dematel approach. In *Recent Advances in Mechanical Engineering*, pages 31–45. Springer, 2020.

51. Weiquan Zhang and Yong Deng. Combining conflicting evidence using the dematel method. *Soft computing*, 23(17):8207–8216, 2019.
52. Lazim Abdullah, Norsyahida Zulkifli, Huchang Liao, Enrique Herrera-Viedma, and Abdullah Al-Barakati. An interval-valued intuitionistic fuzzy dematel method combined with choquet integral for sustainable solid waste management. *Engineering Applications of Artificial Intelligence*, 82:207–215, 2019.
53. Liguo Fei, Yuqiang Feng, and Luning Liu. Evidence combination using owa-based soft likelihood functions. *International Journal of Intelligent Systems*, 34(9):2269–2290, 2019.
54. Peilin Li and Liguo Fei. On combination rule in dempster–shafer theory using owa-based soft likelihood functions and its applications in environmental impact assessment. *International Journal of Intelligent Systems*, 2019.