

## Primes of the Middle

(137&13)

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Article details the unique position of 137 & 13 relative to their nearest perfect squares.

(n)atural number,  $n > 0$

$$\lfloor \sqrt{n} \rfloor = \text{floor } (r)oot$$

$$(r + 1)^2 - n = w$$

$$\frac{w+1}{2} \sqrt{n - r^2} = x$$

$$n - (x^w + w + x) = 0$$

$$n=137$$

$$\lfloor \sqrt{137} \rfloor = 11$$

$$(11 + 1)^2 - 137 = 7$$

$$\frac{7+1}{2} \sqrt{137 - 11^2} = 2$$

$$137 - (2^7 + 7 + 2) = 0$$

$$n=13$$

$$\lfloor \sqrt{13} \rfloor = 3$$

$$(3 + 1)^2 - 13 = 3$$

$$\frac{3+1}{2} \sqrt{13 - 3^2} = 2$$

$$13 - (2^3 + 3 + 2) = 0$$

137 & 13 only satisfactory natural numbers up to at least  $150^2$

See tables.

If; the natural numbers are restricted to the “middle” such that:

$$\left(\frac{1}{2} + \frac{1}{16}\right) < \frac{n-r^2}{(n+1)^2-r^2} < \left(\frac{1}{2} + \frac{1}{4}\right)$$

Then; 137 & 13 only satisfactory natural numbers of the middle to infinity.

See tables.

137 & 13 are also the only satisfactory primes of the middle to infinity.

| n  | r | $n - (x^w + w + x)$ |
|----|---|---------------------|
| 1  | 1 | -2                  |
| 2  | 1 | -2                  |
| 3  | 1 | -2                  |
| 4  | 2 | -1                  |
| 5  | 2 | -1                  |
| 6  | 2 | -1.24...            |
| 7  | 2 | -1.40... low, r=2   |
| 8  | 2 | -1                  |
| 9  | 3 | 2                   |
| 10 | 3 | 2                   |
| 11 | 3 | 1.56...             |
| 12 | 3 | .64...              |
| 13 | 3 | 0 low, r=3          |
| 14 | 3 | .52...              |
| 15 | 3 | 2                   |
| 16 | 4 | 7                   |
| 17 | 4 | 7                   |
| 18 | 4 | 6.44...             |
| 19 | 4 | 5.05...             |
| 20 | 4 | 3.33...             |
| 21 | 4 | 1.96...             |
| 22 | 4 | 1.85... low, r=4    |
| 23 | 4 | 3.95...             |
| 24 | 4 | 7                   |
| 25 | 5 | 14                  |

|    |   |                  |
|----|---|------------------|
| 26 | 5 | 14               |
| :  | : | :                |
| 31 | 5 | 4.37...          |
| 32 | 5 | 3.32... low, r=5 |
| 33 | 5 | 4.55...          |
| :  | : | :                |
| 35 | 5 | 14               |
| 36 | 6 | 23               |

|    |   |                    |
|----|---|--------------------|
| 37 | 6 | 23                 |
| :  | : | :                  |
| 44 | 6 | 5                  |
| 45 | 6 | 4.95... low, r = 6 |
| 46 | 6 | 11.21...           |
| :  | : | :                  |
| 48 | 6 | 23                 |
| 49 | 7 | 34                 |

|    |   |                    |
|----|---|--------------------|
| 50 | 7 | 34                 |
| :  | : | :                  |
| 58 | 7 | 6.89...            |
| 59 | 7 | 5.42... low, r = 7 |
| 60 | 7 | 7.02...            |
| :  | : | :                  |
| 63 | 7 | 34                 |
| 64 | 8 | 47                 |

|    |   |                  |
|----|---|------------------|
| 65 | 8 | 47               |
| :  | : | :                |
| 75 | 8 | 6.02...          |
| 76 | 8 | 5.81... low, r=8 |
| 77 | 8 | 9.63...          |
| :  | : | :                |

|    |   |    |
|----|---|----|
| 80 | 8 | 47 |
| 81 | 9 | 62 |

|     |    |                  |
|-----|----|------------------|
| 82  | 9  | 62               |
| :   | :  | :                |
| 93  | 9  | 6.76...          |
| 94  | 9  | 4.70... low, r=9 |
| 95  | 9  | 6.26...          |
| :   | :  | :                |
| 99  | 9  | 62               |
| 100 | 10 | 79               |

|     |    |                  |
|-----|----|------------------|
| 101 | 10 | 79               |
| :   | :  | :                |
| 114 | 10 | 3.73...          |
| 115 | 10 | 3.04... low r=10 |
| 116 | 10 | 6.88...          |
| :   | :  | :                |
| 120 | 10 | 79               |
| 121 | 11 | 98               |

|     |    |               |
|-----|----|---------------|
| 122 | 11 | 98            |
| :   | :  | :             |
| 136 | 11 | 2.91...       |
| 137 | 11 | 0 low, r = 11 |
| 138 | 11 | 1.12...       |
| :   | :  | :             |
| 143 | 11 | 98            |
| 144 | 12 | 119           |

| r  | n   | $n - (x^w + w + x)$ |  |
|----|-----|---------------------|--|
| 12 | 144 | 119                 |  |
| :  | :   | :                   |  |
| 12 | 152 | 82.94...            |  |

|    |     |          |   |                     |
|----|-----|----------|---|---------------------|
| :  | :   | :        |   |                     |
| 12 | 160 | 2.22...  | + | $r^2 + r + 4 = n$   |
| 12 | 161 | -2.85... | - |                     |
| 12 | 162 | -4.35... |   |                     |
| 12 | 163 | -.96...  | - | $(r + 1)^2 - 6 = n$ |
| 12 | 164 | 8.92...  | + |                     |
| :  | :   | :        |   |                     |
| 12 | 168 | 119      |   |                     |

|    |     |           |   |                     |
|----|-----|-----------|---|---------------------|
| 17 | 298 | 254       |   |                     |
| :  | :   | :         |   |                     |
| 17 | 310 | .637      | + | $r^2 + r + 4 = r$   |
| 17 | 311 | -13.22... | - |                     |
| :  | :   | :         |   |                     |
| 17 | 315 | -48.25... |   |                     |
| :  | :   | :         |   |                     |
| 17 | 318 | -11.95... | - | $(r + 1)^2 - 6 = n$ |
| 17 | 319 | 21.21...  | + |                     |
| :  | :   | :         |   |                     |
| 17 | 323 | 254       |   |                     |

| r  | $r^2 + r + 4$ | $n - (x^2 + w + x)$ |
|----|---------------|---------------------|
| 14 | 214           | 1.241...            |
| 15 | 244           | .930...             |
| 16 | 276           | .731...             |
| 17 | 310           | .637...             |
| 18 | 346           | .644                |
| 19 | 384           | .746                |
| 20 | 424           | .939                |
| 21 | 466           | 1.219               |

| r  | n    | $n - (x^w + w + x)$ |                       |
|----|------|---------------------|-----------------------|
| 35 | 1225 | 1154                |                       |
| :  | :    | :                   |                       |
| 35 | 1264 | 12.60...            | + $r^2 + r + 4 = n$   |
| 35 | 1265 | -37.80...           | -                     |
| :  | :    | :                   |                       |
| 35 | 1277 | -564.88...          |                       |
| :  | :    | :                   |                       |
| 35 | 1290 | -1.18...            | - $(r + 1)^2 - 6 = n$ |
| 35 | 1291 | 204.07...           | +                     |
| :  | :    | :                   |                       |
| 35 | 1295 | 1154                |                       |

|    |      |            |                       |
|----|------|------------|-----------------------|
| 36 | 1296 | 1223       |                       |
| :  | :    | :          |                       |
| 36 | 1336 | 13.86...   | + $r^2 + r + 4 = n$   |
| 36 | 1337 | -38.47...  | -                     |
| :  | :    | :          |                       |
| 36 | 1350 | -625.29... |                       |
| :  | :    | :          |                       |
| 36 | 1362 | -176.12... | - $(r + 1)^2 - 7 = n$ |
| 36 | 1363 | 3.42...    | +                     |
| :  | :    | :          |                       |
| 36 | 1368 | 1223       |                       |

|    |      |             |                     |
|----|------|-------------|---------------------|
| 65 | 4225 | 4094        |                     |
| :  | :    | :           |                     |
| 65 | 4294 | 68.66...    | + $r^2 + r + 4 = n$ |
| 65 | 4295 | -39.59...   |                     |
| :  | :    | :           |                     |
| 65 | 4322 | -2957.88... |                     |
| :  | :    | :           |                     |
| 65 | 4349 | -269.07...  | $(r + 1)^2 - 7 = n$ |
| 65 | 4350 | 407.21...   |                     |
| :  | :    | :           |                     |

|    |      |      |  |
|----|------|------|--|
| 65 | 4355 | 4094 |  |
|----|------|------|--|

|    |      |              |                     |
|----|------|--------------|---------------------|
| 94 | 8836 | 8647         |                     |
| :  | :    | :            |                     |
| 94 | 8934 | 148.99... +  | $r^2 + r + 4 = n$   |
| 94 | 8935 | -15.62... -  |                     |
| :  | :    | :            |                     |
| 94 | 8977 | -7317.04...  |                     |
| :  | :    | :            |                     |
| 94 | 9018 | -10.97... -  | $(r + 1)^2 - 7 = n$ |
| 94 | 9019 | 1449.16... + |                     |
| :  | :    | :            |                     |
| 94 | 9024 | 8647         |                     |

|    |      |               |                     |
|----|------|---------------|---------------------|
| 95 | 9025 | 8834          |                     |
| :  | :    | :             |                     |
| 95 | 9124 | 152.11... +   | $r^2 + r + 4 = n$   |
| 95 | 9125 | -14.46... -   |                     |
| :  | :    | :             |                     |
| 95 | 9167 | -7421.30...   |                     |
| :  | :    | :             |                     |
| 95 | 9208 | -1326.07... - | $(r + 1)^2 - 8 = n$ |
| 95 | 9209 | 5.868... +    |                     |
| :  | :    | :             |                     |
| 95 | 9215 | 8834          |                     |

|     |       |             |                   |
|-----|-------|-------------|-------------------|
| 106 | 11236 | 11023       |                   |
| :   | :     | :           |                   |
| 106 | 11346 | 187.69... + | $r^2 + r + 4 = n$ |
| 106 | 11347 | -.35... -   |                   |

|     |       |             |                       |
|-----|-------|-------------|-----------------------|
| :   | :     | :           |                       |
| 106 | 11394 | -9497.06... |                       |
| :   | :     | :           |                       |
| 106 | 11441 | -1445.37... | - $(r + 1)^2 - 8 = n$ |
| 106 | 11442 | 229.93...   | +                     |
| :   | :     | :           |                       |
| 106 | 11448 | 11023       |                       |

|     |       |             |                       |
|-----|-------|-------------|-----------------------|
| 107 | 11449 | 11234       |                       |
| :   | :     | :           |                       |
| 107 | 11561 | 1.03...     | + $r^2 + r + 5 = n$   |
| 107 | 11562 | -190.16...  | -                     |
| :   | :     | :           |                       |
| 107 | 11609 | -9803.27... |                       |
| :   | :     | :           |                       |
| 107 | 11656 | -1455.55... | - $(r + 1)^2 - 8 = n$ |
| 107 | 11657 | 253.917...  | +                     |
| :   | :     | :           |                       |
| 107 | 11663 | 11234       |                       |

|     |       |              |                       |
|-----|-------|--------------|-----------------------|
| 124 | 15376 | 15127        |                       |
| :   | :     | :            |                       |
| 124 | 15505 | 27.36...     | + $r^2 + r + 5 = n$   |
| 124 | 15506 | -197.20...   | -                     |
| :   | :     | :            |                       |
| 124 | 15561 | -13650.53... |                       |
| :   | :     | :            |                       |
| 124 | 15617 | -1561.48...  | - $(r + 1)^2 - 8 = n$ |
| 124 | 15618 | 758.75...    | +                     |
| :   | :     | :            |                       |
| 124 | 15624 | 15127        |                       |

|     |       |          |                     |
|-----|-------|----------|---------------------|
| 149 | 22201 | 21902    |                     |
| :   | :     | :        |                     |
| 149 | 22355 | 74.13... | + $r^2 + r + 5 = n$ |



|     |       |             |   |                     |
|-----|-------|-------------|---|---------------------|
| 149 | 22356 | -199.59...  | - |                     |
| :   | :     | :           |   |                     |
| 149 | 22424 | -20866.188  |   |                     |
| :   | :     | :           |   |                     |
| 149 | 22492 | -1521.90... | - | $(r + 1)^2 - 8 = n$ |
| 149 | 22493 | 1855.62...  | + |                     |
| :   | :     | :           |   |                     |
| 149 | 22499 | 21902       |   |                     |

|     |       |       |  |
|-----|-------|-------|--|
| 150 | 22500 | 22199 |  |
|-----|-------|-------|--|

| r   | n     | $\frac{n - r^2}{(n + 1)^2 - r^2}$ |
|-----|-------|-----------------------------------|
| 3   | 13    | .571...                           |
| 11  | 137   | .695...                           |
| 36  | 1336  | .547...                           |
| 36  | 1363  | .917...                           |
| 95  | 9124  | .518...                           |
| 95  | 9209  | .963...                           |
| 149 | 22355 | .515...                           |
| 149 | 22493 | .976...                           |