The Fisher-Tully Law solely with 1915 General Relativity and Dark Energy

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Abstract

Observation of the distribution of the velocity of galactic rotation curves differed from their expected centripetal form and lead to the notion of Dark Matter or modifications to Newtonian and General Relativity, such as MOND, TeVeS and the like and even Quantised Inertia. We aim to show that General Relativity with Dark Energy/the Cosmological Constant is all that is needed, with the proviso that the Cosmological Constant can increase in the presence of a gravitational field and become gravitating to account for the hypothesis of Dark Matter Haloes.

Keywords: Cosmological Constant, Dark Matter Haloes, Dark Energy, Einstein Field Equations, Fisher-Tully Law, Galactic Rotation Curves, MOND, Quantised Inertia, TeVeS, Virtual Particles

1. Introduction

General Relativity (GR) is a pearl of science found on simple intuitive principles by Einstein and graceful, economic mathematics by Hilbert. Its classically complete and clean structure only permits a free constant, the Cosmological Constant (CC)[1]. The non-relativistic limit follows as Newtonian Gravity. It came as a surprise (not chronologically) observations of the "Pioneer Anomaly"[2] and the distribution of galactic rotation curves. The former has been explained by radiation pressure from a heat source on the Pioneer craft[3] and dispensed with the need for some Modified Newtonian Dynamics (MOND)[4] at the outer planetary system scale. It would have beggared belief - ad-hoc empirical corrections to well-founded laws based on good mathematical principles.

Similarly, though at the outer galactic scale, rather large and obvious discrepancies have been observed in the distribution of velocities in galactic rotation curves, the so-called Fisher-Tulley Law[5, 6].

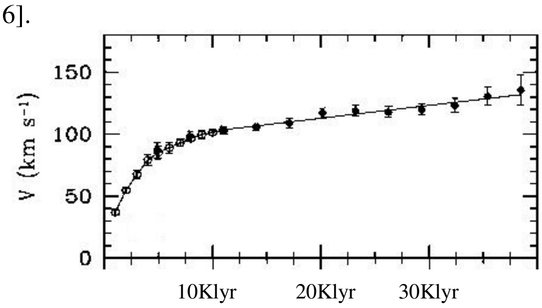

Figure 1 – Distance from centre of M33

Figure 1 (from [5]) shows a typical velocity distribution curve for a galaxy, in this case, M33 ("Triangulum Galaxy", some 30 Klyr in radius). There seems to be a perhaps linear increase after 10 kilo-light years or perhaps some other power law.

A simple consideration with the mass of the galaxy concentrated at the centre and the centripetal force would arrive at $v \propto r^{-0.5}$, even if we had a somewhat centrally distributed mass in the galaxy (the initial slow rise of the graph). Use of Gauss' Law and a uniform, spherical distribution of matter would give $v \propto r$. A general distribution of matter (for non-relativistic considerations: Newtonian Gravity (NG), which is mostly the case) would involve this integral:

$$\int_V Gm \frac{\rho(r)}{\left(r-r_p\right)^2} d^3r = m\frac{v_p^2}{r_p} \quad \Rightarrow \quad v_p = \sqrt{G\int_V \rho(r)\frac{r_p}{\left(r-r_p\right)^2} d^3r} \qquad \text{eqn. 1}$$

If the motion around the mass distribution is assumed circular, where r_p and v_p concern the position and velocity of the point in question. This has been done for various distributions of mass $\rho(r)$: dust, gas, stars, etc. but to obtain the Fisher-Tully Law, additional "dark matter" is speculated as an extra gravitating source, other than matter already accounted for. This matter is most peculiar and only acts via the gravitational force. Some have postulated that dark matter may be primordial black holes[7]. Further speculation exists as to whether it is cold or hot and its distribution is expected to be a halo around galaxies out to several galactic radii.

Other researchers have gilded the lily of the mathematical structures of Newtonian Gravity and General Relativity[4, 8] (and more, a detailed literature review is needed). These typically have been found wanting for the following reasons: non-local; inconsistency with known results, such as star formation theory; observations of colliding galaxies proving incompatible with such theories[9].

Yet another researcher has tried to add some underpinning to MOND by "quantising inertia"[10], positing that some minimum possible acceleration is responsible for the effects. McCulloch believes that accelerating bodies experience two horizons: aft being a Rindler Horizon[1] and fore, the Cosmological Horizon[1], with the intention of modelling a Casmir Cavity, such that inertia is described by radiation pressure.

The current author is not aware how this would create a minimum acceleration figure but believes the idea is flawed, in the first instance, by equating the Unruh temperature, the intensity of the radiation by the Stefan-Boltzmann Law and the radiation pressure from the said intensity of radiation – it cannot explain inertia. It is miniscule and should have been picked up sooner by peer review[†](DARPA grant indeed[11]). (Worth mentioning too is that purported propulsion device developed by Mulloch[12], from these ideas, falls foul of hidden momentum considerations[13, 14], for the quantised inertia ideas are surely wrong.) No further mention of this hypothesis is needed.

Our contribution is to aim for purity and parsimony by respecting the mathematical and physical structure of GR, whilst looking at the Cosmological Constant.

2. Is there truly a need for Dark Matter?

The author's earlier paper[15] on fitting and matching the "zeropoint energy" of quantum field theory (QFT) to the Cosmological Constant, achieved the gulf of 120 orders of magnitude in a final audacious step of postulating that it was 9 orders of magnitude higher than calculated by QFT. This allowed it to fit into a Taylor expansion of the Einstein Field Equation at second order in frequency, recognising that, technically, zeropoint energy is a fluctuation. The paper in question asked if there was interaction energy between the modes and performed a semi-classical calculation that gave a ball-park figure. This seems well-founded, though a more detailed QFT calculation is required as further work, for the creation of e^-e^+ pairs (etc.) present in the electrical fields of the other field modes.

Similarly in this paper we will now ask if a shift upwards of the zeropoint level is possible in and around galaxies and if it can become gravitating too (positive rather than negative pressure). GR measures or "responds" to absolute energy, whilst the rest of physics deals with differences in energy levels. We postulate here, without much detail at the moment with back of the envelope calculations, whether the astronomical vacuum energy (derivable from observations of the Hubble Constant) of some $5.4\text{x}10^{-10}$ J/m^3 might increase by a few orders of magnitude, in the said spatial environment and whether this energy over a few galactic radii provides the mass attributed to Dark Matter:-

A few factors help with a rough calculation which we'll improve later on: A typical galaxy like the Milky Way hold some 10^{11} stars, of which the sun is fairly typical at some $2\text{x}10^{30}$ kg mass. The galactic radius is some 100,000 lyr ($9.5\text{x}10^{20}$ m) and the dark matter halo is meant to be three radii in diameter and 90% the mass of the galaxy itself[16-18]. A figure for the mass we can see would be about $2\text{x}10^{41}$ kg and the dark matter halo, assuming a spherical distribution, would be $2\text{x}10^{58}$ J. The energy density of space, as inferred from astronomical measurements is around $5.4\text{x}10^{-10}$ J/m^3 and an increase in this by a factor of up to 10^6, near the core, is postulated in section 3 of this paper by electromagnetic interactions[15] and gravitational contraction.

However the mathematical structure of general relativity is preserved and the machinery to affect the Fisher-Tully Law is placed in the free constant, the Cosmological "constant". Metrics of the galaxy, considered as a Schwarzschild metric and a FLRW metric[1] shows that the non-g_{00} components of the former "swamp" the latter, such that near or within the galaxy, only matter, EM radiation and the energy density of some modified ρ_{vac} gravitate, with no expansion of space. The situation is akin to what is shown in figure 2.

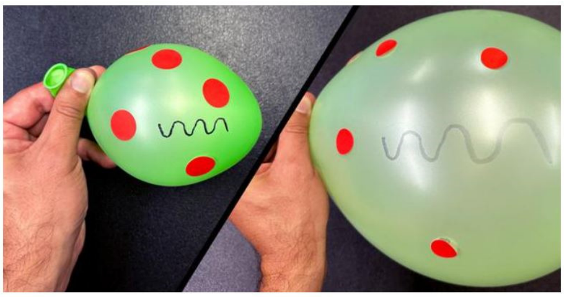

Figure 2 – Model of expanding universe with balloons and stickers (jpl.nasa.gov)

You may observe a slight puckering near the stickers and a "ballooning"/bulging effect away from them. The stickers are less elastic than the balloon material and this is a good analogue to space not expanding as much within galaxies (due

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† $F = -\int_A P d\vec{A}\cdot\vec{n} = -\int_A \frac{2\pi^4\hbar a^4}{15c^7} d\vec{A}\cdot\vec{n}, \quad \vec{n} = \frac{\vec{a}}{|\vec{a}|}$ where a is the acceleration and $d\vec{A}\cdot\vec{n}$ is an area element normal to it.

to gravitating sources overcoming the tendency for space to expand by Dark Energy) but expanding outside galaxies or galactic clusters.

3. Modulation of Dark Energy

Our earlier paper[15] postulated an electromagnetic interaction between the modes of the zeropoint to give a figure 10^9 times higher than the generally accepted value but gave good agreement with the measured Cosmological Constant. We can put this on a slightly firmer footing by postulating that the e^+e^- pairs form a degenerate Fermi-Dirac gas[19]. This provides the means for the zeropoint fields to interact.

The earlier paper[15] showed the work of Pauli and how the photon field is responsible for the vast bulk of zeropoint energy. Nethertheless the Uncertainty Principle allows the temporary creation of e^+e^- pairs. Figure 3 shows how for the creation energy of the process how the lifetime of these created pairs is pretty constant across non-relativistic energies.

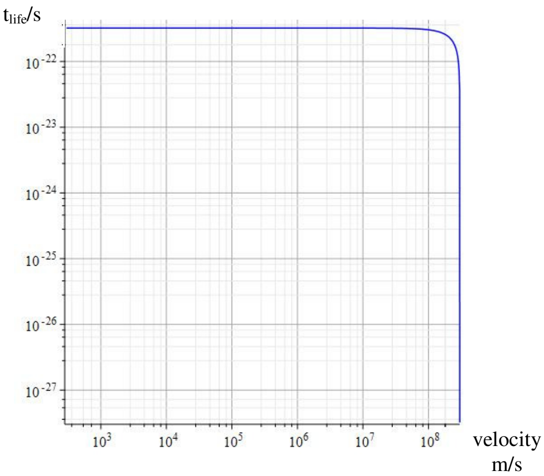


Figure 3 – Lifetime of virtual e^-e^+ pairs (Appendix 1 has Maple™ script)

3.1 The Virtual Particle debate

There is much discussion on virtual particles: are they real or mathematical artefacts? Indeed only perturbative methods use them and some formulations of Quantum Mechanics don't; often the same result can be calculated by either means.

Virtual particles aren't real; no direct measurement will yield detection in the rest frame of the source. They can't be said to propagate like regular excitation of the field but are more like a disturbance of the field‡, with some of the quantum numbers of a regular field disturbance. As such the reality of these objects is definitely true and "real", in the sense that their influence can be indirectly felt to affect real particles.

A probably loose analogy is that naked protons don't exist in solvated chemistry and can't be isolated but no-one would deny the existence of the Oxonium ion, $H_3O^+.(H_2O)_n$ - whereas balance equations have no such need. Appealing as such, where potentials/sources exist, there is no problem in invoking discussion of virtual particles.

3.2 The need for fermions from Dark Energy in our theory and the mechanism for modulation of it by gravity

The pressure of Bose and Fermi gases is very different near or at absolute zero due to Pauli Exclusion[19]. The radiation pressure of a Bose gas will be zero but Fermi gases have a "degeneracy pressure". Dark Energy behaves as an ideal superfluid in the stress-energy tensor,

$$\mathbf{T}_{\mu\nu} = \begin{pmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \qquad \text{eqn. 2}$$

However this pressure cannot be due to virtual photons at absolute zero but must be due to virtual electrons. Furthermore, where space is expanding the system's internal energy increases adiabatically (by the interaction between zeropoint modes[15]),

$$dU = TdS - PdV \qquad \text{eqn. 3}$$

Thus the fluid has negative pressure there is no contradiction in saying that the degeneracy pressure of the vacuum pairs is positive and we might account for it by equating the cosmological energy (or pressure) to the fluctuation caused by a (cold) totally degenerate virtual electron gas[19]:-

$$P = \frac{\left(\frac{6\pi^2}{g}\right)^{2/3} \hbar^2 \left(\frac{N}{V}\right)^{5/3}}{5m_e} \qquad \text{eqn. 4}$$

Where $g = 2S + 1$ and S is the spin of the particle, in this case ½ for the electron and m_e is the mass of the electron, N/V is the number per unit volume. For the value of the Cosmological Constant this works out at a density, ρ of 0.5×10^{17} e^+e^- pairs per cubic meter. Consideration of the mean-free-path $(\rho\sigma)^{-1}$ and the scattering cross-section, σ, for electrons (given by the classical electron radius[20], so some 10^{-30} m^2 to relativistic[21], some 10^{-20} m^2) of 10^{-13} m to 10^{-3} m in the extreme relativistic case. Given figure 3, we argue that most of the virtual particles not created in the relativistic regime would not have enough time to thermalize in the 10^{-22} seconds of their existence (they would

‡ They are precisely solutions of the wave equation (Green's function) with point sources.

need to be able to travel the mean-free-path several times), whilst those that were ultra-relativistic would be a very small percentage of the distribution given in figure 3. It is safe to say that the zeropoint spectrum is non-thermal with an undefined temperature.

There is no problem in using the non-relativistic form of eqn. 4, as figure 3 shows that the life-time of the virtual pairs drops precipitously near relativistic velocities. It is noted too that this pair creation process can happen in any frame (as by the frame invariance of the zeropoint energy) and so it is correct to consider the pairs as being created in all frames at all energies.

More correctly, if eqn. 4 is equated to the Cosmological Constant (the energy density of the vacuum multiplied by |c|, see [15]) divided by κ^3, so that it is considered a fluctuation at second order in an expansion of the field equations[15], we find a density of virtual e^+e^- pairs of some $8.85\text{x}10^{98}$ per cubic meter, though this is properly considered a fluctuation and not literal particles.

3.3 The mechanism for modulation of Dark Energy by gravity

The zeropoint superfluid pervades everything and no material can compress it. The only thing that can compress it is the contraction of space itself. This would seem to be at odds with General Covariance: apart from tidal forces, nothing in a general inertial frame is aware that it is in a gravity field. However, viewed in global coordinates, the region in question has more space and hence more dark energy.

3.3.1. Dark-Energy can expand space or gravitate dependent on coordinate viewpoint

In this section we shall show that dark-energy upon looking towards the centre of a gravity-well adds to the mass-energy causing the well but when looking away from the well, causes the expansion of space. This happens at several nested layers, the effect increasing with more expanse of space.

Thus: near the edges of a solar system the dark-energy contributes to the mass-energy of the system; nearby stars of the same group red-shift somewhat. At the edges of the star system the dark-energy contributes to holding it together but when looking at other star systems, they are observed to red-shift somewhat more. At the edge of a galaxy the dark-energy helps to hold it together and looking away at other galaxies, they are observed to be even more red-shifted.

Once again with Local Groups dark-energy helps to hold it together and looking away, other groups of galaxies are even more red-shifted. On the largest scales Super Groups are held together, observation of other Super Groups finds them fleeing at high red-shift. A "God's eye view" of the whole Universe would find it held together with the assistance of dark-energy. Presumably God would see his other universes rushing away too – unless he went up a scale and stood outside it all again. None of this is paradoxical and is to do with the relative scales of the expansion between the different levels and the volume of space (and dark-energy contained therein or outside), as we shall now show.

The spherical metric in flat space is used as a comparison to the Schwarzschild metric in curved space resulting from a gravitating body,

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad \text{eqn. 5}$$

$$ds^2 = -\left(1-\frac{2GM}{c^2r}\right)c^2dt^2 + \left(1-\frac{2GM}{c^2r}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad \text{eqn. 6}$$

Using the -+++ signature. Both these metrics have the same differential 4-volume element given by,

$$\begin{aligned} dV &= \sqrt{|-g|}\,dtdrd\theta d\phi \\ &= r^2\sin\theta dtdrd\theta d\phi \end{aligned} \quad \text{eqn. 7}$$

The three dimensional metric can be derived from the four dimensional metric thus[1, 22],

$$h_{\alpha\beta} = \left(g_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta}}{g_{00}}\right) \quad \text{eqn. 8}$$

(The signs are reversed for the +--- signature.) However then the 3-volume differential elements are different for these static metrics,

$$dV_{3,flat} = r^2\sin\theta dtdrd\theta d\phi \quad \text{eqn. 9}$$

$$dV_{3,Schwarz} = \left(1-\frac{2GM}{c^2r}\right)^{-1} r^2\sin\theta dtdrd\theta d\phi \quad \text{eqn. 10}$$

The flat space metric (consider a globe) has the usual shrinking of the volume element, as shown in figure 4, by the angles subtended from the radius

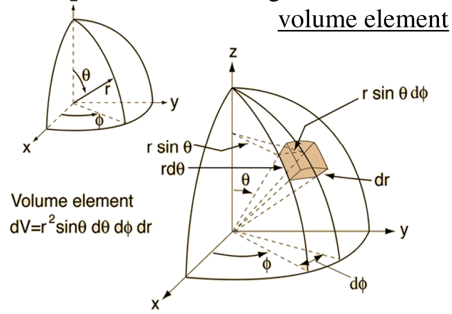


Figure 4 – differential volume element

However an additional real shrinkage factor is caused by gravitational length contraction of $\left(1-\frac{2GM}{c^2 r}\right)^{-1/2}$, which of course acts radially. There is more space near a gravitating body and up to the Schwarzschild radius $r = 2GMc^{-2}$ (anything beyond this requires an interior coordinate system/metric[1] for this is just a coordinate singularity and it is beyond relevance here for the general argument, such radius are small and well inside a gravitating body) an observer further out from the centre of the gravity-well will see an object squeezed and elongated, its volume apparently increasing as it heads towards the centre. However, gravity causes volume contraction.

This seems paradoxical until we realise that in flat space and spherical coordinates, a retreating cube doesn't really become smaller – it is a trick of perspective. Yet when the ratio of volume elements for an observer at the edge of the gravity-well (or the edge of our spherical coordinate domain) is taken with the subject, some co-ordinate distance r into both and comparing ratios of volume elements,

$$\frac{dV_{3,Schwarz}(r)\big|_{subject}}{dV_{3,flat}(r)\big|_{subject}} = \frac{\sqrt{\left|-h_{3,Schwarz}\right|}}{\sqrt{\left|-h_{3,flat}\right|}} = \left(1-\frac{2GM}{c^2 r}\right)^{-1/2} \quad \text{eqn. 11}$$

The Schwarzschild volume element is bigger than the volume element of flat space, which doesn't change size at all with position and conclude that space really has contracted in size by the factor $\left(1-\frac{2GM}{c^2 r}\right)^{1/2}$. Incidentally, the author has an earlier paper[23] which attempts to put the effects of both Special and General Relativity on a mechanistic rather than a phenomenological basis, by the variation of masses of particles due to their velocity or position in a gravity field: that time dilation and length contraction (bond lengths governed by virtual or bound particles) are real physical effects.

Now if we consider the ideal dark-energy gas between an observer near the edge of the gravity-well and a subject inside the well, the pressure and volume changes between the two cases obviously relates as,

$$P_{Ob}V_{Ob} = P_{Sub}V_{Sub} \quad \text{eqn. 12}$$

The volumes can be differentiated with respect to the polar co-ordinate,

$$P_{Sub} = P_{Ob} \, {}^{dV_{Ob}}\!/_{dr} \left({}^{dV_{Sub}}\!/_{dr}\right)^{-1} \quad \text{eqn. 13}$$

Near the periphery of the well ${}^{dV_{Ob}}\!/_{dr}$ is positive and near unity but ${}^{dV_{Sub}}\!/_{dr}$ in the direction towards the centre is negative <u>and here is our point</u>: Looking towards the centre, the usual negative pressure of dark-energy becomes positive and so contributes to the mass-energy of the whole system (we'd know this by taking the trace of the stress-energy tensor and obtaining the scalar curvature) and *gravitates*. However, looking out of the well the pressure is negative and so the observer will see other gravitating bodies red-shifting away, especially with the large amount of negative pressure accrued over astronomical volumes.

3.3.2. <u>The variation of pressure of the degenerate electron gas with volume</u>

Having discussed gravitational contraction of volume, it the follows that there is a change in the degenerate pressure of the virtual e^+e^- pairs given by eqn. 4 by the indirect action on the virtual photon gas. Differentiation results in the following,

$$\frac{dP}{dV} = \frac{\left(\frac{1}{3}\right)\left(\frac{6\pi^2}{g}\right)^{2/3}\hbar^2\left(\frac{N}{V}\right)^{5/3}\left(\frac{1}{V}\right)}{m_e}$$

$$\Rightarrow \Delta P = \frac{\left(\frac{1}{3}\right)\left(\frac{6\pi^2}{g}\right)^{2/3}\hbar^2\left(\frac{N}{V}\right)^{5/3}\left(\frac{\Delta V}{V}\right)}{m_e}$$

$$\therefore \Delta P = \frac{\left(\frac{1}{3}\right)\left(\frac{6\pi^2}{g}\right)^{2/3}\hbar^2\left(\frac{N}{V}\right)^{5/3}\left(\frac{V - {}^{V}\!/_{\sqrt{g_{rr}}}}{V}\right)}{m_e} \quad \text{eqn. 14}$$

And the change in pressure, as considered far from the gravitational-well by an observer is,

$$\Delta P = \frac{\left(\frac{1}{3}\right)\left(\frac{6\pi^2}{g}\right)^{2/3}\hbar^2\left(\frac{N}{V}\right)^{5/3}\left(1-\left(1-\frac{2GM}{c^2 r}\right)^{1/2}\right)}{m_e} \quad \text{eqn. 15}$$

Utilising the figures from earlier, we can model the dark-energy contribution of a Milky-Way sized galaxy with an external Schwarzschild metric[§], that is, with a mass some 10^{11} solar masses, so $2\text{x}10^{41}$ kg and with the *fluctuation density* for N/V from eqn. 15 calculated earlier as $8.8\text{x}10^{98}$ virtual pairs per m^3. We then multiply ΔP by κ^3, the Einstein constant in front of the field equations cubed[15], to show that it is a 2nd order term and obtain the following graph of what a distant observer would reckon the dark-energy pressure/energy to be, looking into the galaxy and noting the volume contraction.

§ The internal metric contribution is negligible, see Appendix 1

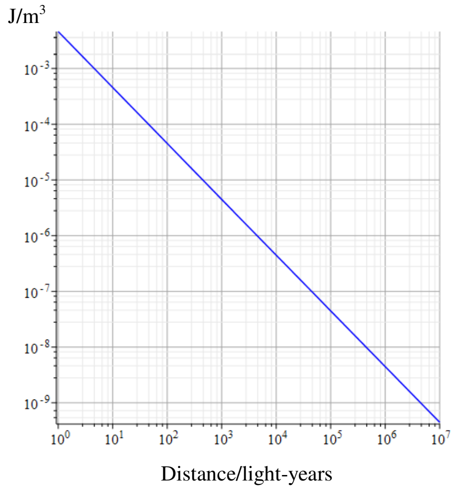


Figure 5 – Dark-energy density vs. distance from galactic centre in light-years seen by a distant observer (Appendix 1 has Maple™ script)

When this is integrated with respect to proper volume, the additional mass of the galaxy from dark-energy is,

$$E = \int_{r_1}^{r_2} \kappa^3 \Delta P(\rho_{fluct}) \frac{r^2 \sin\theta}{\sqrt{1 - r_{Schwarz}/r}} dr d\theta d\phi, \quad r_{Schwarz} = \frac{2GM}{c^2} \qquad \text{eqn. 16}$$

$$= 4\pi \int_{r_1}^{r_2} \kappa^3 \Delta P(\rho_{fluct}) \frac{r^2}{\sqrt{1 - r_{Schwarz}/r}} dr$$

Where, $r_{Schwarz}$ is the Schwarzschild radius and as previously stated, κ^3 has been introduced along with fluctuation density for virtual e^+e^- pairs placed into eqn. 15 (hence figure 5). Figure 6 shows the integration of figure 5 over proper volume,

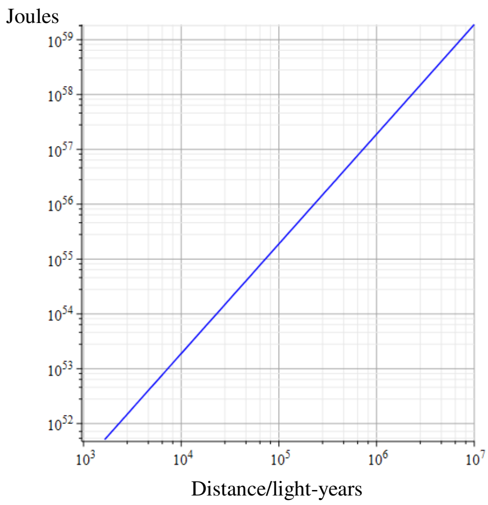


Figure 6 – Total Dark-energy vs. distance from galactic centre in light-years (Appendix 1 has Maple™ script)

There is no point integrating when the vacuum energy becomes that of deep space and limits the process to a few million light years radii – the gravitational effect of the galaxy is minimal at that distance, compared to deep space. The result at 1 Mlyr is comparable to the mass needed (80-90% mass of the galaxy[16-18], some 10^{11} solar masses for the Milky Way) in the galactic halo.

4. Conclusion and discussion

Our understanding of gravity is based on Newtonian Gravity and General Relativity. Explaining the Fisher-Tully Law requires these alternatives:

- Modify GR and lose the mathematical structure of it.
- Add elusive dark matter and hunt for that.
- Modify the Cosmological "Constant" to vary with position/time *and* preserve the mathematical structure of GR.

Dark-matter is still speculation, not known nor measured directly.

This paper posits The Dark Energy Modulation Hypothesis instead and has followed on from an earlier paper[15] by the author which considered Dark-Energy as a fluctuation term in the Einstein Field Equations. The suggestion of making the accepted value some 10^9 times higher with interaction between the zeropoint modes (via e^+e^- virtual pair interaction) seems to account for the "120 orders of magnitude problem" (actually 129 orders). The paper then went on to consider if this fluctuation of dark-energy could be modulated by gravity to substitute the Dark-Matter Haloes hypothesis with our hypothesis. The two hypotheses are meant to dispel clunky modifications to Newtonian or General Relativity by force law or even modification terms of "Modified Newtonian Gravity" (MOND) or "Quantised Inertia" hypotheses.

The motivation for the work was Occam's Razor or indeed, Newton's own "Hypotheses non-fingo" – that is, cutting down on unlikely or clunky material appended onto a graceful theory. The author believes that the suggestion contained herein is reasonable. The elephant in the room or the question being begged, is how an invariant quantity is discerned locally to have changed? To this we might add that locally nothing changes when performing electrodynamic calculations, however concerns at the global level (in a global co-ordinate system) with the stress-energy tensor (that is, the effect of fluctuations at 2^{nd} order) may manifest. Perhaps the tensorial/scale-invariant nature of General Relativity, at the largest scales, is not strictly true but the form can be kept, as already said, by "sweeping the dirt under the rug" into the Cosmological Constant.

Appendix 1 – Maple Script (copied as images but available as supplemental file)

restart
with(*ScientificConstants*) :

using Physics
using Physics

$g\_s := 2 \; = 2$

Constant(*Planck_constant_over_2pi*)
$Constant(\hbar)$

$\hbar := GetValue(\mathbf{(2)})$
$\hbar := 1.054571800\ 10^{-34}$

Constant(*Planck_time*)
$Constant(t_P)$

$$\omega_{Planck} := \frac{2 \cdot \mathrm{pi}}{GetValue(\mathbf{(4)})} = 3.709778432\ 10^{43}\ \pi$$

Constant(*speed_of_light_in_vacuum*)
$Constant(c)$

$c := GetValue(\mathbf{(5)})$
$c := 299792458$

Constant(*electron_mass*)
$Constant(m_e)$

$m\_e := GetValue(\mathbf{(7)})$
$m\_e := 9.109383560\ 10^{-31}$

Constant(*Boltzmann_constant*)
$Constant(k)$

$k\_Boltz := GetValue(\mathbf{(9)})$
$k\_Boltz := 1.380648510\ 10^{-23}$

Constant(*Stefan_Boltzmann_constant*)
$Constant(\sigma)$

$\sigma := GetValue(\mathbf{(11)})$
$\sigma := 5.670366658\ 10^{-8}$

Constant(*Newtonian_constant_of_gravitation*)
$Constant(G)$

$G := GetValue(\mathbf{(13)})$
$G := 6.67408\ 10^{-11}$

$$\kappa := \frac{8\,\pi\,G}{c^4}$$
$\kappa := 2.076578992\ 10^{-43}$

$\rho\_vac := 0.54\ 10^{-9}$
$\rho\_vac := 5.400000000\ 10^{-10}$

$T\_cmb := 2.725$
$T\_cmb := 2.725$

$Lyr := 9.460e15$
$Lyr := 9.460\ 10^{15}$

$Mass\_Sun := 2e30$
$Mass\_Sun := 2.\ 10^{30}$

$Mass\_MW := 1e11$
$Mass\_MW := 1.\ 10^{11}$

$$r\_Schwarz := \frac{2 \cdot G \cdot Mass\_Sun \cdot Mass\_MW}{c^2} = 2.970366194\ 10^{14}$$

$$ZPE := \frac{10^9 \cdot \rho\_vac}{\kappa^3}$$
$ZPE := 6.030431540\ 10^{127}$

$$Pressure\_ZP := (N, V) \mapsto \frac{6^{2/3} \cdot \left(\frac{\pi^2}{g\_s}\right)^{2/3} \cdot \hbar^2 \cdot \left(\frac{N}{V}\right)^{5/3}}{5 \cdot m\_e}$$

$$Pressure\_ZP := (N, V) \mapsto \frac{6^{2/3} \cdot \left(\frac{\pi^2}{g\_s}\right)^{2/3} \cdot \hbar^2 \cdot \left(\frac{N}{V}\right)^{5/3}}{5 \cdot m\_e}$$

$$dP\_zp := (N, V, dV) \mapsto \left(\frac{\mathrm{d}}{\mathrm{d}V} Pressure\_ZP(N, V) \cdot dV\right) = (N, V, dV) \rightarrow \left(\frac{\partial}{\partial V} Pressure\_ZP(N, V)\right) dV$$

$$DegenTemp := (N, V) \mapsto \frac{6^{2/3} \cdot \left(\frac{\pi^2}{g\_s}\right)^{2/3} \cdot \hbar^2 \cdot \left(\frac{N}{V}\right)^{2/3}}{3 \cdot m\_e \cdot k\_Boltz} = (N, V) \rightarrow \frac{1}{3}\, \frac{6^{2/3} \left(\frac{\pi^2}{g\_s}\right)^{2/3} \hbar^2 \left(\frac{N}{V}\right)^{2/3}}{m\_e\ k\_Boltz}$$

Note that this is exact

$$Pressure := (N, V, T) \mapsto \frac{N \cdot k\_Boltz \cdot T \cdot \left(1 + \frac{\pi^{3/2} \cdot N \cdot \hbar^3}{2 \cdot g\_s \cdot V \cdot (m\_e \cdot k\_Boltz \cdot T)^{3/2}}\right)}{V} =$$

$$= (N, V, T) \rightarrow \frac{N\, k\_Boltz\, T \left(1 + \frac{1}{2} \frac{\pi^{3/2} N \hbar^3}{g\_s\, V\, (m\_e\, k\_Boltz\, T)^{3/2}}\right)}{V}$$

$$dP := (N, V, T, dV) \mapsto \left(\frac{d}{dV} Pressure(N, V, T) \cdot dV\right)$$

$$= (N, V, T, dV) \rightarrow \left(\frac{\partial}{\partial V} Pressure(N, V, T)\right) dV$$

$$solve\left(Pressure\_ZP(N, 1) - \frac{10^9 \cdot \rho\_vac}{\kappa^3} = 0, N\right)$$

$= 1.766138047\ 10^{99}$ # Considered a fluctuation

$$solve\left(Pressure\_ZP(N, 1) - \frac{10^9 \cdot \rho\_vac}{1} = 0, N\right)$$

$=$

$2.619701853\ 10^{22}$

 # Considered as not a fluctuation

$$solve\left(Pressure\_ZP(N, 1) - \frac{\rho\_vac}{1} = 0, N\right) =$$

$1.042922093\ 10^{17}$

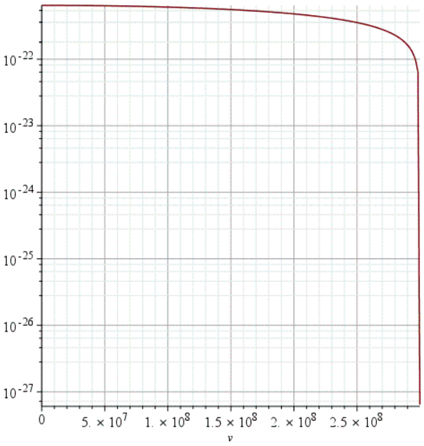
Considered as not a fluctuation and without a 10^9 increase in energy density

 # Lifetime of virtual $e^+ e^-$ pairs

$$tlife := (v) \mapsto \frac{\hbar}{2} \left(\frac{m\_e \cdot c^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}\right)^{-1} =$$

$$v \rightarrow \frac{1}{2} \frac{\hbar \sqrt{1 - \frac{v^2}{c^2}}}{m\_e\, c^2}$$

$$plot(tlife(v), v = 0..c)$$



Schwarzschild radius so we know the limits for the inner and outer metric

$r\_Schwarz = 2.970366194\ 10^{14}$

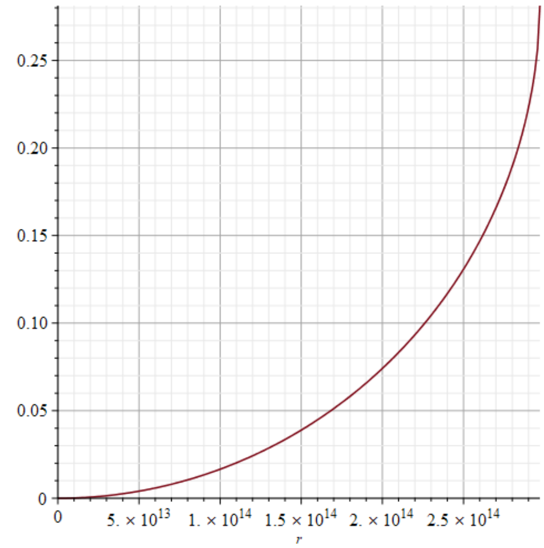
$$\frac{Lyr}{31.85} = 2.970172684\ 10^{14}$$

Energy density for inner metric calculation

$$dP\_zp\_internal := (r) \mapsto \kappa^3 \cdot \frac{\left(\frac{1}{3}\right) \cdot (3 \cdot \pi^2)^{\left(\frac{2}{3}\right)} \cdot \hbar^2 \cdot (8.85e98)^{\left(\frac{5}{3}\right)}}{m\_e} \cdot \left(1 - \left(1 - \frac{r^2}{r\_Schwarz^2}\right)^{\frac{1}{2}}\right) =$$

$$r \rightarrow \frac{2.719260806\ 10^{164}\, \kappa^3\, 3^{2/3}\, (\pi^2)^{2/3}\, \hbar^2 \left(1 - \sqrt{1 - \frac{r^2}{r\_Schwarz^2}}\right)}{m\_e}$$

$$plot\left(dP\_zp\_internal(r), r = 0..\frac{Lyr}{31.85}\right)$$



$$Inner\_DE = int\left(\frac{dP\_zp\_internal(r)\cdot r^2}{\operatorname{sqrt}\left(1-\frac{r^2}{r\_Schwarz^2}\right)}, r = 1e\text{-}4..\frac{Lyr}{31.85}\right)$$

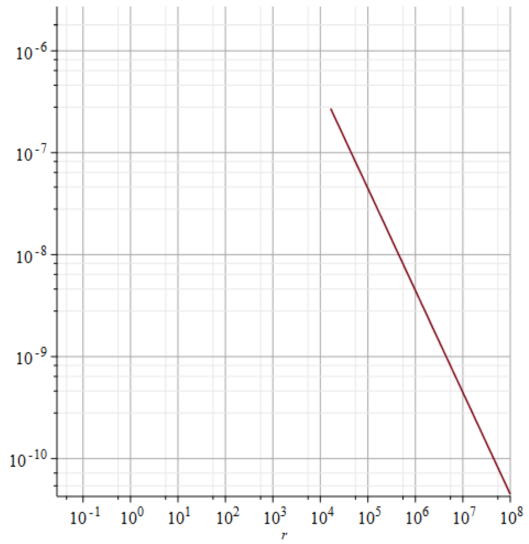
$$= Inner\_DE = 3.286211130\ 10^{42}$$

So this is the integral of the energy density of the inner metric right up to the Schwarzschild radius

Energy density for outer metric calculation

$$dP\_zp2 := (r) \mapsto \kappa^3\cdot\frac{\left(\frac{1}{3}\right)\cdot(3\cdot\pi^2)^{\left(\frac{2}{3}\right)}\cdot\hbar^2\cdot(8.85e98)^{\left(\frac{5}{3}\right)}}{m\_e}\cdot\left(1-\left(1-\frac{r\_Schwarz}{r}\right)^{\frac{1}{2}}\right) =$$
$$r \rightarrow \frac{2.719260806\ 10^{164}\ \kappa^3\ 3^{2/3}\ (\pi^2)^{2/3}\ \hbar^2\left(1-\sqrt{1-\frac{r\_Schwarz}{r}}\right)}{m\_e}$$

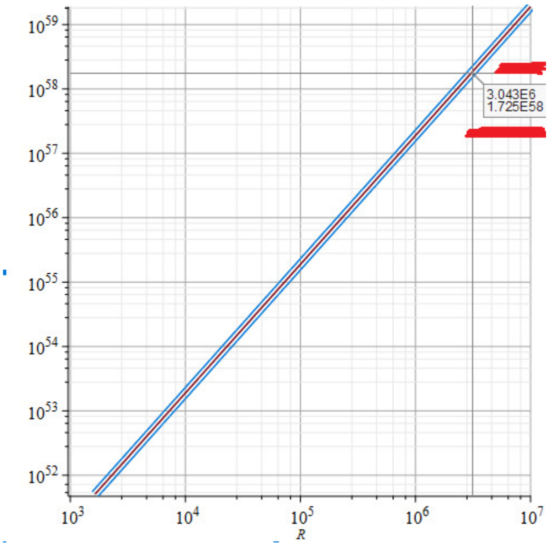
$$plot\left(dP\_zp2(r\cdot Lyr), r = \frac{1}{31.85}..1e8\right)$$



$$int\_zp2 := (R) \mapsto int\left(\frac{dP\_zp2(r)\cdot(r)^2}{\operatorname{sqrt}\left(1-\frac{r\_Schwarz}{r}\right)}, r = 1e\text{-}3..R\right)$$

$$= R \rightarrow \int_{0.001}^{R}\frac{dP\_zp2(r)\ r^2}{\sqrt{1-\frac{r\_Schwarz}{r}}}\,dr$$

$$plot\left(int\_zp2(R\cdot Lyr), R = \frac{1}{31.85}..1e7\right)$$



# The energy / mass ~= c^2 note dark energy from inner Schwarzschild metric,
Inner_DE = 3.286211130*10^{42}, is negligible

$$Mass\_Sun\cdot Mass\_MW\cdot c^2 = 1.797510357\ 10^{58}$$

So out to about 3 lightyears from the galaxy, the Dark-Energy is comparable to the mass of the galaxy. QED.

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