PILOT WAVE MECHANICS: A QUANTUM MECHANICAL ALTERNATIVE

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Abstract: The Pilot Wave Mechanics (PWM) presents an intriguing alternative to mainstream quantum mechanics, offering fresh perspective on the behavior of quantum particles. This paper provides a comprehensive overview of PWM, elucidating its fundamental concepts, theoretical framework, and implications for our understanding of quantum phenomena. By introducing the concept of a guiding wave that determines the motion of particles, PWM seeks to reconcile the probabilistic nature of standard quantum mechanics with deterministic dynamics. I examine the historical development of PWM, discussing its key proponents and significant milestones. Furthermore, I delve into the mathematical formulation and equations that underpin the theory, emphasizing its compatibility with quantum experiments and observed phenomena. Additionally, I explore the unique predictions and potential experimental tests of PWM, highlighting its ability to offer new insights into the double-slit experiment, entanglement, and the measurement problem. Finally, I discuss the broader implications of PWM for the foundations of quantum mechanics and its potential to bridge the gap between classical and quantum realms. Through a thorough investigation of PWM, this paper aims to contribute to the ongoing discourse surrounding the nature of quantum mechanics, offering researchers and theorists a valuable resource for further exploration and analysis.

Keywords: Pilot Wave Mechanics, Quantum mechanics, Wave-particle duality, Determinism, Pilot Wave function. Relativistic Pilot Wave Mechanics.

INTRODUCTION

The field of quantum mechanics has revolutionized our understanding of the microscopic world, providing a mathematical framework to explain the behavior of particles at the atomic and sub-atomic levels. However, the standard interpretation of quantum mechanics, known as the Copenhagen interpretation, is still marked by the unresolved conceptual and philosophical questions. In recent years, alternative theories have emerged, offering fresh perspectives and interpretations of quantum phenomena. One such alternative is the Pilot Wave Mechanics (PWM), which presents a novel approach to quantum mechanics.

PWM challenges the probabilistic nature of quantum mechanics by introducing the notion of a "pilot wave". In contrast to Copenhagen interpretation's reliance on wave—particle duality and indeterminacy, PWM posits that particles are guided by an underlying wave that determines their trajectory and behavior. This deterministic aspect of the theory seeks to restore a sense of causality and determinism to quantum phenomena, while still incorporating the wave-particle duality displayed in experiments.

Historically, the roots of PWM can be traced back to the work of Louis de Borglie, David Bohm and others in the early 20th Century. These pioneering researchers proposed that quantum particles, such as electrons, are not mere statistical entities but possess definite positions and velocities defined by an associated wave. While the interpretation struggled to gain widespread acceptance due to the dominance of the Copenhagen interpretation, the ideas of PWM have sparked renewed interest and investigation in recent decades. [1]

PWM introduces a new and different mathematical formulation which is based on the observable, along with their operators. Together, these equations offer a unified description of particles and waves, allowing for the calculation of dynamic behavior in a manner consistent with experimental observations. This unique framework enables PWM to make predictions that parallel those of traditional quantum mechanics, while providing a different conceptual framework for understanding and interpreting the underlying physics. Furthermore, PWM offers intriguing potential applications and implications. By extending the reaching of classical mechanics and quantum realm. PWM has been able to offer new heights into the double slit experiment, elucidating the behavior and interactions of quantum particles in a deterministic manner. It also presents alternative explanations for the phenomenon of entanglement and the measurement problem, shedding light on the underlying mechanics that govern these intriguing mechanics that govern these intriguing quantum behaviors.

In conclusion, the Pilot Wave Mechanics challenges the prevailing interpretation of quantum mechanics, offering a compelling alternative that seeks to reconcile the probabilistic nature of quantum phenomena with deterministic dynamics. The historical development, mathematical framework and broader implications of PWM provide researchers and theorists with new lens through which to explore the foundations of quantum mechanics. By studying PWM, I aim to broaden our understanding of the quantum world and its intricate workings, ultimately deepening our appreciation for the underlying principles that shape our universe.

THEORETICAL FRAMEWORK

I. WAVE—PARTICLE DUALITY:

Wave—particle duality is the physical concept which posits that matter has both particle and wave natures. According to PWM, particles are guided by a pilot wave, which determines their behavior. The pilot wave is a wave function that evolves deterministically. This wave function guides the particles and is responsible for the wave-like behavior. In PWM, particles still have definite positions and velocities, unlike Copenhagen interpretation of quantum mechanics. The pilot wave guides the particles along well-defined trajectories, causing them to exhibit particle—like behavior. However, the pilot wave itself exhibits wave—like properties, allowing for wave interference phenomena. [2]

II. QUANTUM DETERMINISM

PWM is a deterministic microphysical which proposes that there exists a pilot wave that determines the motion of particles. These particles are described by their positions and velocities, which are influenced by the pilot wave. The concept of quantum determinism in Pilot Wave Mechanics suggests that there is a deterministic evolution of the system in accordance with the wave equation and particles' positions. Pilot Wave Mechanics posits that the outcome of a measurement can be determined precisely if all relevant information about the system is known.

This notion challenges the conventional understanding of quantum mechanics, which often portrays the inherent uncertainty associated with measurement outcomes. Pilot Wave Mechanics provides an alternative framework that allows for a deterministic understanding of quantum phenomena. [3]

III. QUANTUM SPACE

In PWM, space is not considered as a fixed background or passive container, but rather as an active dynamic entity. The pilot wave associated with particles not only guides their motion, but also influences the structure of space itself. The presence of particles creates a "quantum potential" that affects the curvature and properties of space.

In PWM, particles are not confined to point-like entities but are accompanied by a spreading wave that extends through space. This implies that the behavior of particles is intrinsically connected to the spatial environment they inhabit. The interaction between particles and pilot wave modifies the distribution of matter, shaping the spatial landscape. As a result, the concept of quantum space in Pilot Wave Mechanics goes beyond the traditional notion of a static stage for quantum phenomena. It acknowledges the dynamic interplay between particles, waves and the underlying spatial structure. This perspective helps to explain various quantum phenomena and offers potential insight into the nature of quantum entanglement, nonlocality, and the wave—particle duality. [4]

IV. QUANTUM TIME

In PWM, the evolution of quantum systems is described by the pilot wave function and position of particles which depend explicitly on time. The equation of motion incorporates the influence of the pilot wave and position of particles. The pilot wave serves as a guiding field that determines the motion of particles. It propagates through space and time, influencing the behavior of particles in a deterministic manner. The position of the particles are then determined by the interaction between the pilot wave and the particles themselves, evolving as time progresses.

MATHEMATICAL FRAMEWORK

Relation to Gravity:

$$G_{uv} = 2\left(\frac{A_{sph}(\hbar\kappa)}{c^4 m^2 t}\right)T_{uv}$$

Proof:

V= u+gt

For uniformly accelerated motion from rest, u=0;

V=gt------(1)

Also, $g = \frac{Gm}{R^2}$--------(2)

Substituting equation (2) into (1) gives;

$v = \frac{Gmt}{R^2}$--------(3)

From General Relativity,

$$G_{uv} = \frac{8\pi G}{c^4}T_{uv}$$

Make ‘G’ the subject;

$G = \frac{c^4}{8\pi}G_{uv}T_{uv}^{-1}$----------(4)

Substituting equation (4) into (3) gives;

$$v = \frac{mt}{R^2} \times \frac{c^4}{8\pi}G_{uv}T_{uv}^{-1}$$

$v = \frac{c^4 mt}{8\pi R^2}G_{uv}T_{uv}^{-1}$----------(5)

From de Broglie equation;

$v = \frac{h}{m\lambda}$----------(6)

Substituting equation (6) into (5);

$$\frac{h}{m\lambda} = \frac{c^4 mt}{8\pi R^2}G_{uv}T_{uv}^{-1}$$

Making G_{uv} the subject gives;

$$G_{uv} = \frac{8\pi R^2 h}{c^4 m^2 \lambda t}T_{uv}$$

But $4\pi R^2$=Area of a sphere (A_{sph})

$\lambda^{-1} = \frac{\kappa}{2\pi}$

Finally,

$$G_{uv} = 2\left(\frac{A_{sph}(\hbar\kappa)}{c^4 m^2 t}\right)T_{uv}$$

$$G_{uv} = \frac{A_{sph}(\hbar\kappa)^2}{c^4 m^3 x}T_{uv}$$

$Proof$:

$v^2 = u^2 + 2gx$

For uniformly accelerated motion from rest, u=0;

$v^2 = 2gx$---------(1)

Also, $g = \frac{Gm}{R^2}$--------(2)

Substituting equation (2) into (1) gives;

$v^2 = \frac{2Gmx}{R^2}$-----------(3)

From General Relativity,

$$G_{uv} = \frac{8\pi G}{c^4}T_{uv}$$

Make ‘G’ the subject;

$G = \frac{c^4}{8\pi}G_{uv}T_{uv}^{-1}$----------(4)

Substituting equation (4) into (3) gives;

$v^2 = \frac{2mx}{R^2}(\frac{c^4}{8\pi}G_{uv}T_{uv}^{-1}$-)

$v^2 = \frac{c^4 mx}{4\pi R^2}G_{uv}T_{uv}^{-1}$----------(5)

From de Broglie's equation;

$v = \frac{h}{m\lambda}$

Squaring both sides;

$v^2 = \frac{h^2}{m^2\lambda^2}$-----------(6)

Substituting equation (6) into (5);

$\frac{h^2}{m^2\lambda^2} = \frac{c^4 mx}{4\pi R^2}G_{uv}T_{uv}^{-1}$

Making G_{uv} the subject gives;

$$G_{uv} = \frac{4\pi R^2 h^2}{c^4 m^3 \lambda^2 x}T_{uv}$$

But $4\pi R^2$=Area of a sphere (A_{sph})

$\lambda^{-2} = \frac{\kappa^2}{4\pi^2}$

Finally,

$$G_{uv} = \frac{A_{sph}(\hbar\kappa)^2}{c^4 m^3 x}T_{uv}$$

$$R_{uv} - {}^1\!/_2\, R_s g_{uv} + \Lambda g_{uv} = 2\left(\frac{A_{sph}(\hbar\kappa)}{c^4 m^2 t}\right)T_{uv}$$

Proof:
from Einstein Field Equation;

$R_{uv} - {}^1\!/_2\, R_s g_{uv} + \Lambda g_{uv} = G_{uv}$----(1)

but $G_{uv} = 2\left(\frac{A_{sph}(\hbar\kappa)}{c^4 m^2 t}\right)T_{uv}$----------(2)

substituting equation (2)*into* (1)*gives*
;$R_{uv} - {}^1\!/_2\, R_s g_{uv} + \Lambda g_{uv} = 2\left(\frac{A_{sph}(\hbar\kappa)}{c^4 m^2 t}\right)T_{uv}$

$$R_{uv} - {}^1\!/_2\, R_s g_{uv} + \Lambda g_{uv} = \frac{A_{sph}(\hbar\kappa)^2}{c^4 m^3 x}T_{uv}$$

Proof:
from Einstein Field Equation;

$R_{uv} - {}^1\!/_2\, R_s g_{uv} + \Lambda g_{uv} = G_{uv}$------(1)

but $G_{uv} = \frac{A_{sph}(\hbar\kappa)^2}{c^4 m^3 x}T_{uv}$----------(2)

substituting equation (2)*into* (1)*gives*
;$R_{uv} - {}^1\!/_2\, R_s g_{uv} + \Lambda g_{uv} = \frac{A_{sph}(\hbar\kappa)^2}{c^4 m^3 x}T_{uv}$

<u>Definition of Mathematical Terms:</u>

$\hbar$= Reduced Planck's constant
A_{sph}= Area of quantum system
m= mass of quantum system
n= principal quantum number
t= time of measurement
x= position of quantum system
G_{uv}= Einstein Tensor
t=time of measurement (quantum system)
x=position of quantum system
k=wave vector of quantum system
R_{uv} =Ricci Tensor
R_s =Ricci Scalar
Λ =Cosmological Constant
g_{uv} =metric Tensor
T_{uv} =Stress-energy Tensor

Deterministic Relations:

$$\mathrm{E} = \frac{\hbar\kappa \mathrm{x}}{\mathrm{t}}$$

Proof:

$v^2 = u^2 + 2gx$

For uniformly accelerated motion from rest, u=0;

$v^2 = 2gx$---------(1)

Also, $g = \frac{Gm}{R^2}$--------(2)

Substituting equation (2) into (1) gives;

$v^2 = \frac{2Gmx}{R^2}$-----------(3)

From General Relativity,

$$G_{uv} = \frac{8\pi G}{c^4} T_{uv}$$

Make 'G' the subject;

$G = \frac{c^4}{8\pi} G_{uv} T_{uv}^{-1}$----------(4)

But $G_{uv} = \frac{8\pi R^2 h}{c^4 m^2 \lambda t} T_{uv}$------(5)

Substituting equation (5) into (4) gives;

$G = \frac{c^4}{8\pi} (\frac{8\pi R^2 h}{c^4 m^2 \lambda t} T_{uv}) T_{uv}^{-1}$

$G = \frac{R^2 h}{m^2 \lambda t}$---------(6)

Substituting equation (6) into (3) gives;

$v^2 = \frac{2mx}{R^2} \times \left(\frac{R^2 h}{m^2 \lambda t}\right)$

$v^2 = \frac{2hx}{m\lambda t}$---------(7)

Multiply equation (7) by $\frac{1}{2}m$;

$\frac{1}{2} m v^2 = \frac{2hx}{m\lambda t} \times \frac{1}{2} m$

Therefore;

$\frac{1}{2} m v^2 = \frac{hx}{\lambda t}$---------(8)

But E=$\frac{1}{2} m v^2$------(9)

Substitute equation (9) into (8);

$$\mathrm{E} = \frac{\mathrm{hx}}{\lambda \mathrm{t}}$$

But $\lambda^{-1} = \frac{\kappa}{2\pi}$

Finally,

$$\mathrm{E} = \frac{\hbar \kappa \mathrm{x}}{\mathrm{t}}$$

$$P = \frac{\hbar \omega t}{x}$$

Proof:

$E = \hbar\omega$------(1)

$E = \frac{\hbar k x}{t}$------(2)

Comparing eqns 1 and 2;

$\frac{\hbar k x}{t} = \hbar\omega$

$\frac{kx}{t} = \omega$

Make k the subject;

$k = \frac{\omega t}{x}$-------(3)

But,

$P = \hbar k$------(4)

Substituting eqn 3 into 4;

$P = \hbar \left(\frac{\omega t}{x}\right)$

Finally,

$$P = \frac{\hbar \omega t}{x}$$

$$L = \frac{\hbar \omega t}{\theta}$$

Proof:

$L = p \times r$-------(1)

$P = \frac{\hbar \omega t}{x}$---------(2)

Substituting eqn. 2 into 1;

$L = \left(\frac{\hbar \omega t}{x}\right) \times r$

$L = \frac{\hbar \omega t r}{x}$ but, $\frac{r}{x} = \frac{1}{\theta}$

$L = \frac{\hbar \omega t}{\theta}$

$$P = \frac{(\hbar\kappa)^2 t}{2mx}$$

Proof:

V= u+gt

For uniformly accelerated motion from rest, u=0;

V=gt------(1)

Also, $g = \frac{Gm}{R^2}$--------(2)

Substituting equation (2) into (1) gives;

$v = \frac{Gmt}{R^2}$--------(3)

From General Relativity,

$$G_{uv} = \frac{8\pi G}{c^4} T_{uv}$$

Make 'G' the subject;

$G = \frac{c^4}{8\pi} G_{uv} T_{uv}^{-1}$----------(4)

But,

$G_{uv} = \frac{4\pi R^2 h^2}{c^4 m^3 \lambda^2 x} T_{uv}$------(5)

Substituting equation (5) into equation (4);

$$G = \frac{c^4}{8\pi}\left(\frac{4\pi R^2 h^2}{c^4 m^3 \lambda^2 x} T_{uv}\right) T_{uv}^{-1}$$

$G = \frac{R^2 h^2}{2m^3 \lambda^2 x}$------(6)

Substituting equation (6) into (3) gives;

$$v = \left(\frac{R^2 h^2}{2m^3 \lambda^2 x}\right) \times \left(\frac{mt}{R^2}\right)$$

$v = \frac{h^2 t}{2m^2 \lambda^2 x}$-----------------(7)

Multiply equation (7) by m;

$$mv = \frac{h^2 t}{2m^2 \lambda^2 x} \times m$$

Therefore,

$mv = \frac{h^2 t}{2m \lambda^2 x}$----------(8)

But Momentum(P)= mass(m) × velocity(v);

Finally,

$$P = \frac{h^2 t}{2m \lambda^2 x}$$

But $\lambda^{-2} = \frac{\kappa^2}{4\pi^2}$

Finally,

$$P = \frac{(\hbar\kappa)^2 t}{2mx}$$

$$L = \frac{(\hbar\kappa)^2 t}{2m\theta}$$

Proof:

$P = \frac{(\hbar\kappa)^2 t}{2mx}$--------(1)

But for objects performing rotational motion,

$P = \frac{L}{r}$---------(2)

And also,

x=r θ--------(3)

substituting equations (2) and (3) into (1) gives;

$\frac{L}{r} = \frac{(\hbar\kappa)^2 t}{2m\text{r}\ \theta}$

Multiply both sides by r;

$$r \times \frac{L}{r} = \frac{(\hbar\kappa)^2 t}{2m\text{r}\ \theta} \times r$$

Finally,

$$L = \frac{(\hbar\kappa)^2 t}{2m\theta}$$

$$\omega = \frac{\hbar\kappa^2}{2m}$$

Proof:

$P = \frac{\hbar\omega t}{x}$-------(1)

$P = \frac{(\hbar\kappa)^2 t}{2mx}$-----(2)

Comparing eqn 1 and 2,

$$\frac{\hbar\omega t}{x} = \frac{(\hbar\kappa)^2 t}{2mx}$$

Finally,

$$\omega = \frac{\hbar\kappa^2}{2m}$$

$$F = \frac{\hbar\omega}{x}$$

Proof:

$F = \frac{P}{t}$-------(1)

$P = \frac{\hbar\omega t}{x}$------(2)

Substituting eqn 2 into 1;

$$F = \left(\frac{\hbar\omega t}{x}\right) \times \frac{1}{t}$$

Finally,

$$F = \frac{\hbar\omega}{x}$$

$$V = \frac{\hbar\omega}{\theta}$$

Proof:

$F = \frac{P}{t}$--------(1)

$P = \frac{\hbar\omega t}{x}$-----(2)

$F = \frac{\hbar\omega}{x}$------(3)

$$F = \frac{V}{r}$$

$$\frac{V}{r} = \frac{\hbar\omega}{x}$$

$$V = \frac{\hbar\omega r}{x}$$

But,

$$\frac{r}{x} = \frac{1}{\theta}$$

Finally,

$$V = \frac{\hbar\omega}{\theta}$$

<u>QUANTIZED TIME</u>

$$t_n = n\left(\frac{\theta}{\omega}\right)$$

<u>Proof:</u>

$$mvr = n\hbar$$

But $mv = p = \frac{(\hbar\kappa)^2 t}{2mx}$

Therefore,

$$\frac{(\hbar\kappa)^2 t}{2mx} \times r = n\hbar$$

$$\frac{\hbar k^2 tr}{2mx} = n$$

Make ‘t’ the subject;

$$t_n = n\left(\frac{2m}{\hbar k^2} \times \frac{x}{r}\right) = n\left(\frac{2m\theta}{\hbar k^2}\right)$$

But,

$$\frac{2m}{\hbar k^2} = \frac{1}{\omega}$$

Finally,

$$t_n = n\left(\frac{\theta}{\omega}\right)$$

QUANTIZED SPACE (POSITION)

$$x_n = \frac{1}{n}(\omega tr)$$

Proof:

$$mvr = n\hbar$$

But $mv = p = \frac{(\hbar\kappa)^2 t}{2mx}$

Therefore,

$$\left(\frac{(\hbar\kappa)^2 t}{2mx}\right) \times r = n\hbar$$

$$\frac{\hbar k^2 tr}{2mx} = n$$

Make 'x' the subject;

$$x_n = \frac{1}{n}\left(\frac{\hbar k^2 tr}{2m}\right)$$

But,

$$\omega = \frac{\hbar k^2}{2m}$$

Finally,

$$x_n = \frac{1}{n}(\omega tr)$$

DOUBLE SLIT FORMULAE

$$d \sin\theta = \frac{n\pi x}{\omega t}$$

Proof:

$$2d \sin\theta = n\lambda$$

$$but\ \lambda = \frac{2\pi}{k}$$

$$2d \sin\theta = \frac{2n\pi}{k}$$

$$d \sin\theta = \frac{n\pi}{k}$$

$$k = \frac{\omega t}{x}$$

Therefore,

$$\frac{1}{k} = \frac{x}{\omega t}$$

$$d \sin\theta = n\pi\left(\frac{x}{\omega t}\right)$$

Finally,

$$d \sin\theta = \frac{n\pi x}{\omega t}$$

$$Y_n = \frac{2L}{d}\left(\frac{n\pi x}{\omega t}\right)$$

$$Y_n d = nL\lambda$$

$$but\ \lambda = \frac{2\pi}{k}$$

$$Y_n d = nL\left(\frac{2\pi}{k}\right)$$

$$Y_n d = \frac{2Ln\pi}{k}$$

But,

$$\frac{1}{k} = \frac{x}{\omega t}$$

$$Y_n d = 2Ln\pi\left(\frac{x}{\omega t}\right)$$

Rearranging,

$$Y_n d = 2L\left(\frac{n\pi x}{\omega t}\right)$$

Make Y_n the subject;

$$Y_n = \frac{2L}{d}\left(\frac{n\pi x}{\omega t}\right)$$

PILOT WAVE FIELD METRIC

$$\boldsymbol{ds^2 = -\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2 k^3}{m^2\theta}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2 k^3}{m^2\theta}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)}$$

Proof:

From the Schwarzschild metric;

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

From Pilot Wave Field Theory;

$$\boldsymbol{G = \frac{r^2\hbar k}{m^2 t}}\text{-------(1)}$$

$$\boldsymbol{G = \frac{(r\hbar k)^2}{2m^3 x}}\text{-------(2)}$$

Substitute eqn.1 into the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{c^2 r}\left(\frac{r^2\hbar k}{m^2 t}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2M}{c^2 r}\left(\frac{r^2\hbar k}{m^2 t}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Let M=m;

$$ds^2 = -\left(1 - \frac{2}{c^2}\left(\frac{r\hbar k}{mt}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2}{c^2}\left(\frac{r\hbar k}{mt}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

For quantization;

$$t = 2n\left(\frac{m\theta}{\hbar k^2}\right)$$

Therefore;

$$ds^2 = -\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2 k^3}{m^2\theta}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2 k^3}{m^2\theta}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$ds^2 = -\left(1 - \frac{2n}{c^2}\left(\frac{\hbar}{mt}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2n}{c^2}\left(\frac{\hbar}{mt}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Proof:

From the Schwarzschild metric;

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

From Pilot Wave Field Theory;

$$G = \frac{(r\hbar k)^2}{2m^3 x}\text{-------(1)}$$

Substitute eqn.1 into the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{c^2 r}\left(\frac{(r\hbar k)^2}{2m^3 x}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2M}{c^2 r}\left(\frac{(r\hbar k)^2}{2m^3 x}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Let M=m;

$$ds^2 = -\left(1 - \frac{1}{c^2}\left(\frac{r(\hbar k)^2}{m^2 x}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{1}{c^2}\left(\frac{r(\hbar k)^2}{m^2 x}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

For quantization;

$$x = \frac{1}{2n}\left(\frac{\hbar k^2 tr}{m}\right)$$

Therefore;

$$ds^2 = -\left(1 - \frac{2n}{c^2}\left(\frac{\hbar}{mt}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2n}{c^2}\left(\frac{\hbar}{mt}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

PILOT WAVE FIELD METRIC FOR A GROUND STATE HYDROGEN ATOM.

$$ds^2 = -\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2k^3}{m^2\theta}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2k^3}{m^2\theta}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

But;

$$hk = p = mv$$

$$ds^2 = -\left(1 - \frac{1}{nc^2}\left(\frac{rm^2v^2k}{m^2\theta}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{1}{nc^2}\left(\frac{rm^2v^2k}{m^2\theta}\right)\right)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Therefore;

$$ds^2 = -\left(1 - \frac{v^2}{nc^2}\left(\frac{rk}{\theta}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{v^2}{nc^2}\left(\frac{rk}{\theta}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

For a hydrogen atom in the ground state (n=1);

$$v = 2.19 \times 10^6 ms^{-1}$$

$$\boldsymbol{Mds^2} = -\left(1 - \left(\frac{2.19 \times 10^6 ms^{-1}}{c}\right)^2\left(\left(\frac{r_1k_1}{\theta_1}\right)\right)\right)c^2dt^2 + \frac{1}{\left(1 - \left(\frac{2.19 \times 10^6 ms^{-1}}{c}\right)^2\left(\left(\frac{r_1k_1}{\theta_1}\right)\right)\right)}dr^2 + r_1{}^2(d\theta^2 + \sin^2\theta_1\, d\phi^2)$$

But; $\left(\frac{2.19\times10^6 ms^{-1}}{c}\right) = \boldsymbol{\alpha}$(fine structure constant)

Therefore;

$$ds^2 = -\left(1 - \alpha^2\left(\frac{r_1k_1}{\theta_1}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \alpha^2\left(\frac{r_1k_1}{\theta_1}\right)\right)}dr^2 + r_1{}^2(d\theta^2 + \sin^2\theta_1\, d\phi^2)$$

Meaning/Definition of the Fine Structure Constant.

Fine structure constant can then be defined as the fundamental constant that determines the formation of stable Hydrogen atoms via space-time interval fields.

PILOT WAVE MECHANICS & THE DIMENSIONALITY OF REALITY.

According to Pilot Wave Mechanics, the dimensionality of reality extends from four to nine, to encompass everything in reality.

Aside the usual four (3+1) dimensions of space and time, the other four extra dimensions are as follows:

Frequency dimension (5th dimension), Mass dimension (6th dimension), Force dimension (7th dimension), Energy dimension (8^{th} dimension) and Power dimension (9^{th} dimension).

The metric that describes the 9-Dimensional Reality is given by;

<u>For flat surfaces:</u>

$$ds^2 = \left[\left(\frac{(r\hbar)^2k^3}{EFmc}\right)dP^2 + \left(\frac{(r\hbar)^2k^5}{Fm^2cf}\right)^2 dE^2 + \left(\frac{k^3r^2\hbar}{(fm)^2c}\right)^2 dF^2 + \left(\frac{r^2cf}{E}\right)^2 dm^2 + \left(\frac{r^2}{c}\right)^2 df^2 + c^2dt^2\right] - dr^2$$

<u>For curved surfaces:</u>

$$ds^2 = -\left(1 - \frac{1}{c^2}\left(\frac{r(\hbar k)^2}{2m^2x}\right)\right)\left[\left(\frac{(r\hbar)^2k^5}{Fm^2cf}\right)^2 dE^2 + \left(\frac{k^3r^2\hbar}{(fm)^2c}\right)^2 dF^2 + \left(\frac{r^2cf}{E}\right)^2 dm^2 + \left(\frac{r^2}{c}\right)^2 df^2 + c^2dt^2\right] + \frac{1}{\left(1 - \frac{1}{c^2}\left(\frac{r(\hbar k)^2}{2m^2x}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

OPERATOR RELATIONS

MOMENTUM OPERATOR

$$\Psi = Ae^{i(kx-\omega t)}$$

From,

$$E = \frac{\hbar kx}{t}$$

$kx = \frac{Et}{\hbar}$----------(1)

Also,

$$P = \frac{\hbar\omega t}{x}$$

$\omega t = \frac{px}{\hbar}$--------(2)

Substitute eqn 1 and 2 into the wave equation;

$$\Psi = Ae^{i\left(\frac{Et}{\hbar}-\frac{px}{\hbar}\right)}$$

$$\frac{\partial\Psi}{\partial t} = \frac{iE}{\hbar}\Psi$$

$$E = \frac{\hbar kx}{t}$$

$$\frac{\partial\Psi}{\partial t} = \frac{i}{\hbar}\left(\frac{\hbar kx}{t}\right)\Psi$$

$$\frac{\partial\Psi}{\partial t} = \frac{ikx}{t}\Psi$$

$$k = \frac{p}{\hbar}$$

$$\frac{\partial\Psi}{\partial t} = \frac{ix}{t}\left(\frac{p}{\hbar}\right)\Psi$$

$let\ t = t_o$

$$\frac{\partial\Psi}{\partial t} = \frac{ipx}{\hbar t_o}\Psi$$

$make$ $p\Psi$ the subject;

$$p\Psi = -i\frac{\hbar t_o}{x}\frac{\partial\Psi}{\partial t} = \frac{Et_o}{x}\Psi$$

Finally,

$$\hat{p} = -i\frac{\hbar t_o}{x}\frac{\partial}{\partial t}$$

ENERGY OPERATOR

$$\Psi = Ae^{i\left(\frac{Et}{\hbar}-\frac{px}{\hbar}\right)}$$

$$\frac{\partial\Psi}{\partial x} = -\frac{ip}{\hbar}\Psi$$

But,

$$P = \frac{\hbar\omega t}{x}$$

$$\frac{\partial\Psi}{\partial x} = -\frac{i}{\hbar}\left(\frac{\hbar\omega t}{x}\right)\Psi$$

$$\frac{\partial\Psi}{\partial x} = -\frac{i\omega t}{x}\Psi$$

But,

$$\omega = \frac{E}{\hbar}$$

$$\frac{\partial\Psi}{\partial x} = -\frac{iEt}{\hbar x}\Psi$$

$let\ x = x_o$

$make\ E\Psi$ the subject;

$$E\Psi = \frac{i\hbar x_o}{t}\frac{\partial\Psi}{\partial x} = \frac{px}{t}\Psi$$

$$\hat{E} = \frac{i\hbar x_o}{t}\frac{\partial}{\partial x}$$

In three dimensions,

$$\hat{E} = \frac{i\hbar x_o}{t}\nabla$$

ANGULAR MOMENTUM OPERATOR

$$\Psi = Ae^{i(kx-\omega t)}$$

From,

$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$

$$\mathrm{kx} = \frac{Et}{\hbar}\text{---------}(1)$$

Also,

$$P = \frac{\hbar\omega t}{x}$$

$$\omega t = \frac{px}{\hbar}\text{-------}(2)$$

Substitute eqn 1 and 2 into the wave equation;

$$\Psi = Ae^{i\left(\frac{Et}{\hbar}-\frac{px}{\hbar}\right)}$$

$$\frac{\partial\Psi}{\partial t} = \frac{iE}{\hbar}\Psi$$

$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$

$$\frac{\partial\Psi}{\partial t} = \frac{i}{\hbar}\left(\frac{\hbar \mathrm{kx}}{\mathrm{t}}\right)\Psi$$

$$\frac{\partial\Psi}{\partial t} = \frac{i\mathrm{kx}}{\mathrm{t}}\Psi$$

$$\mathrm{k} = \frac{p}{\hbar}$$

$$\frac{\partial\Psi}{\partial t} = \frac{i\mathrm{x}}{\mathrm{t}}\left(\frac{p}{\hbar}\right)\Psi$$

$$\frac{\partial\Psi}{\partial t} = \frac{ip\mathrm{x}}{\hbar \mathrm{t}}\Psi$$

$$but\ p = \frac{L}{r}$$

$$\frac{\partial\Psi}{\partial t} = \frac{i\mathrm{x}}{\hbar \mathrm{t}}\left(\frac{L}{r}\right)\Psi$$

$$\theta = \frac{x}{r}$$

$$\frac{\partial\Psi}{\partial t} = \frac{iL\theta}{\hbar \mathrm{t}}\Psi$$

make $L\Psi$ the subject;

$$L\Psi = -i\frac{\hbar t_o}{\theta}\frac{\partial\Psi}{\partial t} = \frac{Et}{\theta}\Psi$$

Finally,

$$\hat{L} = -i\frac{\hbar t_o}{\theta}\frac{\partial}{\partial t}$$

TIME OPERATOR

$$\Psi = Ae^{i\left(\frac{Et}{\hbar}-\frac{px}{\hbar}\right)}$$

$$\frac{\partial\Psi}{\partial x} = -\frac{ip}{\hbar}\Psi$$

But,

$$P = \frac{\hbar\omega t}{x}$$

$$\frac{\partial\Psi}{\partial x} = -\frac{i}{\hbar}\left(\frac{\hbar\omega t}{x}\right)\Psi$$

$$\frac{\partial\Psi}{\partial x} = -\frac{i\omega t}{x}\Psi$$

make $t\Psi$ the subject;

$$t\Psi = \frac{ix}{\omega}\frac{\partial\Psi}{\partial x} = \frac{px}{E}\Psi$$

$$let\ x = x_o$$

Finally,

$$\hat{t} = \frac{ix_o}{\omega}\frac{\partial}{\partial x}$$

In three dimensions,

$$\hat{t} = \frac{ix_o}{\omega}\nabla$$

LINEAR POSITION OPERATOR

$$\Psi = Ae^{i(kx-\omega t)}$$

From,

$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$

$$\mathrm{kx} = \frac{Et}{\hbar}\text{---------}(1)$$

Also,

$$P = \frac{\hbar\omega t}{x}$$

$$\omega t = \frac{px}{\hbar}\text{-------}(2)$$

Substitute eqn 1 and 2 into the wave equation;

$$\Psi = Ae^{i\left(\frac{Et}{\hbar}-\frac{px}{\hbar}\right)}$$

$$\frac{\partial\Psi}{\partial t} = \frac{iE}{\hbar}\Psi$$

$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$

$$\frac{\partial \Psi}{\partial t} = \frac{i}{\hbar}\left(\frac{\hbar \mathrm{kx}}{\mathrm{t}}\right)\Psi$$
$$\frac{\partial \Psi}{\partial t} = \frac{i\mathrm{kx}}{\mathrm{t}}\Psi$$

$make\ x\Psi$ the subject;

$let\ t = t_o$

$$x\Psi = -\frac{it_o}{k}\frac{\partial \Psi}{\partial t}$$

Finally,

$$\hat{x} = -\frac{it_o}{k}\frac{\partial}{\partial t}$$

ANGULAR POSITION OPERATOR

$$\Psi = Ae^{i(kx-\omega t)}$$

From,

$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$
$\mathrm{kx} = \frac{Et}{\hbar}$----------(1)

Also,

$$P = \frac{\hbar\omega t}{x}$$
$\omega t = \frac{px}{\hbar}$-------(2)

Substitute eqn 1 and 2 into the wave equation;

$$\Psi = Ae^{i\left(\frac{Et}{\hbar}-\frac{px}{\hbar}\right)}$$
$$\frac{\partial \Psi}{\partial t} = \frac{iE}{\hbar}\Psi$$
$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$
$$\frac{\partial \Psi}{\partial t} = \frac{i}{\hbar}\left(\frac{\hbar \mathrm{kx}}{\mathrm{t}}\right)\Psi$$
$$\frac{\partial \Psi}{\partial t} = \frac{i\mathrm{kx}}{\mathrm{t}}\Psi$$

But $x = r\theta$

$$\frac{\partial \Psi}{\partial t} = \frac{i\mathrm{kr}\theta}{\mathrm{t}}\Psi$$

$make\ \theta\Psi$ the subject;

$let\ t = t_o$

$$\theta\Psi = -\frac{it_o}{kr}\frac{\partial \Psi}{\partial t}$$

Finally,

$$\hat{\theta} = -\frac{it_o}{kr}\frac{\partial}{\partial t}$$

FORCE OPERATOR

$$\Psi = Ae^{i(kx-\omega t)}$$

$\frac{\partial \Psi}{\partial t} = -i\omega\Psi$-------(1)

From,

$P = \frac{\hbar\omega t}{x}$------(2)

$F = \frac{P}{t}$-------(3)

$F = \frac{\hbar\omega}{x}$------(4)

$\omega = \frac{Fx}{\hbar}$----(5)

Substitute eqn 5 into 1;

$$\frac{\partial \Psi}{\partial t} = -i\left(\frac{Fx}{\hbar}\right)\Psi$$

Rearranging;

$$F\Psi = \frac{i\hbar}{x}\frac{\partial \Psi}{\partial t}$$

Finally,

$$\hat{F} = \frac{i\hbar}{x}\frac{\partial}{\partial t}$$

POTENTIAL ENERGY OPERATOR

$$\Psi = Ae^{i(kx-\omega t)}$$

$\frac{\partial \Psi}{\partial t} = -i\omega\Psi$-------(1)

From,

$P = \frac{\hbar\omega t}{x}$------(2)

$F = \frac{P}{t}$-------(3)

$F = \frac{\hbar\omega}{x}$------(4)

$\omega = \frac{Fx}{\hbar}$----(5)

But,

$F = \frac{V}{r}$----(6)

$$\omega = \left(\frac{V}{r}\right)\frac{x}{\hbar}$$

$\theta = \frac{x}{r}$----(7)

$\omega = \frac{V\theta}{\hbar}$------(8)

Substitute eqn 5 into 1;

$$\frac{\partial\Psi}{\partial t} = -i\left(\frac{V\theta}{\hbar}\right)\Psi$$

Rearranging;

$$V\Psi = \frac{i\hbar}{\theta}\frac{\partial\Psi}{\partial t} = \frac{E}{\theta}\Psi$$

Finally,

$$\hat{V} = \frac{i\hbar}{\theta}\frac{\partial}{\partial t}$$

PILOT WAVE HAMILTONIAN

$$E = \frac{p^2}{2m} + V$$

But,

$$\hat{p} = -i\frac{\hbar t_o}{x}\frac{\partial}{\partial t}$$

Therefore,

$$\hat{p}^2 = -\left(\frac{\hbar t_o}{x}\right)^2\frac{\partial^2}{\partial t^2}$$

Therefore,

$$E = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2\frac{\partial^2}{\partial t^2} + V$$

Multiply through by Ψ;

$$E\Psi = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2\frac{\partial^2\Psi}{\partial t^2} + V\Psi$$

But,

$$\hat{H} = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2\frac{\partial^2}{\partial t^2} + V$$

Therefore,

$$E\Psi = \hat{H}\Psi$$

$$E\Psi = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2\frac{\partial^2\Psi}{\partial t^2} + V\Psi$$

*This is the **Coordinate-independent Pilot Wave Equation (CIPWE)**.*

But;

$$E\Psi = \frac{i\hbar x_o}{t}\frac{\partial\Psi}{\partial x}$$

Therefore,

$$\frac{i\hbar x_o}{t}\frac{\partial\Psi}{\partial x} = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2\frac{\partial^2\Psi}{\partial t^2} + V\Psi$$

*This is the **Coordinate -dependent Pilot Wave Equation (CDPWE)**.*

PILOT WAVE TRANSPORT EQUATION

$$\Psi = Ae^{i\left(\frac{Et}{\hbar} - \frac{px}{\hbar}\right)}$$

$$\frac{\partial\Psi}{\partial t} = \frac{iE}{\hbar}\Psi$$

$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$

$$\frac{\partial\Psi}{\partial t} = \frac{i}{\hbar}\left(\frac{\hbar \mathrm{kx}}{\mathrm{t}}\right)\Psi$$

$$\frac{\partial\Psi}{\partial t} = \frac{i\mathrm{kx}}{\mathrm{t}}\Psi$$

But,

$$\omega = \frac{kx}{t}$$

Therefore,

$$\frac{\partial\Psi}{\partial t} = i\omega\Psi \text{—(1)}$$

$$\frac{\partial\Psi}{\partial x} = -\frac{ip}{\hbar}\Psi$$

But,

$$P = \frac{\hbar\omega t}{x}$$

$$\frac{\partial\Psi}{\partial x} = -\frac{i}{\hbar}\left(\frac{\hbar\omega t}{x}\right)\Psi$$

$$\frac{\partial\Psi}{\partial x} = -\frac{i\omega t}{x}\Psi$$

Make "$i\omega\Psi$" the subject;

$$i\omega\Psi = -\frac{x}{t}\frac{\partial\Psi}{\partial x}\text{—(2)}$$

Comparing eqns 1 and 2;

$$\frac{\partial\Psi}{\partial t} = -\frac{x}{t}\frac{\partial\Psi}{\partial x}$$

But, $\frac{x}{t} = \frac{\omega}{k} = \left(\frac{E}{\hbar} \times \frac{\hbar}{p}\right) = \frac{E}{p}$

$$\frac{\partial \Psi}{\partial t} = -\frac{E}{p}\frac{\partial \Psi}{\partial x}$$

Rearranging finally gives;

$$\frac{\partial \Psi}{\partial t} + \frac{E}{p}\frac{\partial \Psi}{\partial x} = 0$$

MISCELLANEOUS

$$E\Psi(t,x) = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2 \frac{\partial^2 \Psi(t,x)}{\partial t^2} + V\Psi(t,x)$$

$$\Psi(t,x) = Ae^{i\left(\frac{Et}{\hbar} - \frac{px}{\hbar}\right)}$$

Since Potential energy (V) has been shown to be a function of time, we can readily separate variables in $\Psi(t,x)$;

$$E\psi(t)e^{-i\left(\frac{px}{\hbar}\right)} = \left[-\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2 \frac{\partial^2}{\partial t^2} + V(t)\right]\psi(t)e^{-i\left(\frac{px}{\hbar}\right)}$$

$$E\psi(t) = \left[-\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2 \frac{\partial^2}{\partial t^2} + V(t)\right]\psi(t)$$

$$E\psi(t) = \widehat{H}\psi(t)$$

In general, there will be many solutions of the *PIPWE*, each corresponding to a different value of $\psi(t)$ and its corresponding eigenvalue, *E*. We therefore attach subscripts to distinguish the different $\psi_n(t)$ and to correlate them with their corresponding eigenvalues, E_n.

The different values of E_n are the eigenvalues and the corresponding values of $\psi_n(t)$ are called eigenfunctions. It is also possible that some eigenfunctions can share the same eigenvalue, in which case the eigenfunction is said to be degenerate.

SUPERPOSITION THEOREM

Thus, a general solution to the *PIPWE*, call it $\psi(t)$, the absence of a subscript signifying that it is not an eigenfunction, may be written

$$\psi(t) = \sum_{n=1}^{\infty} a_n \psi_n(t) \qquad (1)$$

Where, because $\psi(t)$ may be complex, as may be the expansion coefficients, the a_n.

Suppose that $\psi(t)$, as given by the above equation, represent the total wave function at $x = 0$, that is,

$$\Psi(t,x) = \psi(t) = \sum_{n=1}^{\infty} a_n \psi_n(t) \qquad (2)$$

Then, using the universal space dependence, it is a simple matter to write the wave function for all space. We have,

$$\Psi(t,x) = \sum_{n=1}^{\infty} a_n \psi_n(t)\, e^{-i\left(\frac{p}{\hbar}\right)x} \qquad (3)$$

This is a special kind of superposition called **Temporal Superposition**.

Equations (1) and (3) represent one of the most important theorems in Pilot Wave Mechanics, the Superposition Theorem.

Because$\Psi(t,x)$, as given in equation (3), is a solution of the *PDPWE,* it is said that the system is in a superposition of states. Moreover, the expansion represented by equation (3) is a coherent superposition of states in sense that, the “components” of the expansion have definite phase with respect to each other as contained in the expansion coefficients and the spatial dependence.

DYNAMIC STATES

It refers to any state $\Psi(t, x)$ for which the expansion, Equation 3, consists of a single term. Thus, the intensity of the pilot wave $|\Psi(t,x)|^2$ reduces to

$$|\Psi(t,x)|^2 = \left(\psi_n(t)e^{-i\left(\frac{p}{\hbar}\right)x}\right)^* \psi_n(t)e^{-i\left(\frac{p}{\hbar}\right)x}$$

$$|\Psi(t,x)|^2 = {\psi_n}^*(t)e^{i\left(\frac{p}{\hbar}\right)x}\psi_n(t)e^{-i\left(\frac{p}{\hbar}\right)x}$$

$$|\Psi(t,x)|^2 = {\psi_n}^*(t)\psi_n(t) \text{ (Functions of t only)}$$

Solutions that can be written as;

$$\Psi(t,x) = \psi_n(t)e^{-i\left(\frac{p}{\hbar}\right)x}$$

are called Dynamic.

Because the intensity of the pilot wave is dependent on time, it is defined as a "dynamic state".

STATIONARY STATES

It refers to any state $\Psi(t, x)$ for which the expansion, consists of a single term. Thus, the intensity of the pilot wave $|\Psi(t,x)|^2$ reduces to

$$|\Psi(t,x)|^2 = \left(\psi_n(x)e^{i\left(\frac{E}{\hbar}\right)t}\right)^* \psi_n(x)e^{i\left(\frac{E}{\hbar}\right)t}$$

$$|\Psi(t,x)|^2 = {\psi_n}^*(x)e^{-i\left(\frac{E}{\hbar}\right)t}\psi_n(x)e^{i\left(\frac{E}{\hbar}\right)t}$$

$$|\Psi(t,x)|^2 = {\psi_n}^*(x)\psi_n(x) \text{ (Functions of x only)}$$

Solutions that can be written as;

$$\Psi(t,x) = \psi_n(x)e^{i\left(\frac{E}{\hbar}\right)t}$$

are called Stationary.

Because the intensity of the pilot wave is dependent on position, it is defined as a "stationary state".

INTERPRETATION OF THE WAVE FUNCTION

If the intensity of radiation at any point is proportional to the square of the amplitude of an electromagnetic wave at that point, then the intensity of a pilot wave at a given point is proportional to the squared modulus of the wave function.

NORMALIZATION

The intensity of a pilot wave (Φ), described by $\psi_n(x)$, in a given interval of space, say x=a and x=d, is the sum of the intensities Φdx over all parts of that region. The sum is equivalent to the integral;

$$\Phi(\mathrm{a < x < d}) = \int_a^d \Phi(x)dx = \int_a^d \psi^2(x)dx$$

If the limits are extended to $\pm\infty$, the integral must be equal to unity.

$$\int_{-\infty}^{\infty} \psi^2(x)dx = \int_{-\infty}^{\infty} {\psi_n}^*(x)\psi_n(x)\, dx = 1$$

Wave functions meeting this requirement are said to be normalized.

POSTULATES OF PILOT WAVE MECHANICS

Postulate I: "All information that can be obtained about the state of a system is contained in the Pilot wave function Ψ_p.

Postulate II: "The Pilot wave function Ψ_p obeys the position-dependent (or coordinate-dependent) Pilot Wave Equation;

$$\hat{H}\Psi = \frac{i\hbar x_o}{t}\frac{\partial\Psi}{\partial x} \quad or \quad \frac{i\hbar x_o}{t}\nabla\Psi$$

Postulate III: "To every dynamical variable, there is a mathematical operator"

Postulate IV: "If a mechanical variable A is measured without experimental error, the only possible measured values of the variable A are the eigenvalues of the operator Â that corresponds to A.

Postulate V: "The state of a system always corresponds to the Pilot wave function that is the eigenfunction of Â.

APPLICATIONS

FREE PARTICLE:

$$\widehat{H} = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2 \frac{\partial^2}{\partial t^2} \qquad (1)$$

The Pilot Wave Equation $E\Psi = \widehat{H}\Psi$ thus takes the form;

$$E\Psi = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2 \frac{\partial^2 \Psi}{\partial t^2} \qquad (2)$$

Or where $\hbar = \frac{h}{2\pi}$;

$$\frac{\partial^2 \Psi}{\partial t^2} + \frac{8\pi m x^2}{h^2 t_o^2} E\Psi = 0 \qquad (3)$$

Equation (3) is a linear differential equation with constant coefficients, which is a type of equation having two solutions namely;

$$\Psi_1 = Ae^{i\left(\frac{2\pi x (2mE)^{1/2}}{h t_o}\right)t}$$ and

$$\Psi_2 = Be^{-i\left(\frac{2\pi x (2mE)^{1/2}}{h t_o}\right)t}$$

With $\omega = \frac{2\pi x (2mE)^{1/2}}{h t_o}$

POTENTIAL BARRIER:

$$\widehat{H} = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2 \frac{\partial^2}{\partial t^2} + V \qquad (1)$$

The Pilot Wave Equation $E\Psi = \widehat{H}\Psi$ thus takes the form;

$$E\Psi = -\frac{1}{2m}\left(\frac{\hbar t_o}{x}\right)^2 \frac{\partial^2 \Psi}{\partial t^2} + V\Psi \qquad (2)$$

Or where $\hbar = \frac{h}{2\pi}$;

$$\frac{\partial^2 \Psi}{\partial t^2} + \frac{8\pi m x^2}{h^2 t_o^2} (E - V)\Psi = 0 \qquad (3)$$

Equation (3) is a linear differential equation with constant coefficients, which is a type of equation having two solutions namely;

$$\Psi_1 = Ae^{i\left(\frac{2\pi x (2m(E-V))^{1/2}}{h t_o}\right)t}$$ and

$$\Psi_2 = Be^{-i\left(\frac{2\pi x (2m(E-V))^{1/2}}{h t_o}\right)t}$$

With $\omega = \frac{2\pi x (2m(E-V))^{1/2}}{h t_o}$

Pilot Wave Intensity along the x-axis

The pilot wave intensity is $|\psi|^2 = \psi^*\psi$ and the function

$$|\psi|^2 = \psi^*\psi = A^* e^{-i\left(\frac{2\pi x (2mE)^{1/2}}{h t_o}\right)t} A e^{i\left(\frac{2\pi x (2mE)^{1/2}}{h t_o}\right)t} = A^*A = |A|^2$$

Similarly, for the function Ψ_2, the pilot wave intensity is $|B|^2$. Both of these quantities are independent of both x and t, so that there is equal pilot wave intensity at any distance along the x-axis, at any given moment in time. The pilot wave intensity is therefore said to be **spatio-temporally nonlocalized**.

Allowed values of p_t

$$-i\frac{\hbar t_o}{x}\frac{\partial}{\partial t}\psi = p_t\psi$$

If $\psi = \psi_1$;

$$-i\frac{\hbar t_o}{x}\frac{\partial}{\partial t}\left(Ae^{i\left(\frac{2\pi x (2mE)^{1/2}}{h t_o}\right)t}\right) = p_t\left(Ae^{i\left(\frac{2\pi x (2mE)^{1/2}}{h t_o}\right)t}\right)$$

$$i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)\left(-i\frac{ht_o}{2\pi x}\right)\left(Ae^{i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t}\right) = p_t\left(Ae^{i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t}\right)$$

$$(2mE)^{1/2}\left(Ae^{i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t}\right) = p_t\left(Ae^{i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t}\right)$$

$$p_t = \sqrt{2mE}$$

For Pilot Wave systems,

$$\mathrm{E} = \frac{\hbar \mathrm{kx}}{\mathrm{t}}$$

$$p_t = \sqrt{2m\left(\frac{\hbar \mathrm{kx}}{\mathrm{t}}\right)}$$

$$p_t = \sqrt{\frac{2\hbar \mathrm{kmx}}{\mathrm{t}}}$$

$$p_{wave} = \hbar \mathrm{k},\ p_{particle} = \frac{\mathrm{m}x}{t}$$

$$p_t = \sqrt{2p_{wave} \times p_{particle}}$$

Similarly with ψ_2, it is found that

$$p_t = -\sqrt{2p_{wave} \times p_{particle}}$$

So that the two possible solutions are;

$$p_t = \pm\sqrt{2p_{wave} \times p_{particle}}$$

THE PARTICLE IN A BOX:

Quantization does however, appear if the particle is not permitted to travel an infinite distance but is confined to certain region of space at a specific time period. In three dimensions, this problem is referred to as the particle in a box.

Consideration of One Dimensional Problem:

The particle moves along the x axis over a distance from 0 to a.

The potential energy V is taken to be zero within the box $0 < x < a$, and infinity for $x < 0$ and $x > a$.

Within the box, the wave equation is equation (3) as for the free particle. However, acceptable wave function must satisfy certain boundary conditions, which in this case are that $\psi = 0$ at the walls.

Neither ψ_1 nor ψ_2 as given by these equations can individually satisfy these boundary conditions, because the condition that ψ must be zero at x=a, requires both A and B to be zero.

However, this dilemma is avoided by taking a linear combination of ψ_1 and ψ_2, which is also a solution of the wave equation.

It is perfectly general to take $\psi_1 + \psi_2$ as the linear combination, since A and B are in any case adjustable.

$$\psi = Ae^{+i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t} + Be^{-i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t}$$

[Since the particle can be reflected at the walls, the most general wave solution representing it must contain waves going in both directions, as seen in the equation above.]

The boundary condition that $\psi = 0$ when $x = 0$ requires that $A + B = 0$, so that $B = -A$.

The equation could be rewritten as;

$$\psi = A\left[e^{i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t} - e^{-i\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t}\right]$$

Applying Euler's theorem,

$$\left(e^{iy} = \cos y + i\sin y;\ e^{iy-iy} = 2i\sin y\ \right)$$

$$\psi = 2iA\sin\left[\left(\frac{2\pi x(2mE)^{1/2}}{ht_o}\right)t\right]$$

$$\psi = 2iA\sin\left[\left(\frac{2\pi x(2mE)^{1/2}}{h}\right)\left(\frac{t}{t_o}\right)\right]$$

But, $\frac{t}{t_o} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Therefore

$$\psi = 2iA\sin\left[\left(\frac{2\pi x(2mE)^{1/2}}{h}\right)\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)\right]$$

With respect to gravity,

$$\psi = 2iA\sin\left[\left(\frac{2\pi x(2mE)^{1/2}}{h}\right)\left(\frac{t}{t_o}\right)\right]$$

But, $\frac{t}{t_o} = \frac{1}{\sqrt{1-\frac{2gR}{c^2}}}$

Therefore,

$$\psi = 2iA\sin\left[\left(\frac{2\pi x(2mE)^{1/2}}{h}\right)\left(\frac{1}{\sqrt{1-\frac{2gR}{c^2}}}\right)\right]$$

The second boundary condition that $\psi = 0$ when $x = a$ gives;

$$0 = 2iA\sin\left[\left(\frac{2\pi a(2mE)^{1/2}}{ht_o}\right)t\right]$$

The factor $2iA$ cannot be zero, but the sine of an angle is zero when the angle is an integral multiple of π. Thus,

$$\left(\frac{2\pi a(2mE)^{1/2}}{ht_o}\right)t = \pm n\pi$$

Where n=1,2,3……$n > 0$.

For $n = 0$, we have $\psi = 0$ which would mean the particle flux is zero.

Making substitution;

$$\psi_n = \pm 2iA\sin\left(\frac{n\pi x}{a}\right)$$

To determine the value of 2iA, we use the normalization condition;

$$\int_0^a \psi_n{}^*\psi_n\,dx = \pm 4A^2\int_0^a \sin^2\left(\frac{n\pi x}{a}\right)\,dx = 1$$

The value of the integral is $a/2$ for integer values of n;

$$\pm 4A^2\frac{a}{2} = 1$$

$$A = \pm\sqrt{\frac{\pm 1}{2a}}$$

To make the wave function real, we select the negative sign within the square root;

$$A = \pm i\sqrt{+\frac{1}{2a}}$$

$$\psi_n = \pm 2iA\sin\left(\frac{n\pi x}{a}\right)$$

$$\psi_n = \pm 2i\times i\sqrt{+\frac{1}{2a}}\sin\left(\frac{n\pi x}{a}\right)$$

$$\psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

For relativistic particle in a box;

$$\psi = \sqrt{\frac{2}{a}}\sin\left[\left(\frac{2\pi x(2mE)^{1/2}}{h}\right)\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)\right]$$

For a particle in a gravitational potential well;

$$\psi = \sqrt{\frac{2}{a}}\sin\left[\left(\frac{2\pi x(2mE)^{1/2}}{h}\right)\left(\frac{1}{\sqrt{1-\frac{2gR}{c^2}}}\right)\right]$$

From $\left(\frac{2\pi a(2mE)^{1/2}}{ht_o}\right)t = \pm n\pi$,

$$E_n = \pm\frac{n^2h^2t_o^2}{8ma^2t^2}$$

$$E_n = \pm\frac{n^2h^2}{8ma^2}\left(\frac{t_o}{t}\right)^2$$

For a relativistic particle in a box;

$$\frac{t_o}{t} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$E_n = \pm\frac{n^2h^2}{8ma^2}\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^2$$

$$E_n = \pm\left(1 - \frac{v^2}{c^2}\right)\frac{n^2h^2}{8ma^2}$$

$$E_n = \pm\left(1 - \frac{v^2}{c^2}\right)\frac{n^2h^2}{8ma^2} \quad n = 1,2,3 \ldots..$$

$n \neq 0$

For a particle in a gravitational potential well;

$$E_n = \pm\frac{n^2h^2t_o^2}{8ma^2t^2}$$

$$E_n = \pm\frac{n^2h^2}{8ma^2}\left(\frac{t_o}{t}\right)^2$$

But,

$$\frac{t_o}{t} = \sqrt{1 - \frac{2gR}{c^2}}$$

$$E_n = \pm\frac{n^2h^2}{8ma^2}\left(\sqrt{1 - \frac{2gR}{c^2}}\right)^2$$

$$E_n = \pm\left(1 - \frac{2gR}{c^2}\right)\frac{n^2h^2}{8ma^2}$$

$$E_n = \pm\left(1 - \frac{2gR}{c^2}\right)\frac{n^2h^2}{8ma^2} \quad n = 1,2,3 \ldots..$$

$n \neq 0$

EXPRESSING THE DOUBLE SLIT FORMULA IN TERMS OF THE SOLUTION TO THE PILOT WAVE EQUATION

From the Double Slit formula;

$$d\sin\theta = \frac{n\pi x}{\omega t}$$

But, from the solution to the Pilot Wave Equation;

$$\omega = \frac{2\pi x(2mE)^{1/2}}{ht_o}$$

$$d\sin\theta = \frac{n\pi x}{t} \times \frac{ht_o}{2\pi x(2mE)^{1/2}}$$

$$2d\sin\theta = \frac{nht_o}{t\sqrt{2mE}}$$

$$But\ p = \sqrt{2mE}$$

$$2d\ sin\,\theta = \frac{nh}{p}\left(\frac{t_o}{t}\right)$$

For a relativistic particle,

$$\frac{t_o}{t} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$2d\ sin\,\theta = \frac{nh}{p}\sqrt{1 - \frac{v^2}{c^2}} \qquad (1a)$$

For a particle in a gravitational potential well,

$$2d\ sin\,\theta = \frac{nh}{p}\left(\frac{t_o}{t}\right)$$

But,

$$\frac{t_o}{t} = \sqrt{1 - \frac{2gR}{c^2}}$$

$$2d \sin\theta = \frac{nh}{p}\sqrt{1-\frac{2gR}{c^2}} \qquad (1b)$$

Also,

$$Y_n = \frac{2L}{d}\left(\frac{n\pi x}{\omega t}\right)$$

But, from the solution to the Pilot Wave Equation;

$$\omega = \frac{2\pi x(2mE)^{1/2}}{ht_o}$$

$$Y_n = \frac{2L}{d}\left(\frac{n\pi x}{t} \times \frac{ht_o}{2\pi x(2mE)^{1/2}}\right)$$

$$Y_n = \frac{2L}{d}\left(\frac{n\pi x}{t} \times \frac{ht_o}{2\pi x(2mE)^{1/2}}\right)$$

$$Y_n = \frac{L}{d}\left(\frac{nht_o}{t\sqrt{2mE}}\right)$$

$$But\ p = \sqrt{2mE}$$

$$Y_n = \frac{L}{d}\left(\frac{nh}{p}\right)\left(\frac{t_o}{t}\right)$$

For a relativistic particle,

$$\frac{t_o}{t} = \sqrt{1-\frac{v^2}{c^2}}$$

$$Y_n = \frac{L}{d}\left(\frac{nh}{p}\right)\sqrt{1-\frac{v^2}{c^2}} \qquad (2a)$$

For a particle in a gravitational potential well,

$$Y_n = \frac{L}{d}\left(\frac{nh}{p}\right)\left(\frac{t_o}{t}\right)$$

But,

$$\frac{t_o}{t} = \sqrt{1-\frac{2gR}{c^2}}$$

$$Y_n = \frac{L}{d}\left(\frac{nh}{p}\right)\sqrt{1-\frac{2gR}{c^2}} \qquad (2b)$$

EXPERIMENTAL SUPPORT

BOUNCING OIL DROPLET EXPERIMENT

The Yves Couder bouncing droplet experiment is a fascinating and thought-provoking demonstration that blends elements of fluid dynamics and quantum-like behavior on a macroscopic scale. It was pioneered by Yves Couder and Emmanuel Fort at the University of Paris, and it offers a unique way to explore the relationship between classical mechanics and quantum mechanics.

In the experiment, a small droplet of silicone oil is placed on the surface of a vibrating bath of the same oil. The bath is vertically vibrated at a specific frequency, causing waves to form on the surface. What's remarkable is that the droplet is able to bounce and travel across the surface of the vibrating bath, propelled by the interaction of its own waves.

The key insight behind this experiment lies in the concept of "pilot waves." As the droplet bounces on the surface, it generates ripples or waves that extend outward from the droplet. These waves have a wavelength that is determined by the droplet's velocity and the frequency of the bath's vibrations. The intriguing part is that the droplet appears to be guided by its own wave field. The interaction between the droplet and its associated wave creates a self-sustaining system where the droplet's motion is intricately linked to the properties of the waves it generates.

The behavior of the bouncing droplet exhibits some similarities to quantum mechanics, particularly the concept of wave-particle duality. Just as in quantum systems, the droplet seems to exhibit both particle-like behavior (localized bouncing) and wave-like behavior (extended wave field) simultaneously. This is a remarkable example of classical objects seeming displaying quantum-like features.

Researchers have used the bouncing droplet experiment to study a variety of phenomena, including single and double-slit interference patterns. When a barrier with two slits is introduced in the path of the bouncing droplet, it produces an interference pattern on the other side of the barrier, reminiscent of the interference patterns seen in quantum systems. This behavior provides an insightful perspective on how classical systems can exhibit interference phenomena.

The Yves Couder bouncing droplet experiment has sparked intense debate and discussion in the scientific community. Some view it as a promising analog for understanding certain quantum behaviors, while others emphasize the fundamental differences between macroscopic classical systems and microscopic quantum systems. [5]

Regardless of the ongoing debates, the experiment remains a captivating illustration of the interconnectedness between classical and quantum mechanics, pushing the boundaries of our understanding of the nature of reality.

THOUGHT EXPERIMENT

1. **Experiment Name**: Temporal Interference Probe (TIP)
2. **Objective**:
   To investigate the potential existence of temporal superposition within the framework of Pilot Wave Mechanics.
3. **Setup:**
   a. **Particle Source:**
   Use a source that emits particles according to the principles of Pilot Wave Mechanics. The source should allow for the creation of particles in specific states at different temporal instances.
   b. **Spatially Separated Detectors:**
   Position detectors at different locations in space, each equipped with the ability to measure the properties of particles. These detectors should be spatially separated to observe the evolution of the particle's states at distinct positions.
   c. **Temporal Control Mechanism:**
   Implement a mechanism that controls the temporal evolution of the particles. This mechanism should allow for the creation of particles in superposition across different temporal states.
   d. **Interference Screen:**
   Place a screen with multiple slits between the particle source and its detectors. The interference pattern on the screen can reveal information about the temporal aspects of the particle's states.
4. **Procedure:**
   i. **Temporal Superposition Initialization:**
   Use the temporal control mechanism to initialize particles in a superposition of different temporal states.
   This involves creating particles with specific properties at different moments in time.
   ii. **Particle Propagation:**
   Allow particles to propagate toward the interference screen and detectors.
   The particles are in superposition across various temporal states, and their evolution should lead to a distinctive interference pattern.
   iii. **Interference Pattern Analysis:**
   Examine the interference pattern on the screen. The pattern's temporal structure should provide insights into how the particle evolve over time. Differences in the interference pattern compared to traditional non-temporal superposition experiments could indicate the influence of temporal superposition.
   iv. **Detector Measurement:**
   Simultaneously, detectors at different spatial locations measure the properties of the particles as they pass through. The measurements at each detector should reflect the temporal superposition, potentially showing variations in the observed properties over time.
5. **Expected Outcomes:**
   i. **Temporal Interference Pattern:**
   If temporal superposition exists, the interference pattern on the screen should exhibit a temporal structure, indicating the simultaneous presence of particles in different temporal states.
   ii. **Spatial-Temporal correlations:**
   Measurements at the spatially separated detectors should reveal correlations between the spatial and temporal aspects of the particle's properties, consistent with the predictions of Pilot Wave Mechanics.
6. **Considerations:**
   i. **Control Experiments:**
   Include control experiments where temporal superposition is absent to distinguish the effects specifically associated with temporal aspects.
   ii. **Quantum Coherence Verification:**
   Develop methods to verify the quantum coherence of particles across different temporal states during the experiment.

DEFINITION AND SOLUTION OF THE SPACE-TIME SINGULARITY.

Definition

The space-time singularity as defined by the Schwarzschild metric occurs when the physical radius (r) is set to zero. It is mathematically illustrated below:

$$R_s = \frac{2GM}{c^2}$$

$$ds^2 = -\left(1 - \frac{R_s}{r}\right)c^2dt^2 + \frac{1}{\left(1 - \frac{R_s}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$if\ r = 0;$

$$ds^2 = -(\infty)c^2dt^2 + \frac{1}{(\infty)}dr^2$$

This is what is termed as a space-time singularity, a point in space-time in which gravitational forces cause matter to have an infinite density.

Solution

The space-time singularity problem can be solved using the Pilot Wave Field Metric. It is mathematically illustrated below;

$$ds^2 = -\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2k^3}{m^2\theta}\right)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{1}{nc^2}\left(\frac{r\hbar^2k^3}{m^2\theta}\right)\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$if\ r = 0;$

$$ds^2 = -\left(1 - \frac{1}{nc^2}(0)\right)c^2dt^2 + \frac{1}{\left(1 - \frac{1}{nc^2}(0)\right)}dr^2$$

$$ds^2 = -\big(1 - (0)\big)c^2dt^2 + \frac{1}{\big(1 - (0)\big)}dr^2$$

$$ds^2 = -(1)c^2dt^2 + \frac{1}{(1)}dr^2$$

$$ds^2 = -c^2dt^2 + dr^2$$

The above calculation informs us that information about what goes on inside a black hole and what went on at the period of the creation of the universe (Big Bang) can be perfectly modelled using the Pilot Wave Field Metric.

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