Relation between Dark Matter and Dark Energy in Galaxy Clusters and its successful implications

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1. ABSTRACT

This paper develops the Dark Matter by Gravitation theory, DMbG theory hereafter, in clusters of galaxies. Originally this theory was developed by the author for galaxies, especially using MW and M31 rotation curves.
An important results got by the DMbG theory is that the total mass associated to a galactic halo depend on the square root of radius, being its dominion unbounded. Apparently, this result would be absurd because of divergence of the total mass. However it is the Dark energy the responsible to counterbalance the DM. As the DE is negligible at galactic scale it is needed to extend the theory to clusters in order to study the capacity of DE to counterbalance to DM. Put in brief the main goal of this paper has been to develop the DMbG theory at cluster scale.
In this paper the DMbG theory finds unexpected theoretical results because it is at cluster scale where DM and DE are counterbalanced mutually.
In this work it is defined, the total mass as baryonic matter plus DM and the gravitating mass as the addition of the total mass plus the negative mass associated to dark energy.
In clusters it is defined the zero gravity radius (R_{ZG} hereafter) as the radius needed by the dark energy to counterbalance the total mass. It have been found, that the ratio $R_{ZG} / R_{VIRIAL} \approx 7.3$ and its Total mass associated at R_{ZG} is $\approx 2.7 \cdot M_{VIRIAL}$. In addition it has been calculated that the sphere with the extended halo radius $R_E = 1.85 \cdot R_{ZG}$ has a ratio DM density versus DE density equal to 3/7 and its total mass associated at R_E is $\approx 3.67 \cdot M_{VIRIAL}$
This works postulates that the factor 3.67 may equilibrate perfectly the strong imbalance between the Local mater density parameter (0.08) versus the current Global matter density one (0.3). Currently this fact is a big conundrum in cosmology.
Also it has been found that the zero velocity radius, R_{ZV} hereafter, i.e. the cluster border because of the Hubble flow, is $\approx 0.6 \cdot R_{ZG}$ and its gravitating mass is $\approx 1.5\ M_{VIR}$
By derivation of gravitating mass function it is calculated that at $0.49 \cdot R_{ZG}$ this function reaches its maximum whose value is $\approx 1.57 \cdot M_{VIR}$
Throughout the paper some of these results have been validated with recent data published for MW or M31 galaxies and the Virgo or Coma clusters.
By other side, these new theoretical findings offer to scientific community a wide number of tests to validate or reject the theory. The validation of DMbG theory would mean to know the nature of DM that at the present it is an important challenge for the astrophysics science.

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2. INTRODUCTION

The bases of this paper are developed in [1] Abarca, M. 2023, so it is highly recommended to read it to understand the meaning of this paper. The dark matter by gravitation theory, DMbG theory hereafter, is an original theory developed since 2013 through more than 20 papers, although in [1] Abarca, M. 2023 is published the best version as physical as mathematically. Therefore is not possible to understand this paper if reader have not at least a general knowledge about the DMbG theory.
The DM by gravitation theory was introduced in [20] Abarca, M.2014. *Dark matter model by quantum vacuum.* The hypothesis of DMbG theory is that the DM is generated by the own gravitational field. In order to study purely the DM phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. For example, for MW galaxy the radius must be bigger than 30 kpc and bigger than 40 kpc for M31 galaxy. For galactic clusters, the radius must be bigger than its virial radius.

This hypothesis has two main consequences: the first one is that the law of dark matter generation, in the halo region, has to be the same for all the galaxies and clusters; the second one is that the DM haloes are unlimited so the total dark matter goes up without limit.

The DMbG theory has been developed assuming the hypothesis that DM is a quantum gravitational effect. However, it is possible to remain into the Newtonian framework to develop the theory. In my opinion there are two factors to manage the DM conundrum with a quite simple theory.
The first one, that it is developed into the halo region, where baryonic matter is negligible. The second one, that the mechanics movements of celestial bodies are very slow regarding velocity of light, which is supposed to be the speed of gravitational bosons. It is known that community of physics is researching a quantum gravitation theory since many years ago, but does not exist yet; however my works in this area support strongly that DM is a quantum gravitation phenomenon.

Use a more simple theory instead the general theory is a typical procedure in physics.
For example the Kirchhoff ´s laws are the consequence of Maxwell theory for direct current and remain valid for alternating current, introducing complex impedances, on condition that signals must have low frequency. However these laws do not work for electromagnetic microwaves because of its high frequency.

In [1] Abarca, M. 2023, in the framework of DMbG, is demonstrated mathematically that the total mass (baryonic plus DM) enclosed by a sphere with a specific radius is given by the Direct mass into the galactic halo and that the direct mass formula goes up proportionally to the root square of radius.

It is well known that DE may be modelled as a constant density of negative mass in the whole space, see [9] Chernin,A.D. et all.2013, therefore the total amount of DE grows up with the cubic power of the sphere radius and this way it is clear that DE is able to counterbalance the total mass of the clusters, which grows up more slowly. Precisely, the main goal of this paper is to study the relation between both phenomenons in clusters. Namely in cluster haloes.

This paper explores the mutual counterbalance between DM and DE in the framework of DMbG theory and the result got have been fructiferous, with a dozen of new formulas never published before.

In the following paragraphs will be introduced the paper structure:

The newness of the important results got in this paper are due to the possibility to approximate the virial radius to R_{200} and the virial mass to M_{200} , the chapter 3 is dedicated to validate this approximation using recent data published for some important clusters such as Virgo, Coma or some others.

The chapter 4 is dedicated to extend the direct mass formula to clusters. The direct mass $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ has only one parameter a^2 whose units are $m^{5/2}/s^2$. Using the approximation R_{200} as virial radius and M_{200} as virial mass

into the direct mass formula it is got the formula $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$ which is on the basis for some important results got in the following chapters.

The chapter 5 contains three main concepts:
Firstly, it is defined the zero gravity radius, R_{ZG}, as the sphere radius where the total mass is counterbalanced by the DE. It is found that the ratio $R_{ZG} / R_{VIRIAL} \approx 7.3$ is universal and its Total mass at R_{ZG} is $\approx 2.7 \cdot M_{VIRIAL}$ as a universal law as well.
Secondly, it is defined the gravitating mass as the addition of total mass plus the dark energy and the gravitating mass function using a dimensionless parameter f = Radius / R_{ZG}. It is found that for any cluster at $\approx 0.5 \cdot R_{ZG}$ is reached the maximum of gravitating mass and its value is $M_G(< R_M) \approx 1.57 \cdot M_{VIR}$
Finally it is defined the concept of extended halo (R_E) as the spherical region where the ratio $\frac{Density_{TM}^{SPHERE}(<R_E)}{\rho_{DE}} = 3/7$ i.e. the local ratio of such densities is equal to the current global ratio one, and it is found that $R_E \approx 1.85\ R_{ZG}$

In the sixth chapter it is defined the zero velocity radius, R_{ZV}, as the sphere radius where the escape velocity is zero because of the Hubble flow. It is demonstrated that the ratio $R_{ZV} / R_{ZG} \approx 0.602$ is universal and it is found that its gravitating mass associated at such radius is $M_G(< R_{ZV}) \approx 1.5\ M_{VIR}$ The zero velocity radius is the effective border of the cluster because for bigger distances the Hubble flow velocity is bigger than escape velocity regarding the cluster.

In the seventh chapter, it is validated the gravitating mass formula into the Virgo cluster for a couple of radius. Namely at 7.3 Mpc and at 3.4 Mpc. The calculus made with the formula of gravitating mass is compared successfully with recent results published in 2020. Also it is validated the theoretical result of $R_{ZV} / R_{ZG} \approx 0.602$ with result of measures published.
Two of the most important results got in this work are the formula $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR}$ being U ≈ 2.7 and the formula $M_{TOTAL}(< R_E) \approx 3.67 \cdot M_{VIR}$ associated to zero gravity radius and the extended radius respectively.

Thanks these formulas this work suggest the possibility to solve the current discrepancy between the local parameter of matter density, $\Omega_m^{local} = 0.08$, see [12] Karachentsev et al. 2014, and the current global one $\Omega_m^{global} = 0.3$. This discrepancy is an open problem for the current cosmology.

As this work contains a number of formulas totally original never published before, the chapter 8 is dedicated to summarize all of them. Finally the chapter 9 is devoted to the concluding remarks.

3. VIRIAL MASS AND VIRIAL RADIUS IN CLUSTER OF GALAXIES

As the reader knows it is a good estimation about virial radius and virial mass for cluster of galaxies to consider Rvir = R_{200} and Mvir= M_{200}. Where R_{200} is the radius of a sphere whose mean density is 200 times bigger than the critic density of Universe $\rho_C = \frac{3H^2}{8\pi G}$ (3.1) and M_{200} is the total mass enclosed by the radius R_{200}.
In this epigraph will be shown some data published by several researchers that confirm this approximation.

Considering the spherical volume formula, it is right to get the following relation between both concepts

$R_{VIR}^3 \approx R_{200}^3 = \frac{G \cdot M_{200}}{100 \cdot H^2}$ (3.2) or $M_{VIR} \approx M_{200} = \frac{100 H^2 R_{200}^3}{G}$ (3.3)

The checking process will begin with the bigger cluster in the Local Universe. The graph below comes from [7] Seong –A Oh.2023.They inform that virial radius is 2.8 Mpc, so using the above formula it is got Mvirial = $2.5 \cdot 10^{15} M_\odot$, that match with mass published. In this work will be used the value H= 70 Km/s/Mpc

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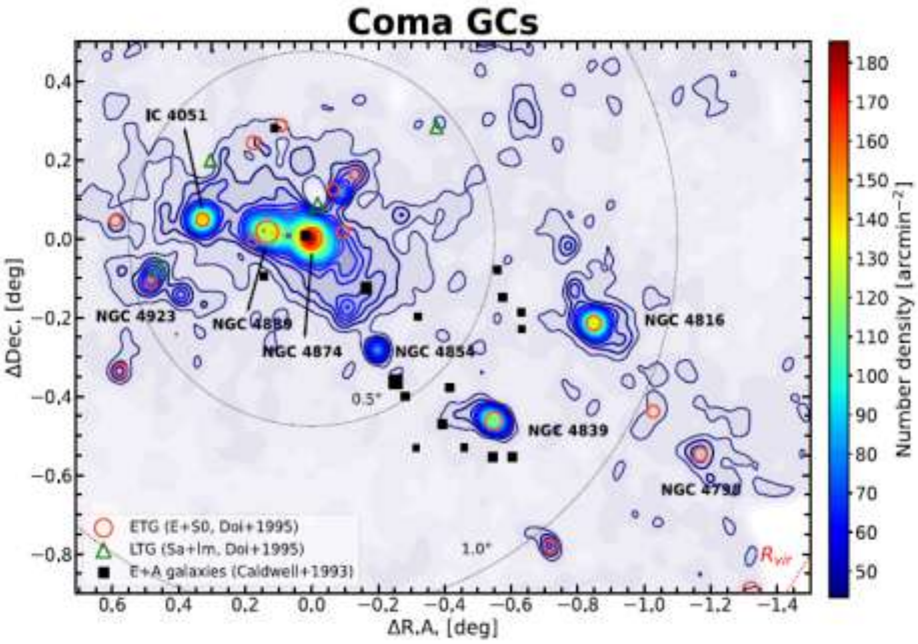


Figure 3. Spatial number density contour map of GCs in the Coma field including NGC 4839 and NGC 4816 (see S. Oh et al. 2023, in preparation, for details). Dotted line circles represent $R = 0°.5$, $1°.0$, and R_{vir}(=2.8 Mpc) from NGC 4874 at the Coma center. Red circles and green triangles mark early-type galaxy members, and late-type galaxy members (Doi et al. 1995). Black boxes mark E+A galaxies (Caldwell et al. 1993). The contour levels denote $2\sigma_{bg}$ and higher with an interval of $1\sigma_{bg}$ where σ_{bg} denotes the background fluctuation. The contour maps were smoothed using a Gaussian filter with $\sigma_G = 1'$. The color bar represents the GC number density.

Coma Cluster parameters according [7] Seong –A Oh.et al. 2023

Parameter	Main Cluster
Heliocentric galaxy velocity, v_h	7167 km s^{-1}
Heliocentric group velocity, v_h	6853 km s^{-1}
Velocity dispersion, σ_v	1082 km s^{-1}
Virial mass (dynamics),[b] M_{vir}	$2.7 \times 10^{15} M_\odot$
Weak-lensing mass,[c] M_{WL}	$1.2 \times 10^{15} M_\odot$

Table 1

Data from Dynamic method

M_{VIR} = 2.7E15 $M_\odot$ and R_{VIR} = 2.8 Mpc

Using in formula (3.3) the value 2.8 Mpc as R_{200} is got $M_{200} = 2.5\cdot10^{15} M_\odot$ whose relative difference versus M_{VIR} is 7.4%

The author [7] Seong, shows that the formula to calculate Virial mass is $M_{VIR} = 1.5\cdot10^{6}\cdot h^{-1}\cdot\sigma_V^3\ M_\odot$ where σ_V =1082 km/s So the formula gives the mass in $M_\odot$ units on condition that velocity dispersion σ_V units are km/s and h = 0.7

3.1 CHECKING THE VIRIAL MASS APROXIMATION ON A SAMPLE OF CLUSTERS AND GROUP OF G.

Data [4] R.Ragusa et al.2022			Table 2	
Group of galaxies G. Or Clusters C.	Virial Radius	Virial Mass	Mass calculated	Relative diff for M
Name	Mpc	x10^{13} $M_\odot$	x10^{13} $M_\odot$	%
Antlia C.	1,28	26,3	2,39E+01	-9,21E+00
NGC596/584 G.	0,5	1,55	1,42E+00	-8,18E+00
NGC 3268 G.	0,9	8,99	8,30E+00	-7,67E+00
NGC 4365Virgo SubG.	0,32	0,4	3,73E-01	-6,73E+00
NGC 4636 Virgo SubG.	0,63	3,02	2,85E+00	-5,73E+00
NGC 4697Virgo Sub G.	1,29	26,9	2,44E+01	-9,14E+00
NGC 5846 G.	1,1	16,6	1,52E+01	-8,71E+00
NGC 6868 G.	0,6	2,69	2,46E+00	-8,57E+00

Data beside in green have been taken from [4] R. Ragusa et al. 2022 and using the formula $M_{200} = \frac{100H^2R_{200}^3}{G}$ it is calculated its mass associated for each radius. The yellow column shows the relative difference for masses, always under 10 %. The mass calculated are lower than mass published. With these examples it is shown that the consideration of R_{200} and M_{200} as virial radius and virial mass is acceptable for group of galaxies or clusters.

As the Virgo cluster is the nearest between the big clusters it is crucial to check the approximation for virial mass and radius with its data.

According [12] Karachentsev I.D. et al. 2014.In page 5 it is shown that Rvir = 1.8 Mpc and Mvir = 7E14 M_{Θ}. And using formula (3.3) it is got Mvir = 6.64E14 M_{Θ} which is quite close to published value.

According [13] Olga Kashibadze, I. Karachentsev 2020, see pag 9, Rg=Rvir= 1.7 Mpc and Mvir= (6.3±0.9)E14 M_{Θ}. Using formula (3.3) with R_{200} =1.7 Mpc it is got M_{200} = 5.59E14 M_{Θ} which match with mass published if it is considered the range of errors.

In table 3 are summarized the results for the two most prominent cluster of galaxies.

Table 3	Virial Radius	Virial mass	Calculated M_{200}	Mass Relative diff.
Cluster of galaxies	Mpc	x 10^{14} M_{Θ}	x 10^{14} M_{Θ}	%
Virgo [13]Kashibadze 2020	1.7	6.3±0.9	5.59	11
Coma [7] Seong-A. 2023	2.8	27	25	7.4

In conclusion R_{200} and M_{200} are a very good estimation for Virial radius and Virial mass for galaxies, group of galaxies and cluster of galaxies, when they are in dynamical equilibrium, as it is well known by the astrophysicist.

4. VIRIAL THEOREM AS A METHOD TO GET THE DIRECT MASS FORMULA IN CLUSTERS

In chapter 9, of paper [1]Abarca,M.2023 was demonstrated that direct formula $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ (4.1) is the most suitable formula to calculate the total mass (baryonic and DM) depending on radius in the galactic halo region.

So (4.1) may be written as $M_{TOTAL}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ where the units of parameter a^2 are $m^{5/2}/s^2$

This chapter is based on the 16 chapter in [1]Abarca,M.2023, where the Direct mass formula is extended to clusters.

4.1 PARAMETER a^2 FORMULA DEPENDING ON VIRIAL RADIUS AND VIRIAL MASS

Due to the fact that the Direct mass formula has one parameter only, is enough to know the mass associated to a specific radius to be able to calculate parameter a^2. That is the situation when it is known the virial mass and the virial radius for a cluster of galaxies.

This formula is only a way to estimate parameter a^2 because outside the virial radius always there will be a fraction of the galaxies belonging to cluster. Anyway, this method may estimate a lower bound of parameter a^2 associated to clusters.

If it is considered that the virial radius is the border of halo cluster where galaxies are in dynamical equilibrium and at the same time is negligible the amount of Baryonic matter outside the sphere with such radius, then according DMbG theory is possible to do an equation between $M_{VIRIAL}(< R_{VIRIAL}) = M_{DIRECT}(< R_{VIRIAL})$ (4.2.1) i.e.

$M_{VIRIAL} \equiv M_{VIRIAL}(<R_{VIRIAL}) = \frac{a^2 \cdot \sqrt{R_{VIRIAL}}}{G}$ (4.2.2) and clearing up $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ (4.3), this formula will be called parameter $a^2(M_{VIR}, R_{VIR})$ because depend on both measures.

4.2 PARAMETER a^2 FORMULA DEPENDING ON VIRIAL MASS ONLY

In chapter 3 was got this formula $R_{VIR}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ (3.2) as a good approximation between virial mass and virial radius .

So using that formula and by substitution of virial radius in $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ (4.3) it is right to get parameter a^2 depending on M_{VIR} only $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$ (4.4) This formula will be called parameter a^2 (M_{VIR}) as depend on M_{VIR} only.

With the virial data for some important clusters such as Virgo or Coma cluster will be calculated its parameter $\mathbf{a^2}$ with the formula (4.3) i.e. a^2(M_{VIR} , R_{VIR}) and with formula (4.4) i.e. a^2 (M_{VIR})

The last formula is an approximation of the previous formula as it is supposed that $R_{VIR} = R_{200}$. Below are calculated both formulas and fortunately its relative difference is negligible.

Table 4	Virial Radius	Virial mass	Parameter a^2(M_{VIR} , R_{VIR})	Parameter a^2 (M_{VIR})	Relative diff.
Clusters	Mpc	$\cdot 10^{14}\ M_{\odot}$	I.S. units $m^{5/2}/s^2$	I.S. units $m^{5/2}/s^2$	%
Virgo	1.7	6.3±0.9	3.6527E23	3.581E23	2
Coma	2.8	27	1.2198E24	1.2042E24	1.3

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh,2023

Notice how close are both results for parameter a^2 especially when relative differences for masses are 11% and 7.4% as it was shown in table 3.

4.3 M_{200} FORMULA AND COMPARISON WITH M_{200} PUBLISHED ABOUT MILKY WAY

As the most of the important results of this paper are based on the direct mass formula,(4.1) and this formula was got studying the rotation curves of MW and M31, in this epigraph and the following one it will be shown a procedure to validate the results got through that formula with the virial masses published about MW and M31.

From formula (4.4) it is right to clear up the Mass $M_{VIR} \approx M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$ (4.5) .These formulas are based on direct mass formula (4.1) which is one of the most important result of DMbG, developed mainly in galaxies. The best version of this theory can be consulted in [1] Abarca, M. 2023.

This formula gives a new possibility to check the DMbG with recent works published, using the following procedure:

In [16] Abarca, M. 2023, was found the optimal value to fit the rotation curve in halo region, being this parameter $\mathbf{a}_{M\text{-}W} = 3.9 \cdot 10^{10}\ m^{5/4}/s$ (4.6) for the MW galactic halo. This value was calculated with data from [6] Sofue,Y.2020, but using only the halo region data from 30 kpc up to 95 kpc.

Using the central hypothesis of the theory was got the direct mass, which is a formula with radius unbounded as according DMbG theory the DM is linked to gravitational field.

So comparing the value of M_{200} got by (4.5) with the value published as a result of sophisticated astrophysical survey of Milky Way, it is possible to check the rightness of calculus and consequently the rightness of the DMbG theory in this test.

Firstly it will be calculated M_{200} and afterwards R_{200} . As M_{200} depends on parameter H, considering H= 70 km/s/Mpc, and $\mathbf{a}_{M\text{-}W} = 3.9 \cdot 10^{10}\ m^{5/4}/s$, (See 4.6), so it is right to get $M_{200} = 8.976 \cdot 10^{11}\ M_{\odot}$ and using $R_{200}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ it is right to get R_{200} = 199 kpc.

Notice that using R_{200} into the Direct mass formula it is got rightly the M_{200} . In other words, formula (4.5) is mathematically equivalent to Direct mass at R_{200} i.e. $M_{DIRECT}(< R_{200}) \equiv M_{200}$.

For example $M_{MW}(< 199 \text{ kpc}) = 8.976 \cdot 10^{11}\ M_{\Theta}$,using Direct mass, $\mathbf{a}_{M\text{-}W} = 3.9 \cdot 10^{10}\ m^{5/4}/s$ and $R_{200} = 199$ kpc

There are published a lot of studies about DM in Milky Way, but it has been select two recent works whose authors are well known astrophysicist. Namely the paper sources for the comparison are: [17] Jeff Shen , G. M. Eadie et al. and [18] E.V. Karukes, M.Benito et al.

M_{200} COMPARISON WITH RESULT GOT BY JEFF SHEN TEAM`S

The clipping text below belong to the abstract of the paper [17] Jeff Shen.2022

Abstract

The mass of the Milky Way is a critical quantity that, despite decades of research, remains uncertain within a factor of two. Until recently, most studies have used dynamical tracers in the inner regions of the halo, relying on extrapolations to estimate the mass of the Milky Way. In this paper, we extend the hierarchical Bayesian model applied in Eadie & Juri to study the mass distribution of the Milky Way halo; the new model allows for the use of all available 6D phase-space measurements. We use kinematic data of halo stars out to 142 kpc, obtained from the H3 survey and Gaia EDR3, to infer the mass of the Galaxy. Inference is carried out with the No-U-Turn sampler, a fast and scalable extension of Hamiltonian Monte Carlo. We report a median mass enclosed within 100 kpc of $M(<100 \text{ kpc}) = 0.69^{+0.05}_{-0.04} \times 10^{12}\ M_{\odot}$ (68% Bayesian credible interval), or a virial mass of $M_{200} = M(<216.2^{+7.5}_{-7.5} \text{ kpc}) = 1.08^{+0.12}_{-0.11} \times 10^{12}\ M_{\odot}$, in good agreement with other recent estimates. We analyze our results using posterior predictive checks and find limitations in the model's ability to describe the data. In particular, we find sensitivity with respect to substructure in the halo, which limits the precision of our mass estimates to $\sim$15%.

Beside is remarked two important data: M_{200} and R_{200} $M_{200} = M(<216.2^{+7.5}_{-7.5} \text{ kpc}) = 1.08^{+0.12}_{-0.11} \times 10^{12}\ M_{\odot}$,as $R_{200} = 216.2$ kpc and $M_{200} = 1.08 \cdot 10^{12}\ M_{\Theta}$ then it is possible to clear up the value for H because $R_{200}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$
Namely this value is H= 67.817 km/s/ Mpc. As it is known the value of H is lightly different depending on technique used to measure it, and at the present there are not consensus about its exact value.
Now it will be calculated this value $M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$ using H= 67.817 km/s/ Mpc and $\mathbf{a}_{M\text{-}W} = 3.9 \cdot 10^{10}\ m^{5/4}/s$, then $M_{200} = 9.09 \cdot 10^{11}\ M_{\Theta}$ and $R_{200} = 204.1$ kpc
The value for M_{200} is only 6% lower regarding the lower mark got by Jeff Shen. Clearly both results are very close, because a relative difference of 6% regarding M_{200} is totally negligible.

From these results it is possible to state one main conclusion: The direct mass formula for total mass with its dominion unbounded works perfectly at 204 kpc. It is important to remember that the parameter $\mathbf{a}_{M\text{-}W} = 3.9 \cdot 10^{10}$ was got thanks [6] Sofue,Y.2020 paper, using the rotation data set from 30 kpc up to 95 kpc, whereas M_{200} is the total mass calculated at 204 kpc.

M_{200} COMPARISON WITH RESULT GOT BY EKATERINA KARUKES TEAM`S

The clipped texts below belong to the paper [18] E.V. Karukes. 2020, placed in pages 10 and 11.

sity, HPD, interval, i.e., the shortest interval containing 68% of posterior probability). Our estimate of the total mass of the Milky Way –the sum of baryons and dark matter– within the virial radius, is $\log_{10} M_{\text{tot}}/M_{\odot} = 11.95^{+0.04}_{-0.04}$ or on a linear scale:

$$M_{\text{tot}} = 8.9^{+1.0}_{-0.8} \times 10^{11}\ M_{\odot}. \tag{3.2}$$

As the reader may see, this time the agreement is perfect between the result of Karukes team`s $M_{200} = 8.9 \cdot 10^{11}\ M_{\Theta}$

and the calculus got by the DMbG theory, using the Direct mass formula $M_{200} = 9.09 \cdot 10^{11}\ M_{\Theta}$.

Newly arises one main conclusion: The direct mass formula for total mass with its dominion unbounded works perfectly at 204 kpc.

4.4 CHEKING M_{200} FORMULA WITH M_{200} PUBLISHED ABOUT M31

The clipped text below is placed in the abstract of paper [19] Xiangwei Zhang.2024.

determined rotation curve, we have constructed a mass distribution model for M31. Our measurement of the M31 virial mass is $M_{vir} = 1.14^{+0.51}_{-0.35} \times 10^{12} M_{\odot}$ within $r_{vir} = 220 \pm 25$ kpc.

In page 9 of [19] Zhang.2024.is mentioned that it has been considered H = 70 km/s/Mpc and as virial mass the value M_{200}. Now it will be calculated M_{200} and R_{200} using (4.5) and (3.2) respectively, in the framework of DMbG theory.

Using the parameter $a_{M31} = 4.727513 \cdot 10^{10}$ m $^{5/4}$ / s, (4.7), published in the paper [16] Abarca, M. 2023, and using H = 70 km/s/Mpc it is right to get the value for $M_{200} = 1.42 \cdot 10^{12} M_{\Theta}$ and $R_{200} = 232.14$ kpc.

So reader can check that the value calculated match fully with the data interval marked in the clipped text.
One more time the DMbG theory has overcome this test, this time using M31, the second best well known galactic rotation curve.

4.5 THE VIRIAL MASS OF THE LOCAL GROUP USING THE PARAMETER a^2

With the previous epigraph it has been validated the parameter a^2 as an acceptable way to calculate the virial masses of the galaxies.

As the formula of Direct mass $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ (4.1) depend on the parameter a^2 it is right to deduce that the parameter a^2 associated to a set of galaxies is got adding the parameters a^2 associated to each galaxy.

Table 5 Galaxies	Parameter a^2 Unit m $^{5/2}$ / s^2	Virial mass units M_{Θ}
M31	2,235E+21	$1.4245 \cdot 10^{12}$
MW	1,527E+21	$9.0185 \cdot 10^{11}$
M33	4E+20	$1.8072 \cdot 10^{11}$
LMC	1,1881E+20	$4.2107 \cdot 10^{10}$
Parameter a^2 Local Group	4,28E+21	$3.1064 \cdot 10^{12}$

Now adding the four parameters a^2 associated to MW, M31, LMC and M33 it is possible to postulate that such value will be a good estimation of parameter a^2 associated to Local Group.

It is important to explain that in the framework of DMbG the DM depend on the baryonic matter i.e. the parameter a^2 associated to a dwarf galaxy will be negligible in comparison with the previous four galaxies quoted.

There are around 100 dwarf galaxies into the L.G. but according DMbG its influence in the total mass of L.G. is negligible.

In paper [1] Abarca, M. 2023. It was calculated the parameter a^2 associated to the four ones galaxies before mentioned. See table. So using this formula $M_{VIR} \approx M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$ (4.5) it is calculated each virial mass. With the formula (3.2) using the value of M_{200} got for the Local Group, it is right to calculate $R_{200} = 301$ kpc.

Notice that adding the virial mass of the four galaxies it is got $2.55 \cdot 10^{12}\ M_{\Theta}$, being this value 18% lower regarding the virial mass associated to the one calculated with parameter a^2 of Local Group. This result is because of the factor $a^{12/5}$ in the M_{200} formula, whose exponent is 0.4 bigger than 2.

5. DARK MATTER IS COUNTER BALANCED BY DARK ENERGY AT ZERO GRAVITY RADIUS

This chapter is based on the 17 chapter in [1] Abarca,M.2023

The basic concepts about DE on the current cosmology can be studied in [9] Chernin,A.D.

According [11] Biswajit Deb. Plank satellite data (2018) give a new updated, Hubble constant, H = 67.4 $\pm$ 0.5 km/s/Mpc and a new $\Omega_{DE} =$ 0.6889 $\pm$ 0.0056. However currently there is a tension regarding Hubble constant as there are well known works with others measures for H bigger than 70 Km/s/Mpc. In this paper will be used H =70 Km/s/Mpc and $\Omega_{DE} = 0.7$ as the fraction of Universal density of DE.

5.1 ZERO GRAVITY RADIUS DEPENDING ON PARAMETER a^2 FORMULA

According [9] Chernin,A.D. in the current cosmologic model ΛCDM , dark energy has an effect equivalent to antigravity i.e. the mass associated to dark energy is negative and the dark energy have a constant density for all the Universe equal to $\varphi_{DE} = \varphi_C \bullet \Omega_{DE} = -6.444 \cdot 10^{-27}\, kg/m^3$ being $\Omega_{DE} = 0.7$ and $\rho_C = \dfrac{3H^2}{8\pi G}$ = 9.205E-27 kg/m^3 the critic density of the Universe.

As DE density is constant, the total DE mass is proportional to Radius with power 3, whereas DM mass grows with radius power 0.5 so it is right to get a radius where DM is counter balanced by DE.

According [9] Chernin, A.D.The mass associated to DE is $M_{DE}(< R) = -\dfrac{\rho_{DE}\, 8\pi R^3}{3}$,(5.1) , or equivalently

$M_{DE}(< R) = -\varphi_{DE} \dfrac{8\pi R^3}{3} = \dfrac{-H^2 \cdot \Omega_{DE}}{G} \cdot R^3$ (5.2). Notice that the author multiplies by two the volume of a sphere by reasons explained in his work, that reader can consult.

[9] Chernin defines gravitating mass $M_G(<R) = M_{DE}(<R) + M_{TOTAL}(<R)$ (5.3), where M_{TOTAL} is baryonic plus dark matter mass, and defines R_{ZG} , Radius at zero Gravity as the radius where $M_{DE}(< R_{ZG}) + M_{TOTAL}(< R_{ZG}) = 0$. i.e. where the gravitating mass is zero.

According DMbG theory this definition leads to equation $M_{TOTAL}(< R_{ZG}) = \varphi_{DE} \dfrac{8\pi R_{ZG}^3}{3}$,(5.4).

Using (4.1) formula $M_{TOTAL}(< R_{ZG}) = \dfrac{a^2 \cdot \sqrt{R_{ZG}}}{G}$ the expression (5.4) leads to $\dfrac{a^2 \cdot \sqrt{R_{ZG}}}{G} = \rho_{DE} \cdot \dfrac{8\pi \cdot R_{ZG}^3}{3}$, (5.5),

where it is possible to clear up $R_{ZG} = \left[\dfrac{3a^2}{8\pi G \rho_{DE}} \right]^{2/5}$ (5.6) and as $\varphi_{DE} = \dfrac{3 \cdot H^2}{8\pi G} \Omega_{DE}$ (5.7) then by substitution

$R_{ZG} = \left[\dfrac{a^2}{H^2 \cdot \Omega_{DE}} \right]^{2/5}$ (5.8) This formula will be called R_{ZG} (parameter a^2).

As the radius R_{ZG} is the distance to cluster centre where is zero the gravitating mass, it is right to consider R_{ZG} as the halo radius and its sphere defined as the halo cluster.

For example, in epigraph 4.5 was shown that the parameter $a^2 = 4.28 \cdot 10^{21}$ (I.S. units) is associated to Local Group, adding the four ones associated to MW, M31,LMC and M33, that as it is known they are the most massive galaxies in the Local Group. So using such value, the R_{ZG} for Local Group is 2.19 Mpc

5.2 ZERO GRAVITY RADIUS FORMULA DEPENDING ON VIRIAL MASS

In previous chapter was got the value for $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$,(4.4), depending on M_{VIR} as local parameter solely, so by substitution in R_{ZG} formula it is right to get $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ (5.9) where the only local parameter is M_{VIR} so this formula will be called $R_{ZG}(M_{VIR})$ and $R_{ZG} = K \cdot (M_{VIR})^{1/3}$, where $K = \frac{G^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \cdot \Omega_{DE}^{2/5}} \approx 3.683309948 \cdot 10^8$ (I. S. of units)

Below are calculated R_{ZG} by two ways: R_{ZG}(parameter a^2),(see 5.8) and R_{ZG} (M_{VIR}),(see 5.9) . It is remarkable how both calculus are mathematically equivalents as it was expected, i.e. when it is used the parameter $a^2(M_{VIR})$ to calculate R_{ZG} (parameter a^2) then match mathematically with R_{ZG} (M_{VIR}). See in table below how the grey values match perfectly.

Table 6	Virial mass	Parameter a^2 (M_{VIR})	R_{ZG} (parameter a^2)	R_{ZG} (M_{VIR})	Relative diff.
Clusters	$\cdot 10^{14}$ $M_\odot$	I.S. units $m^{5/2}/s^2$	Mpc	Mpc	%
Virgo	6.3±0.9	3.581E23	12.871	12.871	0
Coma	27	1.2042E24	20.9069	20.9069	0
Local G.	0.031064	4.28E21	2.19	2.19	

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh,2023. With these important cluster of galaxies, it has been illustrated how the total mass, calculated by $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$, is counter balanced by dark energy at mega parsecs scale, and precisely this Radius at zero gravity determines the region size where the cluster has gravitational influence.

5.3 ZERO GRAVITY RADIUS VERSUS VIRIAL RADIUS

From formula (3.2) $R_{VIR}^3 \approx R_{200}^3 = \frac{G \cdot M_{200}}{100 \cdot H^2} \approx \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ it is right to get $R_{VIR} = \left(\frac{G \cdot M_{VIR}}{100 \cdot H^2}\right)^{1/3}$. In previous epigraph was got $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ (see 5.9). So it is right to get the ratio $\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5} \approx 7.277$ (5.10) which is universal as it does not depend on virial mass associated to a specific cluster. This is a very important result because it can provide a simple method to validate the DMbG theory through cosmological measures.

Table 7	Virial Radius	Virial Mass	Zero Gravity R.	Ratio
Celestial Body	Mpc	1E13 $M_\odot$	Mpc	R_{ZG} / R_{VIR}
Antlia cluster	1,28	26,3	9,62E+00	7,52E+00
NGC596/584	0,5	1,55	3,74E+00	7,49E+00
NGC 3268	0,9	8,99	6,73E+00	7,47E+00
NGC 4365	0,32	0,4	2,38E+00	7,45E+00
NGC 4636	0,63	3,02	4,68E+00	7,42E+00
NGC 4697	1,29	26,9	9,69E+00	7,51E+00
NGC 5846	1,1	16,6	8,25E+00	7,50E+00
NGC 6868	0,6	2,69	4,50E+00	7,50E+00

Columns in blue come from [4] R. Ragusa et al.2022

The second and third columns show the virial radius and the virial mass for each cluster.

The column in green shows the R_{ZG} using the formula by the virial mass $R_{ZG} = K \cdot (M_{VIR})^{1/3}$ (see 5.9) where K=3.683309948·10^8 (I.S.)

The column in pink is the ratio of radius.

It is clear that the ratio R_{ZG}/R_{VIR} got in this sample of celestial bodies match very well with the value got by the theory.

Table 8	VirialRadius	Zero Grav R	Ratio
Clusters	Mpc	Mpc	R_{ZG} / R_{VIR}
Virgo C.	1.7	12.871	7.57
Coma C.	2.8	20.9069	7.467

For Virial radius see table 4, for Zero gravity rad. see table 6

The results got for the most prominent clusters match perfectly with the theoretical ratio of radius as well.

5.4 TOTAL MASS ASSOCIATED TO A CLUSTER OF GALAXIES

5.4.1 TOTAL MASS ASSOCIATED TO THE SPHERE WITH ZERO GRAVITY RADIUS

In epigraph 5.1 was shown how this equation $M_{TOTAL}(<R_{ZG}) = \rho_{DE}\cdot\frac{8\pi\cdot R_{ZG}^3}{3}$ (see 5.4) is used to define R_{ZG}. By simplification it is got $M_{TOTAL}(<R_{ZG}) = \frac{H^2\cdot\Omega_{DE}}{G}\cdot R_{ZG}^3 = M\cdot R_{ZG}^3$ (5.11) being M= $5.3984\cdot10^{-26}$ (I.S.) So that is the total mass formula, using the equation between total mass and dark energy at zero gravity radius.

In table below are shown calculus of total mass for Virgo, for Coma cluster and the Local Group of galaxies.

Table 9	Radius ZG	Parameter a^2 (M_{VIR})	$M_{TOTAL}(<R_{ZG})=M\cdot R^3_{ZG}$
Clusters	Mpc	$m^{5/2}/s$ (I.S.)	M_{Θ}
Virgo	12.871	3.581E23	1.699945E15
Coma	20.9069	1.2042E24	7.28353E15
Local group	2.19073	4.28E21	8.379931E12

5.4.2 TOTAL MASS AT ZERO GRAVITY RADIUS USING THE VIRIAL MASS

Using rightly the direct mass formula (see 4.1) at R_{ZG} it is got $M_{TOTAL}(<R_{ZG}) = \frac{a^2\cdot\sqrt{R_{ZG}}}{G}$ (5.12)

As in epigraph 4.2 was got $a^2 = (G\cdot M_{VIR})^{5/6}\cdot(10\cdot H)^{1/3}$ and in epigraph 5.2 was got $R_{ZG} = \frac{(G\cdot M_{VIR})^{1/3}\cdot\sqrt[15]{100}}{H^{2/3}\Omega_{DE}^{2/5}}$

by substitution in $M_{TOTAL}(<R_{ZG}) = \frac{a^2\cdot\sqrt{R_{ZG}}}{G}$ (see 5.12) it is got $M_{TOTAL}(<R_{ZG}) = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}\cdot M_{VIR}$ (5.13) calling $U = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}$ (5.14) then $M_{TOTAL}(<R_{ZG}) = U\cdot M_{VIR}$ (5.15) being U≈ 2.6976.

So may be stated that according Dark matter by gravitation theory, the total mass (baryonic plus DM) enclosed by the sphere with radius R_{ZG} is equivalent to 2.7 times the Virial Mass.

Below are compared the masses for Virgo and Coma clusters and the local Group, using this formula and the previous one by the R_{ZG} to the cubic power. It is clear that both are mathematically equivalents.

Table 10	Virial mass	$M_{TOTAL}(<R_{ZG})$= U·M_{VIR}	$M_{TOTAL}(<R_{ZG})$ =M·R^3_{ZG}	Relatif diff.
Clusters	$\cdot 10^{14}$ M_{Θ}	M_{Θ}	M_{Θ}	%
Virgo	6.3±0.9	1.6994E15	1.6994E15	0
Coma	27	7.2835E15	7.2835E15	0
Local G.	0.031064	8.3799E12	8.3799E12	0

Green data come from [13] Olga Kashibadze.2020 and yellow data come from [7] Seong –A Oh, 2023.

Using the previous calculus, by the direct formula for total mass, may be calculated easily the total mass associated to a radius, which is a fraction of R_{ZG}, with the following property: if $(R = f\cdot R_{ZG})$ then

$M_{TOTAL}(< R) = M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f \cdot R_{ZG}} \cdot \frac{a^2}{G} = \sqrt{f} \cdot U \cdot M_{VIR}$.(5.16)

because $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR}$.(See 5.15)

For example, if R= 0.5·R_{ZG} then $M_{TOTAL}(< 0.5 \cdot R_{ZG}) = \sqrt{0.5} \cdot U \cdot M_{VIR} \approx 0.707 \cdot U \cdot M_{VIR} \approx 1.9 \cdot M_{VIR}$

A second example using the data for Virgo cluster R_{ZG} = 12.87 and R_{VIR} =1.7 Mpc, so R_{VIR} / R_{ZG} = f = 0.13209 and by the formula (5.16) $M_{TOTAL}(< R_{VIR}) = M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR} \approx 0.98 M_{VIR}$.

The answer is not exactly Mvir because Rvir =1.7 Mpc is the experimental data. If Rvir is calculated by the formula $R_{VIR}^3 \approx R_{200}^3 = \frac{G \cdot M_{200}}{100 \cdot H^2}$, using the Mvir as M_{200} then R_{VIR} = 1.7687 Mpc and using this value $M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR} \approx 0.999997 M_{VIR}$

5.4.3 CHECKING THE TOTAL MASS FORMULA INTO THE COMA CLUSTER

According [9] Chernin,A.D. they have estimated $R_{ZG} \approx 20\ Mpc$, and M_M (< R_{ZG}) = 6.2·10^{15} Msun. In Chernin´s paper, the M_M represents baryonic matter and DM, which is called total mass in this paper and it is calculated by the direct mass formula.
In epigraph 5.2 was calculated $R_{ZG} \approx 20.9\ Mpc$. which match perfectly with Chernin data. According the formula got in previous epigraph $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR}$ In paper [9] Chernin,A.D, the authors do not give virial mass, so it will be considerate virial mass given by [7] Seong –A Oh,2023. M_{VIR} = 2.7·10^{15} Msun so multiplying by factor U≈ 2.6976 then $M_{TOTAL}(< R_{ZG}) = U \cdot M_{VIR}$ = 7.29·10^{15} Msun and by the formula (5.16) using f= 20/20.9 = 0.9569378

$M_{TOTAL}(< 20\ Mpc) = M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR}$ it is got that $M_{TOTAL}(< 20 Mpc) = 7.131 \cdot 10^{15} M_{\Theta}$ whose relative difference versus Chernin value is 15%. Clearly with this relative difference may be stated that there is an acceptable match between both results and newly the DMbG theory overcome successfully this test.

5.5 TOTAL DARK ENERGY AT ZERO GRAVITY RADIUS

Using equation (5.2) at R_{ZG} it is got $M_{DE}(< R_{ZG}) = -\frac{\rho_{DE} 8\pi R_{ZG}^3}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3$ (5.17) that it is just the opposite value to $M_{TOTAL}(< R_{ZG}) = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}} \cdot M_{VIR}$ (see 5.13) because the total gravitating mass enclosed into the sphere of zero gravity radius is zero by definition . M_G(<R_{ZG})=M_{TOTAL} (<R_{ZG})+ M_{DE} (<R_{ZG}) = 0 (5.18). See epigraph 5.1

Therefore $M_{DE}(< R_{ZG}) = -\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}} \cdot M_{VIR} = -U \cdot M_{VIR}$ (See 5.15), being U≈ 2.6976 and joining both formulas it is got

$$M_{DE}(< R_{ZG}) = -\frac{\rho_{DE} 8\pi R_{ZG}^3}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -U \cdot M_{VIR} \quad (5.19)$$

This formula may be used to calculate the mass associated to DE if the radius is a fraction of R_{ZG} : if $(R = f \cdot R_{ZG})$ then (See 5.1) $M_{DE}(< R) = -\frac{\rho_{DE} 8\pi R^3}{3} = M_{DE}(< f \cdot R_{ZG}) = \frac{-f^3 \cdot H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -f^3 \cdot U \cdot M_{VIR}$ (5.20)

For example using the Virgo data R_{VIR} = 1.7 Mpc and R_{ZG} =12.871 then f= R_{VIR}/ R_{ZG} = 0.13208 and $M_{DE}(< f \cdot R_{ZG}) = -f^3 \cdot U \cdot M_{VIR} = -0.00621 M_{VIR}$ So at this radius the DE may be considered negligible. However for bigger radius the DE begins to increase and finally at R_{ZG} is able to cancel the total mass (baryonic and DM).

5.6 GRAVITATING MASS FUNCTION

In the epigraph 5.1 was defined the gravitating mass $M_G = M_{DE} + M_{TOTAL}$, where M_{TOTAL} is baryonic plus dark matter mass and M_{DE} is the negative mass associated to DE. As the M_{DE} and M_{TOTAL} depend on the radius, it is got the function of gravitating mass depending on the radius by addition of both types of masses, the M_{DE} as a negative quantity and M_{TOTAL} as a positive quantity.
The best way to calculate the gravitating mass is using the formulas got in epigraph 5.3 and 5.4 where are calculated the both types of masses associated to a radius, which is a fraction of R_{ZG}, i.e. $R = f \cdot R_{ZG}$, these formulas are:

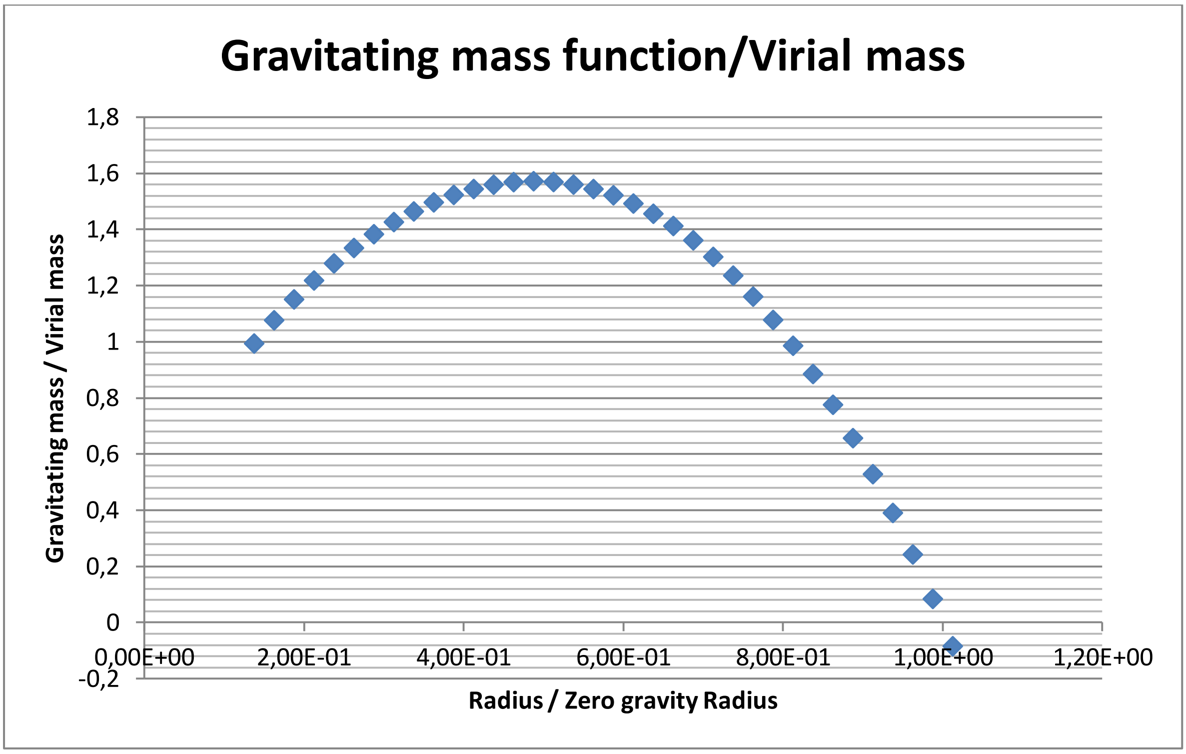
$M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR}$ (See 5.16) and $M_{DE}(< f \cdot R_{ZG}) = \frac{-f^3 \cdot H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -f^3 \cdot U \cdot M_{VIR}$ (See 5.20)

and joining both formulas it is got $M_G(< R) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{VIR}$ (5.21) where $U \approx 2.6976$.
This way the gravitating mass depends on the dimensionless factor f.

DOMINION OF GRAVITATING MASS FUNCTION

Although its mathematical dominion ranges from zero ad infinitum, the real dominion finish at R_{ZG} because at R_{ZG} the gravitating mass is zero so the gravitating mass depending on f finish at 1, and as the gravitating mass is defined into the halo cluster, its dominion begins at R_{VIR} so the gravitating mass depending on f begins at R_{VIR}/R_{ZG}

The formula (5.10) $\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5}$ =7.277 therefore it is concluded that the dominion of the gravitating mass function depending on $f = R/R_{ZG}$ is the interval [0.13732, 1] as the lowest value will be $f = R_{VIR} / R_{ZG}$



Above is represented the ratio gravitating mass/virial mass versus the ratio R/R_{ZG} in its dominion and close to 0.5 there is the function maximum that will be calculated below.

5.7 CALCULUS FOR THE MAXIMUM OF GRAVITATING MASS

Given $M_G(< R) = -\varphi_{DE}\dfrac{8\pi R^3}{3} + \dfrac{a^2\cdot\sqrt{R}}{G}$ It is clear that such function is zero at R=0 and at $R_{ZG} = \left[\dfrac{a^2}{H^2\cdot\Omega_{DE}}\right]^{2/5}$ (See epigraph 5.1) therefore there will be a maximum for the gravitating mass function, that it is found easily by derivation, being $R_M = \frac{1}{\sqrt[5]{36}}\cdot R_{ZG} \approx 0.48836\cdot R_{ZG}$. (5.22) or $f = \frac{1}{\sqrt[5]{36}}$

By substitution of this value $R_M = \frac{1}{\sqrt[5]{36}}\cdot R_{ZG}$ where $f = \frac{1}{\sqrt[5]{36}} \approx 0.48836$ into the mass gravitating formula $M_G(< R_M) = M_G(< f\cdot R_{ZG}) = \left[\sqrt{f} - f^3\right]\cdot U\cdot M_{VIR}$ being $U \approx 2.6976$ it is got

$M_G^{MAXIMUM}(< R_M) = 1.57\cdot M_{VIR}$ (5.23) .

Put in brief may be stated one interesting result, got by the Dark Matter by Gravitation theory:

For any cluster of galaxies at a half of R_{ZG}, it is reached the maximum of gravitating mass which is 1.57 M_{VIR} .

For example: With that formula it is possible to estimate easily the gravitating mass associated to Local Group. Beside table are collected that data shown in tables 5 and 6.

Local Group
$M_{VIR} = 3.1\cdot 10^{12} M_\Theta$
R_{ZG} = 2.19 Mpc

$\left(R_M^{LG} = f\cdot R_{ZG}\right)$ = 0.48836· R_{ZG} = 1.07 Mpc $and\ M_{Gravitating}^{MAXIMUM}\left(< R_M^{LG}\right) = 4.88\cdot 10^{12}\ M_\Theta$

This result is compatible with the one published by [21] Patel &Mandel.2023. They quote at the conclusions that currently the total mass of L.G. ranges between 4 and $4.5\cdot 10^{12}$ M_Θ although they do not mention the radius considered. The mass that they call total mass, is named in this paper gravitating mass because it is the mass calculated by dynamical studies into the L.G.

5.7.1 TOTAL MASS AT RADIUS WHERE GRAVITATING MASS HAS A MAXIMUM

In order to compare the gravitating mass with the total mass, now it is calculated the total mass using the direct mass formula with the property shown in epigraph 5.3. if $(R = f\cdot R_{ZG})$ then $M_{TOTAL}(< R) = \sqrt{f}\cdot U\cdot M_{VIR}$.

Now $R_M = \frac{1}{\sqrt[5]{36}}\cdot R_{ZG}$ So $M_{TOTAL}(< R_M) = \sqrt{36^{-1/5}}\cdot U\cdot M_{VIR} \approx 1.885 M_{VIR}$ (5.24). Comparing this result with the previous one $M_G^{MAXIMUM}(< R_M) \approx 1.571\cdot M_{VIR}$ may be deduced that the DE enclosed into the sphere with radius R_M is $M_{DE}(<R_M)$ = - 0.314 M_{VIR} . (5.25) Now it will be calculated that value with the formula (5.20), using as $f = \frac{1}{\sqrt[5]{36}}$ so $M_{DE}\left(< f\cdot R_{ZG}\right) = -f^3\cdot U\cdot M_{VIR} = -0.314\cdot M_{VIR}$ that match perfectly with result (5.25).

5.8 DENSITY OF THE TOTAL MASS INTO THE HALO CLUSTER THEOREM

Assuming the hypothesis that Rvir = R_{200} and Mvir= M_{200}, i.e. the virial sphere $Density_{VIR} = \frac{\text{Mvir}}{Vol_{VIR}} = 200\rho_C$ being $\rho_C = \dfrac{3H^2}{8\pi G}$ the critical density of Universe.

Then into the halo cluster sphere $Density_{TOTAL\ MASS}(< R_{ZG}) = 1.4\cdot\rho_C$ (5.26). Being R_{ZG} the halo radius.
Proof.

$$Density_{TOTAL\ MASS}^{CLUSTER} = \frac{M_{TOTAL}(< R_{ZG})}{Vol_{CL}(< R_{ZG})} = \frac{M_{VIR}\cdot U}{Vol_{vir}\cdot\left(\frac{R_{ZG}}{R_{VIR}}\right)^3} = \frac{M_{VIR}\cdot U}{Vol_{vir}\cdot U^6} = \frac{200\cdot\rho_C}{U^5} = \frac{200\cdot\rho_C\cdot\Omega_{DE}}{100} = 1.4\cdot\rho_C$$

In that chained equalities has been used two main properties before got: (5.15) and (5.10)

5.8.1 DARK ENERGY DENSITY INTO THE HALO CLUSTER

As into the current ΛCDM model the DE density has a constant value, $\rho_{DE} = 0.7\rho_C$, it is not necessary to demonstrate such property. Nevertheless, in this epigraph it will be checked such value.

So, according [9] Chernin, A.D. The mass associated to DE is $M_{DE}(< R) = -\frac{\rho_{DE} 8\pi R^3}{3}$,(5.1).

By other side, according DMbG theory $M_{DE}(< R_{ZG}) = -\frac{\rho_{DE} 8\pi R_{ZG}^3}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -U \cdot M_{VIR}$ (5.19)

So, at a specific radius such formula may be rewritten $M_{DE}(< R) = -2 \cdot \rho_{DE} \cdot Volume$ (5.27) and by similar reasons that previous epigraph may be got that

$$Density_{DAR\ ENERGY}^{CLUSTER} = \frac{M_{DE}(<R_{ZG})}{2 \cdot Vol_{CL}(<R_{ZG})} = \frac{-U \cdot M_{VIR}}{2 \cdot Vol_{CL}(<R_{ZG})} = -0.7 \cdot \rho_C = \rho_{DE} \quad (5.28)$$

Although ρ_{DE} is negative, in the scientific literature is referred as positive, so usually is written $\rho_{DE} = 0.7 \cdot \rho_C$

COROLARIUS

The ratio density of total mass versus density of DE into the halo cluster is

$$\frac{Density_{TM}^{HALO}(<R_{ZG})}{\rho_{DE}} = 2 \quad (5.29)$$

5.9 EXTENDED HALO WHERE THE RATIO TOTAL MASS VERSUS DARK ENERGY IS EQUAL TO 3 / 7

At the present, it is accepted that $\frac{\Omega_M}{\Omega_{DE}} = 3/7$ as a global average in the current Universe, so it is worth to calculate an extension of halo cluster, R_E , where it is reached such ratio. Below, it will be calculated the radius of a sphere that verify $\frac{Density_{TM}^{SPHERE}(<R_E)}{\rho_{DE}} = 3/7$ (5.30)

$As\ M_{TM}(< R) = Density_{TM}^{SPHERE} \cdot Volume$ and by (5.27) $M_{DE}(< R) = 2 \cdot \rho_{DE} \cdot Volume$ then by substitution into (5.30) it is got $\frac{Density_{TM}^{SPHERE}}{\rho_{DE}} = \frac{2 \cdot M_{TM}(<R)}{M_{DE}(<R)}$ = 3/ 7 (5.31)

Using (5.10) and (5.20) it is right to get the second one equality below.

$$\frac{Density_{TM}^{SPHERE}(<R)}{\rho_{DE}} = 2 \cdot \frac{M_T(<R)}{M_{DE}(<R)} = \frac{2 \cdot \sqrt{f}}{f^3} \quad (5.32)$$

Using the condition (5.30) it is got $\frac{2 \cdot \sqrt{f}}{f^3} = \frac{3}{7}$ whose solution is $f = \left(\frac{14}{3}\right)^{2/5} \approx 1.85$ (5.33)

Therefore the radius of extended halo searched is $R_E \approx 1.85 \cdot R_{ZG}$ (5.34)

In order to validate such calculus is enough to check that the Density of total mass with R_E radius is $0.3 \cdot \rho_C$

$Density_{TOTAL\ MASS}^{EXT.\ HALO}(< R_E) = \frac{M_{VIR} \cdot U \cdot \sqrt{1.85}}{Vol_{vir} \cdot \left(\frac{R_{ZG}}{R_{VIR}}\right)^3 \cdot 1.85^3} = \frac{200 \cdot \rho_C \cdot \Omega_{DE}}{100 \cdot 1.85^{5/2}} = \frac{1.4}{1.85^{5/2}} \cdot \rho_C = 0.3 \cdot \rho_C$ (5.35) as it was expected.

For example for the Virgo cluster $R_E = 1.85 \cdot 12.9$ Mpc= 23.9 Mpc

6. ZERO VELOCITY RADIUS BECAUSE OF THE HUBBLE FLOW

It is defined the zero velocity radius as the distance to the cluster centre, where the escape velocity from gravitation field is equal to Hubble flow velocity. i.e. $V_E = V_{HF}$ (6.1).

From classical dynamic it is taken the formula $\frac{V_E^2}{2} = -V(R)$ (6.2) i.e. the kinetic energy associated to escape velocity compensates the potential energy getting zero as total energy ad infinitum.

It is not possible to use the classical escape velocity formula $V_E^2 = \frac{2GM}{R}$ (6.3) because of two reasons:

1º The gravitational potential in DMbG theory is different to $V = -\frac{GM}{R}$

2º The border of the halo cluster is R_{ZG} where the gravitating mass is zero.

6.1 GRAVITATIONAL POTENTIAL INTO THE HALO CLUSTER

For classical dynamic, the gravitational potential is a line integral from a point up to infinitum, but according DMbG theory the real border of the cluster is R_{ZG} so the result formula is $V = \int_R^{R_{ZG}} \left[\frac{-GM_G(<r)}{r^2}\right] \cdot dr$ (6.4)

The gravitating mass $M_G(< r) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{VIR}$ (5.21) will be used to do the integral, doing some little changes.

As f= r/ R_{ZG} and calling K= U·M $_{VIR}$, by substitution in (5.21) it is got $M_G(< r) = K \cdot \left[\frac{\sqrt{r}}{\sqrt{R_{ZG}}} - \frac{r^3}{R_{ZG}^3}\right]$ (6.5) that by substitution in (6.4) results $V(R) = -GK \int_R^{R_{ZG}} \left[\frac{r^{-3/2}}{\sqrt{R_{ZG}}} - \frac{r}{R_{ZG}^3}\right] \cdot dr$ (6.6) whose result is

$V(r) = \frac{5GK}{2 \cdot R_{ZG}} - GK \cdot \left[\frac{2}{\sqrt{r} \cdot \sqrt{R_{ZG}}} + \frac{r^2}{2 \cdot R_{ZG}^3}\right]$ (6.7) where its dominion ranges from R_{VIR} up to R_{ZG}.

Notice that $V(R_{ZG}) = 0$ and V(r) is negative inside its dominion.

6.2 EQUATION FOR ZERO VELOCITY RADIUS

In this epigraph will be developed the equation $V_E(r) = V_{HF}(r)$ (6.1) to calculate the R_{ZV}

From (6.2) $V_E^2 = -2 \cdot V(R)$ So $-2 \cdot V(R) = V_{HF}^2 = H^2 \cdot R^2$ (6.8) and by substitution of potential formula (6.7) it is got $\frac{-5GK}{R_{ZG}} + 2GK \cdot \left[\frac{2}{\sqrt{r} \cdot \sqrt{R_{ZG}}} + \frac{r^2}{2 \cdot R_{ZG}^3}\right] = H^2 \cdot r^2$ (6.9) reorganising that equation it is got:

$\frac{4GK}{\sqrt{R_{ZG}}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{GK}{R_{ZG}^3} - H^2\right] \cdot r^2 = \frac{5GK}{R_{ZG}}$ (6.10) and multiplying that equation by this factor $\frac{R_{ZG}}{GK}$ it is got a better expression $4\sqrt{R_{ZG}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{1}{R_{ZG}^2} - \frac{H^2 \cdot R_{ZG}}{GK}\right] \cdot r^2 = 5$ (6.11) which is the equation for zero velocity radius.

That equation is not possible to solve with algebraic methods, but is quite easy to solve numerically for specific data.

6.3 ZERO VELOCITY RADIUS FOR VIRGO

Table 11	Virial Radius	Virial mass	R_{ZG}
Cluster	Mpc	$\cdot 10^{14}\ M_\Theta$	
Virgo	1.7	6.3±0.9	12.87 Mpc

Using the data M_{VIR} and R_{VIR} for Virgo cluster from [13] Olga Kashibadze.2020, it is right to calculate the coefficients of that equation.

$4\sqrt{R_{ZG}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{1}{R_{ZG}^2} - \frac{H^2 \cdot R_{ZG}}{GK}\right] \cdot r^2 = 5$ (6.11) As U≈2.7 and K= U·M_{VIR} then GK= 2.259E+35 (I.S. units) ,

$\frac{H^2 \cdot R_{ZG}}{GK}$ = 9.047E-48 m^{-2} , $\frac{1}{R_{ZG}^2}$ = 6.3397E-48 m^{-2} and $4\sqrt{R_{ZG}}$ = 2.5208E+12 $m^{1/2}$

The equation (6.11) is written as S1+S2 = 5 in order to solve numerically. In table below has been tabulated both terms S1 and S2. Using f = Radius / R_{ZG} as dimensionless parameter it is got that f = 0.602 is a very good approximation for the solution. Therefore the R_{ZV} = f· R_{ZG} = 7.75 Mpc

Table 12 f	Radius	S1	S2	S1+S2
dimensionless	m	dimensionless	dimensionless	dimensionless
5,990E-01	2,379E+23	5,168E+00	-1,532E-01	5,015E+00
6,000E-01	2,383E+23	5,164E+00	-1,538E-01	5,010E+00
6,010E-01	2,387E+23	5,160E+00	-1,543E-01	5,005E+00
6,020E-01	2,391E+23	5,155E+00	-1,548E-01	5,001E+00
6,030E-01	2,395E+23	5,151E+00	-1,553E-01	4,996E+00
6,040E-01	2,399E+23	5,147E+00	-1,558E-01	4,991E+00
6,050E-01	2,403E+23	5,143E+00	-1,563E-01	4,986E+00

Table 13	R_{ZG}	R_{ZV}	V_{HF}
Cluster	Mpc	Mpc	Km/s
Virgo	12.87	7.75	542.5

6.4 ZERO VELOCITY RADIUS THEOREM

Thesis: According DMbG theory the ratio R_{ZV} / R_{ZG} is universal and its value is R_{ZV} / R_{ZG} ≈ 0.602

In epigraph 6.2 was got the equation for R_{ZV} , $4\sqrt{R_{ZG}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{1}{R_{ZG}^2} - \frac{H^2 \cdot R_{ZG}}{GK}\right] \cdot r^2 = 5$ (6.11) where K= U·M_{VIR} being $U = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}$ (5.14) and $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ (5.9)

Using the values for K, U and R_{ZG} , doing some algebraic substitutions and transformations it is not difficult to get the following equation. $4\sqrt{R_{ZG}} \cdot \frac{1}{\sqrt{r}} + \frac{1}{R_{ZG}^2}\left[\frac{\Omega_{DE} - 1}{\Omega_{DE}}\right] \cdot r^2 = 5$ (6.12) Defining f= r/ R_{ZG} the equation (6.12) becomes

$4 \cdot \frac{1}{\sqrt{f}} + \left[\frac{\Omega_{DE} - 1}{\Omega_{DE}}\right] \cdot f^2 = 5$ (6.13) and considering $\Omega_{DE} = 0.7$ becomes:

$4 \cdot \frac{1}{\sqrt{f}} + \left[\frac{-3}{7}\right] \cdot f^2 = 5$ (6.14) By elementary algebraic operations it is got this equivalent equation:

$9 \cdot f^5 + 210 \cdot f^3 + 1225 \cdot f - 784 = 0$
Thanks Wolfram alpha software, this is its only real solution f= 0.602016

By this new formulation of the equation for R_{ZV} calculus, it has been demonstrated that the ratio f = R_{ZV} / R_{ZG} is universal and depends on Ω_{DE} solely.

6.5 GRAVITATING MASS AT ZERO VELOCITY RADIUS

Using a procedure similar used to calculate the maximum of the gravitating mass, see epigraph 5.7, being $R_{ZV} = f \cdot R_{ZG}$ where $f \approx 0.602$ by substitution at formula $M_G(< R_{ZV}) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{VIR}$ (5.21) where $U = \frac{100^{1/5}}{\Omega_{DE}^{1/5}} \approx 2.6976$ it is right to get that $M_G(< R_{ZV}) \approx 1.5 \cdot M_{VIR}$ (6.12)

Notice that $M_G^{MAXIMUM}(< R_M) \approx 1.57 \cdot M_{VIR}$ (5.23) where $R_M \approx 0.488 \cdot R_{ZG}$ and $M_G(< R_{ZV}) \approx 1.5 M_{VIR}$ where $R_{ZV} \approx 0.602 \cdot R_{ZG}$. Remember the epigraph 5.6 where is shown that gravitating function is decreasing in the interval $[R_M, R_{ZG}]$.

7. VALIDATION OF THE THEORY WITH RESULTS PUBLISHED ABOUT VIRGO CLUSTER

In this chapter some result got by DMbG theory will be validated with three result published about the Virgo cluster, that it is the closest between the big clusters, so it is the perfect benchmark to test the theory.

The first test is relative to the R_{ZV} and its associated mass. These results were studied in chapter 6.

The second test is relative to the gravitating mass associated to the twice of virial radius.

In the third test, the most important, it is postulated that DMbG theory is able to multiply by the factor $U \cdot \sqrt{1.85} = 2.7 \cdot 1.36 = 3.67$ the current parameter of local matter density $\Omega_m^{local} = 0.08$ reaching 0.294 which match with the value $\Omega_m^{Global} = 0.3$ accepted currently by the scientific community.

7.1 GRAVITATING MASS ASSOCIATED UP TO THE ESTIMATED ZERO VELOCITY RADIUS AT 7.3 Mpc

Clipped text of [13] O. Kashibadze.2020

7. Concluding remarks

The analysis of galaxy motions in the outskirts of the Virgo cluster makes it possible to measure the radius of the zero-velocity surface, $R_0 = 7.0 - 7.3$ Mpc (Karachentsev et al. 2014, Shaya et al. 2017, Kashibadze et al. 2018), corresponding to the total mass of the Virgo cluster $M_T = (7.4 \pm 0.9) \times 10^{14} M_\odot$ inside the R_0. The numerical simulated trajectories of nearby galaxies with accurate distance estimates performed by Shaya et al. (2017) confirmed the obtained estimate of the total mass of the cluster. The virial mass of the cluster, being determined independently at the scale of $R_g = 1.7$ Mpc from the internal motions, is nearly the same - $M_{VIR} = (6.3 \pm 0.9) \times 10^{14} M_\odot$. The agreement of

In this epigraph are compared the result published in [13] O. Kashibadze.2020 with the calculus got in chapter 6 about the zero velocity radius and its gravitating mass associated.

The matching is very good especially for the zero velocity radius.

At the concluding remarks [13] O. Kashibadze, the authors gives the interval for [7, 7.3] Mpc for R_{ZV}.

According DMbG theory, the zero velocity radius R_{ZV} =7.75 Mpc, see epigraph 6.3, so the relative difference is only 6% if it is considered the upper value of the interval[7, 7.3] . This is a very good match between experimental results and theoretical results by DMbG theory.

At the concluding remarks [13] O. Kashibadze, the authors give for the total mass $M_T (< R_0) = (7.4 \pm 0.9) \cdot 10^{14} M_\Theta$. As this value is got by dynamical measures in fact this value must be considered as gravitational mass.
The theoretical value (6.12) calculated in epigraph 6.4 is $M_G (<R_{ZV}) \approx 1.5 \cdot M_{VIR}$= 9.48E14 M_Θ which is 14 % bigger regarding the value given by the authors, if it is considered the upper value of the interval. So both results may be considered compatibles.

In table 15 are summarized and compared the observational results and theoretical results.

Table 15 Virgo cluster	[13] O. Kashibadze	DMbG theory	Relative difference
R_{ZV}	[7, 7.3] Mpc	7.75 Mpc	6% - Very good
M_{VIR}	$(6.3 \pm 0.9)\cdot 10^{14}\ M_{\Theta}$		
$M_G(< R_{ZV})$	$(7.4 \pm 0.9)\cdot 10^{14}\ M_{\Theta}$	$1.5\cdot M_{VIR}=9.45\cdot 10^{14} M_{\Theta}$	14% - Compatibles

7.2 GRAVITATING MASS ASSOCIATED UP TO THE TWICE OF VIRIAL RADIUS

As R_{VIR} = 1.7 Mpc its twice value is 3.4 Mpc. As R_{ZG} = 12.9 Mpc then f =3.4/12.9= 0.2635 and

$M_G(< 2 \cdot R_{VIR}) = \mathbf{1.335} \cdot M_{VIR}$= $8.4\cdot 10^{14}\ M_{\Theta}$. This value match perfectly with the interval of masses given below in the clipped text.

As mentioned above, the Planck Collaboration (2016) performed a detailed study of the Virgo cluster through Sunyaev-Zeldovich effect and found the total mass of warm/hot gas to be $(1.4 - 1.6) \times 10^{14} M_{\odot}$. Assuming the cosmic value for the baryon fraction, $f_b = \Omega_b/\Omega_m = 0.1834$, they found that the total mass of the cluster would be $(7.6 - 8.7) \times 10^{14} M_{\odot}$ on a scale up to 2 times larger than the virial radius.	Clipped text from page 9 of [13] O. Kashibadze.2020

The total mass mentioned in Kashibadze paper it is considered in this paper as gravitating mass because in the whole paper of Kashibadze et al. they use always the concept of total mass as a result of dynamical measures so it is more suitable to interpret his total mass as gravitating mass.
Anyway, considering the total mass given by the formula (5.16) $M_{\boldsymbol{TOTAL}}(< 2R_{VIR}) = \mathbf{1.386} \cdot M_{VIR}$= $8.73\cdot 10^{14}\ M_{\Theta}$ that match with the upper value of mass range given by the authors.

7.3 SOLVING THE CONUNDRUM: LOCAL DENSITY MATTER VERSUS GLOBAL DENSITY MATTER

Below is the clipped text of a paper published for a team of well known astrophysicist.

As it has been noted by different authors (Vennik 1984, Tully 1987, Crook et al. 2007, Makarov & Karachentsev 2011, Karachentsev 2012), the total virial masses of nearby groups and clusters leads to a mean local density of matter of $\Omega_m \simeq 0.08$, that is 1/3 the mean global density $\Omega_m = 0.24 \pm 0.03$ (Spergel et al. 2007). One possible explanation of the disparity between the local and global density estimates may be that the outskirts of groups and clusters contain significant amounts of dark matter beyond their virial radii, beyond what is anticipated from the integrated light of galaxies within the infall domain. If so, to get agreement between local and global values of Ω_m, the total mass of the Virgo cluster (and other clusters) must be 3 times their virial masses. A measure of this missing
Clipped text from introduction of paper [12] Karachentsev I.D.,R. Brent Tully, et al. 2014

In page 3 of that paper, they state that at the nearby clusters the mean local density of matter is $\Omega_m^{local} = 0.08$, whereas the global mass density in the Universe is $\Omega_m^{Global} = 0.24$. (data year 2007)
Currently the data updated for scientific community is $\Omega_m^{Global} = 0.3$
The authors suggest that a possible solution for this tension would be that the total mass for cluster haloes must be three times the virial mass. That is justly what is found in this paper studying the DM at cluster scale as universal law in the framework of DMbG theory.

In chapter 5, the formula (5.13) $M_{TOTAL}(< R_{ZG}) = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}} \cdot M_{VIR} = U \cdot M_{VIR}$, shows that the total mass (baryonic and DM) enclosed into the halo cluster is U ≈ 2.7 times the virial mass equal to $M_{TOTAL}(<R_{VIR}) \approx M_{200}$

However in the epigraph 5.9 has been calculated an extension of the halo cluster up to radius $R_E = 1.85 \cdot R_{ZG}$ to obtain a ratio $\frac{\Omega_{TM}^{LOCAL}}{\Omega_{DE}^{LOCAL}} = \frac{\Omega_{TM}^{GLOBAL}}{\Omega_{DE}^{GLOBAL}} = 3/7$. Now using the formula (5.16) it is right to calculate the total mass enclosed by such sphere $M_{TOTAL}(< R_E) = M_{TOTAL}(< f \cdot R_{ZG}) = \sqrt{f} \cdot U \cdot M_{VIR}$ being f= 1.85 the factor $\sqrt{f} \cdot U = 3.67$
then $M_{TOTAL}(< R_E) = 3.67 \cdot M_{VIR}$ (7.1) .

Therefore with such factor the parameter $\Omega_m^{local} = 0.08$ is increased up to a $\Omega_m^{local} = 0.08 \cdot 3.67 = 0.2936$ (7.2) because according [12] Karachentsev et al. 2014, the coefficient $\Omega_m^{local} = 0.08$ is calculated considering the virial masses of clusters into the Local Universe, and as according DMbG such total mass is increased by the factor 3.67 if it is considered an extended halo with radius $R_E = 1.85 \cdot R_{ZG}$ then the coefficient $\Omega_m^{local} = 0.08 \cdot 3.67 = 0.293$ match perfectly with $\Omega_m^{global} = 0.3$

This result enables an experimental test to validate these theoretical findings: If at the present Universe, the average distance between clusters is about its R_E associated to each one, then the DMbG theory would explain the current $\Omega_m^{GLOBAL} = 0.3$

In other words, considering that almost the total baryonic matter at the current Universe is enclosed inside the virial radius of the clusters, as it is confirmed by multiples measures, the DMbG is able to justify that $\Omega_m^{LOCAL} = \Omega_m^{GLOBAL}$ on condition that the average distance between clusters is the extended radius R_E

8. SUMMARY

Thanks to the approximation of virial radius and virial mass for R_{200} and M_{200} it has been possible to get some impressive general results for clusters in the framework of Dark Matter by Gravitation theory.
This is the summary for the main formulas got in the paper.

In chapter 3 using the formula of Universal critic density it got this relation $M_{200} = \frac{100 H^2 R_{200}^3}{G}$ (3.3) and using the approximation of virial radius and virial mass by R_{200} and M_{200} it is stated $M_{VIR} \approx M_{200} = \frac{100 H^2 R_{200}^3}{G}$ (3.3) This approximation has been checked into a sample of clusters.

In (4.1) is introduced the direct mass formula as an important formula got in previous papers, originally for galactic haloes and extended to galactic clusters.

In 4.2 using formula (3.3) into the direct mass formula $M_{DIRECT}(< r) = \frac{a^2 \cdot \sqrt{r}}{G}$ (4.1) allows getting the parameter a^2

$a^2 = (G \cdot M_{200})^{5/6} \cdot (10 \cdot H)^{1/3}$ (4.4) or conversely the virial mass depending on a^2

$M_{200} = \frac{a^{12/5} \cdot (10 \cdot H)^{-6/15}}{G}$ (4.5)

The formula (4.4) deeply related with direct mass formula (4.1) is on the base of all the important formulas in this paper.

In 4.3 and 4.4 is compared the value M_{200} calculated by the formula (4.5), with the value one published recently about M_{200} in MW and M31. This comparison has been a successful test for the DMbG theory.

In 5.1 and 5.2 is defined R_{ZG} as the radius where the total mass is counterbalanced by the negative mass associated to DE, and it is got a formula for Radius of Zero gravitating mass, being $R_{ZG} = \frac{(G \cdot M_{VIR})^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ (5.9)

In 5.3 it is found a remarkable formula for this ratio $\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5}$ ≈7.277 (5.10) being universal as it does not depend on the virial mass associated to a specific cluster.

In 5.4.1 it is got the formula for total mass $M_{TOTAL}(< R_{ZG}) = \frac{H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = M \cdot R_{ZG}^3$ (5.11) being M= $5.3984 \cdot 10^{-26}$

In 5.4.2 is got the total mass associated to R_{ZG} as $M_{TOTAL}(< R_{ZG}) = \frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}} \cdot M_{VIR} = U \cdot M_{VIR}$ (5.13) Being U≈ 2.7

In 5.5 it is got the mass associated to DE at zero gravitating mass $M_{DE}(< R_{ZG}) = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = -U \cdot M_{VIR}$ (5.19)

In 5.6 is introduced the mass gravitating function depending on dimensionless parameter $\boldsymbol{f}$ = Radius / R_{ZG} ,
being $M_G(< R) = M_G(< \boldsymbol{f} \cdot R_{ZG}) = \left[\sqrt{\boldsymbol{f}} - \boldsymbol{f}^3\right] \cdot U \cdot M_{VIR}$ (5.21) and it is plotted on a graph the ratio M_G / M_{VIR} versus the parameter $\boldsymbol{f}$.
In 5.7 is got the radius where the gravitating mass has a maximum, being $R_M = \frac{1}{\sqrt[5]{36}} \cdot R_{ZG} \approx 0.48836 \cdot R_{ZG}$ (5.22)

And it is got the gravitating mass associated to R_M, i.e. $M_G^{MAXIMUM}(< R_M) \approx 1.57 \cdot M_{VIR}$ (5.23)

In 5.8 it is demonstrated that $Density_{TOTAL\ MASS}(< R_{ZG}) = 1.4 \cdot \rho_C$ (5.26) being twice that $\rho_{DE} = 0.7 \cdot \rho_C$

In 5.9 it is calculated an extended radius to the cluster halo R_E ≈$1.85 \cdot R_{ZG}$ (5.34) verifying that ratio

$\frac{Density_{TM}^{SPHERE}(<R_E)}{\rho_{DE}} = 3/7$ (5.30) i.e. the local ratio of such densities is equal to the global ratio ones and consequently $Density_{TOTAL\ MASS}^{EXT.\ HALO}(< R_E) = 0.3 \cdot \rho_C$ (5.35)

In chapter 6 is defined the gravitational potential into the halo cluster as $V = \int_R^{R_{ZG}} \left[\frac{-GM_G(<r)}{r^2}\right] \cdot dr$ (6.4) the solution of this line integral is $V(r) = \frac{5GK}{2 \cdot R_{ZG}} - GK \cdot \left[\frac{2}{\sqrt{r} \cdot \sqrt{R_{ZG}}} + \frac{r^2}{2 \cdot R_{ZG}^3}\right]$ (6.7)

It is got the equation for zero velocity radius depending on specific parameters of a cluster, R_{ZG} and $K = U \cdot M_{VIR}$

$4\sqrt{R_{ZG}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{1}{R_{ZG}^2} - \frac{H^2 \cdot R_{ZG}}{GK}\right] \cdot r^2 = 5$ (6.11) and it is solved for Virgo cluster being R_{ZV} = 7.75 Mpc

It is demonstrated *the zero velocity radius theorem* stating that the ratio f = $R_{ZV} / R_{ZG} \approx 0.602$ is universal.

and it is got his associated gravitating mass $M_G(< R_{ZV}) \approx 1.5 \cdot M_{VIR}$ (6.15)

In chapter 7 are validated some results got by the DMbG theory with some data published recently from Virgo cluster, see [13] O. Kashibadze, 2020.Namely the validated results are:

The relative difference between R_{ZV} estimated by measures and R_{ZV} calculated in this work is 6% and the gravitating mass calculated at R_{ZV} is only 17% bigger versus the data published at the same radius.

Also there is a very good matching between calculus and data published for mass calculated at $2 \cdot R_{VIR}$ = 3.4 Mpc being

$M_G(< 2 \cdot R_{VIR}) = 1.335 \cdot M_{VIR}$

In addition, because of the formula of total mass enclosed into the extended halo $M_{TOTAL}(< R_E) = 3.67 \cdot M_{VIR}$ (7.1) is postulated that the DMbG theory is able to solve the current discrepancy between the density matter coefficient $\Omega_m^{local} = 0.08$, for the Local Universe, and the global density matter coefficient for the whole Universe $\Omega_m^{global} = 0.3$

9. CONCLUDING REMARKS

Thanks direct mass $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ (4.1) and using the approximation of virial mass for M_{200} and the virial radius for R_{200} it is possible to get the formula $a^2 = (G \cdot M_{VIR})^{5/6} \cdot (10 \cdot H)^{1/3}$ (4.4) , which is on the base of all the important formulas got in this paper.

Direct mass formula (4.1) is one of the most important results of DMbG and it was developed studying rotation curves of galaxies.

In chapter 4, the direct mass has been extended to clusters, and it is possible to state that all the new theoretical results obtained in this paper are based on this formula, co working with the well known properties of DE.

- A) The universality of ratio $R_{ZG} / R_{VIR} \approx 7.3$ and its total mass associated, being $M_{TOTAL}(<R_{ZG}) \approx 2.7 \cdot M_{VIR}$
- B) For any cluster at $0.488 \cdot R_{ZG}$ is reached the maximum of gravitating mass and $M_G(<R_M) \approx 1.57 \cdot M_{VIR}$
- C) The universality of the ratio $R_{ZV} / R_{ZG} \approx 0.602$, and its gravitating mass $M_G(<R_{ZV}) \approx 1.5\ M_{VIR}$
- D) The universality of the ratio $R_E / R_{ZG} \approx 1.85$, where the ratio total mass density versus DE density is 3/7 and its total mass $M_{TOTAL}(< R_E) = 3.67 \cdot M_{VIR}$

Finally in chapter 7, are introduced some results published about Virgo cluster that back fully the previous findings:

- 1º Regarding the property D) may be consulted [12] Karachentsev I.D.,R. Brent Tully.2014. to understand more in deep the current tension between the low value for Local mass density parameter $\Omega_m = 0.08$ and the current global matter density parameter $\Omega_m = 0.3$.
- 2º Calculus of the zero velocity radius and its associated gravitating mass of Virgo are compatibles with the result published by [13] Olga Kashibadze et al. 2020, see epigraph 7.1
- 3º Calculus of the mass gravitating at two times the virial radius match fully with the result published by [13] Olga Kashibadze et al. 2020, see epigraph 7.2

It is important to remark that throughout the paper there are a number of papers quoted to test some formulas in MW, M31 galaxies or even the Local Group of galaxies and Virgo or Coma cluster that are able to confirm some theoretical result got thanks DMbG theory and published by first time in this paper.

By other side, these new theoretical findings offer to scientific community a wide number of tests to validate or reject the theory. The validation of DMbG theory would mean to understand that DM is a quantum gravitation effect, giving to scientific community new elements to continue searching a quantum gravitation theory.

10. BIBLIOGRAPHYC REFERENCES

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