Electrogravity: On a scalar field of time and electrogmagnetism

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Abstract

It is possible to describe a universal scalar field of time but not a universal coordinate of time and to attribute its non-geodesic alignment to the electromagnetic phenomena. A very surprising outcome is that not only mass generates gravity, but also electric charge does. Charge is, however, coupled to a non-geodesic vector field and thus is not totally equivalent to inertial mass. Only the entire "Energy-Momentum" tensor has a vanishing divergence. The model can be seen as misalignment of physically accessible events in an observer spacetime and of gravity as a controlling response by volumetric contraction of the observer spacetime in the direction where events bend or accelerate to. This non geodesic acceleration is described by a generalization of the Reeb class vector. Misalignment of events can be described by 1, 2, and 3 such vectors. The paper presents a term with 4 vectors but does not discuss its physical meaning. The paper also discusses particle mass ratios and the Fine Structure Constant where added or subtracted area in relation to a disk does not involve a ratio $\frac{1}{24}$ but $\frac{1}{96}$ due to the physical meaning of the orientation of a space foliation which is perpendicular to a time-like vector α and due to the orientation of a plane which is perpendicular to a time-like vector α and its Reeb class vector η where α is mapped to a 1-Form, $d\alpha = \pm\eta\hat{}\alpha$. This forgotten definition of the Reeb class vector η is not limited to contact manifolds. These two orientations mean that only one side of a 3-dimensional foliation has a physical meaning and only one side of a sub-plane of that foliation has a physical meaning then $\frac{1}{2}\frac{1}{2}\frac{1}{24} = \frac{1}{96}$. Another interpretation of the factor $\frac{1}{4}$ is the Bekenstein - Hawking entropy to area constant. An additional coefficient $\frac{4}{\pi}$ describes an acceleration field strength and has a compelling source in mainstream physics. Other two field strength coefficients have compelling explanations, these are $\frac{95}{96}$ and a critical value due to an imbalance equation between gravity and anti-gravity ~1.556198537190348396563877031439915299.

Keywords: General Relativity, Time, Electromagnetism.

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1. Introduction – measurement of non - geodesic deviation

The Result of the Geroch Splitting Theorem [1] is that a field of time can be defined. In simple geometries such as FRWL, which are Big Bang geometries, such time also has an intuitive meaning; it is a scalar field and not a coordinate of time. It is the maximal time between each event of space-time and the Big Bang as a limit, measured by a physical clock that may experience forces. Such proper time can be measured along different curves and is therefore not traceable, not geodesic under forces and cannot be a coordinate that also requires a 4-direction. The existence of a non – traceable time is not a new idea and was postulated by the philosopher R. Joseph Albo [2] in the 14^{th} century. The approach that will be presented to make peace between General Relativity and Quantum Mechanics is not to describe Space-Time as emergent out of huge matrices and to preserve the particles approach [3], but to replace particles with events. In non-hyperbolic spacetime, a scalar field can still be defined as universal clock but will no longer be an upper limit of measurable time to an event from a Cauchy surface as an interpretation to [1].

What information can a scalar field encode, that is not already predicted by the metric tensor of space time $g_{\mu\nu}$? The answer is non - geodesic motion. The motion equations of the theory of General Relativity predict only geodesic motion. This theory is based on two assumptions,

1) The basic assumption is that matter can be described via acceleration in the gradients of scalar fields, more specifically, the electromagnetic phenomena can be described by a non-zero acceleration of the gradient of a Geroch function [1] P^2 in hyperbolic space-time or PP* if P is complex. This acceleration is known as a Reeb class vector field [4] in odd dimensions but can also be defined in 4 dimensions via a 1-Form α, $d\alpha = \eta\wedge\alpha$ where η is the Reeb class vector.

Important: In odd dimensions, the Reeb class field can be defined in a way that it is not the acceleration of at least one unit vector field [5]. In two dimensions, the generalized Reeb class vector is not geodesic. That is an important difference that has been missed all these years. $d\alpha = \eta{}^\wedge\alpha$ is the forgotten definition of a Reeb class vector which is used in the definition of the Reeb class [6] and which is not limited to contact manifolds but is also defined on Symplectic manifolds.

Important: Another problem with most papers on Reeb class vectors is that they ignore divergence points.
Actions are defined for 1 Reeb class field, "electromagnetic", 2 Reeb class fields "electro-weak", 3 Reeb class fields, "Strong" and 4 Reeb class fields as a "Fifth Force" or massive gravity. A definition can be made also for 4 Reeb class fields but its physical meaning is not discussed in this paper. See appendix C, (65). The motivation to use Reeb class vector fields, including a complex formalism, can be seen in the paper by Yaakov Friedman [7]. To complete assumption 1, energy density is $\frac{a_\mu a^\mu}{8\pi K}$ where K is Newton's constant of gravity and a^μ describes an acceleration of a normalized vector $X = c\frac{p_\mu}{\sqrt{P_\lambda P^\lambda}}$ where $p_\mu = \frac{dp}{dx^\mu}$ where p is a scalar field, x^μ are the coordinates of the spacetime manifold and c is the speed of light. In simple words, what is claimed in this paper is that starting from the field X, which is derived from a Geroch function, a physical test clock which moves along X will continue to move along X also when X is not geodesic. That is to say that a_μ is a field which prohibits geodesic motion. The paper will show a way to define such a field regardless of the direction of motion of the test clock in the field. X and a_μ span only one two-dimensional hyperplane of spacetime. The field must be defined in 4 dimensions. If such a field is the reason for the energy of the electric field, then the components of a_μ must be very small, otherwise acceleration of neutral particles in a strong electromagnetic field would be easily noticeable.

2) The scalar fields quantization is $P = \sum_{k=1}^{\infty} P(k)$ such that $\int_\Omega \frac{P(k)P^*(j)+P(j)P^*(k)}{2}\sqrt{-g}d\Omega = 0$ if $k \neq j$ and $\int_\Omega \frac{P(k)P^*(j)+P(j)P^*(k)}{2}\sqrt{-g}d\Omega = 1$ if $k = j$ where $\sqrt{-g}$ is the volume element of space-time, where g is the determinant of the metric tensor. In other words, instead of a Geroch function, PP^* can be replaced by a scalar PP^* that integrates to 1 on reference spacetime manifold and the Lagrangians of the theory will be defined almost-everywhere in terms of measure theory.

Note: The mathematical foundation of this paper is the Geroch function [1], [2], Reeb class vector fields [4] for encoding trajectory curvature, symplectic geometry directly on spacetime and not on any phase space due to [7], and the idea of physically accessible events in an embedding spacetime, an idea very similar to Hartland Snyder's quantized spacetime [8] but without any assumed non-commutative relation. The Lagrangians of this paper are based only on acceleration vectors of normalized gradients of scalar fields.

Challenges to the reader: The challenges to the reader are to understand Reeb class vectors in their original formalism with the meaning of non-geodesic acceleration, which unlike the usual Reeb vector, is not limited to contact manifolds but describes how much a gradient of a scalar field is not geodesic, to understand how two scalar fields and two Reeb class vectors describe a Scarr – Friedman acceleration matrix as a field and not as a uniform acceleration as originally proposed in their paper, with the differences from the Scarr-Friedman matrix which are +1,-1 handedness of a second acceleration plane, the ability to describe spin through y, z rotation when the acceleration is in the t, x axes plane and the ability to describe zero charge when one divergence of the acceleration in the complex plane is positive and the other is negative so that adding conjugates of divergences nullifies. These are not trivial properties and they do not exist in the Scarr Friedman matrix. Another challenge is to understand how such an acceleration matrix can serve as a Symplectic form that acts directly on spacetime and not on any phase space as is the usual case in mainstream physics. Another challenge, which is somewhat a quantum leap, is to understand the use of non-geodesic geometry of foliations of spacetime and its meaning as matter. The Scarr-Friedman formalism will be discussed shortly in this paper and is essential to the understanding of this paper. This paper does not, however, take the path of Tzvi-Scarr and Yaakov Friedman because the acceleration matrix which is used in this paper has different properties and a different goal. The paper purports to reach a description of a field that rotates the gradients of scalar fields in order to be able to describe spin for example. Most theories in mainstream physics deal with geodesic curves and not with accelerated curves, unlike this paper which speaks of both. Another challenge is to accept that lack of collaboration in solving the field equations of this paper (4), (64) requires educated guess of field strength coefficients for Leptons. It is responsible to say that $\frac{95}{96}$ for the electron is better understood than before and that the Tau field strength coefficient is better understood too though more research and collaboration would greatly benefit the paper. The muon field strength coefficient $\frac{4}{\pi}$ is, however, from a critical field value of Quantum Mechanics and not directly from the presented theory.

We can describe non geodesic integral curves along a field $P_\mu \equiv \frac{dP}{dx^\mu}$ for the coordinates x^μ, also, P_μ need not be time-like in all events of space-time. We now define the square norm for real numbers as $Z \equiv |P_\lambda P^\lambda|$ and its gradient $Z_\mu \equiv \frac{dZ}{dx^\mu}$. We define a geometric object $\frac{U_\mu}{2}$ that will measure how much the field P_μ is not geodesic.

When $c\tau$ describes the evolution of the vector $X = c\frac{p_\mu}{\sqrt{P_\lambda P^\lambda}}$ along the integral curves which are formed by the field X, $\frac{dX}{d\tau}$ must be perpendicular to X because $X_\mu X^\mu = c^2$ and then $\frac{d(X_\mu X^\mu)}{cd\tau} =$

$\frac{1}{c}(\dot{X}_\mu X^\mu + X_\mu \dot{X}^\mu) = 0$ which implies $X^\mu \dot{X}_\mu = 0$ since $d\tau$ is a scalar. Now writing $Z = P_\lambda P^\lambda$ we have

$$\frac{d}{d\tau}\frac{p_\mu}{\sqrt{P_\lambda P^\lambda}} = \frac{d}{d\tau}\frac{p_\mu}{\sqrt{Z}} = \frac{\dot{p}_\mu}{\sqrt{Z}} - \frac{p_\mu \dot{Z}}{2z^{\frac{3}{2}}} = \frac{P_{\mu};_\nu}{\sqrt{Z}}\frac{dx^\nu}{d\tau} - \frac{p_\mu Z;_\nu}{2z^{\frac{3}{2}}}\frac{dx^\nu}{d\tau} = \frac{P_{\mu};_\nu}{\sqrt{Z}}\frac{p^\nu}{\sqrt{Z}} - \frac{p_\mu Z;_\nu}{2z^{\frac{3}{2}}}\frac{p^\nu}{\sqrt{Z}} = \tag{1}$$

$$= \frac{P_\mu;_\nu\, p^\nu}{Z} - \frac{p_\mu Z;_\nu\, p^\nu}{2z^2} = \frac{P_\nu;_\mu\, p^\nu}{Z} - \frac{p_\mu Z;_\nu\, p^\nu}{2z^2} = \frac{Z_\mu}{2Z} - \frac{Z_\nu p^\nu p_\mu}{2z^2}$$

Important: Understanding (1) is all which is needed to understand this paper. There could be one or more such fields as $\frac{U_\mu}{2} = \frac{Z_\mu}{2Z} - \frac{Z_\nu p^\nu p_\mu}{2z^2}$ and there is also a complex numbers formalism of $\frac{U_\mu}{2}$ however, all the Lagrangians in this paper are based on one or more such fields. (1) is consistent with the Reeb class vector, not with the ordinary Reeb vector and it means acceleration of a unit vector in Minkowski spacetime, while P^2 or PP^* in the complex case is a Geroch function [1]. When describing space as a foliation of spacetime, except for a Geroch function, there are also 3 gauge fields the describe the foliation and one additional gauge field due to the fact that acceleration can be described in two perpendicular planes.

Defining: $U_\mu \equiv \frac{Z_\mu}{Z} - \frac{Z_k P^k}{Z^2} P_\mu$ consider,

$$\frac{d}{dx^\nu}\frac{P_\mu}{\sqrt{Z}} - \frac{d}{dx^\mu}\frac{P_\nu}{\sqrt{Z}} = \tag{1.1}$$

$$\frac{P_{\mu},_\nu}{\sqrt{Z}} - \frac{P_\mu Z_\nu}{2Z^{\frac{3}{2}}} - \frac{P_{\nu},_\mu}{\sqrt{Z}} + \frac{P_\nu Z_\mu}{2Z^{\frac{3}{2}}} =$$

$$\frac{P_\nu Z_\mu}{2Z^{\frac{3}{2}}} - \frac{P_\mu Z_\nu}{2Z^{\frac{3}{2}}} =$$

$$\frac{1}{2}\left(\frac{Z_\mu}{Z}\frac{P_\nu}{\sqrt{Z}} - \frac{Z_k P^k}{Z^2}P_\mu \frac{P_\nu}{\sqrt{Z}}\right) - \frac{1}{2}\left(\frac{Z_\nu}{Z}\frac{P_\mu}{\sqrt{Z}} - \frac{Z_k P^k}{Z^2}P_\nu \frac{P_\mu}{\sqrt{Z}}\right) = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$$

But why to use, $\frac{1}{2}U_\mu = \frac{1}{2}\left(\frac{Z_\mu}{Z} - \frac{Z_k P^k P_\mu}{Z^2}\right)$ and not simply, $\frac{Z_\mu}{Z}$? The reason is that $\frac{U_\mu}{2}\frac{P^\mu}{\sqrt{Z}} = 0.$

It is easy to show that $\frac{U_\mu}{2}$ behaves as the acceleration of the unit vector $\frac{P_\mu}{\sqrt{Z}}$. See Appendix D for another way to derive the Reeb class vector. In terms of a 4-acceleration a_μ, it is easy to see:

$$\frac{U_\mu}{2} = \frac{dc^{-1}X^\mu}{cd\tau} = \frac{a_\mu}{c^2} \tag{2}$$

Where c is the speed of light. $\frac{U_\mu}{2}$ is the generalization of a Reeb class vector [4] to 4 dimensions. Can this a_μ have a simple physical meaning of accelerating any neutral mass? There is an experimental way to find out, once we analyze the electric field in the coming sections.

Defining $A_{\mu\nu} \equiv \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$ we get $A_{\mu\nu}\frac{p^\nu}{\sqrt{Z}} = \frac{U_\mu}{2}$ which means that $A_{\mu\nu}$ is a rotation and scaling matrix, however, as a linear operator it acts only on one of two hyper-planes of spacetime.

Hodge star extension: To extend $A_{\mu\nu}$ as a rotation and scaling matrix on the entire tangent bundle $T(M)$ of the spacetime manifold M there is a need to use a contraction of $A_{\mu\nu}$ with an antisymmetric tensor and to sum the result with $A_{\mu\nu}$. The reason for the difference between the Scarr-Friedmann acceleration matrix and a field of acceleration can be viewed in the light of rotations in spacetime when both indices of the acceleration matrix are either lower or upper.

The tangent space at the identity of a Lie group is a Lie Algebra and it follows from a

differentiation of the Lie Group left action at the identity. Consider that $A_{\mu\nu}$ is extended to a second plane in order for $A_{\mu\nu}$ to become a regular matrix so now $A_{\mu\nu} = A_{\mu\nu}(1) + A_{\mu\nu}(2)$ and in local base $A(1) = \begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $A(2) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a \\ 0 & 0 & a & 0 \end{pmatrix}$ for some field a.

For regular orthogonal matrices we have $A(\tau)A^{trasposed}(\tau) = I$ where I is the identity matrix and with orbits crossing $\tau = 0$, differentiating at Tau = 0 and remembering that an exponent of a transposed matrix is the transposed of the exponent, we get from $A(\tau) = e^{\tau A'}$, and $A^{transposed}(\tau) = e^{\tau A'^{trasposed}}$.

$$(A(\tau)A^{trasposed}(\tau))' = (I)' = 0 \qquad (2.1)$$

$$\left(A(\tau)A^{trasposed}(\tau)\right)' =$$

$$A'e^{\tau A'}A^{trasposed}(\tau) + A(\tau)A'^{transposed}e^{\tau A'^{transposed}}$$

Where A' is a Lie Algebra matrix. Setting

$$A^{trasposed}(\tau = 0) = e^{0A'^{transposed}} = A(\tau = 0) = e^{0A'} = I \qquad (2.2)$$

$$A'II + IA'^{transposed}I = A' + A'^{transposed} = 0$$

Which means that the Lie Algebra of orthogonal matrices is antisymmetric matrices.

Cartan subalgebras

There are 6 ways to split the tangent space of spacetime into 2 rotation and acceleration planes.

Without loss of generality, consider for some real numbers a, b:

$$\begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & -b \\ 0 & 0 & b & 0 \end{pmatrix} = \begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -b \\ 0 & 0 & b & 0 \end{pmatrix} = \tag{2.3}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ i & -i & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{pmatrix} \begin{pmatrix} -ai & 0 & 0 & 0 \\ 0 & ai & 0 & 0 \\ 0 & 0 & -bi & 0 \\ 0 & 0 & 0 & bi \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}i & 0 & 0 \\ \frac{1}{2} & \frac{1}{2}i & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & 0 & \frac{1}{2} & \frac{1}{2}i \end{pmatrix}$$

The eigenvectors are the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ i & -i & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{pmatrix} \tag{2.4}$$

and

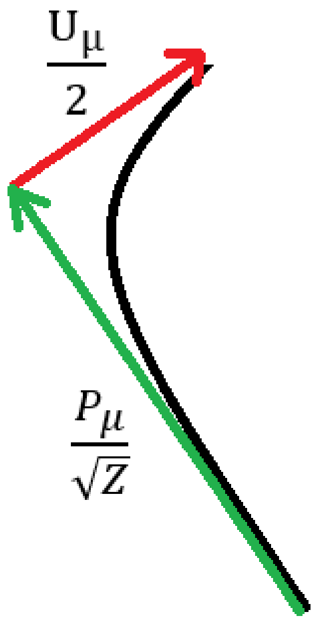
$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ i & -i & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}i & 0 & 0 \\ \frac{1}{2} & \frac{1}{2}i & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & 0 & \frac{1}{2} & \frac{1}{2}i \end{pmatrix}$$

Each one of the six Cartan subalgebra is a maximal abelian set of matrices which are diagonalizable to a purely imaginary trace zero matrix, quite like skew-Hermitian matrices:

$$\begin{pmatrix} -ai & 0 & 0 & 0 \\ 0 & ai & 0 & 0 \\ 0 & 0 & -bi & 0 \\ 0 & 0 & 0 & bi \end{pmatrix} \tag{2.5}$$

Where $i = \sqrt{-1}$. (2.5) chooses rows and columns 0,1 for the first matrix and the rest for the second matrix. There are 6 such choices which make 6 Cartan subalgebras [9].

Fig 1. – The generalized Reeb class vector as an acceleration vector.



To describe a field that accelerates any unit vector, we need an anti-symmetric matrix of acceleration similar to the Tzvi Scarr & Yaakov Friedman's acceleration matrix [10] but with the mentioned important differences.

Considering A(1) in the x^0, x^1 plane, say ct, x in Special Relativity and A(2) in the x^2, x^3 plane, say y, z in Special Relativity, the second rotation and scaling matrix means spin while the x direction is a boost. So looking at a radial source of such a field, the field perpendicular to the radius appears rotating from every angle of view and can have two real valued orientations A(2) and -A(2). Both A(1) and A(2) can be complex, however, there is a problem using skew-Hermitian matrices because skew-Hermitian matrices allow non-zero imaginary diagonal values in the complex plane. Diagonal elements should be zero if A(1), A(2) describe an acceleration field, unless only the real value of $VAV^*$is considered, where A is skew-Hermitian, and V is a complex vector,

$$Real(VAV^*) = \frac{1}{2}(VAV^* + (VAV^*)^*) = \frac{1}{2}(VAV^* + VA^*V^*) = \frac{1}{2}(VAV^* - VAV^*) = 0 \quad (2.6)$$

And therefore, in the skew-Hermitian case VAV^* is purely imaginary.

In that case The matrix $A_{\mu\nu} = \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$ is insufficient for that purpose; however, it can be extended quite easily, by using the Levi-Civita alternating tensor [11], not the alternating Levi-Civita symbol.

The problem of chirality relative to the direction of the acceleration field

Next, we would like to see if there is a mathematical reason to prefer right handedness,

$$A1 = \begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & -a \\ 0 & 0 & a & 0 \end{pmatrix} \tag{2.7}$$

Or left handedness of the acceleration fields $A1\frac{V}{c} = \frac{a}{c^2}$ or $A2\frac{V}{c} = \frac{a}{c^2}$ where $\frac{V}{c}$ is a unit 4-vector in spacetime, $\frac{a}{c^2}$ is its 4-acceleration and c is a speed of light, where $\frac{V}{c}$ is a unit 4-vector in spacetime, $\frac{a}{c^2}$ is its 4-acceleration and c is a speed of light. The action of $A1$, $A2$ takes the form of the Scarr-Friedmann uniform acceleration [10] although it has a very different meaning in this paper.

$$A2 = \begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & a \\ 0 & 0 & -a & 0 \end{pmatrix} \tag{2.8}$$

Both matrices A1 and A2 implement a possible way to extend the matrix $A_{\mu\nu} \equiv \frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}$ from $\begin{pmatrix} 0 & -a \\ -a & 0 \end{pmatrix}$ to a 4-dimensional matrix. The preference of $A1$ is discussed.

Obviously when

$$A = \begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & -b \\ 0 & 0 & b & 0 \end{pmatrix}, \; C = \begin{pmatrix} 0 & -c & 0 & 0 \\ c & 0 & 0 & 0 \\ 0 & 0 & 0 & -d \\ 0 & 0 & d & 0 \end{pmatrix} \tag{2.9}$$

AC-CA = 0, therefore A(B+C)-(B+C)A = AB -BA such that

$$B = \begin{pmatrix} 0 & 0 & -x & -z \\ 0 & 0 & -y & -w \\ x & y & 0 & 0 \\ z & w & 0 & 0 \end{pmatrix} \tag{2.10}$$

To find the root decomposition of the Lie algebra of the skew-symmetric matrices by the Cartan subalgebra which is described by matrix A we need to solve for some eigenvalues λ and eigenvectors of the ad() operator $ad(A)B = [A,B] = AB - BA$.

$$ad(A)B = \lambda B \tag{2.11}$$

$$\begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & -b \\ 0 & 0 & b & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -x & -z \\ 0 & 0 & -y & -w \\ x & y & 0 & 0 \\ z & w & 0 & 0 \end{pmatrix} -$$

$$\begin{pmatrix} 0 & 0 & -x & -z \\ 0 & 0 & -y & -w \\ x & y & 0 & 0 \\ z & w & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & -b \\ 0 & 0 & b & 0 \end{pmatrix} = \lambda \begin{pmatrix} 0 & 0 & -x & -z \\ 0 & 0 & -y & -w \\ x & y & 0 & 0 \\ z & w & 0 & 0 \end{pmatrix} \tag{2.12}$$

$$\begin{pmatrix} 0 & 0 & ay & aw \\ 0 & 0 & -ax & -az \\ -bz & -bw & 0 & 0 \\ bx & by & 0 & 0 \end{pmatrix} -$$

$$\begin{pmatrix} 0 & 0 & -bz & bx \\ 0 & 0 & -bw & by \\ ay & -ax & 0 & 0 \\ aw & -az & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & ay + bz & aw - bx \\ 0 & 0 & -ax + bw & -az - by \\ -ay - bz & ax - bw & 0 & 0 \\ -aw + bx & az + by & 0 & 0 \end{pmatrix} \tag{2.13}$$

So, we have the following equations:

$$\begin{pmatrix} -ay - bz \\ -aw + bx \\ ax - bw \\ az + by \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \tag{2.14}$$

Therefor we need to find solutions to the eigenvectors and values equation:

$$\begin{pmatrix} 0 & -a & -b & 0 \\ b & 0 & 0 & -a \\ a & 0 & 0 & -b \\ 0 & b & a & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \tag{2.15}$$

Bearing in mind that the skew symmetric matrices are a Lie algebra also over the complex numbers, consider the root system of the Cartan subalgebra of skew-symmetric matrices (2.3):

$$S = \begin{pmatrix} -1 & 1 & 1 & -1 \\ -i & -1 & 1 & i \\ -i & 1 & -1 & i \\ 1 & 1 & 1 & 1 \end{pmatrix} \tag{2.16}$$

$$S^{-1} = \begin{pmatrix} -\frac{1}{4} & \frac{i}{4} & \frac{i}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & -\frac{i}{4} & -\frac{i}{4} & \frac{1}{4} \end{pmatrix}$$

$$D = \begin{pmatrix} -i(a+b) & 0 & 0 & 0 \\ 0 & a-b & 0 & 0 \\ 0 & 0 & b-a & 0 \\ 0 & 0 & 0 & i(a+b) \end{pmatrix}$$

$$\begin{pmatrix} 0 & -a & -b & 0 \\ b & 0 & 0 & -a \\ a & 0 & 0 & -b \\ 0 & b & a & 0 \end{pmatrix} = SDS^{-1}$$

And consider the root system over the Cartan sub-algebra of the skew-symmetric matrices,

$$A = \begin{pmatrix} 0 & -a & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & -b \\ 0 & 0 & b & 0 \end{pmatrix} \tag{2.17}$$

$$Root = \begin{pmatrix} 0 & 0 & -x & -z \\ 0 & 0 & -y & -w \\ x & y & 0 & 0 \\ z & w & 0 & 0 \end{pmatrix} \in \tag{2.18}$$

$$\left\{ \begin{pmatrix} 0 & 0 & 1 & i \\ 0 & 0 & i & -1 \\ -1 & -i & 0 & 0 \\ -i & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & -i \\ 0 & 0 & -i & -1 \\ -1 & i & 0 & 0 \\ i & 1 & 0 & 0 \end{pmatrix} \right\}$$

A physical meaning in the classical sense - because roots are linear operators and are therefore acceleration matrices - is that the real part of the resulting matrix should consist of the following representations $\begin{pmatrix} 0 & \dots & -a \\ \dots & \dots & \dots \\ a & \dots & 0 \end{pmatrix}$ but this can only be true if b=a in which case the roots of the Cartan algebra have no real eigenvalues except for zero, as expected from an acceleration matrix $\frac{1}{2}(Root + Root^*)$ where the * operator does not represent transposed conjugation but conjugation over the complex numbers

$$b = a \Longrightarrow \frac{1}{2}(Root + Root^*)\frac{V}{c} = \frac{a}{c^2} \tag{2.19}$$

Where $\frac{V}{c}$ is a unit 4-vector in spacetime, $\frac{a}{c^2}$ is its 4-acceleration and c is a speed of light.

The meaning of (2.19) which leads to (2.7) is that a subspace of the skew-symmetric matrices can be used to represent a physical acceleration in addition to (2.3). This subspace is represented as a direct sum of the Cartan algebra [9] and two roots out of 4. The linear combination of only two roots obviously does not cover all the skew-symmetric matrices over the complex field.

In conventional particle - based physics, there is no meaning to the chirality of an electric field of an electron although existing models do say that an electric charge emits inert photons and obviously such inert photons should have a chirality. However, there should be a fundamental difference between the chirality of a spin and the chirality of the acceleration field, which is discussed in this section.

The result of (2.19), (2.7) is an equivalence between the orientability in space and the asymmetry of time.

An orientation on a manifold is the sign of the determinant of an atlas of coordinate systems. When Dr. Sam Vaknin was shown the result in (2.19) he made an important remark that (2.19) leads to time asymmetry. Obviously due to "Appendix H – Causality conservation theorem", the Geroch time function PP^* can be either monotonically increasing or monotonically decreasing except for a set of measure zero. If we assume that the cosmos is a "Big Bang" cosmos then the Geroch function must be increasing, however, it is preferable not to make such an assumption.

Theorem 0: Time asymmetry special theorem (Suchard - Vaknin): The local time coordinate $\frac{1}{2}\left(P_\mu + P_\mu^*\right)$ must have only one possible direction, when $\frac{u_\mu}{2}$ is not zero, or in the real case, P_μ must have only one possible direction if and only if space is orientable.

Note: Implicitly the theorem assumes that $\frac{u_\mu}{2}$ is smooth and that A1 in (2.7) describes a non-degenerate smooth matrix along the integral curves of P_μ and in small neighborhoods around this curve in each foliation perpendicular to P_μ in the real case or $\frac{1}{2}\left(P_\mu + P_\mu^*\right)$ in the complex case.

Proof: Notice that if we choose $x^0 = \frac{p^\nu}{\sqrt{Z}}, x^1 = \frac{u^\nu}{\sqrt{u_\lambda u^\lambda}}$ or $x^0 = \frac{p^\nu}{\sqrt{Z}}, x^1 = -\frac{u^\nu}{\sqrt{u_\lambda u^\lambda}}$ the acceleration matrix restricted to the plane spanned by x^0, x^1 will be $\begin{pmatrix} 0 & -a \\ a & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$ respectively and due to (2.19), (2.7) is either $A1$ or $-A1$. It is easier to check how the sign of alternating forms changes instead of using determinants. For example, consider $x^0 \wedge x^1 \wedge x^2 \wedge x^3$ with locally perpendicular coordinates x^0, x^1, x^2, x^3. The theorem proof is immediate from the orientability of the space foliation which is perpendicular to $\frac{1}{2}\left(P_\mu + P_\mu^*\right)$ or to P_μ in the real case. For simplicity, proceed with the real case; one direction of the proof is easy, if P_μ is asymmetrical and therefore $-P_\mu$ is not a valid direction of time, then (2.7) dictates the orientation of space when $\frac{u_\mu}{2}$ is not zero, to see why, consider that the alternating form $(\frac{p_\nu}{\sqrt{Z}}\frac{u_\mu}{2}-\frac{p_\mu}{\sqrt{Z}}\frac{u_\nu}{2})dx^\mu{}^\wedge dx^\nu$ defines the orientation also of the perpendicular plane by using the 2-form $dx^2{}^\wedge dx^3$ and by (2.7). We will later write such an extension to the perpendicular plane by using the form $(\frac{p_\nu}{\sqrt{Z}}\frac{u_\mu}{2}-\frac{p_\mu}{\sqrt{Z}}\frac{u_\nu}{2})dx^\mu{}^\wedge dx^\nu$ in a tensorial way. For simplicity, by choosing $x^0=\frac{p^\nu}{\sqrt{Z}}$ and $x^1=\frac{u^\nu}{\sqrt{u_\lambda \mathrm{u}^\lambda}}$, two possible orientations of a perpendicular form $x^2{}^\wedge x^3$ are dictated by (2.7) which depend only on the sign of u_ν, $(\pm)u_\mu$, but that means that the sign of $x^1{}^\wedge x^2{}^\wedge x^3$ can be only one. The 3 cases we need to consider that are dictated by (2.7) are,

$$x^1\text{−>}-x^1\wedge(x^2\text{−>}-x^2\ \vee\ x^3\text{−>}-x^3)\Rightarrow A1=\begin{pmatrix}0&a&0&0\\-a&0&0&0\\0&0&0&a\\0&0&-a&0\end{pmatrix}\qquad(2.20)$$

And

$$x^2\text{−>}-x^2\ \wedge\ x^3\text{−>}-x^3\Rightarrow A1=\begin{pmatrix}0&-a&0&0\\a&0&0&0\\0&0&0&-a\\0&0&a&0\end{pmatrix}\qquad(2.21)$$

In both cases the orientation of the space spanned by x^1, x^2, x^3 is maintained.

The converse starts with space foliations which are orientable so the sign of $x^1{}^\wedge x^2{}^\wedge x^3$ is determined but then if we change the sign of $x^1=\frac{u_\nu}{\sqrt{u_\lambda \mathrm{u}^\lambda}}$ we also must change the sign of either x^2 or x^3 but not both or to swap their order and then by (2.19) which says that a=b, and by the resulting (2.7), the sign of $x^0=\frac{p_\nu}{\sqrt{Z}}$ cannot change which means time asymmetry. Q.E.D.

Note: Not to be ungrateful it is important to mention that (2.16) - (2.18) was checked by using the online Wolfram Equations internet site.

Hodge star spin-like field extension: We have $B^{\mu\nu}=\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ which define an acceleration matrix in a perpendicular plane to the plane spanned by $\frac{P_\mu}{\sqrt{Z}}$ and $\frac{\mathrm{U}_\mu}{2}$. In the complex case we define the acceleration matrix: $F_{\mu\nu}=A_{\mu\nu}+\gamma B_{\mu\nu}$ where $\gamma\epsilon U(1)$. With a vector w^ν, $w^\nu w_\nu=c^2$, we derive its acceleration,

$$F_{\mu\nu}\frac{w^\nu}{c}=\frac{a_{\mu(w)}}{c^2}, B^{\mu\nu}=\mp\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}\qquad(3)$$

$$\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{U_\mu U^\mu}{4}$$

Note: If spacetime could have only one orientation as hinted in (2.19), then either $+\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ would result in $\frac{U_\mu U^\mu}{4} = 0$ or $-\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ would result in $\frac{U_\mu U^\mu}{4} = 0$. In this case, the action could be described differently as $\frac{1}{4}\left(A_{\mu\nu}A^{\mu\nu} \mp B_{\mu\nu}B^{\mu\nu}\right)$. Such a possibility would lead to peculiar particles which obviously do not exist, there are no neutral electrons, Muons and Tau leptons. And neutrinos do not behave as such due to their low mass.

Exercise to the reader: show that the Reeb class vector $\frac{U_\mu}{2}$ of $\frac{P_\mu}{\sqrt{|Z|}}$ is the same as for $\frac{P_\mu}{\sqrt{|Z|}}e^{i\theta}$for $i = \sqrt{-1}$ and a smooth scalar θ. See that you understand the idea of a field of acceleration that maps 4-velocity to 4-acceleration by multiplication with an anti-symmetric matrix [10], $F_{\mu\nu}\frac{w^\nu}{c} = \frac{a_{\mu(w)}}{c^2}$.

In Special Relativity, 4-velocity is perpendicular to 4-accleration and $w^\nu w_\nu = c^2$. $F_{\mu\nu}$ is then an Acceleration Field and it can be deconstructed into the sum of two matrices which act on two perpendicular two-dimensional hyperplanes in spacetime. $B_{\mu\nu} = \frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ and $B_{\mu\nu} = -\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ yield the same result in (3). When reduced to the three-dimensional foliation which is perpendicular to P_μ, If $\frac{U_\mu}{2}$ has a divergence point, say Q, then the choice $B_{\mu\nu} = -\frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$ mean that $B_{\mu\nu} = \frac{P(2)_\nu}{\sqrt{|Z|}}\frac{U(2)_\mu}{2} - \frac{P(2)_\mu}{\sqrt{|Z|}}\frac{U(2)_\nu}{2}$, where $P(2)_\nu P^\nu = 0$, $U(2)_\nu P^\nu = 0$, $U(2)_\nu P(2)^\nu = 0$, and finally the complex numbers case can also yield the following,

$$U_\mu{;}^\mu + U^*{}_\mu{;}^\mu = 0 \tag{3.1}$$

Lagrangian generalization offer and further research offer

It is possible to define a Lagrangian for two independent acceleration vectors that are related to each other by multiplication, here it is presented in a complex formalism, with a volume element $\sqrt{-g}$, and with Reeb class vectors, not Reeb vectors, which are perpendicular to $\frac{P_\mu}{\sqrt{Z}}$,

$$\left|\frac{\zeta^{*\lambda}\zeta_\lambda + \zeta^\lambda\zeta^*{}_\lambda}{8}\right| = \begin{vmatrix} 1 & \frac{\mathrm{P}_k U(1)^{*k} + \mathrm{P}^*{}_k U(1)^k}{2\sqrt{2\mathrm{Z}}} & \frac{\mathrm{P}_k U(2)^{*k} + \mathrm{P}^*{}_k U(2)^k}{2\sqrt{2\mathrm{Z}}} \\ \frac{\mathrm{P}_k U(1)^{*k} + \mathrm{P}^*{}_k U(1)^k}{2\sqrt{2\mathrm{Z}}} & \frac{U^k U_k^* + U^{*k}U_k}{8} & \frac{U(2)^k U_k^* + U(2)^{*k}U_k}{8} \\ \frac{\mathrm{P}_k U(2)^{*k} + \mathrm{P}^*{}_k U(2)^k}{2\sqrt{2\mathrm{Z}}} & \frac{U(2)^k U_k^* + U(2)^{*k}U_k}{8} & \frac{U(2)^k U(2)_k^* + U(2)^{*k}U(2)_k}{8} \end{vmatrix}^{\frac{1}{2}} \sqrt{-g} \tag{3.2.1}$$

The meaning of (3.2.1) is of a squared acceleration which is the Minkowski squared norm of a spacelike vector. In (+,-,-,-) metric convention, a negative sign has to be added, $-\frac{\zeta^{*\lambda}\zeta_\lambda+\zeta^\lambda\zeta^*{}_\lambda}{8}$.

The following norm calculates a physical non-geodesic acceleration, $\sqrt{\frac{\zeta^{*\lambda}\zeta_\lambda+\zeta^\lambda\zeta^*{}_\lambda}{8}}$. Since the 3 forces in Nature seem to be aligned with the electric field, it is reasonable to assume that ζ_λ must be either aligned or anti-aligned with U_λ, or in other words,

$$\zeta_\lambda = f(x^\mu)U_\lambda \tag{3.2.2}$$

$$\frac{\zeta_\lambda + \zeta^*{}_\lambda}{4} = f(x^\mu)\frac{U_\lambda + U^*{}_\lambda}{4}$$

for some scalar function of the coordinates $f(x^\mu)$. A real valued vector is then $\frac{U_\mu+U^*{}_\mu}{4}$ but to assume this expression is the direction of an acceleration vector, by Occam's razor must be inferred from a variation of the Lagrangian L=$\frac{U_\mu U^{*\mu}+U^*{}_\mu U^\mu}{8}\sqrt{-g}$. Such a variation indeed involves the divergence, $\left(\frac{U^\mu+U^{*\mu}}{4}\right)_{;\mu}$ which implies that $\frac{U_\mu+U^*{}_\mu}{4}$ has indeed a meaning of an acceleration of a unit vector. The zeros in (3.2) mean that the acceleration vector $\frac{U_k}{2}$ is perpendicular to the unit vector $\frac{\mathrm{P}^*{}_\mathrm{k}}{\sqrt{Z}}$, $\left(\frac{Z_\mu}{2Z}-\frac{Z_\lambda P^{*\lambda}P_\mu}{2Z^2}\right)\frac{P^{*\mu}}{\sqrt{Z}} = \left(\frac{Z_\mu P^{*\mu}}{2Z}-\frac{Z_\lambda P^{*\lambda}}{2Z^2}Z\right)\frac{1}{\sqrt{Z}} = 0$ and then the term $\frac{\mathrm{P_k}U^{*k}+\mathrm{P}^*{}_\mathrm{k}U^{*k}}{2\sqrt{2\mathrm{Z}}} = 0$. If $\mathrm{P_k}$ and $\mathrm{U_k}$ are perpendicular to $\mathrm{P(2)_k}$ and $\mathrm{U(2)_k}$ then of course $\frac{\mathrm{P_k}U(2)^{*k}+\mathrm{P}^*{}_\mathrm{k}U(2)^{*k}}{2\sqrt{2\mathrm{Z}}} = 0$, however, this Lagrangian can define an action operator even without such an orthogonality as a prerequisite and is therefore more general. The Lagrangian above has symmetry SU(2) and is therefore offered as a generalization of this paper with properties of the "electroweak" field. To summarize the motivation of this section, saying that an energy density can be described as the negative squared norm of an acceleration of unit vectors in (+,-,-,-) metric does not mean such acceleration field can't be a result of other Reeb class fields. The description of the electric field as the simplest example is discussed later.

Could non-geodesic acceleration vectors also explain the gravitational field?

The pseudo-acceleration of a test particle with velocity $\frac{dx^\mu}{d\tau}$ in weak gravity satisfies,

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0 \tag{3.3}$$

$$\frac{d^2x^\mu}{dt^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = 0 \tag{3.4}$$

With weak gravity:

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu} \tag{3.5}$$

$$O\left(\frac{dx^{\mu}}{dt}\right) = O(\epsilon) \tag{3.6}$$

And then space terms are neglected, which reduces the equation of motion to

$$\frac{d^2x^{\mu}}{dt^2} + \Gamma^{\mu}_{\alpha\beta}\frac{dx^0}{dt}\frac{dx^0}{dt} \approx 0 \tag{3.7}$$

While $\frac{dx^0}{dt} = c$, the speed of light, we get,

$$\frac{d^2x^{\mu}}{dt^2} \approx -c^2\Gamma^{\mu}_{00} = -\frac{1}{2}c^2\epsilon({h^{\mu}}_{0,0} + {h^{\mu}}_{0,0} - h_{00},^{\mu}) \tag{3.8}$$

And in a static field

$$\Gamma^{\mu}_{00} = -\frac{1}{2}h_{00},^{\mu} \tag{3.9}$$

$$\frac{d^2x^{\mu}}{dt^2} \approx \frac{1}{2}c^2\epsilon h_{00},^{\mu} \tag{3.10}$$

Consider the following representation of the metric tensor,

$$g_{\mu\nu} = \frac{P(0)_{\mu}P(0)_{\nu}}{Z(0)} - \frac{P(1)_{\mu}P(1)_{\nu}}{Z(1)} - \frac{P(2)_{\mu}P(2)_{\nu}}{Z(2)} - \frac{P(3)_{\mu}P(3)_{\nu}}{Z(3)} \tag{3.11}$$

At this point no full tetradic representation is considered and $P(i)_{\mu}P(j)^{\mu} \neq 0$ for $i \neq j$ or $P(i)_{\mu}P(j)^{\mu} = 0$ for $i \neq j$.

Consider the weak field equation of motion while focusing on the contribution of $\frac{P(0)_{\mu}P(0)_{\nu}}{Z(0)}$ so in that case it is necessary to say that we account for only $\frac{1}{4}$ of the gravity if the contribution from all fields, $P(i)$ is equal, in that case, $P(i)^2$ represents time,

$$-\frac{1}{2}\frac{(p(0)_0)^2}{Z(0)},^{\mu} = -\frac{1}{2}\frac{2p(0)_0}{\sqrt{Z(0)}}\left(\frac{p(0)_0,^{\mu}}{\sqrt{Z(0)}} - \frac{p(0)_0 Z(0),^{\mu}}{2Z^{\frac{3}{2}}}\right) \approx \frac{1}{4}\Gamma^{\mu}_{00} \tag{3.12}$$

$$-\frac{1}{2}\frac{2p(0)_0}{\sqrt{Z(0)}}\left(\frac{p(0)_0,^{\mu}}{\sqrt{Z(0)}} - \frac{p(0)_0 Z(0),^{\mu}}{2Z^{\frac{3}{2}}}\right) = \frac{p(0)_0}{\sqrt{Z(0)}}\left(\frac{p(0)_0 Z(0),^{\mu}}{2Z^{\frac{3}{2}}} - \frac{p(0)_0,^{\mu}}{\sqrt{Z(0)}}\right) \approx \frac{1}{4}\Gamma^{\mu}_{00}$$

$$p(0)_0,^{\mu} \approx p(0)^{\mu},_0 \Longrightarrow \frac{p(0)_0}{\sqrt{Z(0)}}\left(\frac{p(0)_0 Z(0),^{\mu}}{2Z^{\frac{3}{2}}} - \frac{p(0)^{\mu},_0}{\sqrt{Z(0)}}\right) \approx \frac{Z(0),^{\mu}}{2Z} \approx \frac{U(0)^{\mu}}{2} \approx \frac{1}{4}\Gamma^{\mu}_{00}$$

The latter result is due to $p(0)_1, p(0)_2, p(0)_3$ being neglected but not their derivatives and due to (1). The conclusion of (3.12) is that (3.11) can describe weak gravity, however the scalar fields $P(0), P(1), P(2), P(3)$ in this case, do not represent force fields but Gauge fields.

Caveat: An important caveat is that even if the complex formalism of (3.12) is used, 8 complex scalars may not be able to describe gravity. Contribution from additional fields may be needed.

Caveat: Do not confuse pseudo-acceleration and non-geodesic acceleration, which is a generalized Reeb class vector, here used to describe the energy of force fields. In general, geodesic curves are not geodesic when mapped to a flat spacetime. The meaning of (3.12) was simply to show the possibility of using non-geodesic curves as the underlying field that drives gravity too, and not only other force fields. This can be achieved by mapping geodesic curves to non-geodesic curves in flat spacetime. Even the complex formalism may not be sufficient as mentioned in the previous caveat note:

$$g_{\mu\nu} = \frac{P(0)_\mu P^*(0)_\nu + P^*(0)_\mu P(0)_\nu}{2} - \sum_{i=1}^{3} \frac{P(i)_\mu P^*(i)_\nu + P^*(i)_\mu P(i)_\nu}{2} \quad (3.13)$$

Important: $P(0)_\mu$ is interesting when it is not geodesic also in curved geometry, when it is not only pseudo non-geodesic simply by omission of the Christoffel symbols.

What is this paper's goal?

This theory represents energy density as a Lagrangian of accelerations of normalized gradients of scalar fields. The most interesting case is when these scalar fields are over the complex field. The first scalar field P is such that locally PP^*, or P^2 in the real case, can be considered as a time coordinate through a time-like vector $(PP^*)_{,\mu} = \frac{d(PP^*)}{dx^\mu}$. This paper will also offer a Lagrangian for 3 accelerations which result from three scalar functions, $P1, P2, P3$ such that $P1P1^*, P2P2^*, P3P3^*$ are "local coordinates" of the foliation perpendicular to $(PP^*)_{,\mu}$. The offered Lagrangian will be offered such that $P_{,i}$, $P1_{,i}$, $P2_{,i}$, $P3_{,i}$ need not be perpendicular, by getting rid of the non-perpendicular components in the Lagrangian calculation. Such a definition does not require cumbersome spin-connections and simplifies the theory. The case for two accelerations can be defined in two different ways. One is to define $P_{,\mu}$ and its acceleration $\frac{U_\nu}{2}$ and then to use the Levi-Civita tensor to calculate the plane perpendicular to $P_{,\mu}$ and its acceleration $\frac{U_\nu}{2}$. The second is to use a Lagrangian formalism for two perpendicular planes. The first formalism extends the acceleration from one plane to the other. These planes are known as Lagrangian Planes in the theory of Symplectic Geometry. The second case represents two independent accelerations fields. The theory is also well defined if $PP^*, P1P1^*, P2P2^*, P3P3^*$ integrate to 1 on a spacetime manifold as long as the gradients $P_{,i}$, $P1_{,i}$, $P2_{,i}$, $P3_{,i}$ vanish on a set whose measure is zero. In such a case, it is said that $P_{,i}$, $P1_{,i}$, $P2_{,i}$, $P3_{,i}$ are geometric chronon scalar fields. As a last caveat it is important to distinguish between an acceleration of a normalized velocity as in Special Relativity and the acceleration of a normalized gradient of a scalar field as it is described in this paper. In the following expression for normalized acceleration of a moving frame, the dot above the coordinates means derivative in relation to time,

$$\frac{d\frac{\dot{x}^\mu}{c\sqrt{1-\frac{V^2}{c^2}}}}{cd\tau} = \frac{d\frac{(1,\frac{\dot{x}^1}{c},\frac{\dot{x}^2}{c},\frac{\dot{x}^3}{c})}{\sqrt{1-\frac{\dot{x}^{1^2}+\dot{x}^{2^2}+\dot{x}^{3^2}}{c^2}}}}{\sqrt{1-\frac{\dot{x}^{1^2}+\dot{x}^{2^2}+\dot{x}^{3^2}}{c^2}}cdt} = \frac{\frac{1}{c^2}(0,\ddot{x}^1,\ddot{x}^2,\ddot{x}^3)}{1-\frac{\dot{x}^{1^2}+\dot{x}^{2^2}+\dot{x}^{3^2}}{c^2}} + \frac{\frac{1}{c^2}(c,\dot{x}^1,\dot{x}^2,\dot{x}^3)(\frac{\dot{x}^1\ddot{x}^1+\dot{x}^2\ddot{x}^2+\dot{x}^3\ddot{x}^3}{c^2})}{\left(1-\frac{\dot{x}^{1^2}+\dot{x}^{2^2}+\dot{x}^{3^2}}{c^2}\right)^2} \quad (3.14)$$

Which is not solely dependent on the normalized velocity as is, $\frac{\dot{x}^\mu}{c\sqrt{1-\frac{V^2}{c^2}}}$.

From (3) a generalization for multiple event fields $\int_\Omega P(i)P^*(i)d\Omega = 1$ where $P(i)P^*(i)$ is no longer a Geroch function is

$$A_{\mu\nu}(i) = \left(\frac{P_\mu(i)}{\sqrt{Z(i)}}\right)_{,\nu} - \left(\frac{P_\nu(i)}{\sqrt{Z(i)}}\right)_{,\mu} \text{ and } i \in \mathbb{N} \quad (3.15)$$

And the action is

$$L = \frac{1}{8}(\sum_{i,j}(A_{\mu\nu}(i)A^{*\mu\nu}(j) + A^*_{\mu\nu}(i)A^{\mu\nu}(j))(P(i)P^*(j) + P^*(i)P(j))\sqrt{-g} \quad (3.16)$$

It will be clearer, as this paper develops, that the mixed terms in (3.16) are due to the non-covariant classical limit of the electrostatic field, specifically $\|E(1) + E(2)\|^2 = \|E(1)\|^2 + \|E(2)\|^2 + 2\, E(1) \cdot E(2)$ for two non-covariant electric fields $E(1)$ and $E(2)$.

With Einstein Hilbert Langrangian, which should not change also when considering other fields such as will be seen in (64). This is because other fields are emergent from time.

$$L = \frac{1}{2}\sum_{i,j} R(P(i)P^*(j) + P^*(i)P(j))\sqrt{-g} \quad (3.16.1)$$

Where g is the determinant of the metric tensor.

In the real case and from (3),

$$L = \frac{1}{2}\sum_{i,j} A_{\mu\nu}(i)A^{\mu\nu}(j)P(i)P(j)\sqrt{-g} = \frac{1}{4}\sum_{i,j} F_{\mu\nu}(i)F^{\mu\nu}(j)P(i)P(j)\sqrt{-g} \quad (3.17)$$

Or as energy density

$$\frac{c^4}{8\pi K}\frac{1}{4}\sum_{i,j} F_{\mu\nu}(i)F^{\mu\nu}(j)P(i)P(j)\sqrt{-g} \quad (3.18)$$

Electro-gravity

The action of gravity is defined as: $Action = Min\int_\Omega \left(R - \frac{1}{4\beth}U^k U_k\right)\sqrt{-g}\, d\Omega$

The Euler Lagrange equations by the metric $g_{\mu\nu}$, by the scalar field of time P yield, Appendix A or [12]:

$$\frac{1}{4ב}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z}\right) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \quad (4)$$

$$W^\mu;_\mu = \left(-4U^k;_k\frac{P^\mu}{Z} - 2\frac{Z_\nu P^\nu}{Z^2}U^\mu\right);_\mu = 0$$

It is easy to prove without the right hand side that $\frac{1}{4ב}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z}\right);^\nu = 0$ see Appendix B or [12]. (4) assumes $ב = 1$.

Consider $\rho = \frac{1}{2}U^k;_k$ to be stationary along p^μ, with local coordinates such that only p_0 is numerically significant. We will neglect all small terms that are multiplied by U^μ and its derivatives. with $2\left(\frac{p^\mu}{Z}\right);_\mu \approx \frac{Z_\nu p^\nu}{Z^2}$ and $\left(\frac{Z_\nu p^\nu}{Z^2}\right);_\mu U^\mu \ll 1$, the first approximation is the result of $2\left(\frac{p^\mu}{Z}\right);_\mu \approx 2\left(\frac{p^0}{p^0p_0}\right),_0$ and $\frac{Z_\nu p^\nu}{Z^2} \approx \frac{p^0(p^0p_0),_0}{(p^0p_0)^2} \approx 2\left(\frac{p^0}{p^0p_0}\right),_0$ the last approximation $\left(\frac{Z_\nu p^\nu}{Z^2}\right);_\mu U^\mu \ll 1$ is due to $\frac{Z_\nu p^\nu}{Z^2} \approx \frac{p^0(p^0p_0),_0}{(p^0p_0)^2}$ and the fact that U_μ is spacelike. Then,

$$\left(-4U^k;_k\frac{P^\mu}{Z} - 2\frac{Z_\nu P^\nu}{Z^2}U^\mu\right);_\mu = 0 \Longrightarrow 2\rho \approx U^k;_k \quad (4.1)$$

Dynamics: (4.1) implies the dynamics of the electric field of points of divergence $U^k;_k \neq 0$.

Theorem 1: If non-geodesic curves are prescribed to motion in material fields then zero Einstein tensor implies $\frac{1}{2}U_\mu = 0$, i.e. $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0 \Longrightarrow \frac{1}{2}U_\mu = 0$ i.e. geodesic motion.

Proof: We contract both sides of (4) with $U^\mu U^\nu$ so $\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z}\right)U^\mu U^\nu = 0 \Longrightarrow U_\lambda U^\lambda = 0$ because $U^\mu P_\mu = 0$ and now we contract both sides of (4) with $\frac{P^\mu P^\nu}{Z}$ so we have $\frac{P^\mu P^\nu}{Z}\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z}\right) = -\frac{1}{2}U_\lambda U^\lambda - 2U^k;_k = 2U^k;_k = 0$ because $U_\lambda U^\lambda = 0$ and $\frac{P^\lambda P_\lambda}{Z} = 1$ so we get $U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z} = U_\mu U_\nu = 0 \Longrightarrow U_\mu = 0$. In other words, motion must be geodesic and we are done.

Remember $\frac{U_\mu}{2} = \frac{a_\mu}{c^2}$ as acceleration and the equation of gravity by Einstein, using the dust energy momentum tensor from General Relativity,

$$\frac{8\pi K}{c^4}T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \quad (5)$$

in (-,+,+,+) convention, we will use (5) further on, to show unique gravity by electric charge.

$$\frac{1}{4}U^kU_k = \frac{a^ka_k}{c^4} \tag{6}$$

(6) compared to Einstein's tensor means that the energy density in old physics terms can be seen as:

$$\frac{a^ka_k}{8\pi K ב} = EnergyDensity \Longrightarrow \frac{8\pi K}{c^4}EnergyDensity = \frac{a^ka_k}{ב c^4} = \frac{1}{4ב}U^kU_k \tag{7}$$

Where $ב = 1$ relates non geodesic acceleration to geometry, direct outcomes of (7) will be shown in (13) and (43). (7) means that the energy of the classical non-covariant electric field must be hidden in a very weak acceleration field

$$\frac{a^ka_k}{8\pi K ב} \cong \frac{1}{2}\varepsilon_0E^2 \tag{8}$$

ε_0 is the permittivity of vacuum, K is Newton's constant of gravity, which means

$$|a|^2 = 4\pi K\varepsilon_0 ב E^2 \tag{9}$$

and

$$\|a^\mu\| = \sqrt{4\pi K\varepsilon_0 ב}\|E\| \tag{10}$$

Indeed, a very weak acceleration if $ב = 1$. However, there is a surprise:

$$\frac{1}{4ב}(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k\frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{11}$$

Means that $\frac{1}{2ב}U^k;_k = \frac{a^k;_k}{c^2} = \sqrt{\frac{4\pi K\varepsilon_0 ב}{ב^2}}\frac{\rho}{\varepsilon_0c^2} = \sqrt{\frac{4\pi K}{ב\varepsilon_0}}\frac{\rho}{c^2}$ where ρ is charge density.

Now remember the term $\frac{1}{4ב}(-2U^k;_k\frac{P_\mu P_\nu}{Z})$ and the relation $\frac{P^\mu P^\nu}{Z} \approx \frac{V^\mu V^\nu}{c^2}$ where $\frac{P^\mu}{\sqrt{Z}}$ is equivalent to a normalized velocity vector $\frac{V^\mu}{c}$, in Special Relativity $V^\mu = \frac{(c,v_x,v_y,v_z)}{\sqrt{1-v^2/c^2}}$, so we get

$$\frac{1}{8\pi K}\frac{U^\mu;_\mu}{2ב}\frac{P^\mu P^\nu}{Z^2} \approx \frac{1}{8\pi K}\sqrt{\frac{4\pi K ב}{ב^2\varepsilon_0}}\cdot\frac{\rho_{charge}V^\mu V^\nu}{c^4} = \frac{1}{8\pi Kc^4}\sqrt{\frac{4\pi K}{ב\varepsilon_0}}\rho_{charge}V^\mu V^\nu \tag{12}$$

But that can only mean that charge density behaves like mass density except for the fact that $\frac{P^\mu}{\sqrt{Z}}$ is not geodesic and therefore for charge Q:

$$M = \frac{Q}{\sqrt{16\pi K\varepsilon_0 ב}} \tag{13}$$

Assuming $\beth = 1$ where ε_0 is the permittivity of vacuum and K is Newton's constant of gravity, M is a gravitational mass, from (13) $\pm 1\ Coulombs$ is equivalent to $\pm\mathbf{5.802135215 * 10^9}$ **Kg**.

Note: It is easy to see that in a weak electric field, if all the energy is of an electric field, then when taking (13) into account, the component of pseudo-gravitational acceleration g by the divergence of the classical non-covariant electric field E is

$$\|a^\mu\| \approx \sqrt{4\pi K\varepsilon_0}\|E\| \Longrightarrow \|g\| \approx \frac{1}{2}\|a^\mu\|, -g \approx -\frac{1}{2}a^\mu \tag{13.01}$$

Consider that the usable gravitational energy depends on g as it is not part of a preserved gravitational field because charge can be annihilated,

$$\frac{1}{4}\frac{1}{8\pi K}\int \|a^\mu\|^2 dVolume = Usable\ gravitational\ energy \tag{13.02}$$

(13.02) may look vague right now, however, when describing decay processes of charged particles, it is inevitable that the same portion $\frac{1}{4}$ of the added and subtracted area around negative and positive charge, should account for usable gravitational energy. The term "usable energy" is a concept from thermodynamics. Also note that the sign of g is opposite to the sign of the weak electric acceleration.

Caveat: $\frac{P^\mu}{\sqrt{Z}}$ is not geodesic unless $\frac{1}{2}U_\mu = 0$. So $\rho_{charge}\frac{P_\mu P_\nu}{Z}$ does not behave as inertial mass.

Electric field to acceleration from far observer coordinates – the following is not the way to derive the relation between gravitational mass and charge, not only because charge is coupled to a non-geodesic bivector, however, it does serve as an indication that the results are correct.

$$\frac{e}{4\pi\varepsilon_0 r^2}(4\pi\varepsilon_0 K)^{\frac{1}{2}} = \frac{c^2}{r} \tag{13.1}$$

Where the right-hand side stands for acceleration or the norm of the Reeb class vector multiplied by the squared speed of light. 'e' is the charge of the electron, ε_0 the permittivity of vacuum and K is the gravity constant of Newton. (13.1) is a result of (10).

$$\frac{e}{c^2}\left(\frac{K}{4\pi\varepsilon_0}\right)^{\frac{1}{2}} = r \tag{13.2}$$

We will equate the right-hand side to the Schwarzschild radius of some mass,

$$\frac{e}{c^2}\left(\frac{K}{4\pi\varepsilon_0}\right)^{\frac{1}{2}} = \frac{2Km}{c^2} \tag{13.3}$$

From which

$$e\left(\frac{1}{16\pi K\varepsilon_0}\right)^{\frac{1}{2}} = m \qquad (13.4)$$

This is a very surprising result although it is not derived from the Euler Lagrange equations but just agrees with them 100% for the choice $ב = 1$ in (13).

We are now set to derive the inverse Fine Structure Constant from (13) and from a spin term. We sloppily do this by mixing ideas from General Relativity and Quantum mechanics and (13). From Quantum Mechanics, the angular momentum of the electron is,

$$\sqrt{s(s+1)}\hbar \qquad (13.5)$$

where the spin number is $s = \frac{1}{2}$ for the electron, where $\hbar$ is the reduced Planck constant and $\sqrt{s(s+1)}$ is specific to a particle's spin. Suppose that a positive charge with $\frac{1}{8}\left(U^{*\mu}U_\mu + U^\mu U^*{}_\mu\right) = 0$ or in the real case $\frac{1}{4}U^\mu U_\mu = 0$, is spinning near the speed of light at twice the Schwarzschild radius created by the charge in (13), where this radius is known as the radius of the Marginally Bound [unstable] Orbit, then by (13) this radius should be $2\frac{2eK}{\sqrt{16\pi\varepsilon_0 K}c^2} = 2\frac{2Km}{c^2}$, where $m = \frac{e}{\sqrt{16\pi K\varepsilon_0}}$ and e is the charge of the positron. Of course, we need to remember that $\frac{P_\mu}{\sqrt{Z}}$ is not velocity and therefore the interpretation of m is not as the familiar inertial mass, moreover, $\frac{P_\mu}{\sqrt{Z}}$ is not a geodesic vector field if $\frac{U_\mu}{2}$ is not zero. The radial metric coefficient is 1 due to Schwarzschild metric, not Kerr metric, then by (13) and remembering that the angular momentum does not mean a classical rotation, the angular momentum should be,

$$J = \frac{ec}{\sqrt{16\pi\varepsilon_0 K}}\frac{2*2eK}{\sqrt{16\pi\varepsilon_0 K}c^2} = \frac{e^2}{4\pi\varepsilon_0 c} \qquad (13.6)$$

and we ignore any Kerr metric because the spin effect on spacetime is not identical to the classical rotation of a black hole, otherwise positrons would dissipate their spin energy. We also assume that our field is a fundamental field to all charged particles and therefore omit the $\sqrt{s(s+1)}$ which is specific to the spin number s.

Now consider the ratio between J and the spin independent coefficient $\hbar$, we get,

$$\frac{J}{\hbar} = \frac{e^2}{4\pi\varepsilon_0 c\hbar} \qquad (13.6.1)$$

Which is the known term for the Fine Structure Constant as an upper limit on a ratio between classical angular momentum and Quantum angular momentum.

Theorem 2: If the electromagnetic energy is not zero and the charge density $U^k;_k$ is zero in a domain D of space-time then U_0 is never 0 in all events of D.

Proof:

We write the Einstein - Grossmann equation (4) in its dual form, $R_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T_\alpha^\alpha = \frac{1}{4\beth}\left(U_\mu U_\nu - \frac{1}{2} g_{\mu\nu} U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z} - \frac{1}{2} g_{\mu\nu} g^{ij}\left(U_i U_j - \frac{1}{2} g_{ij} U_\lambda U^\lambda - 2U^k;_k \frac{P_i P_j}{Z}\right)\right) = \frac{1}{4\beth}\left(U_\mu U_\nu - \frac{1}{2} g_{\mu\nu} U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z} - \frac{1}{2} g_{\mu\nu} U^\lambda U_\lambda + g_{\mu\nu} U_\lambda U^\lambda + g_{\mu\nu} U^k;_k\right) = \frac{1}{4\beth}(U_\mu U_\nu + U^k;_k (g_{\mu\nu} - 2\frac{P_\mu P_\nu}{Z}))$. If $U_0 = 0$ in D then there exist local coordinates such that only the P_0 component of P_μ is not zero. We assumed $U^k;_k = 0$. Since $U_0 = 0$, $R_{00} = 0$ so the electromagnetic energy is zero. On the other hand, since U_μ is not zero, P_μ cannot be geodesic and therefore P_0 cannot be the only component of P_μ which is not zero along geodesic coordinates. Note: If there is a time-like curve γ around which U_μ is in relative motion in different events of every small D that contains γ, then R_{00} is not zero in D.

Note: There is one obvious peculiarity about charge generated gravity, $\frac{P^\mu}{\sqrt{Z}}$ is not the velocity of the charge. It is dictated by a scalar field of space-time!

Note – physical interpretation: From (10) and (13), if a^μ has a simple physical interpretation as a field that accelerates any neutral mass then we have to take (13) into account as an opposite effect. The result is that a field of 1,000,000 volts over 1 mm distance will accelerate any neutral particle at 8.61 cm * sec^{-2} and with taking into account (13), (13.01) it will be less, due to an opposite gravitational effect, see (14), will be reduced to 4.305 cm * sec^{-2}.

The quantization of P is into a sum of event wave functions and has the physical meaning of Sam Vaknin's realization chronons [13]. The theory is easily expanded to 2 and to 3 Reeb class vectors where the Lagrangian has U(1) SU(2) SU(3) symmetry if orientation is preserved, otherwise the symmetry group contains also reflections, see also an SU(4) Lagrangian, Appendix C. It is important to say that Vaknin's approach [13] is diametrically opposed to that of Jungjai Lee and Hyun Seok Yang [3].

A conformal map vs. gauge transformation

Consider a local gauge transformation G, and the acceleration matrix (1.1),

$$A_{\mu\nu} = \left(\frac{P_\mu}{\sqrt{Z}}\right),_\nu - \left(\frac{P_\nu}{\sqrt{Z}}\right),_\mu \qquad (13.7)$$

And the gauge transformation

$\frac{P_\lambda}{\sqrt{Z}} \longrightarrow G^\lambda{}_\mu \frac{P_\lambda}{\sqrt{Z}}$

Then

$$\left(G^\lambda{}_\mu \frac{P_\lambda}{\sqrt{Z}}\right);_\nu \neq G^\lambda{}_\mu \left(\frac{P_\lambda}{\sqrt{Z}}\right);_\upsilon \tag{13.8}$$

And therefore, the Gauge-Covariant derivative operator
$\left[\begin{smallmatrix} g \\ ; \end{smallmatrix}\right]_\nu$ would be used with the ordinary gauge field $G^\lambda{}_\mu$

$$\left(G^\lambda{}_\mu \frac{P_\lambda}{\sqrt{Z}}\right)\left[\begin{smallmatrix} g \\ ; \end{smallmatrix}\right]_\nu \equiv \left(G^\lambda{}_\mu \Delta_\nu - igS^\lambda{}_{\mu\nu}\right)\left(\frac{P_\lambda}{\sqrt{Z}}\right) \tag{13.9}$$

Where Δ is the covariant derivative and where g is the coupling coefficient and $S^\lambda{}_{\mu\nu}$ is expressed by vectors of the Lie Algebra of G.

When choosing $G_{kj} dx^k \wedge dx^j$ to be an exact form, we do not have a Lie group because multiplication of anti-symmetric matrices is not closed, and the transformation is usually not a gauge transformation but a conformal map when non-degenerate, however, similar to $\left(\left(\frac{P_k}{\sqrt{Z}}\right),_j - \left(\frac{P_j}{\sqrt{Z}}\right),_k\right) dx^k \wedge dx^j$, with $A_{kj} = \left(\frac{P_k}{\sqrt{Z}}\right),_j - \left(\frac{P_j}{\sqrt{Z}}\right),_k = \frac{U_k}{2}\frac{P_j}{\sqrt{Z}} - \frac{U_j}{2}\frac{P_k}{\sqrt{Z}}$

Demanding non-dependence of $\left(\frac{G_{\mu\lambda}P^\lambda}{\sqrt{Z}}\right);^s$ on derivatives of $G_{\mu\lambda} = \omega_\mu;_\lambda - \omega_\lambda;_\mu$ leads to

$$\frac{P^\lambda}{\sqrt{Z}}\left(G_{\mu\lambda};_\nu - G_{\nu\lambda};_\mu\right) = \frac{P^\lambda}{\sqrt{Z}}\left(-G_{\lambda\mu};_\nu + G_{\lambda\nu};_\mu\right) = 0 \tag{13.10}$$

$$G_{\lambda\nu};_\mu - G_{\lambda\mu};_\nu = \omega_\lambda;_\nu;_\mu - \omega_\nu;_\lambda;_\mu - \omega_\lambda;_\mu;_\nu + \omega_\mu;_\lambda;_\nu = \omega_\beta R^\beta_{\lambda\nu\mu} - \omega_\nu;_\lambda;_\mu + \omega_\mu;_\lambda;_\nu$$

$$-\omega_\nu;_\lambda;_\mu = -\omega_\nu;_\lambda;_\mu + \omega_\nu;_\mu;_\lambda - \omega_\nu;_\mu;_\lambda = -\omega_\beta R^\beta_{\nu\lambda\mu} - \omega_\nu;_\mu;_\lambda$$

$$\omega_\mu;_\lambda;_\nu - \omega_\mu;_\nu;_\lambda + \omega_\mu;_\nu;_\lambda = \omega_\beta R^\beta_{\mu\lambda\nu} + \omega_\mu;_\nu;_\lambda$$

$$\omega_\beta\left(R^\beta_{\lambda\nu\mu} - R^\beta_{\nu\lambda\mu} + R^\beta_{\mu\lambda\nu}\right) + \omega_\mu;_\nu;_\lambda - \omega_\nu;_\mu;_\lambda = \omega_\beta\left(R^\beta_{\lambda\nu\mu} + R^\beta_{\nu\mu\lambda} + R^\beta_{\mu\lambda\nu}\right) + \omega_\mu;_\nu;_\lambda - \omega_\nu;_\mu;_\lambda$$

And by the first Bianchi identity $R^\beta_{\lambda\nu\mu} + R^\beta_{\nu\mu\lambda} + R^\beta_{\mu\lambda\nu} = 0$

$$\frac{P^\lambda}{\sqrt{Z}}\left(G_{\lambda\nu};_\mu - G_{\lambda\mu};_\nu\right) = \frac{P^\lambda}{\sqrt{Z}}\left(\omega_\mu;_\nu;_\lambda - \omega_\nu;_\mu;_\lambda\right) = G_{\mu\nu};_\lambda \frac{P^\lambda}{\sqrt{Z}} = 0 \tag{13.11}$$

$$\left(\frac{G_{\mu\lambda}P^\lambda}{\sqrt{Z}}\right);_\nu - \left(\frac{G_{\nu\lambda}P^\lambda}{\sqrt{Z}}\right);_\mu = G_{\mu\lambda}\left(\frac{P^\lambda}{\sqrt{Z}}\right);_\nu - G_{\nu\lambda}\left(\frac{P^\lambda}{\sqrt{Z}}\right);_\mu + G_{\mu\nu};_\lambda \frac{P^\lambda}{\sqrt{Z}}$$

$G_{\nu s}$ acts as rotation and scaling on both indices λ and s which means that,

$$G_{\nu\lambda}G_{\mu s}\frac{P^{\lambda}Z^{s}}{2\sqrt{Z}} - G_{\mu\lambda}G_{\nu s}\frac{P^{\lambda}Z^{s}}{2\sqrt{Z}} = G_{\nu\lambda}G_{\mu s}\left(\frac{P^{\lambda}Z^{s}}{2\sqrt{Z}} - \frac{P^{s}Z^{\lambda}}{2\sqrt{Z}}\right) = w^{2}\tilde{G}_{\nu\lambda}\tilde{G}_{\mu s}\left(\frac{P^{\lambda}Z^{s}}{2\sqrt{Z}} - \frac{P^{s}Z^{\lambda}}{2\sqrt{Z}}\right) \tag{13.12}$$

where $\tilde{G}_{kj}$ are rotation matrices and w is a scalar function.

$$\frac{1}{2}w^{2}\tilde{G}_{\nu k}\tilde{G}_{\mu j}A^{kj}w^{2}A_{sr}\tilde{G}^{s\nu}\tilde{G}^{r\mu} = w^{4}\frac{U_{\mu}U^{\mu}}{4} \tag{13.13}$$

which means that $G_{\mu\nu}$ acts as a scaling on the action by the Reeb class vector as expected from a Scarr-Friedman type of matrix [10] acting twice on a vector. The addition $G_{\mu\nu};_{\lambda}\frac{P^{\lambda}}{\sqrt{Z}}$ was first missed by the author and was later corrected. (3.18) offers coupling between $G_{\mu s}$ tensors that is reducible to a classical non-covariant energy of the electric field.

Now, replacing P_{μ} in which case $\tau = PP^{*}$ is a Geroch function, with ψ such that $\psi\psi^{*}$ integrates to 1. i.e. it is an event wave function, we get a theory like Sam Vaknin's chronon theory [13], providing $\psi_{\mu} = \frac{\partial\psi}{\partial x^{\mu}}$ is Almost Everywhere smooth and non-degenerate. So, this theory lacks bosons as carriers of interactions because the chronons themselves are the reason for interactions.

2. Ceramic capacitors

In this section we will examine gravitational propulsion, not an Alcubierre's warp drive because the Alcubierre [14] extrinsic curvature condition $(K_{i}{}^{i})^{2} - K_{ij}K^{ij} < 0$ will not hold in the same geometry as in the Alcubierre warp drive bubble. However, a negative plate below and a positive plate above, will manifest weak acceleration upwards as the negative gravity will push the positive plate upwards and the negative plate will be pulled by the positive plate above it. The main problem is that due to the dielectric material, the mass of the dielectric material will not be gravitationally repelled by the negative plate. Only a small portion of the mass of the capacitor will be affected in a highly dielectric material. Overcoming the anti-alignment, see Fig 2.A., is a technological challenge which cannot be achieved without a dynamic electric field, see Fig 2.B.

Fig. 2.A. – Only a small portion of the mass, in purple, is affected.

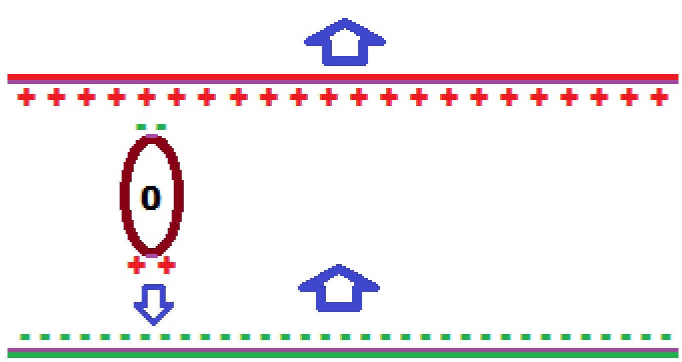
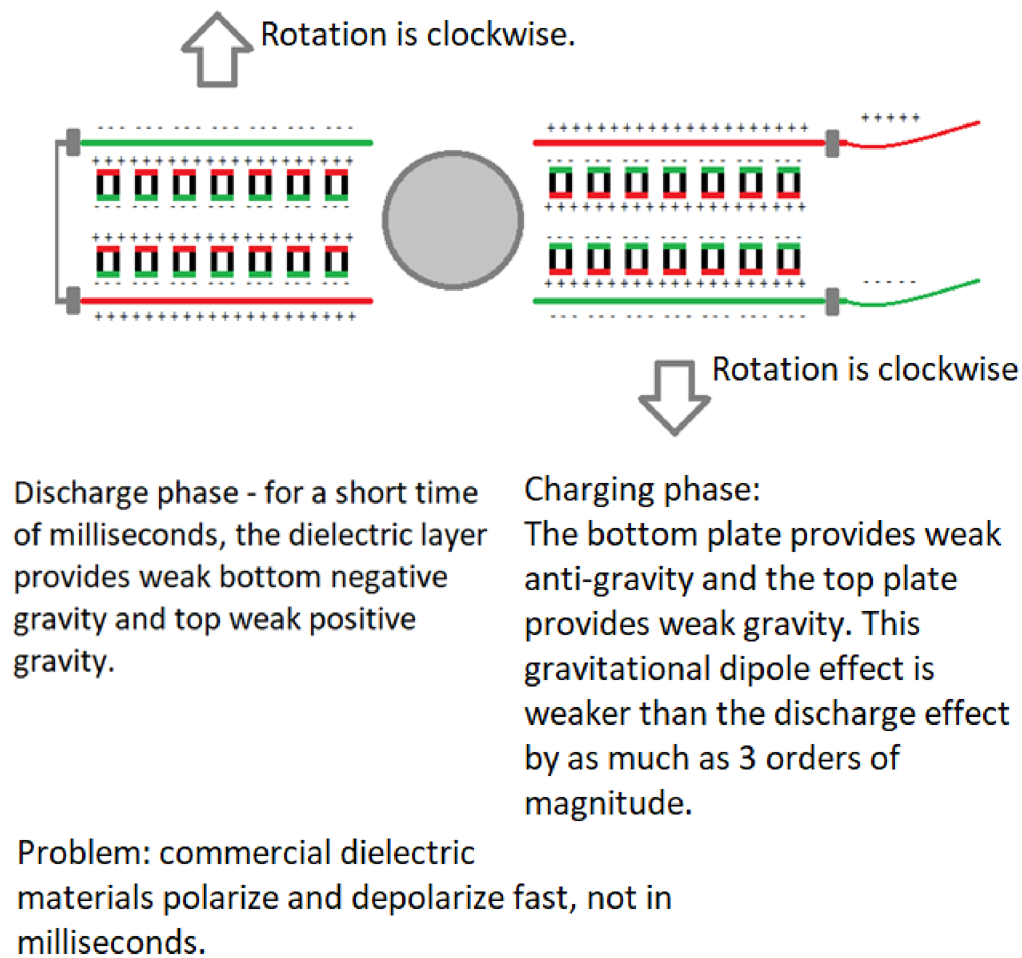


Fig. 2.B. – Electro-gravitational thrust engine with two capacitors and slow anti-alignment dielectric layer, which mitigates the anti-alignment by charging (right) and discharging (left) cycles. The capacitors rotate as depicted in the drawings. The ground direction is bottom. The arrows depict the direction in which the capacitors are rotated by an electric motor. Static field without dielectric anti-alignment requires about $2 * 10^{-4}$ Coulombs / cm^2 in order to accelerate the dielectric layer against the gravity of the Earth. This is why with the current technology, the offered thrust engine is insufficient for a commercial flight. Measurable thrust of up to 1 Newtons is expected with voltage above 2,000,000 volts, relative dielectric constant of above 1000, dielectric polarization time of a millisecond, heavy dielectric layers with mass density similar to Ta_2O_5 but with a higher dielectric constant, electric motor rotation of at least 3000 RPMs and capacitor areas of about 20 x 20 cm^2. Partners in this experiment are Jessica Lynne Suchard and Raviv Yatom. The next figure is **Fig. 2.B,**



It is easy to see from (13) that in the classical limit near the plate, the gravitational field is mostly affected by charge density. By (13) the gravitational acceleration is

$$a \cong \frac{4\pi KQ}{A*\varepsilon*\sqrt{16\pi K\varepsilon_0 \square}} = \frac{V}{d\sqrt{\square}} * \sqrt{\pi K\varepsilon_0} \Longrightarrow \delta Weight \cong \frac{V}{d\sqrt{\square}} * \frac{M_{dielectric}}{g}\sqrt{\pi K\varepsilon_0} = \frac{V\rho A}{g\sqrt{\square}}\sqrt{\pi K\varepsilon_0} \quad (14)$$

where K is Newton's gravitational constant, Q is charge, A is area, ρ is the dielectric layer's density and M is its mass and ε_0 is the permittivity of vacuum, ε is the relative dielectric constant, assuming $\beth = 1$, g is the Earth surface acceleration. (14) is the result of $Q = V\varepsilon_0 \frac{A}{d} = \frac{V}{d}\varepsilon_0 A \Leftrightarrow \frac{Q}{\varepsilon_0 A} = 4\pi \frac{Q}{4\pi\varepsilon_0 A} E$ where E is the classical intensity of the electric field. We saw: $M = \frac{Q}{\sqrt{16\pi K\varepsilon_0 \beth}}$ with $\beth = 1$. The gravitational acceleration by the charge is $a \cong \frac{4\pi KM}{A} = \frac{4\pi KQ}{A*\varepsilon*\sqrt{16\pi K\varepsilon_0 \beth}}$, if we assume an attenuation by the dielectric layer's induced dipoles to be proportional to the attenuation of the electric field by the same induced dipoles. This assumption is problematic because the induced dipoles are the accelerated material by the gravitational dipole of the external plates, and they are in much closer proximity to local charge than to the charge on the external plates. $\delta Weight \cong \frac{V}{d\sqrt{\beth}} * \frac{M_{dielectric}}{g}\sqrt{\pi K\varepsilon_0} = \frac{V\rho A}{g\sqrt{\beth}}\sqrt{\pi K\varepsilon_0}$ is therefore a very optimistic model.

Caution with (14): In reality, the charge of the induced dielectric dipoles is closer to the mass of the dipoles than the external plates. The assumptions of (14) therefore break down and the Inertial Dipole effect is much smaller. One possible technological remedy to this anti-alignment is to add an Alternating Current - AC component to the DC baseline and to disrupt the anti-alignment. Still, even with such a component, a feasible propulsion system may require millions of volts as a baseline. When using voltage above 2 * 511 kV, creation of electron-positron pairs is difficult to avoid (not the Schwinger limit but accelerated electrons through parasitic leakage), and the resulting gamma rays are a serious health hazard. A dynamic voltage and/or current component renders the mathematical description of the Inertial Dipole much more difficult. The following calculations are therefore very optimistic.

Suppose we have a 1000Pf ceramic capacitor and we charge it with 10000 Volts and the area of the plates is 1 cm^2. The charge on the plates is then 10^{-5} Coulombs and its density 10^{-1} Coulombs per square meters. Now we want to calculate the approximate acceleration that the upper positive plate experiences due to the anti-gravity effect from the lower plate. Only a thin portion of the upper layer is affected, where the positive charge accumulates. A calculation shows: 0.48663510306 meters / sec^2. Dividing 0.4866351… meters/sec^2 by 9.81 meters / sec^2 we get 0.049606024776763 which is less than 5 percent relative to the gravity of the Earth. If instead of a dielectric material, an insulator with relative dielectric constant 1 is used for the same charge density of 10^{-5} Coulombs per 1 cm^2, a weight loss of the insulating slab should be measured at about 0.0496 of its weight. With a high relative dielectric constant, the affected mass could be well below 1 milligram, and it will lose 0.0496 of its weight. This renders the measurement of such an effect very hard to achieve unless the dielectric material is saturated and can no longer shield the field of the plates such as in the H4D experiment [15]. In any other case, practically no measurable thrust is expected for an area 1cm^2 with 10,000 Volts and scale resolution worse than 10^{-4} grams. In the case of saturation, at first the inertial dipole is expected to grow with the saturation of the dielectric material and with the amount of charge on the plates. [15] will be discussed later. The H4D lab [15] 69 mm radius and 2mm PMMA thickness capacitor with 20,000 volts, weight loss is at least **0.0015509 grams**, however the thickness of the metal plates is 1mm. It is sufficient to have a low frequency AC ripple from the DC power supply to churn the electrons on the plates such that not only a thin layer of the plates will be charged, also with an AC ripple, of typically 150 VAC for 20000 Volts DC, the induced gravitational field can no longer be considered static. Under such conditions (14) is no longer valid.

3. Thrust from 1000 Pf capacitor with two metallic plates and 10000 volts

Assumptions: Most of the dielectric mass is not completely shielded from the plate fields and the attenuation of the influence of the external dipole on the mass within the induced dipole is by a factor ε^{-1}, where ε is the relative dielectric constant. If this assumption does not hold true then (14) is invalid. Such a problem may occur at least theoretically even if in total the dielectric constant is low only because of low mass density. A second assumption is that dielectric dipoles are evenly distributed within the dielectric layer. A third assumption is a low alternating current – AC component in the power supply and that the influence of the Inertial Dipole on the metal plates is negligible due to the charge concentrating on the metallic surfaces which are in contact with the dielectric material. A high AC component might disrupt electrons alignment on the plates and if the plate's thickness is not negligible then (14) is no longer valid. Also, if the dielectric material reaches saturation and the metallic plates are thick in relation to the dielectric layer, the charge distribution on the plates can no longer be limited to the contact surfaces with the dielectric layers which also results in (14) being no longer valid.

Suppose we have a high voltage ceramic capacitor of 1000Pf of **Ta2O5** [16] with each plate area $1cm^2$ which is charged by 10,000 volts. The permittivity of vacuum is about 8.8541878128 * 10^{-12} Farads*$meter^{-1}$. So we can calculate the distance d between the plates, 8.8541878128 * 10^{-12} Farads * $metere^{-1}$ * 10^{-4} $meters^2$ * d^{-1} * 25 = 10^{-9} Farads. That means d ~ 0.22135469532 * 10^{-1} mm or d ~ 0.22135469532 * 10^{-2} cm. Now we take into account the weight density of the Ta2O5 which is 8.2 grams perm $1cm^3$ volume. So we have 8.2 * 1cm * 1cm * 0.22135469532 * 10^{-2} cm = 0.01815108501624 grams. At 10000 volts the weight loss is of a portion of 0.04960602477676315711411588216388 of the weight of the dielectric material and the inertial dipole is attenuated by the relative dielectric constant 25 just as the electric field is. So we have 0.01815108501624 grams * 0.04960602477676315711411588216388 * 25^{-1} ~ ~**3.60161*10^{-5} grams weight loss**. This estimate can be much lower in a multilayered capacitor where fields cancel out or when the dielectric constant is higher and the dipoles density is not uniform.

4. Martin Tajmar experimental null results analysis

Martin Tajmar [17] used a capacitor of a relative dielectric constant 4500 and a Teflon [18] capacitor with radius 50 mm and Teflon thickness d=1.5 mm and 10,000 Volts. The highly dielectric capacitor weight loss is way below the experiment **scale resolution 3 * 10^{-4} grams** due to division by 4500 of the charge which is 10^{-5} per 1000Pf capacitance. With a radius of 0.5cm, such a capacitor with say 6.02 grams * cm^{-3} density will lose about **2.077389 * 10^{-5} grams**. Next focus is on one of the Teflon capacitors. The gravitational acceleration on the face of the Earth, about g=9.80665 meter * sec^{-2}. By (14), the result is **7.5917876115 * 10^{-6} grams**. This result is smaller than the resolution of 3 * 10^{-4} grams. The results assume $ב = 1$ in (4), (7), (13). It is important to say that unlike Martin Tajmar (sounds as Taymar), the Brazilian H4D experiment [15] used much greater capacitor areas. A significant AC ripple cannot be ruled out.

Important: General research directions for finding astronomical evidence for charge-based gravity and anti-gravity:

1) If a small galaxy collided with a large dust cloud or with another galaxy, it should be positively charged. Due to near electrostatic repulsion, star formation must be very low, however, a Dark Matter effect i.e. unexpected gravity must be higher in comparison to other galaxies, hopefully by more than 10%. On the other hand, collisions between galaxies can emit electrons to neighboring galaxies in the same cluster, which will then manifest a very weak Dark Matter effect due to the excess in negative charge,

2) A large, isolated galaxy with billions of light years of minimal distance to other galaxies should have sufficient time for the electrons it lost to fall back as the galaxy gets older and cooler. If there are such galaxies, then they must have a weak Dark Matter effect or no Dark Matter effect at all, despite the fact that they are big, e.g. the size of the Milky Way.

3) Electrons have a light weight and are easily accelerated to relativistic speeds which helps them to escape the galactic pull. In intergalactic space, they should cause a gravitational repulsion and the expansion of the cosmos. Therefore, cosmic expansion should be faster when there are more free electrons.

5. Particle mass ratios – a reverse engineering ansatz approach

Motivation: solving (4) analytically is extremely hard, let alone, the more general Lagrangians that will be presented in (64) and (65) for complex Reeb class vectors. One possible way to tackle this challenge is to rely on a theorem by Georges Reeb, according to which the restriction of the field to the three-dimensional foliation perpendicular to $\frac{P_\mu}{\sqrt{Z}}$, must have a zero rotor.

Theorem 3 (Reeb): The rotor of η, the acceleration field or as better known as Reeb class, when restricted to the perpendicular foliation to α such that $d\alpha = \pm\eta\wedge\alpha$, $(D\eta)\wedge\alpha$ is zero.

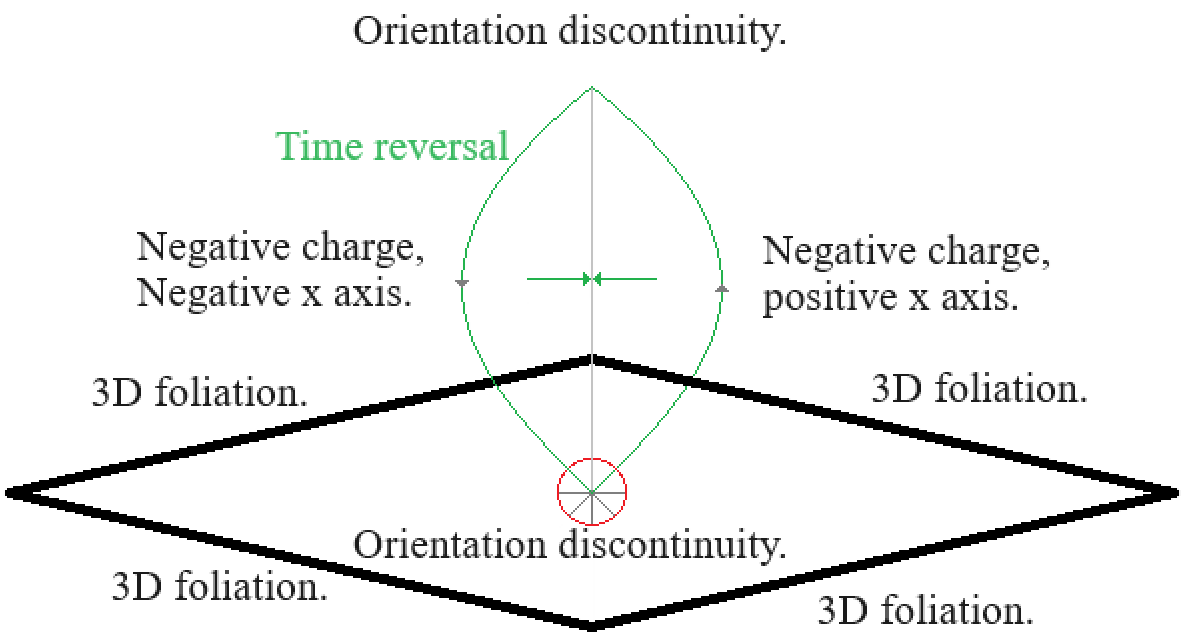
Proof: Using exterior derivative $D\frac{P_\mu}{\sqrt{Z}}dx^\mu = D\alpha = \pm\eta\wedge\alpha = \left(\frac{U_\mu}{2}\frac{P_\nu}{\sqrt{Z}} - \frac{U_\nu}{2}\frac{P_\mu}{\sqrt{Z}}\right)dx^\mu\wedge dx^\nu$

We now take the exterior derivative of $D\alpha = \eta\wedge\alpha$ and get $DD\alpha = (D\eta)\wedge\alpha - \eta\wedge(D\alpha) = 0$ because $D\alpha$ is an exact form. $DD\alpha = (D\eta)\wedge\alpha - \eta\wedge(D\alpha) = (D\eta)\wedge\alpha - \eta\wedge\eta\wedge\alpha = 0$ but $\eta\wedge\eta = 0$ so $\eta\wedge\eta\wedge\alpha = 0$ and therefore $DD\alpha = (D\eta)\wedge\alpha = 0$ Q.E.D. Let the lower indices denote covariant vector components, not derivatives and comma will denote derivatives, then $D\eta = (\eta_{\mu,\nu} - \eta_{\nu,\mu})dx^\mu\wedge dx^\nu$ and $(D\eta)\wedge\alpha = (\eta_{\mu,\nu} - \eta_{\nu,\mu})\alpha_\lambda dx^\mu\wedge dx^\nu\wedge dx^\lambda$ which means that the restriction of the rotor of $\eta_{\mu,\nu} - \eta_{\nu,\mu}$ to the foliation perpendicular to α_λ is zero and therefore the projection of $\frac{U_\mu}{2}$ on the foliation perpendicular to $\frac{P_\mu}{\sqrt{Z}}$ is of a conserving field.

Corollary 4 to theorem 3: $(D\eta)\wedge\alpha = 0 \Longrightarrow D((D\eta)\wedge\alpha) = 0 \Longrightarrow (DD\eta)\wedge\alpha + (D\eta)\wedge D\alpha = 0 \Longrightarrow (D\eta)\wedge D\alpha = (D\eta)\wedge\eta\wedge\alpha = 0$, which means that $(D\eta)$ must not span the Hodge star of $\eta\wedge\alpha$ or in other words $\frac{U_{\mu,\nu} - U_{\mu,\nu}}{2}$ is a bivector that must depend on $\frac{P_\mu}{\sqrt{Z}}$ if it is not zero, because if it doesn't then by theorem 3, $\frac{U_{\mu,\nu} - U_{\mu,\nu}}{2}$ would be 0, Q.E.D.

Corollary 4 implies a time reversal violation at foliations points where $\frac{U_{\mu;}{}^{\mu}}{2} \neq 0$ or $\frac{1}{2}(\frac{U_{\mu;}{}^{\mu}}{2} + \frac{U^*{}_{\mu;}{}^{\mu}}{2}) \neq 0$ in the complex case. $\frac{U^\mu}{2}$ or $\frac{1}{2}(\frac{U^\mu}{2} + \frac{U^{*\mu}}{2}) = \frac{a^\mu}{c^2} = \frac{d^2x^\mu}{(d\tau)^2} = \frac{d^2x^\mu}{(-d\tau)^2}$ and therefore, time reversal does not change the vector $\frac{a^\mu}{c^2}$ where c is the speed of light. It is obvious that to change the sign of U^μ it is not sufficient to change only $d\tau$ to $-d\tau$. What does change sign, is velocity, $\frac{dx^\mu}{(-d\tau)} = -\frac{dx^\mu}{d\tau}$ but the field a^μ does not change sign, which means integration along the same line, one with the time and one back in time should be zero. Consider the coordinates map $x^\mu \longrightarrow -x^\mu$ which combines time reversal and spatial reflection, a.k.a Parity. In that case spin direction is maintained and charge is maintained. Under time reversal alone, charge is maintained. This is not surprising because this theory is not a hermitian theory but a geometry based theroy and the discussion here is of a single source of Reeb class divergence, not of interactions between particles. The Hodge star extension of $\eta\wedge\alpha$ is the extension of the field to angular acceleration and can have two different signs in the real case, left handed and right handed. Parity transormations $x^\mu \longrightarrow -x^\mu, \mu = 1,2,3$ do affect this sign.

Fig. 3. – Spatial reflection (Parity) and time reversal



Time asymmetry – the difference between this model and the Hermitian representation of Quantum Mechanics

Roberts makes a two-step argument to show time reversal in Quantum Mechanics. The first is based on Uhlhorn theorem and on Wigner's theorem.

Roberts shows that time reversal can only be anti-unitary [19].

Uhlhorn theorem: Let T denote a linear bijection on the projection space of separable Hilbert space H with dimension greater than 2. Then if $\varphi \perp \phi \Leftrightarrow \langle\varphi,\phi\rangle = \langle T\varphi, T\phi\rangle$, there exists a unique operator $\tilde{T}$ up to a constant, which implements T, $\tilde{T}: H \longrightarrow H$ such that, $\tilde{\varphi} \in \varphi \Leftrightarrow \tilde{T}\tilde{\varphi} \in T\varphi$ and which satisfies $|\langle T\tilde{\varphi}, T\tilde{\phi}\rangle| = |\langle\tilde{\varphi},\tilde{\phi}\rangle|$ for all $\tilde{\varphi},\tilde{\phi} \in H$ [20].

Uhlhorn's idea is that independent states or mutually exclusive ones, must not depend on the direction of time.

Wigner's Theorem: For any operator T satisfying the Uhlhorn Theorem, there is a Hilber space operator $\tilde{T}$ that implements T which is either unitary or anti-unitary [21].

Then Roberts shows in sufficiently simple Hamiltonians that time reversal is equivalent to the action of an anti-unitary operator, which does not include the electroweak Hamiltonian.

Regarding this paper, the closest consideration to Roberts [20] and Wigner's theorem is to replace $f(P)$ in Appendix H by $f(P) = e^{-iP}$ or $f(P) = e^{iP}$ and see that the Lagrangian of the

Reeb Class vector does not change, however, the Hodge star of $\frac{P_\mu}{\sqrt{Z}} \wedge \frac{U_\nu}{2}$ does change sign with time reversal see $B^{\mu\nu} = \frac{1}{2} E^{\mu\nu\alpha\beta} A_{\alpha\beta}$ after (3). Multiply $E^{\mu\nu\alpha\beta}$ by -1 for time reversal.

From theorem 3, since this model means the acceleration field $\frac{U_\mu}{2}$ is a representation of the electric field, in order to derive the dynamics of charge as we know it from classical mechanics, we need to contract equation (4) twice by $\frac{P_\mu}{\sqrt{Z}}$. In other words, the field must have drains and sources, by which the divergence of the field is not zero. The result of this theorem is that as the far observer $r \to 0$ in source or drain of the field, particles formation is inevitable and linearization of (4) as an approximation should be considered. This section will try to find a relationship between an acceleration as $\sqrt{\left|a^\mu a_\mu\right|} = \xi' \frac{c^2}{r}$ for some ξ' and the norm of the Reeb class field $\sqrt{\frac{1}{8}\left|U^\mu U^*{}_\mu + U^{*\mu} U_\mu\right|} = \xi' \frac{1}{r}$ for some ξ'. In fact, this section considers (4) as $r \to 0$ as an attempt to avoid the extremely hard analytic solutions. The relationship between $\frac{1}{r}$ and $\sqrt{\frac{1}{8}\left|U^\mu U^*{}_\mu + U^{*\mu} U_\mu\right|}$ is not bases on rigorous mathematics although there are rational explanations to these relationships.

The following section will try to reach the Reeb class field strengths of the electron, Muon and Tau Lepton. It will also try to reach the Reeb class field strength for the W and Z bosons. As we shall see, for the first 3 values, the assessment is $\frac{95}{96}, \frac{4}{\pi}$ and ~1.5561985371903483965638770314399…

As we shall see $\frac{95}{96} = 1 - \frac{1}{64} + \frac{1}{192} = \frac{193}{192} + \frac{63}{64} - 1$, which can be interpreted as the summation of two fundamental states of the field. However, to keep an open mind, other possible reasons, although less plausible, are also brought into the discussion. The only value that does not come directly from this theory is $\frac{4}{\pi}$. It has a compelling Quantum Mechanics source; however, other less plausible explanations are also considered. The last value, ~1.55619853719, is derived from maximal imbalance between gravity and anti-gravity. For the W boson, two possible field strength coefficients are discussed $\frac{4}{\pi}$ and $\frac{4}{3}$. The latter yields a higher mass for the W boson although the author tends to accept $\frac{4}{\pi}$ and not $\frac{4}{3}$.

In this section, equation (4) is explored in a small infinitesimal sphere, where we assume a linear relation between a far observer radius r and acceleration $\frac{a^\mu}{c^2} = \frac{U^\mu}{2} = \frac{Z^\mu}{2Z} - \frac{Z^k P_k P^\mu}{2Z^2}$, see (1), (2). Our goal is to reduce (4) from a four-dimensional Minkowsky geometry to a three-dimensional Riemannian geometry and then to a two-dimensional Riemannian geometry of surfaces.

We make the following assumption:

$$\frac{\|a^{\mu}\|}{c^2} = \frac{\xi}{rx} \tag{15}$$

Where, c is the speed of light, ξ is a coefficient that depends on the field as $r \to 0$ and the variable x changes with the density of the field as it passes through a two-dimensional sphere. x is required because space-time curvature can cause such a sphere to be less than or more than $4\pi r^2$.

Note: A natural question due to (10) is, when does the acceleration $\frac{\xi c^2}{r} = \frac{e}{4\pi\varepsilon_0 r^2}\sqrt{4\pi\varepsilon_0 k}$, such that e is the charge of the electron, k is Newton's gravity constant, ε_0 is the permittivity of vacuum and c is the speed of light? The answer is $r = \frac{e}{\xi c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}$ and for $\xi = 1$, $r = \frac{e}{c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}$,which by the order of the inverse of the square root of the Fine Structure Constant is smaller than the Planck length, $\frac{e}{c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}\alpha^{-\frac{1}{2}} = \frac{e}{c^2}\sqrt{\frac{k}{4\pi\varepsilon_0}}\sqrt{\frac{4\pi\varepsilon_0\hbar c}{e^2}} = \sqrt{\frac{\hbar k}{c^3}}$, where $\hbar$ is the reduced Planck constant. This calculation of course, assumes that in such a strong field, the permittivity is that of vacuum and is not affected by virtual electric fields that attenuate the electric field. It is also limited to the far observer coordinates system.

We also make other assumptions as follows:

1) Assumption 1: In small radii, the energy of the gravitational field depends on the area around the source of gravity. This assumption is consistent with the paper of Ted Jacobson [22].

2) Assumption 2: The area ratio that has a physical meaning is between a disk to which the unit vector $\frac{P^{\mu}}{\sqrt{Z}}$ points to and the 1 weighted Euclidean sphere $\lambda * \pi r^2$ so $\lambda = 4$. The area loss of a disk is $\frac{\pi}{24} Rr^4$, where R is obtained by contracting Einstein's tensor twice with a time-like vector $\frac{P^{\mu}}{\sqrt{Z}}$ and r is an infinitesimal radius. However, we consider $\frac{1}{4}\frac{\pi}{24} Rr^4 = \frac{\pi}{96} Rr^4$. As we divide this area by Euclidean disk area, we get $\frac{\pi}{96} Rr^4 * (\pi r^2)^{-1} = \frac{1}{96} Rr^2$. Following are explanations to the factor $\frac{1}{4}$.

   The primary explanation and its contender: Ratio between a length atom and the square root of an area atom: It is easy to see that for (13.02) to hold, a factor $\frac{1}{4}$ must be considered, when describing electric charge as a sphere and by Occam's razor that would be a closed case argument. However, there is an explanation which is more interesting. In the complex version of equation (4), the Geroch function [1] PP^* can be replaced with a probability density function of an event $\psi\psi^*$ and except for a set of measure zero, $\psi_\mu = \frac{d\psi}{dx^\mu}$ is not zero. In that case $\int \psi\psi^* \sqrt{-g}dx^0dx^1dx^2dx^3 = 1$ and the constraint should be added to the Lagrangian, $Action = Min \int_\Omega \left(R - \frac{1}{4ℶ}U^kU_k + \lambda\psi\psi^*\right)\sqrt{-g}\, d\Omega$ where λ is a constant of units $\frac{1}{Length^2}$, which implies the existence of an atom of area. The relation between a length atom and an area atom should be $\frac{1}{2}\sqrt{Area} = Length \Longrightarrow \frac{1}{4}Area = Length^2$. There is more than one reason for this relation. The simplest is that an area is measured around an event in spacetime in a sub-plane of spacetime. The time is then an exception because it has a direction and only one direction around an event can be considered. The relation between a 4-volume to such length unit is then $Area^2 = (2Length)^3Length = 8Length^4$, providing that a time differential $cdt = dLength$ is treated as delta length.

Caveat: do not confuse the use of the term Area in a 4-volume relation to length, with an Area in area to length ratio, it is not the same term as in $\frac{1}{4}Area = Length^2$. The reason for the term $Area^2$ can be seen in “Hodge star spin-like field extension” and is used in (3). (3) describes two planes of acceleration, one of a boosting acceleration and one of a rotation. The relations, $\frac{1}{2}\sqrt{Area} = Length \Longrightarrow \frac{1}{4}Area = Length^2$ and $Area^2 = (2Length)^3Length = 8Length^4$ are a result of discussions with a colleague Aryeh Aldema who unfortunately passed on 30/December/2022. This idea had led to an assessment of the inverse Fine Structure constant. Aryeh was not sure about the exact ratio between atoms of lengths, areas and 4-volumes but insisted that such ratios must be used in this paper.

The simplest explanation: The simplest explanation is that the portion of the gravitational energy that can be used when particles decay, is only from the time-like field $\frac{P(0)_\mu}{Z(0)}$ in (3.12) from which the factor $\frac{1}{4}$ comes from. This explanation is however problematic even if $\frac{P(i)_\mu}{Z(i)}$ are complex functions as in (3.13) because 8 scalar functions may not be sufficient to describe gravity.

Blackhole thermodynamics - Bekenstein and Hawking entropy and area: see the relation $S_{BH} = \frac{1}{4}K_B\frac{A}{{\ell_p}^2}$ [23] where A is area, ℓ_p is the Planck length, and K_B is Boltzmann’s constant. We assume entropy is related to particles decay.

Mathematically and physically compelling explanation: We return to the principles of the chronon field by Sam Vaknin [13] in which the time arrow is defined via spin and thus via orientation: There are two orientations to be considered. The first is the orientation of the foliation that is perpendicular to $\frac{P^\mu}{\sqrt{Z}}$. The second is the plane within that foliation which is perpendicular to $\frac{P^\mu}{\sqrt{Z}}$ and to $\frac{U^\mu}{2}$. In each case only one side of a 3D foliation and one side of a plane can be related to energy and $\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$.

Causal triangulation explanation: A polygonal graph is a graph in which vertices on a circle relate to edges and each vertex is also connected to the center. So, for the m vertices of the polygon and one vertex of the center, the graph has m+1 vertices. We also assume m=2n for some natural number n. The graph has 2m = 4n edges, m connecting the polygon vertices, each vertex to 2 neighbors and m connecting the polygon vertices to the center. Using graph theory techniques, it is easy to see that a random walk for a large n on such polygonal graph reaches a probability $\frac{1}{4}$ at the center and $\frac{3}{4m}$ at each polygon vertex. The probability of moving from a vertex on the polygon to one if its two neighbors is $\frac{1}{3}$ for each neighbor and to the center $\frac{1}{3}$. The probability of reaching one node of the polygon from the center is $\frac{1}{m}$. Seeing a particle as a loop with or without a center is beyond the scope of this paper, however, such a model under random walk reaches the unique probability $\frac{1}{4}$ at the center and is worth mentioning as another approach to area related to energy as $\frac{\pi}{96} R r^4$ instead of $\frac{\pi}{24} R r^4$, where R obtained by contracting Einstein's tensor twice with a timeline vector $\frac{P^\mu}{\sqrt{Z}}$ and r is an infinitesimal radius. The python code for the random walk calculations is brought here:

```
import numpy as NP
import numpy.linalg as LA

print('Random walk on 24-Polygonal  graph with a center.')
matrix = NP.zeros((25, 25), dtype=NP.float64)
a = 1/24
b = 1/3
for i in range(1, 25):
    matrix[0, i] = b
    matrix[i, 0] = a
    k = i + 1 if i < 24 else 1
    matrix[i, k] = b
    k = i - 1 if i > 1 else 24
    matrix[i, k] = b
    w, v = LA.eig(matrix)
    scale = v[:, 0].sum()
    v[:, 0] /= scale

print('Eigenvector of probability:')

for i in range(25):
    print(f'v[{i}]={v[i, 0]}')
```

```
    print(f'Eigenvalue {w[0]}')

The output is:
Random walk on 24-Polygonal graph with a center.
Eigenvector of probability:

v[0]=0.24999999999999997
v[1]=0.03124999999999989
...
```

The Causal Set interpretation (87)-(90) and its relation to the number 96 and the Fine Structure Constant cannot be ignored!

Non rigid explanation: This idea is derived from a physical principle according to which a spin of a particle always either points to an observer or in the opposite direction. In this manner, the observer can only refer to the disc which is perpendicular to the spin axis and not to an entire sphere. An area ratio $\frac{\pi r^2}{4\pi r^2} = \frac{1}{4}$ means 0 gravity.
This assumption means that the delta area of a curved sphere divided by $4\pi r^2$ is $\frac{\delta \pi r^2}{ℷ * \pi r^2}$ and not $\frac{\delta 4\pi r^2}{4\pi r^2}$. There could be other explanations to this assumption including a choice of $32\pi K$ in (7) instead of $8\pi K$ and $\frac{1}{16}$ instead of $\frac{1}{4}$ in (4), however to the author's opinion, (43) does not support such other explanations.

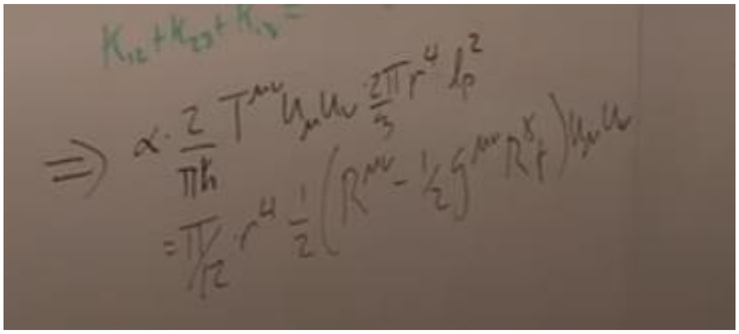
We revisit equation (4) and contract it twice with the unit vector $\frac{P^\mu}{\sqrt{Z}}$ which means a chosen time direction $\frac{1}{4ב}\left(U_\mu U_\nu - \frac{1}{2} g_{\mu\nu} U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right)\frac{P^\mu P^\nu}{Z} = (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu})\frac{P^\mu P^\nu}{Z}$

Since $U_\mu P^\mu = 0$, and assuming $ב = 1$, we have around an electric charge by (15) and an infinitesimal linearization of (15) such that U_μ is space-like and $\frac{U_\lambda U^\lambda}{4} \cong \xi \frac{1}{r}$, where ξ is a field strength coefficient, $r \to 0$ radius and we can write $a = \xi \frac{c^2}{r}$ were c is the speed of light and a represents an acceleration.

$$\frac{1}{ב}(-\frac{1}{2}\frac{U_\lambda U^\lambda}{4} - \frac{1}{2} U^k;_k) = \frac{1}{ב}(-\frac{1}{2}\frac{\xi^2}{r^2 x^2} \mp \frac{\xi}{r^2 x}) = (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu})\frac{P^\mu P^\nu}{Z} \tag{16}$$

We calculated the divergence of a field of a non-geodesic acceleration from intensity $\frac{\xi}{rx}$ to 0 along the distance r. The divergence $U^k;_k$ can be either positive or negative and depends on the sign of the electric charge. We now refer to Seth Lloyd lecture [24],

Fig. 4. Area gain or loss in the direction of a unit vector:



As we see, to get the area loss on a disk which is perpendicular to the unit vector $\frac{P^{\mu}}{\sqrt{Z}}$ due to curvature, we need to multiply (16) by $\frac{\pi}{12}\frac{1}{2}r^4 = \frac{\pi}{24}r^4$.

$$\frac{1}{\text{ב}}\left(-\frac{1}{2}\frac{\xi^2}{r^2x^2} \mp \frac{\xi}{r^2x}\right)\frac{\pi}{24}r^4 = \frac{1}{\text{ב}}\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{\pi}{24}r^2 = AreaLossOfADisk \tag{17}$$

By our second assumption, the following has a physical meaning, where $\text{ג} = 4,\ \text{ב} * \text{ג} = 1 * 4 = 4$

$$\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = \frac{1}{\text{ב}*\text{ג}}\frac{1}{\pi r^2}\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{\pi}{24}r^2 = \frac{AreaLossOfADisk}{\text{ג}*\pi r^2} \tag{18}$$

But x should be a ratio between an area around a charge and Euclidean area, according to assumption 2. If x is greater than 1, then by (17), the non-geodesic acceleration field density is decreased by a factor of $\frac{1}{x}$. If the area ratio is smaller 1 then the non geodesic field density is increased by $\frac{1}{x}$. So, we must have the following equation:

$$x = 1 + \frac{AreaLossOfADisk}{4\pi r^2} \Leftrightarrow x - 1 = \frac{AreaLossOfADisk}{4\pi r^2}$$

And by (10) and (12), (18) becomes:

$$\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = \text{x} - 1 \Leftrightarrow 1 + \left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = x \Leftrightarrow \frac{192x^2 \mp 2\xi x - \xi^2}{192} = x^3 \tag{19}$$

The righthand side is expected to be positive around a negative charge and negative around a positive charge if we consider the H4D experimental qualitative result [15] with imprecise balance.

Important: Only convergent roots $\left(\frac{192{x_n}^2 \mp 2\xi x_n - \xi^2}{192}\right)^{\frac{1}{3}} = x_{n+1}$ with iteration parameter n are expected to have a physical meaning. These are the roots which are closest to 1.

The values of ξ that will be explored are $\frac{95}{96}$, $\frac{4}{\pi}$, ~1.5561985371903483965638770314399 15299.

What are the possible values for ξ if we wish to describe the electron, the Muon and the Tau lepton? It is expected that the lower value for ξ and the upper value will be dictated by minimal and maximal possible values for such a field. The middle ξ value which is the field strength of the Muon should not be dictated by such constraints. For example, a semi classical approach to such a field can come from the understanding that a spinning Reeb class field means that it is stronger in the spin plane and zero in the poles. This approach dictates only one possible value of $\xi = \frac{4}{\pi}$.

Consider the averaging of an acceleration field towards the center that depends on $\frac{U^{*\lambda}U_\lambda + U^\lambda U^*{}_\lambda}{8} = \xi \frac{c^2}{r}$ where c is the speed of light,

Then the intensity of the field with respect to the angle θ with the rotation plane should be $\xi \frac{c^2}{r} \cos(\theta)$. At the poles this angle is $\frac{\pi}{2}$ and $-\frac{\pi}{2}$. In the rotation plane this angle is 0. So the average field yields:

$$\frac{2\int_0^{\frac{\pi}{2}} \xi \frac{c^2}{r} \cos(\theta) 2\pi r \cos(\theta) d\theta}{2\int_0^{\frac{\pi}{2}} 2\pi r \cos(\theta) d\theta} = \frac{4\pi r^2 \xi \frac{c^2}{r}\frac{\pi}{4}}{4\pi r^2} = \frac{c^2}{r} \tag{19.1}$$

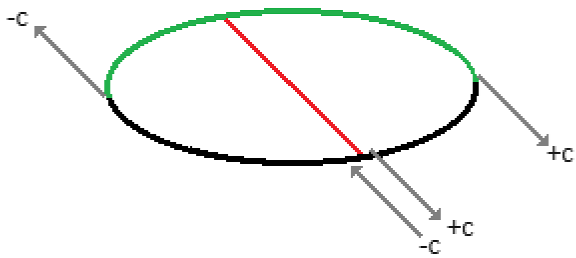
Now we see that in order that the average field will be $\frac{c^2}{r}$ then $\xi = \frac{4}{\pi}$ which means that the field strength in the rotation plane must be stronger than the factor 1 by $\xi = \frac{4}{\pi}$. Without other constraints this should be field strength the defines the Muon, not as the minimal electron and the maximal Tauon in terms of mass. Of course, there are other possible explanations which will all be mentioned but this explanation is by far the simplest. There is another simple explanation.

The primary explanation for the existence of a field strength $\xi = \frac{4}{\pi}$: Another explanation is that changing velocity of from -c to c along half a circle and changing velocity from -c to c along the diameter of the same circle when moving forth and back requires $\xi = \frac{4}{\pi}$ to equate the accelerations.

$$\frac{c-(-c)}{\pi r} = \xi \frac{c-(-c)}{4r} \Longrightarrow \frac{2c}{\pi r} = \xi \frac{2c}{4r} \Longrightarrow \xi = \frac{4}{\pi} \tag{19.2}$$

which explains why the field strength $\xi = \frac{4}{\pi}$ should exist. See the following figure.

Fig. 5.



Caveat: When describing such acceleration as an upper limit on unit vector accelerations, the speed of light does not describe a real physical object.

We will first start with an assumption $\xi = \frac{4}{\pi}$. This assumption is also based on Ettore Majorana's notebook [25] and on the compelling assessment of the critical strength of the Coulomb and the Yukawa potentials [26]. It is also the well-known ratio between a star graph and a Steiner star in Euclidean spaces – *star Steiner ratio* in $\mathbb{R}^d$ [27]. The addition of a middle point in a ball can reduce the length of a star graph in relation to a star where the star graph is defined as straight lines between n-1 points and a single point on the sphere. And a Steiner star connects the points to the center. A physical meaning of such a ratio is that where there is a middle point, divergence of an acceleration field can be defined, where there is no such point, no such divergence can be defined. For such a case, a different value of ξ should be defined. This fact is brought here as thought, not as any proof to why $\xi = \frac{4}{\pi}$. The calculation in (19.1) is much more compelling.

Then (19) yields two solutions as follows,

$$\frac{192{x_1}^2+2\xi x_1-\xi^2}{192} = {x_1}^3 \Longrightarrow \frac{1}{x_1-1} \cong \mathbf{206.75133988502202} \tag{20}$$

This value is surprisingly very close to the mass ratio between the Muon and the electron!

$$\frac{105.6583745\text{MeV}}{0.5109989461\text{MeV}} \cong 206.7682826 \tag{21}$$

The following is an area ratio around a positive charge. The discussion about its meaning is postponed for now.

$$\frac{192{x_2}^2-2\xi x_2-\xi^2}{192} = {x_2}^3 \Longrightarrow \frac{1}{1-x_2} \cong \mathbf{44.63955017596401} \tag{22}$$

Before we continue, we need to prove another theorem which has important implications to Quantum Gravity. The factor $\frac{96}{95}$ is, however, not final in what will be described as Steiner Trees.

Theorem 5: In Riemannian geometry, a computational model for the connection of a finite connected set of points on a sphere S2 and the center with radius r can converge in polynomial time only to a minimal graph of S2 not within radius r but within radius $r\frac{96}{95}$.

Proof: The proof of this theorem is a direct result of the complexity limit of the Minimum Steiner Tree. Finding the minimal length of such a graph is in polynomial time only above $\frac{96}{95}$ of the minimal graph length due to [28]. As a result, to connect all the points in the sphere and its center is possible in polynomial time only for $r\frac{96}{95}$ and we are done. The meaning of this theorem is very deep for most Quantum Gravity theories. For this specific theory, if acceleration depends on r^{-1} then physically the dependence must be on $\frac{95}{96}r^{-1}$. As a caveat, $\frac{96}{95}$ is not believed by the author to be an absolute limit to the hardness of the Steiner Tree problem. Before continuing, a much more compelling explanation for the choice of $\xi = \frac{95}{96}$ for the electron's field strength will be brought. Right now, different options are described.

The primary explanation for the existence of a field strength $\xi = \frac{95}{96}$

By the principle of parsimony, the electron field strength should be explainable by ground state roots of (19). Consider $\xi = x_1$ and $\xi = x_2$ in the following area ratio equations.

$$\xi_1 = x_1 \wedge \left(\frac{1}{2}\frac{{\xi_1}^2}{{x_1}^2} + \frac{\xi_1}{x_1}\right)\frac{1}{96} = x_1 - 1 \Longrightarrow x_1 = \frac{193}{192} \Leftrightarrow \delta x_1 = x_1 - 1 = \frac{1}{192} \tag{22.1}$$

$$\xi_2 = x_2 \wedge \left(\frac{1}{2}\frac{{\xi_2}^2}{{x_2}^2} - \frac{\xi_2}{x_2}\right)\frac{1}{96} = x_2 - 1 \Longrightarrow x_2 = \frac{63}{64} \Leftrightarrow \delta x_2 = x_2 - 1 = \frac{-1}{64} \tag{22.2}$$

Adding these two delta area ratios yields

$$\delta x_1 + \delta x_2 = \frac{1}{192} + \frac{-1}{64} = -\frac{1}{96} \tag{22.3}$$

$$\xi = 1 + \delta x_1 + \delta x_2 = x_1 + x_2 - 1 = \frac{193}{192} + \frac{63}{64} - 1 = \frac{95}{96} \tag{22.4}$$

Definition: $\xi_1 = \frac{193}{192}$ and $\xi_2 = \frac{63}{64}$ will be called Stability Field Strengths and $\xi = \frac{95}{96}$ is called Joint Stability Field Strength. $\xi = \frac{95}{96}$ is the first candidate for the electron field strength that will be used in the Muon/electron mass ratio assessment. It is not difficult to see that for the choice of $\xi = \frac{95}{96}$, also see motivation in Appendix E, (74), (75), (79), and the surprising relation between the Fine Structure Constant and exponential perturbations of $\xi = \frac{95}{96}$ in (81)-(86), the following polynomials yield,

$$\left(-\frac{1}{2}\frac{(\frac{95}{96})^2}{a^2}+\frac{\frac{95}{96}}{a}\right)\frac{1}{96}=a-1 \Longrightarrow \frac{192a^2+2\frac{95}{96}a-(\frac{95}{96})^2}{192}=a^3\ and\ \left(-\frac{1}{2}\frac{\left(\frac{95}{96}\right)^2}{b^2}-\frac{\frac{95}{96}}{b}\right)\frac{1}{96}=b-1 \Longrightarrow$$

$$\frac{192b^2-2\frac{95}{96}b-(\frac{95}{96})^2}{192}=b^3\ and\ \frac{1}{(a-1)(1-b)}\cong \mathbf{12202.8887406646790623199} \qquad (23)$$

Exponential stability of the field strength near $\xi=\frac{95}{96}$: From (22.4), $x_1+x_2-1=a+b-1$ for two different ξ values, $\xi=\frac{193}{192},\ \xi=\frac{63}{64}$, consider replacing $\frac{95}{96}$ in (23) by $(a+b-1)^{\xi}=\xi$. The result is very surprising, $\xi\cong 1-95.9560893107845913618803\,02^{-1}$ instead of $\xi=\frac{95}{96}=1-96^{-1}$ and in that case $\frac{1}{(a-1)(1-b)}\cong 12202.9706958707520243478938\,94$ which is a small error.

Note: Serendipity is not preferable to rigid math but can be an indication. Now consider the following expression:

$$\frac{x}{96}(2^{\frac{x}{95*96}}-1)^{-1} \qquad (23.1)$$

It is easy to see that this expression approximates the inverse Fine Structure Constant for values of x between 1 and 4. Here 2 is considered as an upper limit on the field strength $\xi<2$. The actual maximal value will be explored as $\xi<\frac{\pi}{2}$. When $\xi=2$ we have $\left(-\frac{1}{2}\frac{\xi^2}{a^2}+\frac{\xi}{a}\right)\frac{1}{96}=a-1\Rightarrow a=1$ and then $(a-1)^{-1}$ is undefined.

From (20) and (23.1) we choose $x=log_2\sqrt{\frac{1}{x_1-1}}=-\frac{1}{2}log_2(x_1-1)$. Inserting x into (23.1) we get for $(x_1-1)^{-1}\cong 206.751339885022019871030352078378200531005859375$, a surprising result:

$$x=-\frac{1}{2}log_2(x_1-1)\Longrightarrow x*(96*(2^{\frac{x}{95*96}}-1))^{-1}=\frac{\frac{x}{96}}{(2^{\frac{x}{95*96}}-1)}\cong \qquad (23.2)$$

$$137.03599925379157298.$$

With $(x_1-1)^{-1}$ high sensitivity of two digits after the decimal point, which is surprising but the result in (23.2) depends on negative charge only and therefore cannot be an accurate result.

This value is very close to the accepted inverse Fine Structure Constant, but what can we make of this guess which is not any rigid proof? The expression $(2^{\frac{x}{95*96}}-1)$ is a small delta between 1 and a small power of 2. It is an exponential perturbation around 1. The expression $2^x=2^{-\frac{1}{2}log_2(x_1-1)}=\sqrt{\frac{1}{x_1-1}}$ is the square root of the inverse delta area around a negative charge. It means approximately inverse length. But why in the power expression in the denominator of (23.2), we use $\frac{x}{95*96}$ while in the numerator the expression is $\frac{x}{96}$? This is not a well understood expression. (23.2) can, however, indicate that the Fine Structure Constat is related to 9120 = 95*96 and to powers of 2 or other expressions. A more detailed discussion of these ideas will take place later in this paper.

$(a-1)(1-b)$ answers the question of what happens when the test particle is neutral. To better understand the above expression, it is best to contract the acceleration matrix $A_{\alpha\beta}$ (3), [10] with the Levi-Civita tensor (not symbol), $E^{\mu\nu\alpha\beta}$ but with a possible orientation change from $B_{\mu\nu} = \frac{1}{2}E^{\mu\nu\alpha\beta}A_{\alpha\beta}$. This description is of a second complex plane in which the divergence of a Reeb class-like acceleration vector can be of an opposite sign, $U_\mu;^\mu + U^*{}_\mu;^\mu = 0$ where V_v is a unit vector perpendicular to both $\frac{U^\mu}{2}$ and to $\frac{P^\mu}{\sqrt{Z}}$. One field is then of a positive charge and one of a negative charge which is the explanation for the term $(a-1)(1-b)$ where a denotes the area addition ratio around a negative charge and b is the area loss ratio around a positive charge. One would expect to see $\sqrt{(a-1)(1-b)}$ however, roots will be discussed regarding spin 1 mass ratios.

Roots of such a value also have a meaning, see appendix C, (64). Combining (20) and (23), the following holds:

$$\frac{(x_1-1)\mathbf{105.65837455}MeV}{1+(a-1)(1-b)} \cong \mathbf{0.5109989461}MeV$$

$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2a^{-2}+\left(1-\frac{1}{96}\right)a^{-1}\right)=a$$
$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2b^{-2}-\left(1-\frac{1}{96}\right)b^{-1}\right)=b$$
$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2c^{-2}+\frac{4}{\pi}c^{-1}\right)=c$$
$$MuonMass*(c-1)=ElectronMass+ElectronMass*(a-1)(1-b) \qquad (24)$$

By (23) the ratio is **~206.76828270441461654627346433699131011962890625**

By (13.11) the term $(a-1)(1-b)$ is best interpreted as a coupling between a negative charge and a positive charge and is therefore a neutral term.

Where $ElectronMass*(a-1)(1-b) = \sim\mathbf{41.8752442118608}\ \boldsymbol{eV/c^2}$ looks like a new particle or resonance. Corroboration requires to detect excess in cosmic ~20.9 $\boldsymbol{e}V/c^2$ photons. Verification of this theory by Muon decays can be done by observing rare excess of **20.9376221059304 eV photons**. With electron energy **0.5109989500 MeV** the Muon energy is **~105.658375355 MeV**.

We only needed a small correction to the 2014 Muon energy from 105.6583745 MeV to 105.65837455 MeV with electron energy 0.5109989461055 MeV to arrive at the energy ratio and therefore mass ratio of the Muon and the electron. Is that a mere coincidence? The extremely small ratio error and the choices of $\xi = \frac{4}{\pi}$ and $\xi = \frac{95}{96}$ highly disfavors a mere coincidence. It is important to notice that $1 + \frac{1}{96}\left(-\frac{1}{2}(\frac{193}{192})^2 a^{-2} + \left(\frac{193}{192}\right) a^{-1}\right) = a$ has a biggest root $a = \frac{193}{192} = 1 + \frac{1}{192}$ and $1 + \frac{1}{96}\left(-\frac{1}{2}(\frac{63}{64})^2 b^{-2} - \left(\frac{63}{64}\right) b^{-1}\right) = b$ has a biggest root $b = \frac{63}{64} = 1 - \frac{1}{64}$. The delta $-\frac{1}{64} + \frac{1}{192} = -\frac{1}{96}$ is a delta of energy ratios between the two stable states with field strength coefficients $\xi = \frac{193}{192}$ and $\xi = \frac{63}{64}$ and roots $a = \frac{193}{192}$ and $b = \frac{63}{64}$. 1 plus this delta yields $\frac{95}{96}$, which shows that our choice of $\xi = \frac{95}{96}$ was not at random but is the result of the summation of negative and positive area ratios for which the field strengths are equal to the biggest roots.

Let us define the electron field strength as $\xi = \frac{95}{96}$ and consider a perturbation of this value and its close link to the Fine Structure Constant. Recall (23),

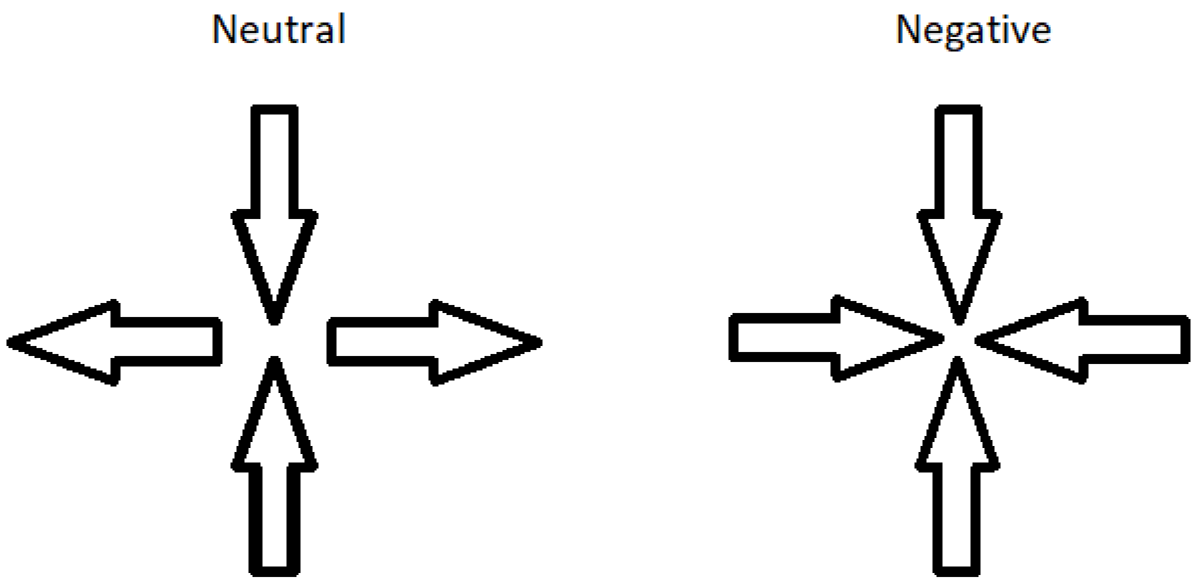
$$1 + \frac{1}{96}\left(-\frac{1}{2}(1 - \frac{1}{96})^2 a^{-2} + \left(1 - \frac{1}{96}\right) a^{-1}\right) = a \qquad (24.1)$$
$$1 + \frac{1}{96}\left(-\frac{1}{2}(1 - \frac{1}{96})^2 b^{-2} - \left(1 - \frac{1}{96}\right) b^{-1}\right) = b$$

Consider a little more accurate result than the one used before in (23):

$$\frac{1}{(a-1)}\frac{1}{(1-b)} \cong 12202.8887406646790623199 \qquad (24.2)$$

Now consider the following perturbation on $\xi = \frac{95}{96} = 1 - \frac{1}{96}$, and raise this $\frac{95}{96}$ to the power $1 + \alpha$ where α is the Fine Structure Constant - FSC. We will take the assessment from (40) of the inverse FSC, about 137.0359990368270075578… and not the larger assessment 137.0359992990990 from the remark after (41). Our new field strength will be $\xi' = (\frac{95}{96})^{1+\alpha}$, which is an exponential perturbation with the help of the Fine Structure Constant. We want to calculate the new value of the neutral area ratio of addition and subtraction in two acceleration planes, one positive and one negative, as expected from the electron Neutrino because it is electrically neutral and see what we get. Before that, please view the following illustration, in reality, it is a 4-dimensional model with two perpendicular planes. The neutral charge is of one positive two-dimensional plane and one negative two-dimensional plane, both defined by two acceleration matrices and two generalizations of two Reeb class vectors to 4 dimensions.

Fig. 6.: this is an over-simplification of two Reeb class fields in two Symplectic Lagrangian planes:



And now we have for $\xi' = (\frac{95}{96})^{1+\alpha}$,

$$\frac{1}{(a\prime-1)}\frac{1}{(1-b\prime)} \cong 12204.188931677483196836 \tag{24.2}$$

And the following shows up:

$$(1 - \frac{(a^{'}-1)(1-b^{'})}{(a-1)(1-b)})^{-\frac{1}{2}} \cong 96.883736818613229502261 7 \tag{24.3}$$

which is remarkably close to $\alpha^{-1}2^{-\frac{1}{2}} \cong 96.899084185613574504714052$, from (40).

The fact that perturbations of the field strength $\frac{95}{96}$ yield the Fine Structure Constant is easy to see in other cases other than (81)-(86) and (24.3). Using a simple datasheet without the accuracy of the Python math libraries, consider $\xi' = \left(\frac{95}{96}\right)^{1+\beta}$ where $\beta \approx 1.00370694$ for which $(\frac{95}{96} - \xi')(\frac{95}{96})^{-1} \cong 25762.75334^{-1}$ then (23) yields $\frac{1}{(a\prime-1)}\frac{1}{(1-b\prime)} \cong 12203.54919$. Now consider the following function $\xi'' = 1 + \ln(\xi')$ for which (23) yields $\frac{1}{(a\prime\prime-1)}\frac{1}{(1-b\prime\prime)} \cong 12204.49567$,

$$\ln(\frac{\ln((a''-1)(1-b''))}{\ln((a'-1)(1-b'))} - 1)^2 \approx 137.0359991 \tag{24.4}$$

In other cases, the inverse Fine Structure Constant can emerge from trigonometric perturbations of a higher field strength. In both cases, fractional powers of roots are involved. This is not surprising if we consider that the Fine Structure Constant must be related to electromagnetic waves, and these should be a result of perturbations of the field strength of elementary particles such as the electron or even of the Tau lepton as an upper limit of an allowed leptonic field strength. Although (24) is not a rigid mathematical proof of the mass ratios between the Muon and the electron, and although only $\xi = \frac{4}{\pi}$ is a well understood field strength, not directly from this paper, one can argue that the result in (24) is too accurate to be ignored, especially if (24.3), (24.4) and (81)-(86) in "Appendix E" are taken into account.

$\xi = 2$ as a field strength is a critical value much higher than the highest field strength for leptons which is offered in this paper, simply because for a negative charge, the gravitational field vanishes.

$\frac{192x^2+2*2*x+2^2}{192} = x^3$ with a stable root x=1, $x(n+1) = \left(\frac{192x(n)^2+2*2*x(n)-2^2}{192}\right)^{\frac{1}{3}}$. But then $\frac{1}{x-1}$ as an added area portion around the negative charge is undefined but with a left limit 0. So, asking whether a logarithmic scale that starts at 2 has a physical meaning is legitimate. We choose our scale to be:

$\{2^{\frac{95*96}{95*96}}, 2^{\frac{95*96-1}{95*96}}, \ldots, 2^{\frac{1}{95*96}}\}$, now consider $y = \left(\left(2^{\frac{1}{95*96}} - 1\right) * 96\right)^{-1}$ ≅137.050820617 and

$\frac{95}{\ln(2)} \cong 137.05602888445$ and it is easy to show that as n grows, $\left(\left(2^{\frac{1}{(n-1)*n}} - 1\right) * n\right)^{-1} \approx \frac{n-1}{\ln(2)}$. It is easy to see a nice result, $\frac{y-137.0359990368270075578}{137.0359990368270075578} \cong 96.1546032^{-2}$ so the relative error to one of the assessment of the inverse Fine Structure Constant, see (40), is nearly expressible as a power of 96. This is one good reason to search for a relation that involves 2 and powers of 96 or of 95*96 as the mathematical term that will yield the Fine Structure Constant, however, such a term should appear out of a perturbation of a field strength because the Fine Structure Constant defines the Quantum electric strength, but which field strength?

Important: A leading idea is that the Fine Structure Constant should be related to perturbations of a maximal allowed field strength for leptons, i.e., the Tau lepton field strength. Any perturbation exceeding this limit must be dissipated as waves.

The exploration which is performed here is not out of analytic solutions to (4) or a complex version of (4) or to the further-on mentioned (64), which may take many years to yield fully analytic solutions. It is a "reverse engineering" of Nature by assessment of (4) and field strengths in an infinitesimal limit. It will require more discussion to reach more comprehensible terms for the inverse Fine Structure Constant.

Another clue to where the Fine Structure constant comes from is the following:

Consider a search for the number 96 and to keep the idea simple and related to the roots of the third order Gravity and Anti-gravity area ratio polynomials.

Consider the following known equation: $\frac{\pi^4}{96} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)^4}$ which results from Parseval's identity when developing the Fourier series of the function $f(x) = |x|$ in $(-\pi, \pi)$. Notice that the fourth root of $\frac{\pi^4}{96}$ is $\frac{\pi}{96^{\frac{1}{4}}} \cong 1.00364948118 \ldots$. We can see what the error of this value in relation to 1 is. $\frac{\pi}{96^{\frac{1}{4}}} - 1 \cong \frac{1}{274.01155134419542\ldots} = \frac{1}{2*137.00577567209771179617192026613}$. We may therefore search for an expression in which twice the inverse Fine Structure Constant appears. If we choose the value from (40), and not the higher value 137.0359992990990 after (41) we get the following error assessment

$$(1 - \frac{137.00577567209771179617192026613}{137.03599903682700755780388135 46}) \cong \frac{2}{(95.227180726406028880040436362512)^2}$$

The residual error is related to a number between 95 and 96 and the factor 2 appears again. Although it is not any mathematical proof, it is still difficult to ignore such a lead in the search for where the inverse Fine Structure comes from.

It is worthy of mentioning that getting the exact Fine Structure Constant in (81)-(86) requires a very small addition $\xi = \frac{95}{96} + \varepsilon$ for some small ε. Then (24) would require the mass of the Muon to be slightly higher than 105.65837455 MeV, about 105.658375 MeV.

The following Python code was used to reach the result in (24),

```
import numpy as np

x1 = 1
third = 1 / 3
f = 4 / np.pi # Ettore Majorana's ring of a disk, potential factor.
f2 = f * f

# Iterate to most stable root.
for i in range(2000):
    x1 = np.power((192 * x1 * x1 + 2 * x1 * f - f2) / 192, third)

a = 1/(x1 - 1) # Negative charge.

print('Xi = 4/Pi, a = %.48f' % a)

x3 = 1
x4 = 1
```

```
f = 95 / 96
f2 = f * f

# Iterate to most stable roots.
for i in range(2000):
    x3 = np.power((192 * x3 * x3 + 2 * x3 * f - f2) / 192, third)
    x4 = np.power((192 * x4 * x4 - 2 * x4 * f - f2) / 192, third)

c = 1/(x3 - 1) # Negative charge.
d = 1/(1 - x4) # Positive charge.
print('Xi = 95/96, c = %.48f, d = %.48f' % (c, d))
print('Xi = 95/96, c * d = %.48f' % (c * d))

print('Approximated mass ratio between the Muon and the electron %.48f'
      % (a * (1 + (x3-1)*(1-x4))))
```

Few words about $\xi = 1 - \frac{1}{96}$. What is so special about $\xi = 1 - \frac{1}{96}$? It is twice the average of an ideal area loss ratio and an ideal area addition ratio + 1. $\frac{1}{192} - \frac{1}{64} = -\frac{1}{96}$ where $x = 1 + \frac{1}{192}$ is the biggest root of $1 + \frac{1}{96}\left(-\frac{1}{2}(1 + \frac{1}{192})^2 x^{-2} - \left(1 + \frac{1}{192}\right) x^{-1}\right) = x$ and $x = 1 - \frac{1}{64}$ is the biggest root of $1 + \frac{1}{96}\left(-\frac{1}{2}(1 - \frac{1}{64})^2 x^{-2} - \left(1 - \frac{1}{64}\right) x^{-1}\right)$.

Null Reeb vectors instead of gauge fields: How about null Reeb class vectors $\frac{U_\mu}{2}\frac{U^\mu}{2} = 0$. It is not difficult to see that in this case, the unit vector $\frac{P_\mu}{\sqrt{Z}}$ should be space-like at least in the near vicinity of the test particle as $r \to 0$ and U^μ may not be all 0 at the center of a sphere but can be a null vector. With $\xi = \frac{4}{\pi}$ and $\xi = \frac{95}{96}$ we have in this case:

$$1 + \frac{1}{96}\left(\pm\frac{4}{\pi}c^{-1}\right) = 1 \pm \frac{c^{-1}}{24\pi} = c \tag{25}$$

and

$$1 + \frac{1}{96}\left(\pm\frac{95}{96}b^{-1}\right) = 1 \pm \frac{95b^{-1}}{96^2} = b \tag{26}$$

From (25)

$$c_1 = \frac{1+\left(1+\frac{1}{6\pi}\right)^{\frac{1}{2}}}{2} \cong 1.0130915\ldots,\ c_2 = \frac{1+\left(1-\frac{1}{6\pi}\right)^{\frac{1}{2}}}{2} \cong 0.986556\ldots \text{ and} \tag{27}$$

From (26)

$$b_1 = \frac{1+\left(1+\frac{95}{96*24}\right)^{\frac{1}{2}}}{2} \cong 1.010204037\ldots,\ b_2 = \frac{1+\left(1-\frac{95}{96*24}\right)^{\frac{1}{2}}}{2} = \frac{95}{96} \text{ and}$$

$$\frac{1}{c_1-1} \cong 76.38530,\ \frac{1}{1-c_2} \cong 74.3845968,\ \frac{1}{\sqrt{(c_1-1)(1-c_2)}} \cong 75.3783115, \tag{28}$$

With $\xi = \frac{4}{3}$, (28) is a bit different:

$$\frac{1}{c_1-1} \cong 72.98648402, \frac{1}{1-c_2} \cong 70.98571137, \frac{1}{\sqrt{(c_1-1)(1-c_2)}} \cong 71.9791462, \tag{28.1}$$

Important: Where does this $\xi = \frac{4}{3}$ come from? The reader is advised to check that the average distance between two points on the Euclidean ring is $\xi = \frac{4}{\pi}$. The average distance between two points on the Euclidean sphere is $\xi = \frac{4}{3}$ and is left as an exercise to the reader. We may say that $\xi = \frac{4}{\pi}$ means a geometric ring field strength and $\xi = \frac{4}{3}$ is a geometric sphere field strength. If we take into account particle decay through Bosons with two different field strengths, $\xi = \frac{4}{\pi}$ if the Muon is involved and $\xi = \frac{4}{3}$ in other cases, then there is a new interaction that is not covered by the W Boson alone!

$$\frac{1}{b_1-1} \cong 98.00042535, \frac{1}{1-b_2} = 96, \frac{1}{\sqrt{(b_1-1)(1-b_2)}} \cong 96.99505572 \tag{29}$$

We now look at:

$$\frac{\sqrt{(b_1-1)(1-b_2)}}{\sqrt{(c_1-1)(1-c_2)}} \cong 1.134361808^{-2} \tag{30}$$

Roots are attributed in this case to spin 1 or 2. It is easy to see that also:

$$(1 + (c_1 - 1)(1 - c_2))\left(\frac{(c_1-1)(1-c_2)}{(b_1-1)(1-b_2)}\right)^{1/4} \cong 1.134561453 \tag{31}$$

$$\approx \frac{91.1876\ GeV}{80.3725\ GeV}$$

Which is remarkably close to the ratio between the energy of the Z boson and the energy of the W boson and for W Boson of $80.3725\ GeV$ the relative error of this ratio is about 1/1528961.689. For where the idea of 4th roots came from, please refer to Appendix C, (65).

Another research direction is to use the inverted value of $\xi = \frac{4}{\pi}$, i.e., $\xi = \frac{\pi}{4}$ in the negative and positive charge area ratio equations as in (24). That yields two new maximal roots $a_1{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{\pi}{4}\right)^2 + \frac{\pi}{4}a_1\right) = a_1{}^3$ and $a_2{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{\pi}{4}\right)^2 - \frac{\pi}{4}a_2\right) = a_2{}^3$ along with the older ones $b_1{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 + \frac{4}{\pi}b_1\right) = b_1{}^3$ and $b_2{}^2 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 - \frac{4}{\pi}b_2\right) = b_2{}^3$. Quite like the ratio in (30), we have, $\frac{\sqrt{(b_1-1)(1-b_2)}}{\sqrt{(a_1-1)(1-a_2)}} \cong \sqrt{\frac{201.6240447 * 86.46523917}{206.7513399 * 44.63955018}} \cong 1.374383282$ which is close to the following mass ratio between a Higgs Boson of 125.3267 GeV and a Z Boson of 91.1876 GeV which yields, 1.37438314, close to 1.374383282. It is interesting though not sufficiently accurate to draw any conclusion at this stage. The idea behind using charge equations without null Reeb class vectors is because the Higgs boson is supposedly responsible for non-zero mass. From (31) and using s instead of c, $\sqrt{(s_1-1)(1-s_2)} \cong 75.3783115\ ...^{-1}$ and 91.1876 GeV * $\frac{\sqrt{(b_1-1)(1-b_2)}}{\sqrt{(a_1-1)(1-a_2)}} * (1+(s_1-1)(1-s_2)) \cong 125.3487702$ GeV. A similar $(1+(s_1-1)(1-s_2))$ value was used in (29) as $(1+(c_1-1)(1-c_2))$. If the reasoning here is correct, the Higgs boson interacts as an electric dipole.

Returning to (22) $\frac{1}{1-x_2} \cong \mathbf{44.63955017596401}$ and written as $\frac{1}{1-c}$,

$$\frac{80372.88 \text{ MeV } (1-c)}{1+\sqrt{(c_1-1)(1-c_2)}} \approx \mathbf{1776.91}\ \boldsymbol{MeV} \quad (32)$$

With $\xi = \frac{4}{3}$ as in (28.1), (32) gets the same result for a higher value of the W Boson mass,

$$\frac{80422.57 \text{ MeV } (1-c)}{1+\sqrt{(c_1-1)(1-c_2)}} \approx \mathbf{1776.91}\ \boldsymbol{MeV} \quad (32.1)$$

The root, $\sqrt{(c_1-1)(1-c_2)}$ can be better understood as a result of taking the root of a determinant of a Gram matrix of two Reeb class vectors in Appendix C or is related to spin 1. The value 1776.91 MeV will be discussed in (36) with a reference. A very surprising relation between Quarks and Leptons with the same $\frac{1}{1-c} \cong 44.63955017596401$ as in (22) is the relation between the **pole energy of the Bottom/Beauty Quark** [29], [30] and the anti-**Muon**, this time we take the Muon value that yields in (24) along with the denominator of (23), the exact mass ratio between the Muon and the electron $105.65837455\ MeV$ instead of the 2014 value $105.6583745\ MeV$,

$$\frac{105.65837455\ MeV}{(1-c)(1+(a-1)(1-b))}(1+\sqrt{(c_1-1)(1-c_2)})$$
$$= 44.63955017596401 * 105.65837455\ MeV$$
$$* (1+75.3783115025728682778607890096 93^{-1}) *$$

$$(1+12202.888740664679^{-1})^{-1} \cong 4{,}778.7223164425585113299 \text{ MeV} \approx \mathbf{4.78\ GeV}$$

Which is equivalent to:

$$\frac{PoleEnergyOfBottomQuark*(1-c)}{(1+\sqrt{(c_1-1)(1-c_2)})} = \frac{MuonEnergy}{(1+(a-1)(1-b))} \quad (33)$$

In which the root in the left denominator is attributed to spin 1. New physics? Looks like it! A.M. Badalian's prediction 4,778 MeV [30] is too close to 4,778.72 MeV to be ignored. The outcome of the Muon being the electro-gravitational energy of the pole energy of the Bottom Quark is as follows:

a) Lepton universality should be broken in decays of anti-Bottom Quark that involve Muons.

b) High energy p-p collisions can no longer be considered for the calculation of the W Boson mass.

Before we proceed, it is worthy of mentioning the following Simon Plouffe identity [31]: consider the functions, $S_n(r) = \sum_{k=1}^{\infty} \frac{1}{k^n e^{\pi r k}-1}$ then there is a well-known relation between π and 96, $\pi = 72S_1(1) - 96S_1(2) + 24S_1(4)$. Notice that the sum of the positive coefficients 72 + 24

= 96 and the negative coefficient is -96. While this identity is not a direct relation between $\xi = \frac{4}{\pi}$ and $\xi = \frac{95}{96} = 1 - \frac{1}{96}$, it does show an example of how π and 96 can be related to each other through Zeta functions in a simple and straight forward manner. A deeper and a very surprising relation will be seen in a note after (40).

6. The exact inverse Fine Structure constant – critical imbalance between gravity and anti-gravity

The following endeavor originated in the search for a field strength coefficient near $\frac{\pi}{2}$ for quite a simple reason. If a motion in a small circle is with the constant velocity c, then after half a circle the velocity will be -c. The difference c-(-c) is 2c and the time between the two velocity measurements is $\frac{\pi r}{c}$ so $2c(\frac{\pi r}{c})^{-1} = \frac{2}{\pi}\frac{c^2}{r}$ while the acceleration of the motion is $\frac{c^2}{r}$. The inferred acceleration $\frac{2}{\pi}\frac{c^2}{r}$ can be interpreted only when the velocity can take one of two values c or -c, or in other words when velocity itself is quantized. The correction in this situation is by a factor $\frac{\pi}{2}$ and $\frac{\pi}{2}\frac{2}{\pi}\frac{c^2}{r} = \frac{c^2}{r}$. Given a radius r and an upper speed limit c, the correction coefficient $\frac{\pi}{2}$ should be considered as a possible upper field strength coefficient. The way a coefficient near $\frac{\pi}{2}$ was found will be discussed along with its relation to the inverse Fine Structure Constant. The fine structure constant is surprisingly reached through the mass ratio between the Tau lepton and the Muon and an interesting perturbation of the field strength of the Tau lepton that will be found in this section. Recommended reading for this section is Appendix E, (70) - (79).

Note: The more advanced parts of this section require basic knowledge of electrical engineering and especially a good understanding of the trivial subject of Dissipation Factor and Loss Tangent and especially of Power Factor [32].

Note: Why dissipation factor? The reason is that any perturbation of the Reeb class field, which behaves as acceleration, above a maximal allowed limit, must be emitted and in mainstream physics, the electromagnetic field is dissipated as photons.

The denominator $1 + \sqrt{(c_1 - 1)(1 - c_2)}$ in (32), (33) and $(1 + (a - 1)(1 - b))$ in (24) can be used together to yield a nice result that seems to be more than just a mathematical coincidence. Consider the following imbalance equation as in (23) of negative and positive charge:

$$1 + \frac{1}{96}\left(-\frac{1}{2}\xi^2 {g_1}^{-2} + \xi {g_1}^{-1}\right) = g_1$$

$$1 + \frac{1}{96}\left(-\frac{1}{2}\xi^2 {g_2}^{-2} - \xi {g_2}^{-1}\right) = g_2$$

$$\text{Such that } (g_1 - 1)^{-\frac{1}{2}} = \frac{1}{2}(1 - g_2)^{-1} \tag{34}$$

With biggest roots $g_1 \cong 1.003629541$ and $g_2 \cong 0.969877163$. g_1 means an area portion $\sim \mathbf{275.51693^{-1}}$ is added around a negative charge and $\sim \mathbf{33.19740^{-1}}$ of the area is subtracted around a positive charge, which reflects a possibly maximal allowed gravitational imbalance between negative and positive charge.

A calculation that uses an electronic datasheet, yields,

$$\xi \cong \mathbf{1.5561985371903484},\ (g_1 - 1)^{-\frac{1}{2}} \cong \mathbf{16.59870203} \tag{35}$$

which is close to the known mass ratio between the Tauon and the Muon, ≅16.817 where ξ denotes a maximal allowed coefficient.

The primary explanation for the existence of a field strength $\xi \cong \mathbf{1.5561985371903484}$

The following regards "Primary explanation: Ratio between a length atom and the square root of an area atom" after (15) and before (16), which is a result of a discussion with a colleague, Aryeh Aldema. The Aldema interpretation as an explanation to (34): (34) can be written in a more illuminating way which reflects an idea of a colleague of mine, Aryeh Aldema, who is unfortunately no longer with us. Aryeh discussed with me the possibility that there exist atoms of length and of area that are not related to each other with simple [root] relationships. For example, one can consider $\sqrt{\delta Area} \neq \delta Length$. Considering that a line segment must have a center means that in Quantum Mechanics, half the wavelength has a physical meaning in length scales and therefore it follows that,

$$\sqrt{\frac{\delta Area_1}{L^2}} = 2\frac{\delta Length}{L} = 2\frac{\delta Area_2}{L^2} \tag{35.1}$$

which is consistent with $\frac{\delta Area_1}{L^2} < \frac{\delta Area_2}{L^2}$, with $\frac{\delta Area_1}{L^2}$ as a lower limit, with a condition,

$$\frac{\delta Length}{L} = \frac{\delta Area_2}{L^2} \tag{35.2}$$

(35.2) with Aryeh Aldema's interpretation from (35.1) in (34), yields,

$$(g_1 - 1)^{\frac{1}{2}} = 2(1 - g_2) \Longrightarrow (\delta g_1)^{\frac{1}{2}} = 2(-\delta g_2) \Longrightarrow$$
$$\left(\frac{\delta Area_1}{L^2}\right)^{\frac{1}{2}} = 2\left(\frac{\delta Length}{L}\right) = 2\frac{\delta Area_2}{L^2} \tag{35.3}$$

for some minimal length unit L. Notice that (35.3) implies an upper limit on the possible field strength and well explains that (34) should describe the greatest possible field strength!

Multiplying this value $\xi \cong \mathbf{1.5561985371903484}$ by $1 + \sqrt{(c_1 - 1)(1 - c_2)}$ from (32), (33) and dividing by $\left(1 + (a-1)(1-b)\right)$ from (24) yields,

$$\frac{Muon\ 105.6583745\ MeV}{\left(1+(a-1)(1-b)\right)} \cong \frac{\sqrt{g_1 - 1}\ Tauon\ 1776.9127923826\ MeV}{\left(1+\sqrt{(c_1-1)(1-c_2)}\right)} \tag{36}$$

Which is $\frac{(1+\sqrt{(c_1-1)(1-c_2)})}{(1+(a-1)(1-b))\sqrt{g_1-1}}$ ≅16.81752914. So, this calculation predicts a Tauon energy of about **1776.9127923826 MeV** which agrees with [33]. Please note the remark after (28.1) for a possible additional W Boson. We now need to check the consistency of (36) with (32) as a test to this theory. We take $\frac{1}{1-x_2}$ from (22) and $\frac{1}{\sqrt{(c_1-1)(1-c_2)}}$ from (28) and check the following:

$$\frac{1776.91279322344...\text{MeV}*(1+\sqrt{(c_1-1)(1-c_2)})}{1-x_2} \cong 80372.8876666694MeV \qquad (36.1)$$

Which is consistent with (32) but less with (32.1) of a higher W Boson energy as the approximation of the W Boson's energy with $\xi = \frac{4}{\pi}$ and a null Reeb class vector. For $\xi = \frac{4}{3}$ the W Boson energy is a bit higher. (35) is strikingly related to (20) and (22). $\frac{192{y_1}^2+2\left(\frac{4}{\pi}\right)y_1-\left(\frac{4}{\pi}\right)^2}{192} = {y_1}^3$ and $\frac{192{y_2}^2-2\left(\frac{4}{\pi}\right)y_2-\left(\frac{4}{\pi}\right)^2}{192} = {y_2}^3$ in the following way:

Assessing the following yields,

$$-\frac{1}{\log(y_1)}\frac{1}{\log(y_2)} \cong 9147.5718747432856616796925 66 \qquad (36.2)$$

and on the other hand from (35),

$$\frac{1}{(g_1-1)}\frac{1}{(1-g_2)} \cong 9146.4461480441150342812761 66 \qquad (36.3)$$

The relative error in these two values in relation to $\frac{1}{(g_1-1)}\frac{1}{(1-g_2)}$ is Relative error ≅ $8124.926018710571952397003770^{-1}$. Please note that for a small d the following holds. $\frac{1}{d} \approx \frac{1}{\log(d+1)}$ and also $\frac{1}{d} \approx -\frac{1}{\log(1-d)}$. This relation alludes to a possible exponential relation between the roots of (20), (22) and the roots of (35) but before we actually check an exponential perturbation on the field strength ξ ≅1.5561985371903483965638770314399 from (35) we notice the following for the same field strength coefficient of (35):

$$\frac{2}{\cos(\xi)} \cong \frac{2}{\cos(1.5561985371903484)} \cong 137.011909869, \qquad (37)$$

$$\tan^{-1}(95^2 96^2 (1-g_2)^{+4}) \cong 1.5561948778250207190765973767615 \qquad (38)$$

remarkably approximate ξ ≅1.5561985371903484 from (34), (35).

$$Error = \frac{\xi-(95^2 96^2 (1-g_2)^4)}{\xi} \cong 425{,}263.601328167905179588241571 33^{-1} \qquad (39)$$

Paul Levy Isoperimetric Theorem and Levy – Gromov Isoperimetric Theorem

Define X, M, μ, d a probability space X with Borel $\sigma - Algebra\ M$, measure μ and a metric distance function $d(x_1, x_2)$ where $x_1, x_2 \in X$. Define $M_\alpha = \{\alpha\ |A \in M,\ \mu(A) = \alpha, \alpha \in (0,1)\}$ now consider $A_\epsilon = \{x \in X, d(x, A) \leq \epsilon\}$. We now consider the Borel $\sigma - Algebra\ M$ of the sphere $S(N) \subset \mathbb{R}^{N+1}$. Consider $A \in M_{\frac{1}{2}}$ such that $\mu(A_\epsilon)$ is minimal when ϵ is sufficiently small, then by Paul Levy's Isoperimetric Theorem for S(N) [34] A is minimal when A is half a sphere. There is a stronger result for domains with smooth boundary on Riemannian manifolds with positive Ricci curvature, see theorem 2.4 in [34] and especially (2.6) in [34]. Contraction of Einstein's tensor twice with the accelerated time-like vector $\frac{p_\lambda}{\sqrt{Z}}$ reduces equation (4) in this paper to an equation in an ordinary Riemannian and not in a Lorentzian geometry, which is the reason why [34] of great interest for this paper. A spherical cap of the sphere $S(N)$ has a maximal measure 1 because $\mu(S(N)) = 1$. The sphere $S(N) \subset \mathbb{R}^{N+1}$ is embedded in the simplest case in Euclidean spaces.

$$\mu\big(Cap(\theta)\big) = \frac{S(N-1,r=1)}{S(N,r=1)} \int_0^\theta \sin(x)^{N-1}\, dx, \tag{39.1}$$

$$\mu\big(Cap(\pi)\big) = \mu(S(N)) = 1$$

In two dimensions $N - 1 = 1$, $\frac{S(N-1,r=1)}{S(N,r=1)} = \frac{2\pi}{4\pi} = \frac{1}{2}$,

$$\mu(Cap(\theta)) = \frac{1}{2}(1 - \cos(\theta)) \tag{39.2}$$

And for half a sphere,

$$\mu\left(Cap\left(\frac{\pi}{2}\right)\right) = \frac{1}{2}(1 - 0) = \frac{1}{2} \tag{39.3}$$

It is immediately apparent that $\frac{\cos(\theta)}{2}$ is the difference between the measure of half a sphere and the cap of angle θ from its geodesic center which is on the sphere.

Why consider half sphere caps? The reason is that the principal circles that pass through the middle of such caps can describe a maximal acceleration. Half these circles have length πr and motion at the speed of light along these half circles enter the plane that cuts the cap at speed of light c and leave the plane at speed $-c$ or vice-versa. If only two speed measurements are done, the acceleration that is measured is $\frac{2c^2}{\pi r}$. In terms of Special Relativity, this acceleration is not of any unit vector because $\sqrt{1 - \frac{v^2}{c^2}}$ becomes zero, however, $\frac{2c^2}{\pi r}$ can still be a value of a non-geodesic acceleration field without c describing any classical or even General Relativistic motion. $\mu\left(Cap\left(\frac{\pi}{2}\right)\right) = \frac{1}{2}$ is of interest because it is the probability measure of a cap which is half a sphere. The term $\frac{2c^2}{\pi r}$ requires a coefficient of $\xi = \frac{\pi}{2}$ and then $\xi \frac{2c^2}{\pi r} = \frac{c^2}{r}$. Values near $\xi = \frac{\pi}{2}$, $\xi < \frac{\pi}{2}$ are therefore of interest as an upper limit for a non-geodesic acceleration field.

$\mu(Cap(\theta)) = \frac{1}{2}(1 - \cos(\theta))$ can be written as

$$\mu(Cap(\theta)) = \frac{1}{2}\left(1 - \cos\left(\frac{\pi}{2} - \epsilon\right)\right) \tag{39.4}$$

for some small ϵ. The area difference from half a sphere, in far observer coordinates, can then be normalized in relation to the entire sphere area,

$$\frac{\cos(\theta)}{2} = \frac{\cos\left(\frac{\pi}{2} - \epsilon\right)}{2} = \frac{\delta Area}{4\pi r^2} \tag{39.5}$$

Area loss or addition in a gravitational field is equivalent to energy. In a relation between an area to the entire sphere it is therefore equivalent to an energy quotient which is smaller or bigger than one. If $\theta \approx 1.5562011034975267$ then $\frac{\cos(\theta)}{2}$ becomes about ~137.0359992349584^{-1}, which is close to the inverse Fine Structure Constant. And $\frac{\pi}{2} \cong 1.5707963267948966$. For this reason, if the Fine Structure Constant can be viewed as a coefficient of energy ratio, e.g. energy dissipation due to area fluctuation near a principal circle one the sphere, when an electron is accelerated, then the equation from which the Fine Structure Constant comes, must have a term $\frac{\cos(\theta)}{2}$ on one of the sides of the equation. For the inverse Fine Structure Constant, the term $\frac{2}{\cos(\theta)}$ must appear in the equation.

Dissipation factor interpretation: In terms of electrical engineering Dissipation Factor and Loss Tangent, we can write, $DF = \frac{95^2 96^2}{\frac{1}{2}(1-g_2)^{-4}} \approx \tan(\xi)$ where the numerator is known as the Resistive Power Loss and the denominator as the Reactive Power Oscillation. It is expected that an oscillating charge will generate oscillation in area due gravity changes, however, it is not expected that the area portion that is lost due to gravity will appear as the power of 4. This is a very rare property that connects between trigonometry and the electro-gravity polynomials (34). We can get from this relation two insights, the first is that if (37) is not a mathematical coincidence, then the inverse Fine Structure constant should come out of a trigonometric function and a numbers relation. The second is that $95^2 96^2 (1 - g_2)^4$ should be part of this equation. We may think that perhaps scaling of the value of ξ in a rational way, will yield the exact inverse Fine Structure Constant. So we want to find some d such that $\frac{2}{\cos\left(1.5561985371903484*(1+\frac{1}{d})\right)}$ will yield the constant we are looking for. We will soon find such d, $d \cong 606400.8$ that complies with [35] and we get, $\frac{2}{\cos\left(1.5561985371903484*(1+\frac{1}{606400.8})\right)} \cong 137.0359990462475253$. The motivation for this endeavor is taken from electrical engineering [32] where the cosine term means a ratio between delivered power and measured power in motors and other electric devices. In our case, we are interested in the ratio between radiation's energy and the energy it delivers upon interaction.

Until now, d is not very interesting because we could not find d out of any new theory. Well, not very accurate. First,

$$d = \frac{1}{2}(1-g_2)^{-4} \cong 607276.5368006824282929 \tag{39.6}$$

and

$$\frac{2}{\cos(1.5561985371903484 * (1+2(1-g_2)^4))} \cong 137.0359643018112763$$

(39.6) is a result of the discussion in "Primary explanation: Ratio between a length atom and the square root of an area atom" after (15) and before (16) and especially of the Aldema interpretation of the relations between 4-volume and length atoms.

Caveat: do not confuse the use of the term $Area$ in a 4-volume relation to length with a surface area to length in the following term, in which $Area^2$ means 4-volume, it is not the same term as in $\frac{1}{4}Area = Length^2$.

$$Area^2 = (2Length)^3 Length = 8Length^4 \tag{39.7}$$

Which means by (34) and (35.3),

$$\left(\frac{1}{2}\sqrt{\frac{\delta Area_1}{L^2}}\right)^3 \sqrt{\frac{\delta Area_1}{L^2}} = 2\left(\frac{\delta Length}{L}\right)^4 \tag{39.8}$$

and can be summarized as the Aldema 4-volume ratio,

$$d^{-1} = 2(1-g_2)^4 = \left(\frac{1}{2}(g_1-1)^{\frac{1}{2}}\right)^3 (g_1-1)^{\frac{1}{2}} = \frac{1}{8}(g_1-1)^2 \cong 607276.5368006824282929^{-1} \tag{39.9}$$

Consider

$$2(1-g_2)^4\, 95^2 96^2 \tag{39.10}$$

as a portion of $n = 95^2 96^2$ energy emission events which occur in a 4-volume unit. In a more illuminating way,

$$n = \left(\frac{95}{96}\right)^2 96^4 \tag{39.11}$$

which from (23) is a multiplication of 96^4 with the smallest field strength coefficient two times, with the electron $\xi = \frac{95}{96}$. In this case, each plane is multiplied by ξ, which has a physical meaning of the field strength depending on a number of events in a 4-volume with a fundamental reference of 96^4 events. (39.9) and (39.11) will both be used in a very surprising assessment of the inverse Fine Structure constant. A resolution of 96 events per each dimension is implied in the factors $\frac{\xi}{96}$, see the left-hand side of (19). Also see (23.1) and (23.2). A portion of energy emission events can be considered as a dissipation factor, which is the motivation in this discussion.

An expected relation: If in (39.11) 96^4 is a maximal, not minimal number of events, and the Tau energy is the maximal energy for leptons, then the event of two annihilating Tau leptons

Must yield 96^4 multiplied by this basic energy. Then by (36),

$$2 * 1776.9127923826\ \mathrm{MeV/c^2} = 96^4 * Basic\ neutral\ particle's\ energy/c^2 \quad (39.12)$$

It turns out that such an energy is **~41.8418788293579 eV/c²**, but this value is very close to the result of (24) $ElectronMass * (a-1)(1-b) = \sim\mathbf{41.8752442118608}\ \boldsymbol{eV/c^2}$ with a relative error ~1255.050626^{-1}. The author's opinion is that, as is, a relative error, slightly smaller than 10^{-3} is not sufficient to be considered as a finding, however, when taken into account along with (24) and (36), and the fact that (39.12) was expected to yield a fundamental unit of mass, (39.12) cannot be ignored. 41.875244211860eV-41.8418788293579eV seems to set a strict neutrino mass bound of ~0.0333653825021eV/c^2 about, **0.033eV/c²**.

If we test the following values for $d \cong 606400.8$, which complies with [35], we get: $\frac{95^4}{d} \cong 134.3181357940161..$, $\frac{96^4}{d} \cong 140.0635619214..$ and the geometric average of these two values is $\left(\frac{95^4}{d}\frac{96^4}{d}\right)^{\frac{1}{2}} = \frac{95^2 96^2}{d} \cong 137.1607689$. It is not difficult to see the following:

As a result of the conclusions of (38), (39), the exact inverse Fine Structure Constant was found by the following, although some aspects of the following calculation are not resolved yet. We put together (20), (22), (34), (35), (37), $\frac{1}{2}(1-g_2)^{-4} \cong 607276.5368006824282929$ from (39.9), and from (35) $\xi \cong 1.5561985371903483965638770314399 15299$,

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 a^{-2}+\frac{4}{\pi}a^{-1}\right)=a \Longrightarrow \frac{1}{a-1} \cong 206.75133988502202$$

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 b^{-2}-\frac{4}{\pi}b^{-1}\right)=b \Longrightarrow \frac{1}{1-b} \cong 44.63955017596401$$

$$d = \left(\frac{1}{2}(1-g_2)^{-4}\right)^{\frac{1}{1+(a-1)(1-b)}} \cong 606401.0372 \approx 606400.8$$

$$\frac{2}{\cos\left(1.5561985371903484*(1+\frac{1}{d})\right)} \cong 137.0359990368270076 \approx 137.035999037 \qquad (40)$$

$1.5561985371903483965638770314399 * (1+\frac{1}{d})$ exceeds the maximal allowed value of the field strength $\xi = 1.5561985371903483965638770314399$ and therefore must account for emission of what we know in mainstream physics as photons.

Note: $p = ((a-1)(1-b))^{-\frac{1}{2}} \cong 96.0691772148863$ is a very special number in the following property that bridges between area ratios and powers as follows, denote $s = \frac{1}{2}(1-g_2)^{-4}$ then $s^{\left(\frac{1}{1+(a-1)(1-b)}\right)} \approx s\left(2-\frac{1}{96^2(a-1)(1-b)}\right)$ or written as numbers 606401.0372 ~ 606401.0194 with a relative error of about 34,109,836.56^{-1}. An exact equality, $s^{\frac{1}{1+p^{-2}}} = s\left(2-\frac{p^2}{96^2}\right) \cong 606401.0371$, follows from replacing $p = \sim 96.0691772148863$ with p = ~96.06917582 with a relative error in $96.0691772148863 = ((a-1)(1-b))^{-1/2}$ of

~1.45953 * 10^{-8}. If the reader still thinks (40) is a fluke of chance, then this note does not agree with such a hypothesis. Also note that p comes from (20), (22) which resulted in (24). See Python code and it's more exact output in Appendix F. Slightly different values are obtained for $\xi\left(1-\frac{1}{s^{\left(\frac{1}{1+(a-1)(1-b)}\right)}}\right)^{-1}$, the reason for a term $\xi\left(1-\frac{1}{d}\right)^{-1}$ instead of $\xi\left(1+\frac{1}{d}\right)$ is important and is discussed in (42.1).

Another result is by finding the variable s where a and b are given in (40) and consider using 95*96 from (23.1), (23.2), and in its second power in (39.11) as follows:

$$\left(\frac{95^2*96^2}{s}\right)^{1+(a-1)(1-b)} = \frac{2}{\cos\left(\xi\left(1+\frac{1}{s^{\left(\frac{1}{1+(a-1)(1-b)}\right)}}\right)\right)} \Longrightarrow \tag{41}$$

$$\left(\frac{95^2*96^2}{s}\right)^{1+(a-1)(1-b)} \cong 137.035999036428876252$$

Very important: Another more accurate algorithm reached 137.03599903642884783039335161447525 and s ~= 607276.54683397442568093538284301757812 and comparing this value to $\frac{1}{2}(1-g_2)^{-4} \cong$ 607276.536800682428292930126190185546875 from (39.9), it is statistically impossible that (41) is a coincidence. The inverse relative error in relation to $\frac{1}{2}(1-g_2)^{-4}$ is,

$$Err^{-1} = \left(\frac{s}{\frac{1}{2}(1-g_2)^{-4}} - 1\right)^{-1} \cong 60526150.002596460282802581787109 \approx 6*10^8 \tag{41.1}$$

This result means that any claim that the results of this paper are by chance, is ridiculous and possibly irresponsible.

Where the term $\frac{1}{1+(a-1)(1-b)}$ is taken from (24) but with $\xi = \frac{4}{\pi}$ as in (40) and not $\xi = \frac{95}{96}$.

A $\left(\frac{95^2*96^2}{s}\right)^{1+\frac{1}{95*96}}$ ≅137.0359992990990, s ≅607280.4243559269234538 and $s^{1/(1+(96*95)^{-1})} \cong$ 606394.43614689458627253770 and $(a-1)(1-b)$ replaced with $(96*95)^{-1}$.

With $a = g1, b = g2$ from (34), $1+(a-1)(1-b)$ yields in (41), $s \cong$ 607279.477540519786998629570007 and $\left(\frac{95^2*96^2}{s}\right)^{1+(a-1)(1-b)} \cong$ **137.035999234958467241085600**

Slightly different values are obtained for $\xi\left(1-\frac{1}{s^{\left(\frac{1}{1+(a-1)(1-b)}\right)}}\right)^{-1}$, the reason for a term $\xi\left(1-\frac{1}{d}\right)^{-1}$ instead of $\xi\left(1+\frac{1}{d}\right)$ is important and is discussed in (42.1).

The latter choice needs to be better explained as follows:

Consider the Airy function Bi, Airy functions Ai and Bi are very popular in Quantum Mechanics [36],

$$Bi(x)=\frac{1}{\pi}\int_0^\infty\left[\exp\left(-\frac{t^3}{3}+xt\right)+\sin\left(\frac{t^3}{3}+xt\right)\right]dt$$

The reason Bi(x) is tested here is because it maps the highest ξ value from (34) near 2 and along with Ai(x) it is used to describe the wave function of a particle in a triangular potential well.

Table 1.

ξ	Airy function $Bi(\xi)$	$\frac{1}{2-Bi(\xi)}$	$\frac{1}{\sqrt{((a-1)(1-b))}}$, see (19), a, b are the largest roots, a>1, b<1.
Electron, $\frac{95}{96}$	1.1977758259 63505749636	1.2465343632 91960163318	110.46668611244152202743862 2899353504180908203125
Muon, $\frac{4}{\pi}$	1.5153444515 1380815816	2.0633210599 2280961894	96.069177214886309457142488 099634647369384765625
Tauon, 1.5561985371903483965638770314399…	1.9895372119 25930116713	95.576818809 7318024431364884	**95.63705426268688825075514614582061767578125**
$\arccos\left(\frac{2}{137.0359990368270078..}\right)$	1.9895424786 76644522794	**95.624954430 323135605547**	
$\arccos\left(\frac{2}{137.0359992349584672..}\right)$	1.9895424787 19955590708	**95.624954826 365255405784**	

We can see a surprising possible relation between $\frac{1}{2-Bi(\xi)}$, $\frac{1}{((a-1)(1-b))}$ and the Fine Structure Constant. This relation is one of the motivations to try the value $1+(a-1)(1-b)$ in (41), where $a=g_1$ and $b=g_2$ in (34).

Another idea is to solve the following equation where s is given by (34) and p is a variable and where $\frac{95^2*96^2}{s}=95^2*96^2*2(1-g_2)^4$ is from (39.10):

$$s=\left(\frac{1}{2}(1-g_2)^{-4}\right)\cong 607276.536800682428292930$$

$$\left(\frac{95^2*96^2}{s}\right)^{1+1/(p*p)}=\frac{2}{\cos\left(\xi\left(1+\frac{1}{s^{\left(\frac{1}{1+1/(p*p)}\right)}}\right)\right)}\Longrightarrow \quad (42)$$

$$\left(\frac{95^2 * 96^2}{s}\right)^{1+1/(p*p)} \cong \mathbf{137.035999035747181551}, p \cong 96.0706666703058402 85$$

$1 + \frac{1}{s^{\left(\frac{1}{1+1/(p*p)}\right)}}$ approximates $(1 - \frac{1}{s^{\left(\frac{1}{1+\frac{1}{p*p}}\right)}})^{-1}$ and

$$\left(\frac{95^2 * 96^2}{s}\right)^{1+\frac{1}{p*p}} = \frac{2}{\cos\left(\xi\left(1 - \frac{1}{s^{\left(\frac{1}{1+\frac{1}{p*p}}\right)}}\right)^{-1}\right)} \Longrightarrow$$

$$\left(\frac{95^2 * 96^2}{s}\right)^{1+\frac{1}{p*p}} \cong \mathbf{137.0359990754972727700078394263982 8},$$

$$p \cong 96.070640530239074905693996697664,$$

$$d = s^{\left(\frac{1}{1+\frac{1}{p*p}}\right)} \cong 606401.063820790150202810764312744140 62$$

The meaning of the term $\xi\left(1 - \frac{1}{s^{\left(\frac{1}{1+\frac{1}{p*p}}\right)}}\right)^{-1}$ is that a 4-volume unit is contracted by a factor of $1 - \frac{1}{s^{\left(\frac{1}{1+\frac{1}{p*p}}\right)}}$ and that the field strength ξ is increased as a result.

For comparison, if we set p=96 in the right-hand side of (42) we get the value 137.035999086935760260530515. Combining (41) and (42) we find a numerical attractor at (42) with $s \cong \mathbf{607276.53680068242829293012 62} \cong (\frac{1}{2}(1 - g_2)^{-4})$, $s^{\left(\frac{1}{1+1/(p*p)}\right)} \cong$ 606401.064296812633983791, $\xi \cong \mathbf{1.5561985371903484}$ from (35). Before we close this discussion, it is nice to mention another relation $(1 - \ln\left(\left(1 + \frac{1}{137.035999035747181551}\right)^{137.035999035747181551}\right))^{-1} \cong 275.4045237287 \approx 275.51693 \cong$ $(g_1 - 1)^{-1}$ in (43.10). That is not a total surprise because $(1 - \ln\left(\left(1 + \frac{1}{z}\right)^{z}\right))^{-1} \approx 2z$ for big z.

Reverse engineering Nature – Looking for simple but not random relations

In this section a much less significant result than (24), (40), remark after (40), (41), (42), will be considered as an interesting course of research. This time, an approximation of the inverse Fine Structure Constant will not be as nearly as accurate and will not be a result of exponential perturbations of a Reeb class field strength.

The search for meaningful field strength coefficients for the electron, Muon and Tau lepton reached the following $\xi \in \{\frac{95}{96}, \frac{4}{\pi}, \sim 1.5561985371903483965638770314399\}$

But these field strength coefficients did not appear out of solutions to equation (4). In fact, there has been no collaboration with mainstream physics to reach such solutions and especially to the complex form of (4). The analytic solutions of such an equation make take decades and without collaboration on solving the Lagrangians in (4), (64), (65), other approaches are required in order to convince the reader that the choices of field strength coefficients are not a mere mathematical pareidolia. The assessment of the mass ratio between the Muon and the electron in (24) is already with a sufficiently small error to trigger interest, especially when considering the simplicity of (24) and that the choice of $\frac{4}{\pi}$ came out of an existing theory [26]. (40), the remark after (40), (41) and (42) are also strong indicators that this research is on the right path. It will be wrong not to mention other findings which are straight forward from the method which had been presented in (16), (17), (18), (19) and the first interesting result (20). In this method, the Reeb class vector term was collapsed with the non-geodesic or accelerated time direction $\frac{P^\mu}{\sqrt{|Z|}}$ and we saw the contraction $\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2} g_{\mu\nu} U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right)\frac{P^\mu P^\nu}{Z}$ that resulted in (20), (22).

With acceleration field $\frac{\xi c^2}{r}$, where $\xi = \frac{4}{\pi}$ denotes the field strength and x is the adjustment factor of the acceleration field because of area loss, we used the term $\left(-\frac{1}{2}\frac{\xi^2}{r^2 x^2} \mp \frac{\xi}{r^2 x}\right)\frac{\pi}{24} r^4 = \left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{\pi}{24} r^2$ to express area loss due to a gravitational field at small r in the far observer coordinates. Now it is time to look at area loss in a direction perpendicular to the direction of time, namely the momentum direction in spacetime, or as expressed through a bivector derived from a unit vector, consider $\frac{U^\mu U^\nu}{U^\lambda U_\lambda}$ and for the sake simplicity, the contraction is not with a complex bivector $\frac{2U^{*\mu} U^{*\nu}}{U^{*\lambda} U_\lambda + U^\lambda U^{*}{}_\lambda}$. From $U^\mu P_\mu = 0$, it is easy to see the following,

$$\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2} g_{\mu\nu} U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right)\frac{U^\mu U^\nu}{U^\lambda U_\lambda} = \frac{1}{8} U_\lambda U^\lambda = \frac{1}{2}\frac{\xi^2}{r^2 x^2} \qquad (42.2)$$

The latter is to achieve a reduction of the curvature calculation from Lorentzian to Riemannian geometry.

Caveat: Notice that using $\frac{U^{\mu}U^{\nu}}{U^{\lambda}U_{\lambda}}$ and not $\frac{U^{\mu}U^{\nu}}{|U^{\lambda}U_{\lambda}|}$ is done here in order to achieve $g_{\mu\nu}\frac{U^{\mu}U^{\nu}}{U^{\lambda}U_{\lambda}} = +1$ as expected from a unit vector in (+,-,-,-) metric convention. The reader may criticize this choice of a bivector because Reeb class vectors in this paper are space-like and not time-like because they represent non-geodesic acceleration as a result of misaligned events in an observer spacetime object.

Multiplying by $\frac{\pi}{24}r^4$ due to [24], see lecture of Seth Lloyd, and dividing by 4 times the area of an Euclidean disk, due to assumption 2 after the note after (15), yields,

$$-\frac{1}{192}\frac{\xi^2}{x_3{}^2} = \frac{1}{2}\frac{\xi^2}{r^2 x_3{}^2}\frac{\pi}{24}r^4\frac{1}{4\pi r^2} = x_3 - 1 \qquad (42.3)$$

From which

$$\frac{192 x_3{}^2 - \xi^2}{192} = x_3{}^3 \qquad (42.4)$$

Which is an iterative equation that converges to the most stable root, a technique that had been used in all previous third order polynomial equations. Solving for $\xi = \frac{4}{\pi}$ as in (20), (22), yields,

$$(x_3 - 1)^{-1} \cong 120.4106111611239198282419238239526748657226562 5 \qquad (42.5)$$

$$(x_1 - 1)^{-1} \cong 206.751339885022019871030352078378200531005859375$$

$$(1 - x_2)^{-1} \cong 44.63955017596401120272275875322520732879638671875$$

And the following calculation yields an interesting result,

$$\ln\left((x_1 - 1)^{-1}(1 - x_2)^{-1}(x_3 - 1)^{-1}\right)^2 2^{-\frac{1}{2}}$$

$$\cong \ln(1111304.06504770903848111629486083984375)^2\, 2^{-\frac{1}{2}} \cong$$

$$137.034102324667713901362731121480464935 3 \qquad (42.6)$$

Which is a surprisingly simple and unexpected approximation of the inverse Fine Structure Constant. The relative error of (42.6) in relation to the result in (40) is about 72249.23316^{-1} which is not even closely significant as (40), (41), (42) or the remark after (40) and yet, if this result joins other approximations of the inverse Fine Structure Constant in this paper, it is not wise to ignore (42.6). In (24.3), (40), (41), (42), (81)-(86), the inverse Fine Structure Constant comes out of exponential field perturbations as in (24.3), (40), (41), (42) or as exponential functions of 2 or $\frac{4}{\pi}$ with coefficients $(95*96)^{-1}$ or 95 and 96 as seen in (81)-(86). Notice that both (24.3) and the last result, involve the square root of 2.

Hypergeometric tests - Dr. Sam Vaknin's suggestion from 2013

A suggestion from Dr. Sam Vaknin regarding the possible solutions of the equations of the Geometric Chronon Field Theory was that they are related to Hypergeometric functions [37]. His idea was lately checked regarding the stable roots of third order polynomials of gravity and anti-gravity, area ratio loss and gain, see (22.1) and (22.2). The stable field strength coefficients were defined as $\xi = \frac{193}{192} = 1 + \frac{1}{192}$ for negative charge and $\xi = \frac{63}{64} = 1 - \frac{1}{64} = 1 - \frac{3}{192}$ for positive charge. The summation of the two deltas $+\frac{1}{192} - \frac{1}{64}$ to 1 yields the field strength coefficient $\xi = 1 + \frac{1}{192} - \frac{1}{64} = 1 + \frac{1}{192} - \frac{3}{192} = 1 - \frac{1}{96} = \frac{95}{96}$. The question is what do these values $\frac{1}{192}$ and $\frac{3}{192}$ teach us about any possible grand theory of particle physics? 1,3 and 192 with 192 in the denominator should hint us about such a theory. As we saw in (40), (41), (42) a key number in the calculation of the positive perturbation over ξ was

$s = 0.5/(1 - g_2)^4 \cong 607276.536800682428292930126190185546875$, see (42).

Can this number be a result of combinatorial mixing by the Gauss hypergeometric function 2F1?

The question is if ${}_2\mathrm{f}_1(\mathrm{a},\mathrm{b},\mathrm{c},\mathrm{z}) = \sum_{k=0}^{\infty} \frac{(a)_n (\mathrm{b})_n}{(c)_n} \frac{z^n}{n!}$, such that $(q)_n = \begin{Bmatrix} 1 & \left| n = 0 \right. \\ q(q+1)(q+2)\ldots(q+n-1) & \left| n \geq 1 \right. \end{Bmatrix}$ can yield such a number in a meaningful way.

If Dr. Sam Vaknin was right, we may be able to find a meaningful z that solves

$$ {}_2\mathrm{f}_1(-3,1,192,\mathrm{z}) = 1 - 2(1 - g_2)^4 \tag{42.6}$$

This is exactly what was done numerically. The result was very surprising, and it is very unlikely that it is a fluke of chance:

$$ z \cong \frac{2}{(137.036271402616947057140350807 4)^2} \tag{42.7}$$

The relative error of $137.0362714026169470571403508074$ from the assessment $137.035999036827007557 8\ldots$ in (40) is about $\frac{137.0359990368270075578 - 137.0359990368270075578\ldots}{137.0359990368270075578\ldots} \cong 503132.19978307740529999136 92^{-1}$

Also, quite near the higher value 137.0359992990990… after (41).

It is quite compelling to say that Dr. Sam Vaknin was right already back then in 2013. There is even stronger evidence in his favor.

Consider a second order perturbation on the hypergeometric coefficients -3, 1:

$$ {}_2f_1\left(-3*\frac{63}{64},1*\frac{193}{192},192,\frac{2}{(137.035999036827007557 \ldots)^2}\right) \cong 1-\frac{1}{607299.79207959282211959362030029296875 0} \tag{42.8} $$

and

$$ {}_2f_1\left(-3*\frac{193}{192},1*\frac{63}{64},192,\frac{2}{(137.035999036827007557 \ldots)^2}\right) \cong 1-\frac{1}{607299.806042340234853327274322509765625} \tag{42.9} $$

Comparing the right hand side denominator to $\frac{1}{2}\frac{1}{(1-g_2)^4} \cong$ 607276.5368006824282929301261901855 46875 from the remark before (40) and from (42), the results in (42.8) and (42.9) are very interesting although not within the ranges of (40)-(42) with a highest value of 137.0359992990990… .

Instead of $\frac{63}{64}$ if we consider all the powers of $-\frac{1}{64}$ we have $\frac{64}{65}=\sum_{k=0}^{\infty}\left(-\frac{1}{64}\right)^k$ and with $+\frac{1}{192}$ we have $\frac{192}{191}=\sum_{k=0}^{\infty}\left(\frac{1}{192}\right)^k$

Consider a second order perturbation on the hypergeometric coefficients -3, 1:

$$ {}_2f_1\left(-3*\frac{64}{65},1*\frac{192}{191},192,\frac{2}{(137.035999036827007557 \ldots)^2}\right) \cong 1-\frac{1}{607135.055724701262079179286956787109375} \tag{42.8.1} $$

and

$$ {}_2f_1\left(-3*\frac{192}{191},1*\frac{64}{65},192,\frac{2}{(137.035999036827007557 \ldots)^2}\right) \cong 1-\frac{1}{607135.069557101931422948837280273437500} \tag{42.9.1} $$

Here is the code in Python for (42.6) and (42.7):

```
import numpy as NP
from scipy.special import hyp2f1 as SCIPY_SPECIAL_hyp2f1

a = 137.035999036827007557803881354629993438720703
q = 607276.536800682428292930126190185546875
```

```
#s = NP.power(q * 2, 0.25)
s = NP.sqrt(NP.sqrt(q * 2))
s = NP.sqrt(s * s * s * 0.25)

print(f's={s:.42f}')

# Was a numerical analysis output:
w = 137.036271402616947057140350807 4
u = 1/(w/a - 1)

print(f'u={u:.42f}')

r = SCIPY_SPECIAL_hyp2f1(-3, 1, 192, 2/(w ** 2))
r = 1/(1-r)
r /= 607276.536800682428292930126190185546875
r = 1/(1-r)
r /= s
print(f'r={r:.42f}')


r = SCIPY_SPECIAL_hyp2f1(-3, 1, 192,
2/137.035999036827007557803881354629993438720703 ** 2)
r = 1/(1-r)
r /= 607276.536800682428292930126190185546875
r = 1/(1-r)
print(f'r={r:.42f}')
```

Here is the code in Python for (42.8), (42.9)

```
import numpy as NP
from scipy.special import hyp2f1 as SCIPY_SPECIAL_hyp2f1

Xi = 1.556198537190348396563877031439915299415588378906

r1 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 63/64, 1 * 193/192, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r1 = 1/(1-r1)
print(f'1/(1-r1)={r1:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha1 = 2/NP.cos(Xi*(1+1/NP.power(r1, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r1) {inverse_alpha1:.33f}')

r2 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 193/192, 1 * 63/64, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r2 = 1/(1-r2)
print(f'1/(1-r2)={r2:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha2 = 2/NP.cos(Xi*(1+1/NP.power(r2, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r2) {inverse_alpha2:.33f}')
```

```
r = r1 / r2
r = 1/(1-r)
print(f'1/(1-r1/r2)={r:.33f}')

r3 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 64/65, 1 * 192/191, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r3 = 1/(1-r3)
print(f'1/(1-r3={r3:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha3 = 2/NP.cos(Xi*(1+1/NP.power(r3, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r3) {inverse_alpha3:.33f}')

r4 = \
    SCIPY_SPECIAL_hyp2f1(-3 * 192/191, 1 * 64/65, 192,
                         2/137.035999036827007557803881354629993438720703 **
                         2)
r4 = 1/(1-r4)
print(f'1/(1-r4)={r4:.33f} compared to'
      f' 607276.536800682428292930126190185546875')

inverse_alpha4 = 2/NP.cos(Xi*(1+1/NP.power(r4, 1/(1+1/(95*96)))))
print(f'Inverse alpha(r4)
{2/NP.cos(Xi*(1+1/NP.power(r4,1/(1+1/(96*96))))):.33f}')

r = r3 / r4
r = 1/(1-r)
print(f'1/(1-r3/r4)={r:.33f}')

print(f'(1/Alpha1+1/Alpha3)/2: '
      f'{(inverse_alpha1+inverse_alpha3)/2:.33f}')
print(f'(1/Alpha2+1/Alpha4)/2: '
      f'{(inverse_alpha2+inverse_alpha4)/2:.33f}')
```

7. The mass hierarchy

By (13) and considering the Planck mass $\sqrt{\frac{\hbar c}{K}}$ and the Fine structure constant Alpha:

$$\sqrt{\frac{\hbar c}{K} * \frac{e^2}{4\pi\varepsilon_0 \hbar c}} = \frac{2e}{2\sqrt{4\pi K\varepsilon_0}} = \frac{2e}{\sqrt{16\pi K\varepsilon_0}} = PlanckMass * \sqrt{Alpha} \tag{43}$$

So, multiplication of the Plank mass by the square root of the Fine Structure Constant yields twice the electro-gravitational mass of a charge e! If we take $\xi \cong 1.5561985371903484$ from (35) to be the maximal allowed field coefficient of an electric charge, then the field around a single charge as a normalized quantity is obtained as

$$\frac{1}{\xi}\frac{1}{2} PlanckMass * \sqrt{Alpha} = \frac{1}{\xi}\frac{e}{\sqrt{16\pi K\varepsilon_0}} \tag{44}$$

Now we recall from (24) the following root a around a negative charge:

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{95}{96}\right)^2 a^{-2}+\left(\frac{95}{96}\right)a^{-1}\right)=a\cong 1+192.0463944^{-1} \quad (45)$$

We take from (24), (40), $(a-1)(1-b)\cong\frac{1}{206.75133988502202*44.63955017596401}$ and calculate

$$\left(\frac{\frac{11}{\xi^2}PlanckMass*\sqrt{Alpha}}{M_e}\right)^{(a-1)(1-b)}\cong 1+192.04864774452^{-1} \quad (46)$$

Where $M_e\cong 0.5109989461\ MeV$, e is the electron's charge $1.602176634\times10^{-19}$ Coulombs, K is Newton's constant of gravity $6.674\times10^{-11}\ m^3\cdot kg^{-1}\cdot s^{-2}$, Planck mass 1.22091×10^{22} MeV, from (40) $Alpha\cong 137.0359990368270076^{-1}$. The relative error between (46) and (45) is $\frac{192.04864774452-192.0463944}{192.0463944}\cong 85{,}227.266539382^{-1}$. We are also led to the following conclusion that $\xi=\frac{95}{96}$ is the Reeb class field strength coefficient of the electron field, $\xi=\frac{4}{\pi}$ is the Muon field strength and from the solution to (35) $\xi=1.5561985371903484$ ... is the field strength coefficient of the Tau lepton. Of course, a lot of work has to be done to achieve exact analytic solutions to (4) and as we shall see also to (64), because only $\xi=\frac{4}{\pi}$ has a compelling source [26]. Serendipity is part of physics and mathematical rigor must follow.

8. Interesting acceleration to radius coefficients relation – the field strength coefficients

Consider the coefficients $\frac{95}{96}$, from (23) $\frac{4}{\pi}$, from (24) and $\xi=$ 1.556198537190348396563877031439915299415588 from (34), (35). Note the following table

Table 2.

ξ of Electron, Muon, Tau	$\xi\left(\frac{4}{\pi}\right)^{-1}$	$\xi\cdot 9\cdot\left(\frac{4}{\pi}\right)^{-1}$
$\frac{95}{96}$	0.7772169325287248897~$\frac{7}{9}$	6.994952392758524
$\frac{4}{\pi}$	1 = $\frac{9}{9}$	9
1.5561985371903483965638770314399	1.22223547299109529~$\frac{11}{9}$	11.0001192569

The numbers in the left column suggest that the field strength coefficients are related to natural numbers. Models of natural numbers occur for example in Heisenberg's XXZ model of spin chains. The normalization factor for L=4 and $\Delta=1$ in such a spin chain is remarkably close to $\frac{4}{\pi}$, and remarkably close to 1.556198537190348… with L=11 and $\Delta=1$. The normalization factor is close to $\frac{95}{96}$ when L=13 and $\Delta = 0$. Here the model is not of any spin chain and spin chains are only brought as an example of how natural numbers can be related to numbers such as this model's field strength coefficients.

The reader can check that 7 * 9 * 11 = 693 is the integer floor of $\left(\frac{1}{96^2(a-1)(1-b)} - 1\right)^{-1} \cong 693.634239847$, see the note after (40) with $2 - \frac{1}{96^2(a-1)(1-b)}$. We have yet to show more compelling evidence the choice of ξ is not by chance. Some readers will remain skeptical no matter what evidence is brought in this paper. This section is not meant for such readers but for readers who agree that serendipity is important for new discoveries in physics. The coefficient $\frac{4}{\pi}$ from (22), (80) is well understood [26], however, $\frac{95}{96}$ from (23), (79), (86) and 1.5561985371903483965638770314399 from the solution to (35) are not well understood. Evidence, except from the previous table and the note after (40), can be found if we look at the polynomial term that means loss or addition of area in relation to 4 times the area of a disk. The factor 4 was thoroughly discussed before (16) and led to the number 96 from $\frac{\frac{1}{4}*\frac{\pi}{24}*R(3)*r^4}{\pi r^2} = \frac{R(3)*r^2}{96}$ where R(3) is obtained by double contraction of the Einstein tensor with a direction of time. We return to (18), $\left(-\frac{1}{2}\frac{\xi^2}{x^2} \mp \frac{\xi}{x}\right)\frac{1}{96} = \frac{\delta Area}{4\pi r^2}$ and consider the following polynomials $\left(-\frac{1}{2}\xi^2 \mp \xi\right)\frac{1}{96}$ of the field strength coefficient ξ. Like before in (23), we consider the terms $\frac{1}{a-1}$ and $\frac{1}{1-b}$ from the biggest and stable roots a, b. Not too surprisingly, these terms are approximated by $\alpha = p1(\xi) = \left(\left(-\frac{1}{2}\xi^2 + \xi\right)\frac{1}{96}\right)^{-1}$ and $\beta = p2(\xi) = \left(\left(-\frac{1}{2}\xi^2 - \xi\right)\frac{1}{96}\right)^{-1}$ which only depend on the field strength coefficients. Consider the following relative error terms $RatioA = \left(\frac{a-1}{\alpha} - 1\right)^{-1}$ and $RatioB = \left(\frac{1-b}{\beta} - 1\right)^{-1}$ or as an output of a python code:

```
Field strength coefficient analysis:

Xi=0.9895833333333334, p1=192.02083559413998, p2=64.89902805794975

Xi=0.9895833333333334, 1/(a-1)=192.04639436012951, 1/(1-b)=63.54135822920768

Xi=0.9895833333333334, RatioA=-7513.91496909199486, RatioB=46.80177527998884

----------------------------------------------------

Xi=1.2732395447351628, p1=207.49126659259227, p2=46.06948110927548

Xi=1.2732395447351628, 1/(a-1)=206.75133988502202, 1/(1-b)=44.63955017596401

Xi=1.2732395447351628, RatioA=279.42137750905704, RatioB=31.21797643232097

----------------------------------------------------
```

```
Xi=1.5561985371903484, p1=278.00172875202145, p2=34.69366870085835

Xi=1.5561985371903484, 1/(a-1)=275.51690891864394, 1/(1-b)=33.19740405023536

Xi=1.5561985371903484, RatioA=110.88003452715024, RatioB=22.18685313217340

-----------------------------------------------------
```

This output shows proximities between functions of the field strength coefficient, ξ or Xi in the Python output. The proximities are p1 of the next ξ to RatioA and p2 of the next ξ to RatioB. The first value ~-7513.91496909199486 is in red as an exception because it is not matched to the value of p1 for the next field strength coefficient $\xi = \frac{4}{\pi}$. Following is the code in Python that was used for the last calculations,

```
import numpy as NP


def function_p(p_x):

    return (-0.5 * p_x * p_x + p_x)/96, -(-0.5 * p_x * p_x - p_x)/96


def function_cubic_viete(a, b, c, d): # If all roots are real.

     # Viete's formula when all roots are real.

     b2 = NP.longdouble(b * b)

     b3 = NP.longdouble(b2 * b)

     a2 = NP.longdouble(a * a)

     a3 = a2 * a

     p = (3 * a * c - b2) / (3 * a2)

     q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)

     offset = b / (3 * a)

     t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) \

                                                  * (3 * q) / (2 * p)) / 3)

     t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \

                                                  (3 * q) / (2 * p)) / 3 - NP.pi / 3)

     t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \

                                                  (3 * q) / (2 * p)) / 3 - 2 * NP.pi /
3)

     x1 = t1 - offset
```

```
     x2 = t2 - offset
     x3 = t3 - offset

     return (x1, x2, x3)

ma_list = [95/96, 4/NP.pi, 1.5561985371903484]

print('Field strength coefficient analysis:')

for ma_x in ma_list:
    ma_tuple = function_p(ma_x)

    ma_a,_,_ = function_cubic_viete(1, -1, -ma_x / 96,
                                    (ma_x * ma_x) / 192)

    ma_b,_,_ = function_cubic_viete(1, -1, ma_x / 96,
                                    (ma_x * ma_x) / 192)

    print('Xi={}, p1={:.14f}, p2={:.14f}'.format(ma_x, 1/ma_tuple[0], 1/ma_tuple[1]))
    print('Xi={}, 1/(a-1)={:.14f}, 1/(1-b)={:.14f}'.format(ma_x, 1/(ma_a-1), 1/(1-
ma_b)))

    ma_a = (ma_a - 1) / ma_tuple[0]
    ma_b = (1 - ma_b) / ma_tuple[1]
    ma_a = 1 / (ma_a - 1)
    ma_b = 1 / (ma_b - 1)

    print('Xi={}, RatioA={:.14f}, RatioB={:.14f}'.format(ma_x, ma_a, ma_b))
    print('------------------------------------------------------')
```

We now return to the field which is smaller than 1, namely to $\xi = \frac{95}{96}$. It is easy to see that if we pick $\xi_1 = \frac{193}{192} = 1 + \frac{1}{192}$ and $\xi_2 = \frac{63}{64} = 1 - \frac{1}{64}$ we get rational roots for the following anti-gravity equation $x_1{}^2 + \frac{1}{96}\xi_1 x_1 - \frac{1}{192}\xi_1{}^2 = x_1{}^3$ and gravity equation $x_2{}^2 - \frac{1}{96}\xi_2 x_2 - \frac{1}{192}\xi_2{}^2 = x_2{}^3$ for which $x_1 = \xi_1$ and $x_2 = \xi_2$, interestingly $1 + \frac{\xi_2 - \xi_1}{2} = \frac{95}{96}$ and $\frac{\xi_1 - \xi_2}{2} = \frac{1}{96}$.

Some nice relation between the roots of gravity and anti-gravity of area ratio polynomials with field strength coefficients $\xi = \frac{95}{96}$ and $\xi \cong 1.556198537190348396563877031 4399$ as in (35) is considered. We saw that for $\xi \cong 1.5561985371903483965638770314399$ the following holds: $\frac{2(1-x_2)}{(x_1-1)^{\frac{1}{2}}} = 1$. There is another relation not less illuminating, $\frac{4(1-x_2)^{\frac{1}{2}}}{x_1-1}$. With low accuracy of a simple datasheet we can see that for $\xi = \frac{95}{96}$ we get $\frac{4(1-x_2)^{\frac{1}{2}}}{x_1-1} \cong 96.36912199$ and for $\xi \cong 1.5561985371903483965638770314399$ as in (35), we get $\frac{4(1-x_2)^{\frac{1}{2}}}{x_1-1} \cong 191.2741085$ which is almost 192=2*96. Multiplying these two values together we have $96.36912199 \ldots * 191.2741085 \ldots \cong 18432.9179 \approx 18432 = 2 * 96^2$, and we can see $(\frac{18432.9179}{2})^{\frac{1}{2}} \cong 96.00239033$.

Conclusion

The presented model predicts gravity not only by mass but also by electric charge. It offers a technological breakthrough by generating inertial dipoles and it offers mass ratios between particles that are not accessible through the Standard Model. (33) and (65) can only be interpreted as the existence of a fifth force of Nature with symmetry SU(4), or by (3.12) is related to gravity, while (24) results in a new neutrally charged particle of energy ~41.8752442118608 eV and (39.12) seems to set a strict electron neutrino mass bound of ~0.033eV/c^2. The muon field strength coefficient is different than the electron's and Tauon field strength, which implies different physics. (33) indicates a deep relation between leptons and hadrons and especially between the Muon and the Bottom Quark.

As for the theoretic approach that this paper took, not any Gauge fields are a blessing. There was a big expectation from Albert Einstein that the Palatini action, which is identical to Einstein-Hilbert action, would be a great insight into Quantum Gravity, especially since spinor equations require tetrads because they are limited to an orthogonal reference frame. However, this paper took a very different approach, to leave the metric tensor as is and instead of using tetrads or Ashtekar variables, to consider the metric as of a reference manifold, like coordinates but as an entire geometric reference object, not as a physically accessible object. Then in this framework, the idea was that time must be the engine of the model and that acceleration of that time in the sense of a generalized Reeb class field - not limited to contact manifolds - will describe the possibility of non-geodesic curves and will predict the electric force. In (64) it becomes an electro-weak-strong action, using indeed 5 fields, but unlike tetrads, time is a meaningful Geroch function, while the other fields are Gauge fields. There is a redundancy in the system because this time can be accounted for by 3 vectors just as Ashtekar variables. This redundancy is cancelled out in action (64), instead of using an ADM formalism or Ashtekar variables, and orthogonality is no longer needed, which renders spin connections redundant. 4 out of the 5 scalar fields describe an additional geometric information to the metric as foliations. The same theory can be written with tetrads and generalized Reeb class vectors of these tetrad fields, but the Einstein-Hilbert action will be the same. On the other hand, action (64) in this case, does add geometric information as non-geodesic alignment of curves and thus of forces. It is a far simpler approach than that of Abhay Ashtekar and it reaches new results. Adding a summation constraint to the action of (64), e.g. that each chronon probability sums to 1, keeps the same action but then PP* is replaced by an event function and the integration of PP* becomes 1. That requires the only constant in the theory except for the speed of light to be with the units of $Length^{-2}$ and then the action is defined *almost everywhere* in terms of measure theory.

Appendix A: Euler Lagrange minimum action equations

We assume $\sigma = 8\pi$ (from the previously discussed term, $-a_\mu a^\mu / 8\pi K$ as an energy density).

$$\mathrm{Z} = N^2 = \mathrm{P}_\mu \mathrm{P}^\mu \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \text{ and } \mathrm{L} = \frac{1}{4}\mathrm{U}^\mathrm{k}\mathrm{U}_\mathrm{k}$$
$$R = Ricci \text{ curvature.}$$
$$Min\,Action = Min\int_\Omega \left(R - \frac{8\pi}{\sigma} L \right)\sqrt{-g}\,d\Omega =$$
$$Min\int_\Omega \left(R - \frac{1}{4}\mathrm{U}^\mathrm{k}\mathrm{U}_\mathrm{k} \right).\sqrt{-g}\,d\Omega \text{ s.t. } \sigma = 8\pi \tag{47}$$

The variation of the Ricci scalar is well known. It uses the Platini identity and Stokes theorem to calculate the variation of the Ricci curvature and reaches the Einstein tensor [38], as follows,

$\delta R = R_{\mu\nu}\delta g^{\mu\nu}$ and $\delta\sqrt{-g} = -\frac{1}{2} g_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}$ by which we infer $\delta(R\sqrt{-g}) = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\delta g^{\mu\nu}$ which will be later added to the variation of $\left((R - \frac{1}{4}U^j U_j)\right)\sqrt{-g}$ by $\delta g^{\mu\nu}$. The following Euler Lagrange equations have to hold,

$$\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial g^{\mu\nu}{}_{,m}} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial g^{\mu\nu}{}_{,m,s}}\left((R - \frac{1}{4}U^j U_j)\sqrt{-g}\right) = 0,$$

$$\frac{\partial}{\partial P} - \frac{d}{dx^m}\frac{\partial}{\partial P_{,m}} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial P_{,m,s}}\left((R - \frac{1}{4}U^j U_j)\sqrt{-g}\right) = 0$$

$U^k U_k = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3}$ which we obtain from the minimum Euler Lagrange equation because $U_\lambda P^\lambda = \frac{Z_\lambda P^\lambda}{Z} - \frac{Z_k P^k P_\lambda P^\lambda}{Z^2} = 0$. In order to calculate the minimum action Euler-Lagrange equations, we will separately treat the Lagrangians, $L = \frac{Z_\mu Z^\mu}{Z^2}$ and $L = \frac{(Z_s P^s)^2}{Z^3}$ to derive the Euler Lagrange equations of the Lagrangian $L = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3} = U_\mu U^\mu$. The Euler Lagrange operator of the Ricci scalar $(\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial(g^{\mu\nu},_m)} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial(g^{\mu\nu},_m,_s)}$.

The reader may skip the following equations up to equation (53). Equations (53), (54) and (55) are however crucial. Note: the relation $\frac{d}{dx^\nu}\sqrt{|g|} = \Gamma^\lambda_{\lambda\nu}\sqrt{|g|}$ is used in the next equations.

$$L = \frac{(P_\lambda Z^\lambda)^2}{Z^3}\ s.t.\ Z = P_\mu P^\mu\ and\ Z_s \equiv Z,_s = \frac{dZ}{dx^s}$$

$$\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m}$$
$$= \Bigg(-2\left(\frac{Z,_s P^s}{Z^3}P_\mu P_\nu P^m\right);_m + 2\left(\frac{Z,_s P^s}{Z^3}\right)\left(\Gamma^{\mathrm{i}}_{\mu\mathrm{m}}P_i P_\nu P^m + \Gamma^{\mathrm{i}}_{\nu\mathrm{m}}P_\mu P_i P^m\right)$$
$$+ 2\left(\frac{Z,_s P^s}{Z^3}\right)(P_\mu P_\nu);_m P^m - 2\left(\frac{Z,_s P^s}{Z^3}\right)\left(\Gamma^{\mathrm{i}}_{\mu\mathrm{m}}P_i P_\nu P^m + \Gamma^{\mathrm{i}}_{\nu\mathrm{m}}P_\mu P_i P^m\right)$$
$$+ 2\left(\frac{Z,_s P^s}{Z^3}\right)Z_\mu P_\nu - 3\frac{(Z,_s P^s)^2}{Z^4}P_\mu P_\nu - \frac{1}{2}\frac{(Z,_s P^s)^2}{Z^3}g_{\mu\nu}\Bigg)\sqrt{-g} =$$

$$\left(-2\left(\frac{Z,_s P^s}{Z^3}P^k\right);_k P_\mu P_\nu - 2\frac{(Z,_s P^s)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - \frac{(Z,_s P^s)^2}{Z^3}\frac{P_\mu P_\nu}{Z} + 2\left(\frac{Z,_s P^s}{Z^3}\right)Z_\mu P_\nu - \frac{1}{2}\frac{(Z,_s P^s)^2}{Z^3}g_{\mu\nu}\right)\sqrt{-g} \quad (48)$$

$$L = \frac{Z^\lambda Z_\lambda}{Z^2}\ s.t.\ Z = P_\mu P^\mu, s.t.\ Z = P_\mu P^\mu\ and\ Z_s \equiv Z,_s = \frac{dZ}{dx^s}$$

$$\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g}\,)}{\partial g^{\mu\nu},_m} = \left(-2\left(\frac{Z^m P_\mu P_\nu}{Z^2}\right);_m + 2\frac{(\Gamma^{\mathrm{i}}_{\mu\mathrm{m}}\mathrm{P_i P_\nu Z^m} + \Gamma^{\mathrm{i}}_{\nu\mathrm{m}}P_i P_\mu Z^m)}{Z^2} + 2\frac{(P_\mu P_\nu);_m Z^m}{Z^2} - \right.$$
$$\left. 2\frac{(\Gamma^{\mathrm{i}}_{\mu\mathrm{m}}\mathrm{P_i P_\nu Z^m} + \Gamma^{\mathrm{i}}_{\nu\mathrm{m}}P_i P_\mu Z^m)}{Z^2} + \frac{Z_\mu Z_\nu}{Z^2} - 2\frac{Z_s Z^s}{Z^3}P_\mu P_\nu - \frac{1}{2}\frac{Z_m Z^m}{Z^2}g_{\mu\nu}\right)\sqrt{-g} = \left(-2\left(\frac{Z^m}{Z^2}\right);_m P_\mu P_\nu - \right.$$
$$\left. 2\frac{Z_s Z^s}{Z^3}P_\mu P_\nu - \frac{1}{2}\frac{Z_m Z^m}{Z^2}g_{\mu\nu} + \frac{Z_\mu Z_\nu}{Z^2}\right)\sqrt{-g} \quad (49)$$

We subtract (48) from (49)

$$Z = P_\mu P^\mu, s.t.\ Z = P_\mu P^\mu\ and\ Z_s \equiv Z,_s = \frac{dZ}{dx^s}, U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2}, L = U^k U_k = \frac{Z_\lambda Z^\lambda}{Z^2} - \frac{(Z_k P^k)^2}{Z^3}$$

$$\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}}-\frac{d}{dx^m}\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu},_m}=\left(+2\left(\frac{Z_mP^m}{Z^3}P^k\right);_k P_\mu P_\nu+2\frac{(Z_mP^m)^2}{Z^3}\frac{P_\mu P_\nu}{Z}-2\frac{Z_mP^m}{Z^3}Z_\mu P_\nu+\frac{1}{2}\frac{(Z_mP^m)^2}{Z^3}g_{\mu\nu}+\frac{(Z_mP^m)^2}{Z^3}\frac{P_\mu P_\nu}{Z}+(-2\left(\frac{Z^m}{Z^2}\right);_m P_\mu P_\nu-2\frac{Z_\lambda Z^\lambda}{Z^2}\frac{P_\mu P_\nu}{Z}-\frac{1}{2}\frac{Z_\lambda Z^\lambda}{Z^2}g_{\mu\nu}+\frac{Z_\mu Z_\nu}{Z^2})\right)\sqrt{-g}=$$

$$\left(\left(+2\left(\frac{Z_mP^m}{Z^3}P^k\right);_k-2\left(\frac{Z^m}{Z^2}\right);_m\right)P_\mu P_\nu+2\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}\frac{P_\mu P_\nu}{z}-2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z}+\frac{1}{2}\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}g_{\mu\nu}-\frac{1}{2}\frac{Z_kZ^k}{Z^2}g_{\mu\nu}+\frac{Z_\mu Z_\nu}{Z^2}-2\left(\frac{Z_sP^s}{Z^3}\right)Z_\mu P_\nu+\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}\frac{P_\mu P_\nu}{Z}\right)\sqrt{-g}=\left(\left(+2\left(\frac{Z_mP^m}{Z^3}P^k\right);_k-2\left(\frac{Z^m}{Z^2}\right);_m\right)P_\mu P_\nu+2\frac{\left(P^\lambda Z_\lambda\right)^2}{Z^3}\frac{P_\mu P_\nu}{z}-2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{z}+U_\mu U_\nu-\frac{1}{2}U^\lambda U_\lambda g_{\mu\nu}\right)\sqrt{-g}=$$

$$\left(U_\mu U_\nu-\frac{1}{2}U^\lambda U_\lambda g_{\mu\nu}-2U^k;_k\frac{P_\mu P_\nu}{z}\right)\sqrt{-g} \tag{50}$$

$$L=\frac{(Z^sP_s)^2}{Z^3}\quad s.t.\ Z=P^\lambda P_\lambda\ and\ Z_m=(P^\lambda P_\lambda),_m$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_\mu,_\nu}=$$

$$\begin{pmatrix} -4(\frac{(Z_sP^s)}{Z^3}P^\mu P^\nu);_\nu+4\frac{(Z_sP^s)}{Z^3}\Gamma_i{}^\mu{}_\nu P^iP^\nu+ \\ +4\frac{(Z_sP^s)}{Z^3}P^\mu;_\nu P^\nu-4\frac{(Z_sP^s)}{Z^3}\Gamma_i{}^\mu{}_kP^iP^k+ \\ +2\frac{Z_mP^mZ^\mu}{Z^3}-6\frac{(Z_mP^m)^2}{Z^4}P^\mu \end{pmatrix}\sqrt{-g}=$$

$$(-4(\frac{(Z_sP^s)P^\nu}{Z^3});_\nu P^\mu+2\frac{Z_mP^mZ^\mu}{Z^3}-6\frac{(Z_mP^m)^2}{Z^4}P^\mu)\sqrt{-g} \tag{51}$$

$$L=\frac{Z^sZ_s}{Z^2}\quad s.t.\ Z=P^\lambda P_\lambda\ and\ Z_m=(P^\lambda P_\lambda),_m$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_\mu,_\nu}=$$

$$\begin{pmatrix} -4(\frac{P^\mu Z^\nu}{Z^2});_\nu+\frac{4}{Z^2}\Gamma_i{}^\mu{}_kP^iZ^k+ \\ +\frac{4}{Z^2}P^\mu;_\nu Z^\nu-\frac{4}{Z^2}\Gamma_i{}^\mu{}_kP^iZ^k+ \\ -4\frac{Z_mZ^m}{Z^3}P^\mu\sqrt{-g} \end{pmatrix}\sqrt{-g}=$$

$$(-4(\frac{Z^\nu}{Z^2});_\nu-4\frac{Z_mZ^m}{Z^3})P^\mu\sqrt{-g} \tag{52}$$

We subtracted the Euler Lagrange operators of $\frac{(Z^sP_s)^2}{Z^3}\sqrt{-g}$ in (48) from the Euler Lagrange operators of $\frac{Z^\lambda Z_\lambda}{Z^2}\sqrt{-g}$ in (49) and got (50) and we will subtract (51) from (52) to get two tensor equations of gravity, these will be (53), and (55). Assuming $\sigma = 8\pi$, where the metric variation equations (47), (48), (49) and (50) yield

$$\mathrm{Z} = N^2 = \mathrm{P}_\mu \mathrm{P}^\mu \text{ , } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \text{ , } L = \frac{1}{4} U_i U^i \text{ and } \mathrm{Z} = \mathrm{P}^\mathrm{k} \mathrm{P}_\mathrm{k}$$

$$\frac{8\pi}{\sigma}\frac{1}{4}\begin{pmatrix} +2((\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mathrm{m} P^m}{Z^3} P^k);_k - 2(\frac{Z^m}{Z^2});_m) P_\mu P_\nu + \\ +2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\ +U_\mu U_\nu - \frac{1}{2} U_\mathrm{k} U^k g_{\mu\nu} \end{pmatrix} =$$

$$\frac{8\pi}{\sigma}\frac{1}{4}(U_\mu U_\nu - \frac{1}{2} U_\mathrm{k} U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$\text{s.t. } R = R_{\mu\upsilon} g^{\mu\upsilon}$$

$$\text{s.t. } R_{kj} = (\Gamma_{jk}{}^P),_p - (\Gamma_{pk}{}^P),_j + \Gamma_{p\mu}{}^P \Gamma_{jk}{}^\mu - \Gamma_{pj}{}^\mu \Gamma_{k\mu}{}^p \tag{53}$$

$R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor [38]. In general, by (4) and $\sigma = 8\pi$, (53) can be written in $(-1,+1,+1,+1)$ metric convention, so $Z = |P_\mu P^\mu|$ as,

$$\frac{1}{4}(U_\mu U_\nu - \frac{1}{2} U_\mathrm{k} U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{54}$$

Charge-less field: The term $-2U^k;_k \frac{P_\mu P_\nu}{Z}$ in (54) can be generalized to:

$-2((U^k;_k + U^{*k};_k)/2)\frac{(P_\mu P^*_\nu + P^*_\mu P_\nu)/2}{Z}$ and can be zero under the following condition:

$$4(A_{\mu\nu};^\mu \frac{P^{*\nu}}{\sqrt{Z}} + A^*_{\mu\nu};^\mu \frac{P^\nu}{\sqrt{Z}}) = U_\mu U^{*\mu} + U^*_\mu U^\mu \Rightarrow U^k;_k + U^{*k};_k = 0$$

Note: The complimentary matrix $B_{\mu\nu} = \frac{1}{\sqrt{2}} E^{\mu\nu\alpha\beta} A_{\alpha\beta}$, see few lines before (3), can be transformed to a real matrix due to the SU(2) x U(1) degrees of freedom and also be imaginary.

From (51), (52) we have, $\frac{d}{dx^\mu}(\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu,\nu}})(U_k U^k \sqrt{-g}) = W^\mu;_\mu \sqrt{-g} = 0$

We recall, $W^\mu = (\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_{\mu,\nu}})(U_k U^k \sqrt{-g})$

$W^{\mu} =$

$(-4(\frac{Z^{\nu}}{Z^2});_{\nu} -4\frac{Z_m Z^m}{Z^3})P^{\mu} + 4(\frac{(Z_s P^s)P^{\nu}}{Z^3});_{\nu} P^{\mu} - 2\frac{Z_m P^m Z^{\mu}}{Z^3} + 6\frac{(Z_m P^m)^2}{Z^4} P^{\mu} =$

$-4(\frac{Z^{\nu}}{Z^2});_{\nu} P^{\mu} - 4\frac{Z_m Z^m}{Z^3} P^{\mu} +$

$+4(\frac{(Z_s P^s)P^{\nu}}{Z^3});_{\nu} P^{\mu} + 4\frac{(Z_m P^m)^2}{Z^4} P^{\mu}$

$-2\frac{Z_m P^m}{Z^2}(\frac{Z^{\mu}}{Z} - \frac{Z_m P^m P^{\mu}}{Z^2}) =$

$-4((\frac{U^k}{Z});_k + \frac{U^k U_k}{Z})P^{\mu} - 2\frac{Z_m P^m}{Z^2} U^{\mu} = 0$

$$W^{\mu};_{\mu} = \left(-4U^{\nu};_{\nu} \frac{P^{\mu}}{Z} - 2\frac{(Z_m P^m)}{Z^2} U^{\mu}\right);_{\mu} = 0 \quad (55)$$

Appendix B: Proof of conservation

Theorem: Conservation law of the real Reeb class vector.

From the vanishing of the divergence of Einstein tensor and (54), we have to prove the following in $(-1,+1,+1,+1)$ metric convention:

$$\frac{1}{4}\left(U_{\mu}U_{\nu} - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_{\mu}P_{\nu}}{Z}\right);^{\mu} = G_{\mu\nu};^{\mu} = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu});^{\mu} = 0 \quad (56)$$

Proof:

From the zero variation by the scalar time field (55)

$$W^{\mu};_{\mu} = \left(-4U^{\nu};_{\nu} \frac{P^{\mu}}{Z} - 2\frac{(Z_m P^m)}{Z^2} U^{\mu}\right);_{\mu} = 0 \quad (57)$$

$$-\left(2U^{\nu};_{\nu} \frac{P^{\mu}}{Z}\right);_{\mu} = \left(\frac{(Z_m P^m)}{Z^2} U^{\mu}\right);_{\mu} \quad (58)$$

$$\left(-2U^k;_k \frac{P^{\mu}P^{\nu}}{Z}\right);_{\mu} = \left(\frac{(Z_m P^m)}{Z^2} U^{\mu}\right);_{\mu} P^{\nu} - \left(2U^k;_k \frac{P^{\mu}}{Z}\right)P^{\nu};_{\mu} =$$

$$\left(\frac{(Z_m P^m)}{Z^2} U^{\mu}\right);_{\mu} P^{\nu} - U^k;_k \frac{Z^{\nu}}{Z} \quad (59)$$

Now let $t \equiv Z_m P^m$

And we have $\left(\frac{t}{Z^2}U^\mu\right);_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z} = (\frac{t}{Z^2});_\mu U^\mu P^\nu + \frac{t}{Z^2}U^\mu;_\mu P^\nu - U^k;_k \frac{Z^\nu}{Z} =$

$$-U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu$$

This is because $-U^\nu = -\frac{Z^\nu}{Z} + \frac{t}{Z^2}P^\nu \Rightarrow -U^\mu;_\mu \frac{Z^\nu}{Z} + \frac{t}{Z^2}U^\mu;_\mu P^\nu = -U^\mu;_\mu U^\nu$. Note that $-U^\nu$ is minus twice the real numbered Reeb class vector. So,

$$(-2U^k;_k \frac{P^\mu P^\nu}{Z});_\mu = -U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu \tag{60}$$

Returning to the theorem we have to prove and using equation (60), we have to show,

$$\begin{aligned}
&\left(U^\mu U^\nu - \frac{1}{2}U_k U^k g^{\mu\nu} - 2U^k;_k \frac{P^\mu P^\nu}{Z}\right);^\mu = \\
&U^\mu;_\mu U^\nu + U^\mu U^\nu;_\mu - \frac{1}{2}(U_k;_\mu U_s + U_k U_s;_\mu)g^{ks}g^{\mu\nu} - \\
&U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = \\
&U^\mu U^\nu;_\mu - \frac{1}{2}(U^s U_s);^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = 0
\end{aligned} \tag{61}$$

Notice that

$$\begin{aligned}
&U^\mu U^\nu;_\mu - \frac{1}{2}U^s U_s;^\nu = \\
&U^\mu\left((\frac{Z_k}{Z});_\mu - (\frac{t}{Z^2});_\mu P_k - (\frac{t}{Z^2})P_k;_\mu\right)g^{k\nu} - \\
&U^s\left((\frac{Z_s}{Z});_k - (\frac{t}{Z^2});_k P_s - (\frac{t}{Z^2})P_s;_k\right)g^{k\nu} = \\
&-U^\mu(\frac{t}{Z^2});_\mu P^\nu
\end{aligned} \tag{62}$$

Since $-(\frac{t}{Z^2});_k P_s U^s = 0$ because the Reeb class vector is perpendicular to the foliation kernel $\frac{P_\lambda}{\sqrt{Z}}$, $\frac{P^k}{\sqrt{Z}}\frac{U_k}{2} = 0$.

Equation (62) is also a result of $\ln(Z),_k;_\mu U^\mu g^{k\nu} = \ln(Z),_s;_k U^s g^{k\nu}$ and of $P_k;_\mu U^\mu g^{k\nu} = P_s;_k U^s g^{k\nu}$.

$$U^\mu U^\nu;_\mu - \frac{1}{2}(U^s U_s);^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = -U^\mu(\frac{t}{Z^2});_\mu P^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = 0 \tag{63}$$

and we are done.

Appendix C: Generalization to more than one generalized Reeb class vector

Considering (3.2.1), (3.2.2), $G_{\mu\nu};_{\lambda}\frac{P^{\lambda}}{\sqrt{Z}}=0$ (13.11) which means other Reeb class vectors can rotate or be constant around P^{λ} and given the previous fields $\frac{\mathrm{P_k}}{\sqrt{\mathrm{Z}}}$ and $\frac{U_{\mu}}{2}$ and additional Reeb class vector fields, not the usual Reeb vectors, $\frac{U(1)_{\mu}}{2}$, $\frac{U(2)_{\mu}}{2}$, $\frac{S_{\mu}}{2}$, $\frac{W_{\mu}}{2}$, $\frac{T_{\mu}}{2}$, such that these fields are perpendicular to $\frac{\mathrm{P_k}}{\sqrt{\mathrm{Z}}}$, the following Lagrangian can be defined with the determinant of the metric g:

$$L=\begin{vmatrix}1 & 0\\ 0 & \frac{U(0)^{k}U(0)_{k}^{*}+U(0)^{*k}U(0)_{k}}{8}\end{vmatrix}\sqrt{-\mathrm{g}}+$$

$$\begin{vmatrix}1 & \frac{\mathrm{P_k}U(1)^{*k}+\mathrm{P^*}_{\mathrm{k}}U(1)^{k}}{2\sqrt{2\mathrm{Z}}} & \frac{\mathrm{P_k}U(2)^{*k}+\mathrm{P^*}_{\mathrm{k}}U(2)^{k}}{2\sqrt{2\mathrm{Z}}}\\ \frac{\mathrm{P_k}U(1)^{*k}+\mathrm{P^*}_{\mathrm{k}}U(1)^{k}}{2\sqrt{2\mathrm{Z}}} & \frac{U^{k}U_{k}^{*}+U^{*k}U_{k}}{8} & \frac{U(2)^{k}U_{k}^{*}+U(2)^{*k}U_{k}}{8}\\ \frac{\mathrm{P_k}U(2)^{*k}+\mathrm{P^*}_{\mathrm{k}}U(2)^{k}}{2\sqrt{2\mathrm{Z}}} & \frac{U(2)^{k}U_{k}^{*}+U(2)^{*k}U_{k}}{8} & \frac{U(2)^{k}U(2)_{k}^{*}+U(2)^{*k}U(2)_{k}}{8}\end{vmatrix}^{\frac{1}{2}}\sqrt{-g}+$$

$$\begin{vmatrix}1 & \frac{p_{\mu}S^{*\mu}+p_{\mu}^{*}S^{\mu}}{2\sqrt{2\mathrm{Z}}} & \frac{p_{\mu}W^{*\mu}+p_{\mu}^{*}W^{\mu}}{2\sqrt{2\mathrm{Z}}} & \frac{p_{\mu}T^{*\mu}+p_{\mu}^{*}T^{\mu}}{2\sqrt{2\mathrm{Z}}}\\ \frac{p_{\mu}S^{*\mu}+p_{\mu}^{*}S^{\mu}}{2\sqrt{2\mathrm{Z}}} & \frac{S_{\mu}S^{*\mu}+S_{\mu}^{*}S^{\mu}}{8} & \frac{S_{\mu}W^{*\mu}+S_{\mu}^{*}W^{\mu}}{8} & \frac{S_{\mu}T^{*\mu}+S_{\mu}^{*}T^{\mu}}{8}\\ \frac{p_{\mu}W^{*\mu}+p_{\mu}^{*}W^{\mu}}{2\sqrt{2\mathrm{Z}}} & \frac{W_{\mu}S^{*\mu}+W_{\mu}^{*}S^{\mu}}{8} & \frac{W_{\mu}W^{*\mu}+W_{\mu}^{*}W^{\mu}}{8} & \frac{W_{\mu}T^{*\mu}+W_{\mu}^{*}T^{\mu}}{8}\\ \frac{p_{\mu}T^{*\mu}+p_{\mu}^{*}T^{\mu}}{2\sqrt{2\mathrm{Z}}} & \frac{T_{\mu}S^{*\mu}+T_{\mu}^{*}S^{\mu}}{8} & \frac{T_{\mu}W^{*\mu}+T_{\mu}^{*}W^{\mu}}{8} & \frac{T_{\mu}T^{*\mu}+T_{\mu}^{*}T^{\mu}}{8}\end{vmatrix}^{\frac{1}{3}}\sqrt{-\mathrm{g}} \qquad (64)$$

Each of the determinants of (64) agrees with the multiplicative rule of (13.12) but unlike (13.12), higher values are achieved when the acceleration vectors are perpendicular. The roots equate units of length to units of area and to units of volume and the components of (64) are higher when the accelerations, which are Reeb class vectors, are also perpendicular to the time-like vector $\frac{p_{\mu}}{\sqrt{Z}}$ as expected from an acceleration vector of a unit field, to be perpendicular to a time-like vector.

The last term of (64) has SU(3) * reflections symmetry, however, when considering the space-like foliation which is perpendicular to $\frac{p_{\mu}}{\sqrt{Z}}$, extremal solutions, not saddle variations, have a physical meaning of rotating fields. See Fig. 7. There could be better action operators than (64), after all, (64) is no more than a research offer although it has its own logic which is not fully explained in this paper.

Uniform gravity and forces formalism, challenges, and an open problem: Apparently the first and the last additives of (64), 2x2 and 4x4 matrices imply that the Einstein-Hilbert action can be written in a tetradic formulation with 4 scalar functions where:

$$e^J{}_\mu = \frac{p^J{}_\mu}{\sqrt{Z(J)}} = \frac{p_{(J),\mu}}{\sqrt{Z(J)}} \text{ and } \eta^{KJ} e_{K\mu} e_{J\nu} = e^J{}_\mu e_{J\nu} = g_{\mu\nu} \text{ or } \frac{1}{2}(e^{*J}{}_\mu e_{J\nu} + e^J{}_\mu e^*{}_{J\nu}) = g_{\mu\nu} \text{ and} \tag{64.1}$$

$$K \neq J \Rightarrow \eta^{KJ} = 0,\ \eta^{00} = -1,\ \eta^{11} = \eta^{22} = \eta^{33} = 1$$

Caveat: $\frac{p^J{}_\mu}{\sqrt{Z(J)}}$ may not be related to any force fields but only to gravity. Please refer to the remark after (3.13).

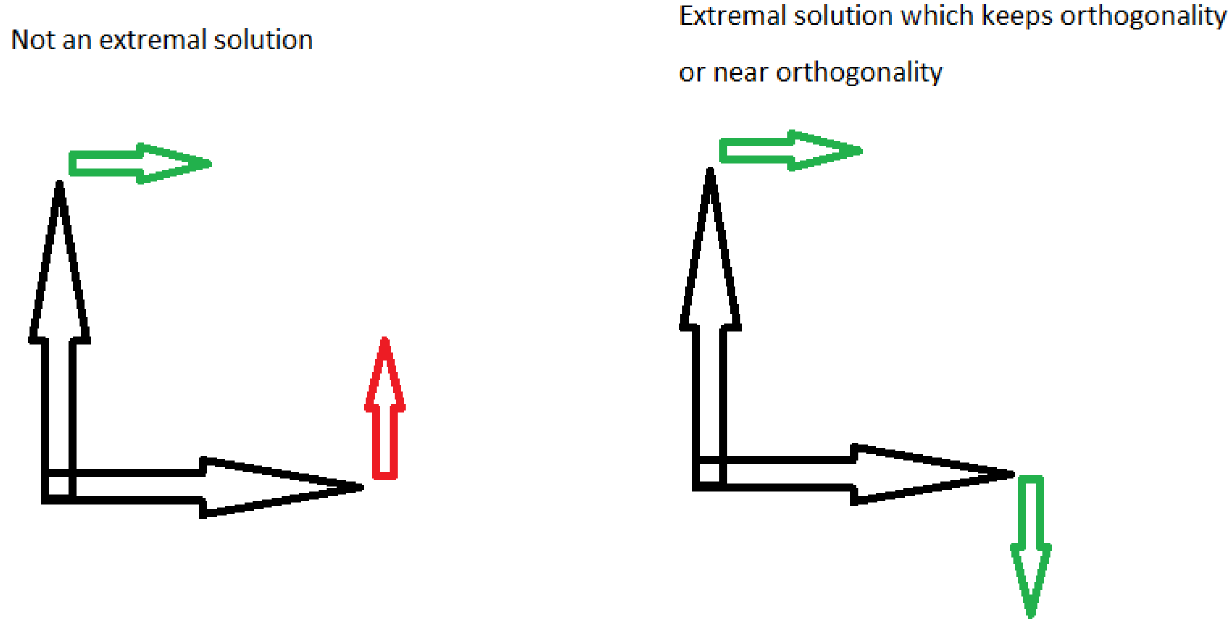
Clarification: Notice that P(0)P*(0) is a Geroch function. Adopting Sam Vaknin's methods [13] in a scalar function formalism, P(0)P*(0), can be a probability density of a single event in 4-volume, instead of time. The good news is that $P(0), P(1), P(2), P(3)$ yield 4 gradients $P(0)_\mu, P(1)_\mu, P(2)_\mu, P(3)_\mu$ and 4 non geodesic accelerations as generalized Reeb class vectors - not limited to contact manifolds, unlike the usual Reeb vectors- $\frac{U_\mu}{2}, \frac{S_\mu}{2}, \frac{W_\mu}{2}, \frac{T_\mu}{2}$ as explained in (64). The much less good news is that 4 complex functions $P(0), P(1), P(2), P(3)$ are 8 real functions which mean that the 2 degrees of freedom in the 10 independent General Relativity equations are gone (without considering the 4 vanishing divergence equations). This problem can be partially mitigated by multiplication of the Tetradic metric tensor by a scalar function ϕ, however, such a solution to a unified formalism of forces and gravity is awkward, $\frac{1}{2}\phi\phi^*(e^{*J}{}_\mu e_{J\nu} + e^J{}_\mu e^*{}_{J\nu}) = g_{\mu\nu}$ is still incomplete. The problem is the following condition:

$\frac{P*_\mu(0)P_\nu(0)+P_\mu(0)P*_\nu(0)}{2\sqrt{Z(0)}} - \sum_{i=1}^{3} \frac{P*_\mu(i)P_\nu(i)+P_\mu(i)P*_\nu(i)}{2\sqrt{Z(i)}} = g_{\mu\nu}$ and from the vanishing of the ordinary covariant derivative: $\frac{1}{2}(e^{*J}{}_\mu e_{J\nu} + e^J{}_\mu e^*{}_{J\nu})_{;\lambda} = g_{\mu\nu;\lambda} = 0$. This is not a good place to be begin with. For the Riemann tensor in the real case, we have:

$R^J{}_{KLM} = e^J((\nabla_L \nabla_M - \nabla_M \nabla_L - [e_L, e_M])e_K)$ and $R = R^{JM}{}_{JM}$ and the volume element is $\sqrt{\left|Deteminant(e^J{}_\mu e_{J\nu})\right|}$ with Greek letters denoting the ordinary Riemann indices.

There are, however, some caveats. $p^{J=0}{}_{\mu=1} = p^0{}_2 = p^0{}_3 = 0$ means that $P(0)P^*(0)$ must be a Geroch time function [1]. The rest of the tetrads need not be perpendicular to each other, $e^J{}_\mu e^{*K\mu} + e^{*J}{}_\mu e^{K\mu} \neq 0$ but using spin connections they may be formulated in such a way that $e^J{}_\mu e^{*K\mu} + e^{*J}{}_\mu e^{K\mu} = 0$.

Fig. 7.



The Aryeh Aldema's offer of a Relative curvature action and it's meaning

Let $\frac{p_\mu(0)}{\sqrt{|Z(0)|}} = \frac{p_\mu}{\sqrt{|Z|}}$ and 3 other scalar fields are defined $\frac{p_\mu(1)}{\sqrt{|Z(1)|}}, \frac{p_\mu(2)}{\sqrt{|Z(2)|}}, \frac{p_\mu(3)}{\sqrt{|Z(3)|}}$ with real numbers. Taking the Gaussian curvature but without demanding that the embedding manifold will be flat, consider

$$K\sqrt{-g} \equiv \frac{\left(\frac{p_i(0)}{\sqrt{|Z(0)|}}\right)_{;\mu}\frac{p^\mu(1)}{\sqrt{|Z(1)|}}\left(\frac{p_j(0)}{\sqrt{|Z(0)|}}\right)_{;\nu}\frac{p^\nu(2)}{\sqrt{|Z(2)|}}\left(\frac{p_k(0)}{\sqrt{|Z(0)|}}\right)_{;\zeta}\frac{p^\zeta(3)}{\sqrt{|Z(3)|}}\frac{p_s(0)}{\sqrt{|Z(0)|}}\epsilon^{ijks}}{\frac{p_a(0)}{\sqrt{|Z(0)|}}\frac{p_b(1)}{\sqrt{|Z(1)|}}\frac{p_c(2)}{\sqrt{|Z(2)|}}\frac{p_d(3)}{\sqrt{|Z(3)|}}\epsilon^{abcd}}\sqrt{-g} \tag{64.2}$$

where ϵ^{ijks} is the Levi-Civita alternating symbols. It is not difficult to prove that this definition does not depend on the choice of $\frac{p_\mu(1)}{\sqrt{|Z(1)|}}, \frac{p_\mu(2)}{\sqrt{|Z(2)|}}, \frac{p_\mu(3)}{\sqrt{|Z(3)|}}$ as long as they are mutually independent and independent of $\frac{p_s(0)}{\sqrt{|Z(0)|}}$. Obviously, $\left(\frac{p_s(0)}{\sqrt{|Z(0)|}}\right)_{;i} p^i(0) = \frac{p_{s;i}(0)}{\sqrt{|Z(0)|}} p^i(0) - \frac{1}{2}\frac{p_s(0)}{|Z(0)|^{\frac{3}{2}}} Z_i(0) p^i(0) = 0$ so there is no need to worry about $p_\mu(1), p_\mu(2), p_\mu(3)$ being within the foliation. A full proof replaces ϵ^{ijks} with the Levi-Civita tensor at first $E^{ijks} = sgn(g)\frac{\epsilon^{ijks}}{\sqrt{-g}} = -\frac{\epsilon^{ijks}}{\sqrt{-g}}$ so we immediately see that

$$K \equiv \frac{\left(\frac{p_i(0)}{\sqrt{|Z(0)|}}\right)_{;\mu}\frac{p^\mu(1)}{\sqrt{|Z(1)|}}\left(\frac{p_j(0)}{\sqrt{|Z(0)|}}\right)_{;\nu}\frac{p^\nu(2)}{\sqrt{|Z(2)|}}\left(\frac{p_k(0)}{\sqrt{|Z(0)|}}\right)_{;\zeta}\frac{p^\zeta(3)}{\sqrt{|Z(3)|}}\frac{p_s(0)}{\sqrt{|Z(0)|}}E^{ijks}}{\frac{p_a(0)}{\sqrt{|Z(0)|}}\frac{p_b(1)}{\sqrt{|Z(1)|}}\frac{p_c(2)}{\sqrt{|Z(2)|}}\frac{p_d(3)}{\sqrt{|Z(3)|}}E^{abcd}} \tag{64.3}$$

Which means that K is the quotient of two scalar functions and is therefore a scalar function.

Without loss of generality,

(64.4)

$$\frac{\left(\frac{p_i(0)}{\sqrt{|Z(0)|}}\right)_{;\mu}\left(\frac{p^\mu(1)+rp^\mu(2)}{\sqrt{\|p^\mu(1)+rp^\mu(2)\|}}\left(\frac{p_j(0)}{\sqrt{|Z(0)|}}\right)_{;\nu}\frac{p^\nu(2)}{\sqrt{|Z(2)|}}\left(\frac{p_k(0)}{\sqrt{|Z(0)|}}\right)_{;\zeta}\frac{p^\zeta(3)}{\sqrt{|Z(3)|}}\frac{p_s(0)}{\sqrt{|Z(0)|}}E^{ijks}}{\frac{p_a(0)}{\sqrt{|Z(0)|}}\frac{p_b(1)+rp_b(2)}{\sqrt{\|p_b(1)+rp_b(2)\|}}\frac{p_c(2)}{\sqrt{|Z(2)|}}\frac{p_d(3)}{\sqrt{|Z(3)|}}E^{abcd}}$$

$$=\frac{\left(\frac{p_i(0)}{\sqrt{|Z(0)|}}\right)_{;\mu}\left(\frac{p^\mu(1)}{\sqrt{\|p^\mu(1)+rp^\mu(2)\|}}\left(\frac{p_j(0)}{\sqrt{|Z(0)|}}\right)_{;\nu}\frac{p^\nu(2)}{\sqrt{|Z(2)|}}\left(\frac{p_k(0)}{\sqrt{|Z(0)|}}\right)_{;\zeta}\frac{p^\zeta(3)}{\sqrt{|Z(3)|}}\frac{p_s(0)}{\sqrt{|Z(0)|}}E^{ijks}}{\frac{p_a(0)}{\sqrt{|Z(0)|}}\frac{p_b(1)}{\sqrt{\|p_b(1)+rp_b(2)\|}}\frac{p_c(2)}{\sqrt{|Z(2)|}}\frac{p_d(3)}{\sqrt{|Z(3)|}}E^{abcd}}$$

$$=\frac{\left(\frac{p_i(0)}{\sqrt{|Z(0)|}}\right)_{;\mu}\left(\frac{p^\mu(1)}{\sqrt{\|p^\mu(1)\|}}\left(\frac{p_j(0)}{\sqrt{|Z(0)|}}\right)_{;\nu}\frac{p^\nu(2)}{\sqrt{|Z(2)|}}\left(\frac{p_k(0)}{\sqrt{|Z(0)|}}\right)_{;\zeta}\frac{p^\zeta(3)}{\sqrt{|Z(3)|}}\frac{p_s(0)}{\sqrt{|Z(0)|}}E^{ijks}}{\frac{p_a(0)}{\sqrt{|Z(0)|}}\frac{p_b(1)}{\sqrt{\|p_b(1)\|}}\frac{p_c(2)}{\sqrt{|Z(2)|}}\frac{p_d(3)}{\sqrt{|Z(3)|}}E^{abcd}}$$

This is because the Levi-Civita symbols are alternating so if the same vector occurs twice with two different indices, this component vanishes.

For example, in the numerator $\left(\frac{p_i(0)}{\sqrt{|Z(0)|}}\right)_{;\mu}\frac{rp^\mu(2)}{\sqrt{\|p^\mu(1)+rp^\mu(2)\|}}$ is annihilated with $\left(\frac{p_j(0)}{\sqrt{|Z(0)|}}\right)_{;\nu}\frac{p^\nu(2)}{\sqrt{|Z(2)|}}$ when these are multiplied by the Levi-Civita tensor. So we find that K does not depend on the choice of $p_\mu(1), p_\mu(2), p_\mu(3)$ and is therefore intrinsic to the foliation perpendicular to $p_\mu(0)$.

Our argument was about the importance of the "relative curvature". My opinion was clear, that the need for $p_\mu(1), p_\mu(2), p_\mu(3)$ is only if they can reveal a value which is not anticipated by the geometry of the 3D foliation because the Reeb class vector $\frac{U_\mu}{2}$ already contains geometric information about the foliation which is perpendicular to the vector $p_\mu = p_\mu(0)$. This information is of how the fields $p_\mu(1), p_\mu(2), p_\mu(3)$ are misaligned and do not make geodesic curves. This was the reason behind (64). Aryeh's input clarified the importance of (64). Unfortunately, Aryeh Aldema, who was my colleague passed in Dec/30/2022.

The vorticity action of 4 Reeb class vectors

Possibly a fifth force of Nature or by (3.12) and mapping curves to a flat spacetime, massive gravity is described by the following SU(4) symmetry Lagrangian of 4 Reeb class vectors: $\frac{א_\mu}{2}, \frac{ב_\mu}{2}, \frac{ג_\mu}{2}, \frac{ד_\mu}{2}$, with Hebrew letters Alef, Beit, Gimmel, Dalet,

$$\begin{vmatrix} \frac{\aleph_\mu\aleph^{*\mu}+\aleph^*_\mu\aleph^\mu}{8} & \frac{\aleph_\mu\beth^{*\mu}+\aleph^*_\mu\beth^\mu}{8} & \frac{\aleph_\mu\gimel^{*\mu}+\aleph^*_\mu\gimel^\mu}{8} & \frac{\aleph_\mu\daleth^{*\mu}+\aleph^*_\mu\daleth^\mu}{8} \\ \frac{\aleph_\mu\beth^{*\mu}+\aleph^*_\mu\beth^\mu}{8} & \frac{\beth_\mu\beth^{*\mu}+\beth^*_\mu\beth^\mu}{8} & \frac{\beth_\mu\gimel^{*\mu}+\beth^*_\mu\gimel^\mu}{8} & \frac{\beth_\mu\daleth^{*\mu}+\beth^*_\mu\daleth^\mu}{8} \\ \frac{\aleph_\mu\gimel^{*\mu}+\aleph^*_\mu\gimel^\mu}{8} & \frac{\gimel_\mu\beth^{*\mu}+\gimel^*_\mu\beth^\mu}{8} & \frac{\gimel_\mu\gimel^{*\mu}+\gimel^*_\mu\gimel^\mu}{8} & \frac{\gimel_\mu\daleth^{*\mu}+\gimel^*_\mu\daleth^\mu}{8} \\ \frac{\aleph_\mu\daleth^{*\mu}+\aleph^*_\mu\daleth^\mu}{8} & \frac{\daleth_\mu\beth^{*\mu}+\daleth^*_\mu\beth^\mu}{8} & \frac{\daleth_\mu\gimel^{*\mu}+\daleth^*_\mu\gimel^\mu}{8} & \frac{\daleth_\mu\daleth^{*\mu}+\daleth^*_\mu\daleth^\mu}{8} \end{vmatrix}^{\frac{1}{4}} \sqrt{-g} \tag{65}$$

The determinant of two Reeb class vectors can help to understand the roots in (30), (31), (32), and (33). It describes accelerations in two perpendicular planes. Three Reeb class vectors describe accelerations in the foliation perpendicular to P_μ. It is not clear whether (65) is related to (3.12) in which case it does not represent a new field but a massive gravitational field.

Appendix D: Another way to derive the Reeb class vector

We may now write the Lie derivative [39] of $\frac{P_i}{\sqrt{Z}}$ with respect to the vector field $\frac{P^{*m}}{\sqrt{Z}}$,

$$Lie\left(\frac{P^{*m}}{\sqrt{Z}},\frac{P_i}{\sqrt{Z}}\right) = \frac{P^{*m}}{\sqrt{Z}}\left(\frac{P_i}{\sqrt{Z}}\right),_m + \left(\frac{P^{*m}}{\sqrt{Z}}\right),_i\frac{P_m}{\sqrt{Z}} \tag{66}$$

In which the second term is positive because the differentiated $\frac{P_i}{\sqrt{Z}}$ vector has a low index.

The first term becomes,

$$\frac{P^{*m}}{\sqrt{Z}}\left(\frac{P_i}{\sqrt{Z}}\right),_m = \frac{P^{*m}P_{i,m}}{Z} - \frac{P^{*m}}{\sqrt{Z}}\frac{P_iZ_m}{2Z^{3/2}} = \frac{P^{*m}P_{i,m}}{Z} - \frac{P^{*m}Z_mP_i}{2Z^2} \tag{67}$$

The second term is,

$$\left(\frac{P^{*m}}{\sqrt{Z}}\right),_i\frac{P_m}{\sqrt{Z}} = \frac{P^{*m},_iP_m}{Z} - \frac{P^{*m}P_mZ_i}{2Z^2} = \frac{P^{*m},_iP_m}{Z} - \frac{Z_i}{2Z} \tag{68}$$

We add (67) and (68) to get (66) and notice that $\frac{P^{*m}P_{i,m}}{Z} + \frac{P^{*m},_iP_m}{Z} = \frac{P^{*m}P_{m,i}}{Z} + \frac{P^{*m},_iP_m}{Z} = \frac{Z_i}{Z}$ from which (66) becomes

$$Lie\left(\frac{P^{*m}}{\sqrt{Z}},\frac{P_i}{\sqrt{Z}}\right) = \frac{Z_i}{Z} - \frac{Z_i}{2Z} - \frac{P^{*m}Z_mP_i}{2Z^2} = \frac{Z_i}{2Z} - \frac{P^{*m}Z_mP_i}{2Z^2} = \frac{U_i}{2} \tag{69}$$

Appendix E: 95/96, the precursor of the inverse Fine Structure Constant and of the muon/electron mass ratio

Results (24), (36), (40), (41), (42), were not reached immediately. There was one finding that was a total serendipity that later lead to these results. The observation was the following, given a scaling factor 1+d of area addition with d=1 as a maximal value, 1+d = 2.

$$(1+\alpha)^{95} < 2 \ \wedge (1+\alpha)^{96} > 2 \tag{70}$$

More precisely

$$\aleph = (2^{\frac{1}{96}} - 1)^{-1} \cong 137.999325615 \tag{71}$$

And

$$\beth = (2^{\frac{1}{95}} - 1)^{-1} \cong 136.5566369 \tag{72}$$

And the geometric average is:

$$\sqrt{\aleph\beth} \cong 137.27608605 \tag{73}$$

Which is close to the result from (40), 137.0359990368270076.

An immediate observation is

$$\aleph = \left(\frac{2-2^{\frac{95}{96}}}{2^{\frac{95}{96}}}\right)^{-1} \tag{74}$$

And

$$\beth = \left(\frac{2^{\frac{96}{95}}-2}{2}\right)^{-1} \tag{75}$$

Where we expressed a power which is close to 1, namely $\xi = \frac{95}{96}$ and $\xi^{-1} = \frac{96}{95}$. as such, ξ was nominated as polynomial coefficient because it was in the range between 0 and 2, unlike $\xi = \frac{4}{\pi}$ which has a geometric interpretation thanks to Ettore Majorana, $\xi = \frac{95}{96}$ seems to have an algebraic meaning.

We continue with a rather surprising relation

$$(2^{\frac{1}{95*96}} - 1)^{-1} \cong 13{,}156.87877924 \tag{76}$$

And it is quite easy to notice the following:

$$\frac{1}{96(1+96^{-2})}(2^{\frac{1}{95*96}} - 1)^{-1} \cong 137.03595126474 \tag{77}$$

which is very close to the inverse Fine Structure Constant. Actually, if we replace the factor $\frac{1}{96(1+96^{-2})}$ by $\frac{1}{n(1+n^{-2})}$ for some integer n, the closest result to the inverse Fine Structure Constant is when n=96

In fact

$$\frac{(2^{\frac{1}{95*96}}-1)^{-1}}{137.0359990368270076} \cong 96.010383196499723 \cong 96(1+96.1546032^{-2}) \qquad (78)$$

See (40). The factor $\frac{1}{95*96}$ can be seen as

$$\frac{1}{95*96} = \frac{95}{96} + \frac{96}{95} - 2 \qquad (79)$$

The factor $95 * 96$ found expression in (41), (42) and is the final missing piece in the puzzle. It is the bridge between trigonometry and electro-gravitational polynomials (35) which resulted in: $\xi \cong 1.556198537190348396563877031439915299415588378906$ and $\frac{1}{2}(1-g_2)^{-4} \cong 607276.536800682428292930126\,2$, provided here with more accuracy if required for further research.

In (78) plugging in $\frac{4}{\pi}$ from (24) instead of 2 and dividing by $2 * 137.0359990368270076^2$ instead of by 137.0359990368270076 we get another indication of a deep theoretical relation,

$$\frac{((\frac{4}{\pi})^{\frac{1}{95*96}}-1)^{-1}}{2*137.0359990368270076^2} \cong 1+(2*95.974269533437)^{-1} \qquad (80)$$

We now explore another approach, exponential perturbation of the field strength coefficient $\frac{95}{96}$.

This approach was not further investigated due to numerical stability issues, but the author finds it quite interesting. The field strength coefficient $\frac{95}{96}$ that appears in (23) is the lowest among 3 coefficients $\frac{95}{96}, \frac{4}{\pi}, 1.5561985371903484 \ldots$. At first this fact was an incentive to search for a relation between the fine structure constant and perturbations around the value $\frac{95}{96}$.

We return to (23):

$$\frac{192a^2+2\frac{95}{96}a-(\frac{95}{96})^2}{192} = a^3 \; and \; \frac{192b^2-2\frac{95}{96}b-(\frac{95}{96})^2}{192} = b^3 \qquad (81)$$

And to the multiplication in (23) $\frac{1}{(a-1)(1-b)} \cong 12202.888740664679$.

We look at the following exponential $\frac{n-1}{n}$ perturbation of the coefficient $\frac{95}{96}$,

$$\frac{192c^2+2\left(\frac{95}{96}\right)^{\frac{n-1}{n}}c-(\frac{95}{96})^{2\frac{n-1}{n}}}{192} = c^3 \; and \; \frac{192d^2-2\left(\frac{95}{96}\right)^{\frac{n-1}{n}}d-(\frac{95}{96})^{2\frac{n-1}{n}}}{192} = d^3 \qquad (82)$$

And we check how relatively close is $(c-1)(1-d)$ to $(a-1)(1-b)$.

The calculation is:

$$Relative\ error = \frac{(c-1)(1-d)}{(a-1)(1-b)} - 1 \quad (83)$$

The strange fact is that

$\alpha^{-1} = \frac{2}{n}\left(\frac{(c-1)(1-d)}{(a-1)(1-b)} - 1\right)$ approximates the inverse fine structure constant. Not as good as (40), (41), (42) but good enough to trigger interest. The last term can be written as in (40) $\alpha^{-1} = \frac{2}{\cos(\eta)}$ for $\eta \equiv cos^{-1}(2\alpha)$. It turns out that α^{-1} is maximal or locally maximal at $n = 96^4 - 805$ or if n is allowed to take real values,

$$n \cong 96^4 - 805.9334 \quad (84)$$

$$\alpha^{-1} \cong 137.0158482935 \quad (85)$$

Putting the terms together:

$$\frac{192a^2 + 2\frac{95}{96}a - (\frac{95}{96})^2}{192} = a^3\ and\ \frac{192b^2 - 2\frac{95}{96}b - (\frac{95}{96})^2}{192} = b^3 \quad (86)$$

$$\frac{192c^2 + 2\left(\frac{95}{96}\right)^{\frac{n-1}{n}} c - (\frac{95}{96})^{2\frac{n-1}{n}}}{192} = c^3\ and\ \frac{192d^2 - 2\left(\frac{95}{96}\right)^{\frac{n-1}{n}} d - (\frac{95}{96})^{2\frac{n-1}{n}}}{192} = d^3$$

$$\max_n \frac{2}{n}\left(\frac{(c-1)(1-d)}{(a-1)(1-b)} - 1\right) \cong 137.015848292861875279413652606308460235595703,$$
$$n \cong 96^4 - 805.933$$

See appendix G for the code in Python for (81)-(86). Consider the same type of perturbation of the field strength $\xi = \frac{4}{\pi}$,

$$\frac{192a^2 + 2\frac{4}{\pi}a - (\frac{4}{\pi})^2}{192} = a^3\ and\ \frac{192b^2 - 2\frac{4}{\pi}b - (\frac{4}{\pi})^2}{192} = b^3 \quad (86.1)$$

$$\frac{192c^2 + 2\left(\frac{4}{\pi}\right)^{\frac{n-1}{n}} c - (\frac{4}{\pi})^{2\frac{n-1}{n}}}{192} = c^3\ and\ \frac{192d^2 - 2\left(\frac{4}{\pi}\right)^{\frac{n-1}{n}} d - (\frac{4}{\pi})^{2\frac{n-1}{n}}}{192} = d^3$$

$$\max_n \frac{2}{n}\left(\frac{(c-1)(1-d)}{(a-1)(1-b)} - 1\right) \cong 136.4^{\frac{1}{2}}$$

Which is close to the square root of the inverse Fine Structure Constant with $n \cong 96^4 - 140631.4697265625$. In both cases, numerical stability issues in (86) and (86.1) made it very difficult to check how close such exponential perturbations of the field strength coefficient can be to the inverse Fine Structure Constant trough the error in the polynomial roots. Numerical stability does exist up to $n = 96^3$. Before we proceed, consider the following, $\xi = (\frac{4}{\pi})^{1+\frac{1}{151.06357822765725984}}$ which is approximately $\frac{4}{\pi}(1 + \frac{1}{624.85524})$, then it is easy to check that

$$\frac{192a^2 + 2\xi a - \xi^2}{192} = a^3 \; and \; \frac{192b^2 - 2\xi b - \xi^2}{192} = b^3$$

$$\frac{192c^2 + 2\frac{2}{\xi}c - \left(\frac{2}{\xi}\right)^2}{192} = c^3 \; and \; \frac{192d^2 - 2\frac{2}{\xi}d - \left(\frac{2}{\xi}\right)^2 \frac{4}{\pi})^{2\frac{n-1}{n}}}{192} = d^3 \Rightarrow$$

$$\frac{(c-1)(1-d)}{(a-1)(1-b)} \cong 1 \qquad (86.2)$$

This result is expected from $\xi = 2^{\frac{1}{2}} = \frac{2}{\xi}$ but not from a field strength so close to $\frac{4}{\pi}$. It is easy to see that from $\xi = 1.25$ to $\xi = 1.5$, (86.2) is very close to 1 within %1 but not as close as when $\xi = (\frac{4}{\pi})^{1+\frac{1}{151.06357822765725984}}$ or when trivially $\xi = 2^{\frac{1}{2}} = \frac{2}{\xi}$.

The Fine Structure Constant as a result of Poisson Distribution of events within radius r:

We proceed with the methods we have discussed until now. Consider the following expression,

$$f(x) = xe^{-x} \qquad (87)$$

which is the Poison distribution for one event and with $\lambda = x$.

Consider the following perturbation equations in two variables in x around 1.

$$\eta = f\left(1 - \frac{1}{a}\right) = f\left(1 + \frac{1}{b}\right) \qquad (88)$$

With the following condition for a wide range of $\eta > 10000$,

$$\alpha^{-2} = (-\ln(\eta) - 1)^{-1} \text{ and } 2\left(\frac{1}{b} + \frac{1}{a}\right)^{-1} \cong \alpha^{-1}2^{-\frac{1}{2}} \qquad (89)$$

Then the system of equations (88), (89) approximates the Fine Structure Constant with the following approximated solution:

$$a \cong 97.2332790992 \qquad (90)$$

$$b \cong 96.56660927693$$

$$\alpha^{-2} = (-\ln(\eta) - 1)^{-1} \cong 18778.86503$$

$$2\left(\frac{1}{b} + \frac{1}{a}\right)^{-1} = \alpha^{-1}2^{-\frac{1}{2}} \cong 96.89879752$$

$$\text{With } \alpha^{-1} \approx 137.03559363$$

These estimates can be greatly improved with better numerical precision than that of an Excel datasheet, however, this paper does not deal with the Causal Set interpretation of the presented theory and chooses to focus on other subjects. Also, (90) depends on the choice of η.

The Causal Set interpretations can be written as Probability(n=k) = $\frac{\xi^k e^{-\xi}}{k!}$ where k is the number of events within a sphere of some small radius r and n is the number of events if this number has the Poisson distribution.

Appendix F: The Python code for (40) and for the remark after (40) and its output

```
import numpy as NP


def function_cubic_viete(a, b, c, d): # If all roots are real.


    # Viete's formula when all roots are real.


    b2 = NP.longdouble(b * b)

    b3 = NP.longdouble(b2 * b)

    a2 = NP.longdouble(a * a)

    a3 = a2 * a


    p = (3 * a * c - b2) / (3 * a2)


    q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)


    offset = b / (3 * a)

    t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) \

                                            * (3 * q) / (2 * p)) / 3)

    t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
```

```
                                             (3 * q) / (2 * p)) / 3 -
                                  NP.pi / 3)
    t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
                                             (3 * q) / (2 * p)) / 3 -
                                  2 * NP.pi / 3)
    x1 = t1 - offset
    x2 = t2 - offset
    x3 = t3 - offset
    return (x1, x2, x3)

def function_fsc_polynomials(): # If all roots are real.

    fp_f, fp_a, fp_b = 1, 1, 1
    fp_start,  fp_end = 1.556, NP.pi / 2

    for i in range(2000):
        # Get the biggest roots. These are the closest to 1.
        # One is above 1 and one is below 1.

        fp_f = (fp_start + fp_end) * 0.5

        fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96,
                                         (fp_f * fp_f) / 192)

        fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96,
                                         (fp_f * fp_f) / 192)

        fp_result_middle = 1/NP.sqrt(fp_a-1) - 0.5/(1-fp_b)
```

```
    if fp_result_middle >= 0:
        fp_end = fp_f
    else:
        fp_start = fp_f

fp_s = 1/(1 - fp_b)
fp_s *= fp_s
fp_s *= fp_s * 0.5
fp_xi = fp_f

print('1/(x1-1): %.42lf\n1/(1-x2): %.42lf' %(1/(fp_a-1), 1/(1-fp_b)))
print('Xi: %.42lf\ns=0.5/(1-x2)^4: %.42lf' %(fp_f, fp_s))

fp_f = 4 / NP.pi
# Get the biggest roots. These are the closest to 1.
# One is above 1 and one is below 1.
fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96, (fp_f * fp_f) / 192)
fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96, (fp_f * fp_f) / 192)
fp_mul = (fp_a - 1) * (1 - fp_b)

fp_inv_fsc = 2 / NP.cos( fp_xi * (1 + 1/NP.power(fp_s,1/(1+fp_mul))))

print('Inv FSC: %.42lf' %(fp_inv_fsc))

fp_p2 = fp_mul
fp_start,  fp_end = fp_mul, fp_mul + 0.00001
```

```
    for i in range(2000):
        # Get the biggest roots. These are the closest to 1.
        # One is above 1 and one is below 1.

        fp_f = (fp_start + fp_end) * 0.5

        fp_result_middle = \
            fp_s * (2 - 1/(96*96*fp_f)) - NP.power(fp_s, 1/(1+fp_f))
        if fp_result_middle >= 0:
            fp_end = fp_f
        else:
            fp_start = fp_f

    fp_p = 1/NP.sqrt(fp_mul)
    fp_miracle_p = 1/NP.sqrt(fp_f)
    fp_relative_p_error = fp_p / (fp_p - fp_miracle_p)

    print('P: %.42lf\nMiracle P: %.48lf\nRelative error in P: %.48lf^-1'
          % (fp_p, fp_miracle_p, fp_relative_p_error))

function_fsc_polynomials()


'''
Output when run from PyCharm and Python 3.6:
1/(x1-1): 275.516908918643935066938865929841995239257812
1/(1-x2): 33.197404050235356010034593055024743080139160
Xi: 1.556198537190348396563877031439915299415588
s=0.5/(1-x2)^4: 607276.536800682428292930126190185546875000000000
```

```
Inv FSC: 137.035999036827007557803881354629993438720703

P: 96.069177214886295246287772897630929946899414

Miracle P: 96.0691758127251773657917510718107223510742187500000

Relative error in P:
68515077.183215767145156860351562500000000000000000000^-1

'''
```

Appendix G: The Python code for (81)-(86)

```
import numpy as NP


def function_cubic_viete(a, b, c, d): # If all roots are real.


    # Viete's formula when all roots are real.


    b2 = NP.longdouble(b * b)

    b3 = NP.longdouble(b2 * b)

    a2 = NP.longdouble(a * a)

    a3 = a2 * a


    p = (3 * a * c - b2) / (3 * a2)


    q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)


    offset = b / (3 * a)

    t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) \
                                            * (3 * q) / (2 * p)) / 3)

    t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
                                            (3 * q) / (2 * p)) / 3 -
```

```
                                           NP.pi / 3)
    t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * \
                                                     (3 * q) / (2 * p)) / 3 -
                                           2 * NP.pi / 3)
    x1 = t1 - offset
    x2 = t2 - offset
    x3 = t3 - offset

    return (x1, x2, x3)

def function_f_polynomials(fp_n=96*96*96*96): # If all roots are real.

    fp_f = 95/96

    fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96,
                                       (fp_f * fp_f) / 192)

    fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96,
                                       (fp_f * fp_f) / 192)

    fp_mul1 = (fp_a - 1)*(1 - fp_b)

    fp_f = NP.power(fp_f, (fp_n-1)/fp_n)

    fp_a, _, _ = function_cubic_viete(1, -1, -fp_f / 96,
                                       (fp_f * fp_f) / 192)

    fp_b, _, _ = function_cubic_viete(1, -1, fp_f / 96,
```

```
                                              (fp_f * fp_f) / 192)

    fp_mul2 = (fp_a - 1)*(1 - fp_b)

    fp_combine = 2/(fp_n *(fp_mul2/fp_mul1-1))

    #print('%.42lf' %fp_combine)

    return fp_combine

def main():
    ma_best_val = 0
    ma_best_m = 0

    #function_f_polynomials(96 * 96 * 96 * 96 - 1)
    #function_f_polynomials(96 * 96 * 96 * 96)
    #function_f_polynomials(96 * 96 * 96 * 96 + 1)

    print('Coarse search:')
    for i in range(-1000, 1000):
        ma_r =  function_f_polynomials(96 * 96 * 96 * 96 - i)
        if ma_best_val < ma_r:
            ma_best_val = ma_r
            ma_best_m = i

    print('Best value %.42lf' %ma_best_val)
    print('Best m = 96^4-%d' % ma_best_m)
```

```
    print('Fine search:')

    ma_best_val = 0.0

    ma_best_m = 0.0


    for i in range(8050000-10000, 8050000+10000):

        ma_d = i/10000

        ma_r = function_f_polynomials(96 * 96 * 96 * 96 - ma_d)

        if ma_best_val < ma_r:

         Fappen   ma_best_val = ma_r

            ma_best_m = ma_d


    print('Best value %.42lf' %ma_best_val)

    print('Best m = 96^4-%.42lf' % ma_best_m)


    '''

    Coarse search:

    Best value 137.015846787740116496934206224977970123291016

    Best m = 96^4-805

    Fine search:

    Best value 137.015848292861875279413652606308460235595703

    Best m = 96^4-805.932999999999992724042385816574096679687500

    '''

if __name__ == '__main__':

    main()
```

Appendix H – Causality conservation theorem

Theorem: If p is real, any monotone function $f(p)$, called causality function will yield the same Reeb class vector. The reader is advised to check the case when p is an imaginary function. Then the Reeb class vector is defined as $\frac{u_\nu}{2} = \frac{z_\nu}{2z} - \frac{z_k}{2z^2} p *^k p_\nu$.

Proof:

We will use capital letters for $P = f(p)$ and as in previous pages, $z = p_\lambda p^\lambda$ and here $Z = P_\lambda P^\lambda$.

$$P = f(p)$$

$$P_\mu = f'(p) p_\mu$$

$$Z = f'(p) p_\mu f'(p) p^\mu = f'(p)^2 z$$

$$\frac{Z_\nu}{Z} = \frac{2f'(p)f''(p)p_\nu z}{f'(p)^2 z} + \frac{f'(p)^2 z_\nu}{f'(p)^2 z} = \frac{2f''(p)p_\nu}{f'(p)} + \frac{z_\nu}{z}$$

$$U_\nu = \frac{2f''(p)p_\nu}{f'(p)} + \frac{z_\nu}{z} - (\frac{2f''(p)p_k}{f'(p)} + \frac{z_k}{z}) \frac{f'(p)p^k f'(p)p_\nu}{f'(p)^2 z}$$

$$U_\nu = \frac{2f''(p)p_\nu}{f'(p)} - \frac{2f''(p)p_\nu}{f'(p)} + \frac{z_\nu}{z} - \frac{z_k}{z^2} p^k p_\nu = \frac{z_\nu}{z} - \frac{z_k}{z^2} p^k p_\nu = u_\nu$$

$$\frac{U_\nu}{2} = \frac{u_\nu}{2} \qquad (91)$$

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