Quantum evidential reasoning rule

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Abstract

In this paper, we propose a quantum evidential reasoning rule in the framework of generalized quantum evidence theory.

Keywords: Generalized quantum evidence theory, Quantum evidential reasoning rule

1. Introduction

We develop a quantum evidential reasoning method in the framework of generalized quantum evidence theory[1] inspired by [2, 3]. A generalized quantum mass function $\mathbb{Q}_{\mathbb{M}_h}$ can be profiled by a quantum belief distribution (QBD), defined as follows

$$\mathbb{Q}_{\mathbb{M}_h} = \left\{ (\theta, \mathbb{P}_{\theta,h}), \forall \theta \subseteq \Theta, \sum_{\theta \subseteq \Theta} |\mathbb{P}_{\theta,h}|^2 = 1 \right\}, \tag{1}$$

where $\mathbb{P}_{\theta,h}$ is a complex number.

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Then, the generalized weighted quantum belief degree (WQBD) is defined as

$$\mathbb{Q}_{\mathbb{M}_{\theta,h}} = \mathbb{Q}_{\mathbb{M}_h}(\theta) = \begin{cases} 0 & \theta = \emptyset \\ w_h \mathbb{P}_{\theta,h} & \theta \subseteq \Theta, \theta \neq \emptyset \\ 1 - w_h & \theta = P(\Theta) \end{cases} \tag{2}$$

where w_h is the weight of generalized quantum mass function $\mathbb{Q}_{\mathbb{M}_h}$.

After the WQBDs are calculated, the recursive combination rule of WQBDs is defined as follows:

$$\mathbb{Q}_{\mathbb{M}_{\theta,e(h)}} = [\mathbb{Q}_{\mathbb{M}_1} \oplus \cdots \oplus \mathbb{Q}_{\mathbb{M}_h}](\theta) = \begin{cases} 0 & \theta = \emptyset \\ \frac{|\hat{\mathbb{Q}}_{\mathbb{M}_{\theta,e(i)}}|^2}{\sum_{D \subseteq \Theta} |\hat{\mathbb{Q}}_{\mathbb{M}_{D,e(i)}}|^2 + |\hat{\mathbb{Q}}_{\mathbb{M}_{P(\Theta),e(i)}}|^2} & \theta \neq \emptyset \end{cases} \tag{3}$$

$$\hat{\mathbb{Q}}_{\mathbb{M}_{\theta,e(i)}} = \left[\mathbb{Q}_{\mathbb{M}_{P(\theta),i}} \mathbb{Q}_{\mathbb{M}_{\theta,e(i-1)}} + \mathbb{Q}_{\mathbb{M}_{P(\theta),e(i-1)}} \mathbb{Q}_{\mathbb{M}_{\theta,i}}\right] + \sum_{B \cap C = \theta} \mathbb{Q}_{\mathbb{M}_{B,e(i-1)}} \mathbb{Q}_{\mathbb{M}_{C,i}}, \quad \forall \theta \subseteq \Theta \tag{4}$$

$$\hat{\mathbb{Q}}_{\mathbb{M}_{P(\Theta),e(i)}} = \mathbb{Q}_{\mathbb{M}_{P(\theta),i}} \mathbb{Q}_{\mathbb{M}_{P(\Theta),e(i-1)}}. \tag{5}$$

Finally, the generalized QER rule is represented as

$$\mathbb{P}_\theta = \mathbb{P}_{\theta,e(L)} = \begin{cases} 0 & \theta = \emptyset \\ \frac{|\hat{\mathbb{Q}}_{\mathbb{M}_{\theta,e(L)}}|^2}{\sum_{B \subseteq \Theta} |\hat{\mathbb{Q}}_{\mathbb{M}_{B,e(L)}}|^2} & \theta \neq \emptyset \end{cases} \tag{6}$$

where L is the length of quantum evidence set for combination.

References

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