A Study on Energy Density in a Universe in Linear Expanding

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Abstracs

The growing interest in dark energy and dark matter has made studies on the energy density in the universe a very current topic. Furthermore, new cosmological measurements are calling into question the validity of the ΛCDM model, and it is necessary to review it in depth. To solve these new challenges, Professor Fulvio Melia has developed a linear expansion universe model, the R_h=ct universe, which is giving very good results in relation to the new cosmological measurements. In this report we have developed, within this model of the universe, an equation that allows us to calculate the value of the energy density as a function of the age of the universe. The result in reference to the current experimental value of the energy density obtained by Mission Planck coincides with the value obtained by our equation $0{,}97.10^{-26}$ Kg/m^3. For this reason, we believe that our equation can be useful when determining energy densities of the universe at earlier and later times. With this wish we present our work.

Keywords: energy density, Rh = ct universe, general relativity.

1.- The linear expansion universe R_h = ct

The standard cosmological model ΛCDM cannot respond to some important new results of modern cosmology. Challenges arise such as Microwave Background Uniformity, the Hubble Stress, the El Gordo collision or impossible galaxies (z > 10) that the standard cosmological model does not resolve. On the other hand, other models are proposed as alternatives.

Professor Fulvio Meliá's linear expansion universe, R_h=ct universe, solves these challenges, where the standard model fails. This model is based on the restriction R_h = ct where Rh is the gravitational horizon, which coincides with the Hubble radius, t the age of the universe and c the speed of light in a vacuum. The model is already theoretically based []4], and constitutes an important tool for analyzing the universe, being today key to its understanding [1]

2.- The value of energy density

We are going to calculate the energy density in the universe R_h=ct; For this, we introduce the equation (1) obtained by us in Annex I. Equation that relates the curvature density parameter to the energy density parameter. Is the next:

$$\Omega_k/\Omega_\rho = 8\pi G/3c^2$$

$$\Omega_k = R/\rho_c$$

$$\Omega_\rho = \rho/\rho_c$$

$$R_h = ct$$

According to the equations obtained in Annex II for the Gaussian curvature K_{gauss}, and the curvature scalar R:

$$R = 2K_{gauss}$$

$$K_{gauss} = GM/c^2r^3$$

(R_s = Schwarzschild radius) $R_s = 2GM/c^2$

Let's calculate the energy density for a value of $r = R_h = R_s$

$$R = 2GM/c^2r^3 = 1/R_h^2$$

$$\Omega_k/\Omega_\rho = R/\rho = 8\pi G/3c^2 = 1/\rho R_h^2$$

$$\boldsymbol{\rho = 3/8\pi G t^2}$$

In the R_h=ct universe it is true that:

$$H=1/t$$

Then:

$$\rho = 3H^2/8\pi G = \rho_c$$

$$\boldsymbol{\Omega_\rho = \rho/\rho_c = 1}$$

3.- Discussion

We have obtained an equation for the energy density of the universe; this equation relates the energy density to the age of the universe and in principle it is valid where the Rh=ct universe is valid. Furthermore, we get that the energy density is equal to the critical energy density at any given moment, being therefore the energy density parameter will always be unity. Therefore, the energy density predicted by our equation for the current age of the universe is $0{,}97.10^{-26}$ Kg/m^3. This result is in agreement with the experimental measurements of Mission Planck [3]. For this reason, we think that this equation found can give correct results when we determine energy densities at other cosmological times and that it provides the R_h=ct universe with a new tool for analyzing the cosmos.

We summarize these results quantitatively through the following Table and graph:

ENERGY DENSITIES IN THE $R_h = ct$ UNIVERSE

Age of the universe	Energy density
685 million years	386.10^{-26} Kg/m^3
761 million years	313.10^{-26} Kg/m^3
856 million years	247.10^{-26} Kg/m^3
979 million years	189.10^{-26} Kg/m^3
1.142 million years	139.10^{-26} Kg/m^3
1.370 million years	96.10^{-26} Kg/m^3
1.713 million years	62.10^{-26} Kg/m^3
2.283 million years	35.10^{-26} Kg/m^3
3.425 million years	15.10^{-26} Kg/m^3
6.850 million years	4.10^{-26} Kg/m^3
13.700 million years	$0,97.10^{-26}$ Kg/m^3

$$\rho = 3/8\pi G t^2 = \rho_c$$

$$\Omega_\rho = 1$$

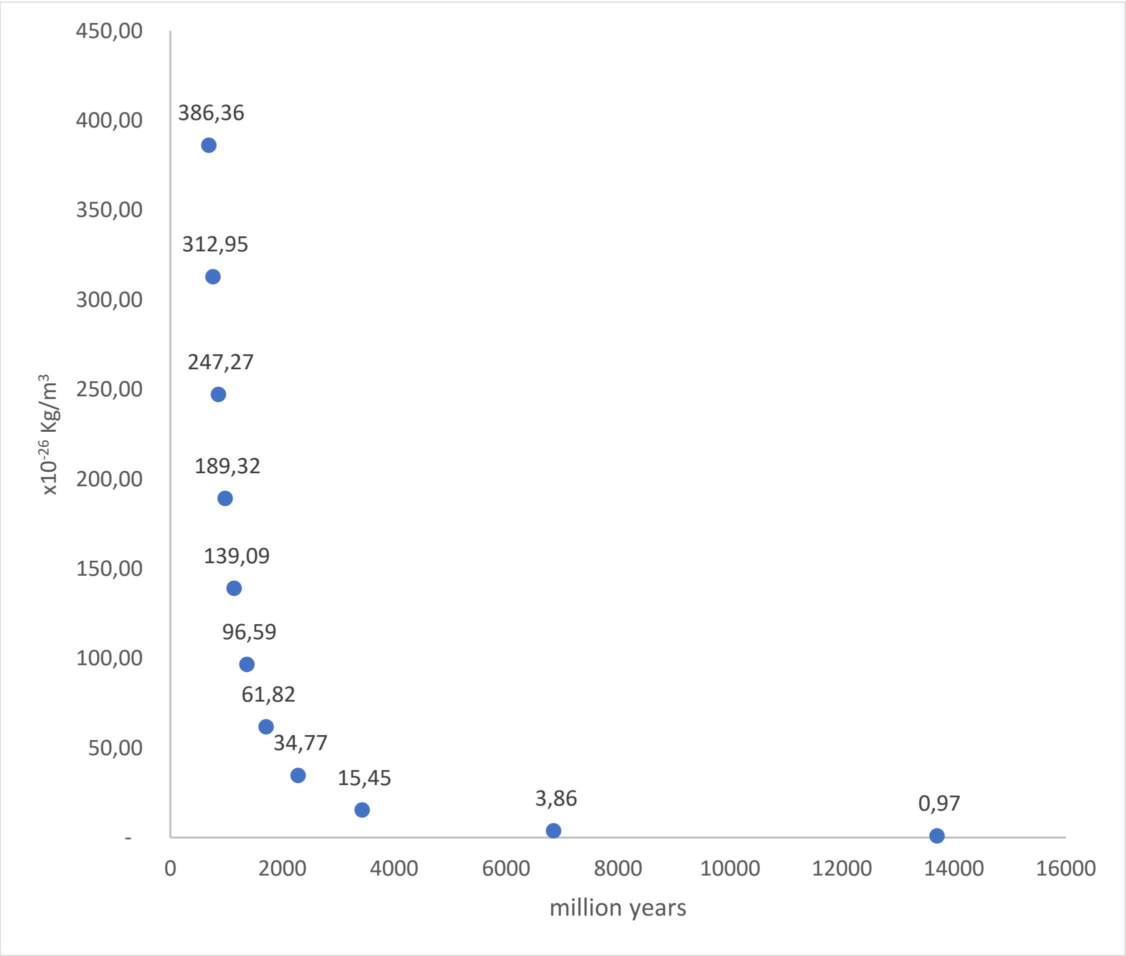

Fig. 1. Energy density of the universe as a function of his age

4. – Conclusions

Introducing the equation found by us, which relates the curvature scalar with the energy density, in the universe of linear expansion R_h=ct, **we have obtained an equation that relates the anergy density of the universe with its age** in a way that allows us to calculate the value of energy density at different cosmic times. **Furthermore, this equation has led us to the conclusion that at all times the energy density is the critical energy density, and therefore the value of the energy density parameter is always unity.** Our equation leads to a current energy density value of $0{,}97.10^{-26}$ Kg/m^3, that matches the experimental value given by the Planck Mission [3] in 2018. Thus, this equation can be very useful in cosmological calculations and will always be a tool for analyzing the cosmos.

Annex I

In this annex we deduce the equation that relates the energy density to curvature.

1. - The cosmic spacetime

We are going to study a uniform and isotropic spacetime from a physical point of view, this is equivalent from a geometric point of view to being invariant under translations and rotations.

According to Professor Fulvio Meliá in reference [1], we define "cosmic spacetime" as the set of points (t, r, Ø Φ) that satisfy the FLRW metric, that is, that satisfy the equation:

$$ds^2 = c^2dt^2 – a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right)$$

We define each of the "3D hypersurfaces" of cosmic spacetime as the set of points that have the same temporal coordinate. Thus, cosmic spacetime will have a different hypersurface for each time t. As we have defined them, these hypersurfaces do not intersect, that is, they have no common points and the set of all of them constitutes cosmic spacetime.

It is in these 3D hypersurfaces where we are going to calculate the curvature scalar that constitute the object of this Annex

2. - Calculating the curvature scalar in the 3D hypersurfaces of cosmic spacetime

First, we are going to calculate the curvature scalar of a 3D hypersurface of our homogeneous and isotropic cosmic spacetime with a matter density ρ.

2.1- Birkhoff–Jebsen theorem

We make a brief comment on this theorem of mathematics applied to the theory of generalized relativity. First, we summarize Professor Fulvio Melia in reference [2] to explain it.

“If we have a spherical universe of mass-energy density ρ and radius r and within it a concentric sphere of radius r_s smaller than r, it is true that the acceleration due to gravity at any point on the surface of the sphere of relative radius r_s to an observer at its origin, depends solely on the mass-energy relation contained within this sphere”.

Thus, according to this, to calculate the curvature of the gravitational field of a point located at a distance "r_s" from the geometric center that we are considering in our continuous universe, it is only necessary to consider its interaction with the points that are at a radius smaller than "r_s", therefore, the mass "m" to be considered will only be that contained in the sphere of radius "r_s".

In general relativity Birkhoff´s theorem states that any spherically symmetric solution of the vacuum field equations must be statically and asymptotically flat. This means, that the outer solution (that is, the spacetime outside a gravitational, non-rotating, spherical body) must be the Schwarzschild metric.

2.2- Calculating the spatial curvature constant

Let's consider our 3D hypersurface and a sphere of radius r inside, the Birkhoff–Jebsen theorem assures us that if we want to calculate the curvature at a point on its surface, we must consider only the interaction with the gravitational mass found inside, the gravitational mass inside for the sphere external point that we are considering behaves as a point mass of equal magnitude to that of the mass of the sphere and located at its central point. In this case we are already in the Schwarzschild model, and we can use its equations to calculate the corresponding curvature.

For all this, we can treat the problem of calculating the curvature scalar in each of the 3D hypersurfaces of our cosmic spacetime as a problem to be solved by the Schwarzschild model and calculate the curvature scalar from that model. In this model, spacetime is reduced to a 2D surface and so Gaussian curvatures are easily calculated; the scalar curvature in this case is twice the Gaussian curvature.

According to Annex II, we have found an equation that relates the Gaussian curvature K_{gauss} of the spacetime of the Schwarzschild model, with the cosmological parameters mass M and universal gravitation constant G. We are going to use this equation to solve our problem. This equation is the following:

$$K_{gauss} = -GM/c^2r^3$$

Since in our case it is a sphere, its mass will be given by

$$M = 4\pi r^3 \rho/3$$

$$K_{gauss} = -4\pi G\ \rho/3c^2$$

The curvature scalar R in bidimensional spaces, 2D surfaces, will be given by twice the Gaussian curvature K_{gauss}, thus:

$$\mathbf{R/\rho = -8\pi G/3c^2} \quad (1)$$

R curvature scalar, spatial curvature constant (m^{-2}) and ρ is the matter density (Kg/m^3)

2.3- Studying the spatial curvature constant

We study the ratio between the curvature parameter Ω_K and the energy density parameter Ω_ρ

$\Omega_\rho = \rho/\rho_c$

$R/\rho = -8\pi G/3c^2$

Dividing the two terms of the fraction by ρ_c, we get:

$(R/\rho_c)\ /\Omega_\rho = 8\pi G/3c^2$

Defining:

$\Omega_k = R/\rho_c$

Result:

$$\mathbf{\Omega_k/\Omega_\rho = 8\pi G/3c^2} \quad (1)$$

Annex II

In this annex we obtain an equation that relates the Gaussian curvature of the Schwarzschild spacetime with several physical parameters.

The Flamm paraboloid, J. Droste's spacetime solution to the problem studied by Schwarzschild, [5], is a 2D surface inserted in an R^3 space. Its geometry allows us to parameterize the paraboloid as a function of the observer's distance from the point mass "r" and the azimuth angle "ϕ". The problem admits a mathematical treatment of differential geometry of surfaces [6], and with it we are going to calculate the Gaussian Curvature. (R_s = Schwarzschild radius $=2GM/c^2$)

Surface parameters (r, ϕ)

$0 \leq r < \infty,\ \ 0 \leq \varphi < 2\pi$

which has this parametric equation:

$x = r\ \cos\phi$

$y = r\ \text{sen}\phi$

$z = 2(Rs\ (r - Rs))^{1/2}$

Vector Equation of the Surface

$f\ (x,y,z) = (r\ \cos\phi,\ \ r\ \text{sen}\phi,\ \ 2(Rs(r - Rs))^{1/2})$

Determination of velocity, acceleration, and normal vectors to the surface

$ðf/ð\phi = (-r\ \text{sen}\phi,\ r\cos\phi,\ 0)$

$ðf/ðr = (\cos\phi,\ \text{sen}\phi,\ (r/Rs - 1)^{-1/2})$

$ð^2f/ð\phi^2 = (-r\ \cos\phi,\ -r\ \text{sen}\phi,\ 0)$

$ð^2f/ðr^2 = (0,\ 0,\ (-1/2R_s).\ (r/Rs - 1)^{-3/2})$

$ðf/ð\phi ðr = (-\text{sen}\phi,\ \cos\phi,\ 0)$

$$n = \frac{(ðf/ð\varphi \times ðf/ðr)}{[\frac{ðf}{ð\varphi} x \frac{ðf}{ðr}]}$$

$(ðf/ð\phi \times ðf/ðr)\ = (r\cos\phi/(r/Rs\ \ -1)^{1/2},\ \ \ r\text{sen}\phi/(r/Rs\ \ -1)^{1/2},\ \ \ -r)$

$[\frac{ðf}{ð\varphi} x \frac{ðf}{ðr}] = r\ ((1/(r/Rs\ \ -1)) + 1)^{1/2}$

Curvature and curvature parameters

Gauss curvature $\quad$ $\mathbf{K_{gauss} = LN\text{-}M^2/EG\text{-}F^2}$

$L = ð^2f/ð\phi^2.\ n = -r(r/Rs)^{-1/2}$

$N = ð^2f/ðr^2.\ n = (1/2Rs)\ (r/Rs)^{-1/2}\ (r/Rs - 1)^{-1}$

$M = (ðf/ð\phi ðr).\ n = 0$

$E = ðf/ð\phi.\ \ ðf/ð\phi = r^2$

$G = ðf/ðr.\ ðf/ðr = 1 + (1/\ (r/Rs - 1))$

$F = ðf/ð\phi.\ ðf/ðr = 0$

$$\mathbf{K_{gauss} = -R_s/2r^3 = - GM/c^2r^3}$$

In 2D problems it is true that the curvature scalar R is twice the Gauss curvature at each point

$$R = 2K_{gauss}$$

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