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THE LAW OF SCALING FOR LARGE NUMBERS: origin of large numbers from the primary large number D_{20} = 1.74349...x 10^20.

Abstract. *The paper solves the problem of mathematical inference of large numbers, which was formulated in 1985 by P. C. W. Davies [1]. The law of scaling of large numbers is derived. The law of scaling gives a new method of obtaining large numbers from dimensionless constants. It complements the known method based on relations of dimensional physical quantities. The law of scaling of large numbers shows that large numbers of scale 10^{39}, 10^{40}, 10^{61}, 10^{122} are only part of the complete family of large numbers. The large numbers are supplemented by new large numbers of scales 10^{140}, 10^{160}, 10^{180}, which are naturally derived from the fundamental parameters of the observable Universe. New coincidences of relations of dimensional quantities on scales 10^{140}, 10^{160}, 10^{180} are found. It is shown that large numbers of different scales are functionally related to each other. The primary large number $D_{20} =(\alpha D_0)$^(1/2) = 1.74349...x 10^{20}, from which large numbers of other scales are formed according to a uniform law, is chosen on the scale of 10^{20}. The primary large number D_{20} = 1.74349...x 10^{20} consists of two dimensionless constants: the fine structure constant alpha and the Weyl number D_0 = 4.16561...x 10^{42}. The coincidences of the relations of the dimensional quantities with large numbers on scales 10^{160} and 10^{180} allowed us to derive simple and beautiful formulas for calculating the Hubble constant H and the cosmological constant Λ. An equation is derived which shows that the constants H and Λ are related. The origin of H and Λ from the fundamental physical constants of the electron is proved. The law of scaling of large numbers makes it possible to calculate analytically the parameters of the observable Universe with high accuracy.A new equation is derived, which unites the 5 most important parameters of the observable Universe: $M_U R_U G \Lambda^2 = H^2$.*

Keywords: *large numbers, large Weyl number, electron constants, Planck constants,* Stoney *constants, Hubble constant, Pioneer-anomaly, cosmological constant Λ, Stewart equation, Dirac equation, Eddington-Weinberg equation, Teller equation, Nottale equation.*

1. Introduction

In physics and cosmology there are many dimensional relations that give large scale numbers 10^{39}, 10^{40}, 10^{42} [1, 2, 3, 4, 5, 6]. Muradyan, R. M. cites dimensional magnitude relations that give large scale numbers 10^{60}, 10^{120} [7]. J. Casado and Scott Funkhouser cite dimensional magnitude relations that give large scale numbers 10^{61}, 10^{122} [8, 9]. Pierre-Henri Chavanis showed that ratios of macroobject and microobject masses to the Planck mass give large numbers on the order of 10^±20, 10^±30 10^±40 10^±60 [10].

The revealed set of coincidences of relations of dimensional quantities finds no explanation. These mysterious numbers have attracted the attention of many famous scientists. It is known that Dirac believed that the coincidence of large numbers hides some yet undiscovered law of nature. H. Weyl was the first to draw attention to the incredibly large number of coincidences of large numbers

[5, 6]. He also obtained the constant $D_0 \sim 10^{42}$ ($D_0 = 4.16561... \times 10^{42}$) as the ratio of the electric force to the gravitational force between two electrons [5, 11]. H. Weyl put the constant D_0 on a par with the fine structure constant alpha in terms of significance. The coincidence of large numbers obtained from the relations of dimensional constants was actively investigated by A. S. Eddington [2, 3].

Many famous physicists tried to find the reasons for the mysterious coincidence of large numbers. The attempts of H. Weyl and Eddington to explain the coincidence of large numbers on the basis of physical principles were not successful. To explain the coincidence of large numbers P. Dirac proposed the hypothesis of large numbers [4]. Dirac's hypothesis of large numbers did not turn into a theory of large numbers and was not recognized. Alternative explanations for the coincidence of large numbers, known as the weak and strong anthropic principles, also did not solve the problem [1]. So this mysterious problem of coincidence of large numbers remained unsolved. So far, it has not been possible to create a "complete theory of cosmology and atomism", which P. Dirac hoped for [12]. It has not been possible to deduce large numbers mathematically, as P. Davis wanted [1].

The coincidences of relations of dimensional quantities leading to large numbers are usually estimated approximately. The Weyl-Eddington-Dirac coincidence of large numbers is usually understood as their coincidences in order of magnitude.It has long been thought that several orders of magnitude would not be of great importance in expressions of scale 10^{40} or larger [13]. At the same time, in order to reveal the relationship within the family of large numbers, it is necessary to compare not only the exponents of degree (N), but also the coefficients before 10^N.

This paper shows that there is a functional dependence and relationship between large numbers of different scales. This functional relationship obeys the strict mathematical law of scaling. The law of scaling of large numbers is deduced, which allowed to obtain with high accuracy their values. It is proved not approximate coincidences of relations of dimensional values by order of magnitude, but coincidences with accuracy close to the accuracy of Newtonian constant of gravitation G. The law of scaling gives a new method of calculation with high accuracy of large numbers from dimensionless constants. Here and further under the exact value we will understand the accuracy close to the accuracy of Newtonian constant of gravitation G. For scales 10^{20} - 10^{180} it is very high accuracy.Compared to order-of-magnitude coincidences, this is also a very high precision.

The paper shows that the large numbers on huge scales 10^{20} - 10^{180} have a single origin. The key to unraveling the mystery of large numbers is given by the number $D_{20} = 1.74349.... \times 10^{20}$. Exact coincidences of relations of dimensional quantities and their equality to large numbers take place on all scales from 10^{20} to 10^{180}. The constant $D_{20} = (\alpha D_0)$^1/2 = 1.74349...x 10^{20} allows us to solve the problem posed by Davis [1]:"*to derive large numbers mathematically*".

2. Equivalent formulas for calculating the Newtonian constant of gravitation G.

In [14] it is shown that the Newtonian constan$_0$t of gravitation G can be calculated analytically using the electron constants, the fine structure constant alpha, and the large number $D_0 = 4.16561...\times 10^{42}$.

$$G = \frac{r_e^3}{t_0^2 \bullet m_e \bullet D_0} = \frac{c^2 r_e}{m_e D_0} = \frac{c^3 r_e^2}{\hbar \alpha D_0} \qquad (1)$$

where: ħ - Planck constant; r_e - classical electron radius; c - speed of light in vacuum; α - fine-structure constant; t_0 - time constant for the electron ($t_0 = r_e/c$); me - electron mass.

A large number of equivalent formulas for the Newtonian constant of gravitation G can be proposed. For example, 10 equivalent formulas are given in Fig. 1:

$$G = \left\{ \begin{array}{cccc} \frac{\hbar \bullet c \bullet \alpha}{D_0 \bullet m_e^2} & \frac{r_e^3}{t_0^2 \bullet m_e \bullet D_0} & & \frac{r_e^5}{t_0^3 \bullet \alpha \bullet \hbar \bullet D_0} \\ \frac{c^3 \bullet l_{pl}^2}{\hbar} & \frac{\hbar \bullet c}{m_{pl}^2} & \frac{c^4 \bullet r_e \bullet t_0}{\alpha \bullet \hbar \bullet D_0} & \frac{c^4}{F_0 \bullet D_0} \\ \frac{E_h \bullet r_e}{\alpha^2 \bullet m_e^2 \bullet D_0} & \frac{4\mu_B^2 \bullet \alpha^2 \bullet 10^{-7}}{r_e^2 \bullet m_e^2 \bullet D_0} & & \frac{c^5 \bullet t_{pl}^2}{\hbar} \end{array} \right\} = \mathbf{6.6743x10^{-11} m^3 kg^{-1} s^{-2}}$$

Fig. 1. Equivalent formulas for the calculation of the Newtonian constant of gravitation G using the fundamental physical constants. ħ - Planck constant; r_e - classical electron radius; c - speed of light in vacuum; α - fine-structure constant; t_0 - time constant for the electron ($t_0 = r_e/c$); m_e - electron mass, D_o - large number, μ_B- Bohr magneton, E_h - Hartree energy, t_{pl} - Planck time, l_{pl} - Planck length, m_{pl} - Planck mass.

In addition to the formulas given in Fig. 1, the Newtonian constant of gravitation G can be represented using the Stoney constants (l_S, t_S, m_S):

$$G = \frac{c^3 l_S^2}{\alpha \bullet \hbar}, \; G = \frac{c^5 t_S^2}{\alpha \bullet \hbar}, \; G = \frac{\hbar \bullet c \bullet \alpha}{m_S^2}, \; G = \frac{c^2 \bullet l_S}{m_S}, \; = \mathbf{6{,}6743\ldots x\ 10^{-11}}\ kg^{-1} m^3 s^{-2} \quad (2)$$

The Newtonian constant of gravitation G can be represented using the force constant F_0 and the Weyl number D_0 [14]:

$$G = \frac{c^4}{F_0 \bullet D_0} = \mathbf{6{,}6743\ldots x10^{-11}}\ kg^{-1} m^3 s^{-2} \quad (3)$$

where: F_0 = 29.0535101 N (4) is the force constant [14].

Since the Newtonian constant of gravitation G is included in the relations of dimensional constants giving large numbers, the set of equivalent formulas leads to the set of coincidences of large numbers.

3. Electrical quantities in the Newtonian constant of gravitation G formula.

The Newtonian constant of gravitation G can be represented not only by mechanical constants, but also by electrical constants:

$$G = \frac{r_e^2 c^3}{e^2 \bullet D_0 \bullet R_0} = \mathbf{6{,}6743\ldots x10^{-11}}\ kg^{-1} m^3 s^{-2} \quad (5)$$

$$G = \frac{c^5}{I_0 \bullet U_0 \bullet D_0} = \mathbf{6{,}6743\ldots x10^{-11}}\ \text{Meter}^5\ \text{Ampere}^{-1}\ \text{Second}^{-5}\ \text{Volt}^{-1} \quad (6)$$

$$G = \frac{4\pi \bullet c^5}{I_0^2 \bullet R_K \bullet \alpha \bullet D_0}$$ = **6,6743…x10^{-11}** Meter^5 Ampere^{-2} Second^{-5} Ohm^{-1} (7)

$$G = \frac{4\pi \bullet c^3 r_e^2}{e^2 \bullet R_K \bullet \alpha \bullet D_0}$$ = **6,6743…x10^{-11}** m^{-5}C^{-2}s^{-3} Ohm^{-1} (8)

where: voltage constant (U_0 = 5.10998949...x10^5 V), current constant (I_0 = 1.70450902...x10^4 A), R_0 - resistance constant (29.9792458 Ohm), D_0 = 4.16561...x 10^{42} - Weyl number, R_K - von Klitzing constant (R_K = 25 812,807 45... Ohm),

Here the dimensionality [Meter^{5}Ampere^{-1}Second^{-5}Volt^{-1}] = [kg^{-1}m^{3}s^{-2}] [34].

For all physical quantities there are their elementary constants. Some elementary constants and formulas for their calculation are given in [15]. Besides elementary charge, elementary length and elementary time, elementary current I_0, elementary resistance R_0, elementary voltage U_0 and other elementary constants are presented in [15].

All the above formulas for G are equivalent. They give the same calculated value of the constant G. And these are not all possible equivalent formulas for calculating the constant G. There are many more equivalent formulas. The reason for the existence of such a large number of exact equivalent formulas for G has no explanation. The Newtonian constant of gravitation G is included in many relations of dimensional constants giving large numbers. Therefore, the set of formulas for calculating the Newtonian constant of gravitation G leads to a large number of relations giving coincidences of large numbers.

4. Formulas of connection of Planck units with electron constants

The equations for the Newtonian constant of gravitation G (1), which include the large number D_0 = 4.16561...x 10^{42} and the fine structure constant alpha, allow us to represent the Planck units by means of the electron constants:

$$m_{pl} = m_e \sqrt{\frac{D_0}{\alpha}}, \quad l_{pl} = \frac{r_e}{\sqrt{\alpha D_0}}, \quad t_{pl} = \frac{t_0}{\sqrt{\alpha D_0}} . \qquad (9)$$

where: t_{pl} - Planck time, l_{pl} - Planck length, m_{pl} - Planck mass, m_e - elementary mass (electron mass); r_e - elementary length (classical electron radius); $t_0 = r_e/c$ - elementary time.

5. Formulas for relating the Stoney constants to the electron constants

The equations for the Newtonian constant of gravitation G (1), containing the large number D_0 = 4.16561..... x 10^{42} allow us to represent the Stoney units by means of the electron constants and the large Weyl number D_0 = 4.16561.... x 10^{42}:

$$m_S = m_e \sqrt{D_0}, \quad l_S = \frac{r_e}{\sqrt{D_0}}, \quad t_S = \frac{t_0}{\sqrt{D_0}} . \qquad (10)$$

where: t_S - Stoney time, l_S - Stoney length, m_S - Stoney mass, m_e - elementary mass (mass of the electron); r_e - elementary length (classical radius of the electron); $t_0 = r_e/c$ - elementary time.

It follows from equations (9) and (10) that the Planck constants and the Stoney constants are related through the fine structure constant alpha (√α).

$$\frac{m_S}{m_{Pl}} = \sqrt{\alpha}\,, \qquad \frac{l_S}{l_{Pl}} = \sqrt{\alpha}\,, \qquad \frac{t_S}{t_{Pl}} = \sqrt{\alpha}\,. \qquad (11)$$

6. The value of the Mass of the observed Universe from the coincidence of large numbers on a scale of 10^{120}.

The value of the Mass of the observed Universe can be obtained from the following relations [1, 7, 9]:

$$\mathbf{GM_U^2/\hbar c} \qquad (12)$$

$$\mathbf{M_U c^2/\hbar H} \qquad (13)$$

where: G - Newtonian constant of gravitation, M_U - mass of the observable Universe, ħ - Planck constant, c - speed of light in vacuum, H - Hubble constant.

Both of these relations (12) and (13) give a large number of scale 10^{120}. For an exact match of the large numbers, the following equality must be satisfied:

$$\mathbf{GM_U^2/\hbar c = MUc2/\hbar H} \qquad \mathbf{(14)}$$

From equation (14) together with Stewart's equation (15) [7, 16]:

$$Gm_e^2 / r_e = \alpha^2 \hbar H \qquad (15)$$

it is easy to obtain an equation relating the mass of the observable Universe to the Newtonian constant of gravitation G

$$\mathbf{M_U = c^3 \hbar \alpha^2 r_e / G^2 m_e^2} \qquad (16)$$

Substituting the value of G from equation (1) into equation (16), we obtain the value of the mass of the observable Universe and a new formula for its calculation:

$$M_U = m_e \alpha D_0^2 = 1.15348... \bullet 10^{53}\, kg \qquad (17)$$

Substituting this value of the mass of the observed Universe into formulas (12) and (13) demonstrates not an approximate coincidence of their order of magnitude, but an exact coincidence:

$$\mathbf{GM_U^2/\hbar c = \alpha^3 D_0^3 = 28{,}088...\ x\ 10^{120}} \qquad \mathbf{(18)}$$

$$\mathbf{M_U c^2/\hbar H = \alpha^3 D_0^3 = 28{,}088...\ x\ 10^{120}} \qquad \mathbf{(19)}$$

7. The Hubble constant problem

The Hubble constant is one of the most important parameters in cosmology. However, the true experimental value of the Hubble constant H has not yet been established. Different measurement methods give different values of the Hubble constant, which vary considerably [17, 18]. The method using the Planck telescope gives a low value of 66.93 km/s/Mpc, the method presented by Carnegie University "1" gives a high value of 74 km/s/Mpc, the NLICOW method gives 71.9 km/s/Mpc, the Carnegie "2" method gives 69.8 km/s/Mpc [19].

The reasons for the significant discrepancy in the experimental H values are unknown [20]. The proponents of different methods defend their value of the Hubble constant and do not agree. The scientific dispute between the proponents of a low value of the Hubble parameter and a high value has not been finalized. At the same time, Pioneer-anomaly [26] indicates that the value of the Hubble parameter should be much higher than the values obtained by different methods.

Until a consistent experimental value of the Hubble constant is obtained, we will use the calculated theoretical value. It can be obtained from the known equations relating the parameters of the observable Universe.

8. Theoretical value of Hubble constant

The value of Hubble constant can be obtained from known equations.Let us use Stewart's equation (15):

$$Gm_e^2 / r_e = \alpha^2 \hbar H$$

From Stewart's equation, the calculated value of Hubble constant is :

$$H = Gm_e^2 / r_e \alpha^2 \hbar = 3.49981... \bullet 10^{-18} s^{-1} \qquad (20)$$

Substituting the value for G from formula (1) into Stewart's equation, we obtain a simple and beautiful formula for the Hubble constant :

$$H = \frac{c}{r_e \bullet \alpha D_0} = 3.49981... \bullet 10^{-18} s^{-1} \qquad (21)$$

where: re - classical electron radius; c - speed of light in vacuum; - fine-structure constant; αD_0 - a large number of scale 10^{40} (αD_0 = 3.03979... x 10^{40}).

Formula (21) gives an unusually high value of the Hubble constant. The high value of the Hubble constant was first noticed by L.Nottale [21]. L. Nottale obtained the value of H ≈ 3 x 10^{-18} s^{-1} from another equation:

$$Gm_e^3 c = \alpha^3 \hbar^2 H \qquad (22)$$

The high theoretical value of the Hubble constant from formulas (20) through (22) needs further verification from other equations. In addition to Stewart's equation and formula (22), we can propose such equations:

$$G\hbar = r_e^3 H c^2 \qquad (23)$$

$$Gm_e / r_e^2 = \alpha H c \qquad (24)$$

The same high value for the Hubble constant follows from these equations:H=3.49981 x 10^{-18} s^{-1}.

Formula (13) gives the same high value of the Hubble constant:

$$\mathbf{H = M_u c^2/\hbar}\alpha^3 D_0^3 \mathbf{= c/r_e}\alpha D_0 = 3.49981\ldots 10^{-18} \text{ s-1} \qquad (25)$$

The formula of J.C. Carvalho [22] (**M = c^3/GH**) gives the same value of H:

$$\mathbf{H = c^3/GM_U = c/r_e \alpha D_0 = 3.49981\ldots 10^{-18} \text{ s-1}} \qquad (26)$$

An additional check of the value of H = 3.49981...10^{-18} s^{-1} on the Planck Force formula obtained using cosmological parameters gives this value of the force:

$$\mathbf{F_{Pl} = M_U R_U H^2 = 1.21025 \times 10^{44}\ N} \qquad (27)$$

This value coincides completely with the value of the Planck force obtained by the well-known formula:

$$\mathbf{F_{Pl} = c^4/G = 1.21025 \times 10^{44}\ N} \qquad \mathbf{(28)}$$

At the same time another value follows from Hoyle's formula [23] ($\mathbf{M_U = c^3/(2GH)}$):

$$H_2 = \frac{c^3}{2 \bullet M_U G} = 1.74990... \bullet 10^{-18} s^{-1} \qquad (29)$$

This value is exactly two times smaller than that obtained from formulas (21) - (26).

The same halved value ($H_2 = 1.74990 \times 10^{-18}\ s^{-1}$) is obtained from the Teller equations [8, 24, 25]:

$$2\frac{G\hbar H}{c^4 l_{Pl}} = 2t_{Pl}H = 2\frac{Gm_{Pl}H}{c^3} = (\sqrt{\alpha D_0})^{-3} \cong \exp(-1/\alpha) \qquad (30)$$

Thus, different equations give two different values of the Hubble constant ($H_2 = 1.74990 \times 10^{-18}\ s^{-1}$) and ($H = 3.49981 \times 10^{-18}\ s^{-1}$). Which of the two theoretical values should be chosen depends on the degree of confidence in the original equations. It should be noted that both values do not disturb the appearance of the known and new equations. In the equations only appears the coefficient "2" in front of the Hubble constant when the smaller value is chosen. Nevertheless, for the final choice of the correct theoretical value of the two possible values, let us use an additional experimental result that may shed light on the problem of the Hubble constant. This is the observed deviation from the predicted accelerations in the Pioneer-10 and Pioneer-11 experiments, which has been called the "Pioneer-anomaly" [26].

9. Pioneer-anomaly

The Pioneer-anomaly was detected during observations of the Pioneer-10 and Pioneer-11 spacecraft. The magnitude of the Pioneer effect is in the range of **(7.41 - 10.07) × $\mathbf{10^{-10}\ m/s^2}$.** In addition to the Pioneer-10 and Pioneer-11 experiment, there are anomalous acceleration data from Galileo and Ulysses [27 - 30]. The acceleration values obtained are as follows:

For Galileo:

$$\mathbf{a_0 = (8 \pm 3) \times 10^{-10}\ m/s^2}$$

For Ulysses:

$$\mathbf{a_0 = (12 \pm 3) \times 10^{-10}\ m/s^2}$$

Two theoretical values of the Hubble constant ($H = 3.49981 \times 10^{-18}\ s^{-1}$ and $H_2 = 1.74990 \times 10^{-18}\ s^{-1}$), give the following values of cosmological acceleration:

$$\mathbf{a_0 = Hc = 10.4922 \times 10^{-10}\ m/s^2} \qquad (31)$$

$$\mathbf{a_{02} = H_2 c = 5.2461 \times 10^{-10}\ m/s^2} \qquad (32)$$

The closest to the experimental values of Pioneer-anomaly ($\mathbf{a_0 = (7.41 - 10.07) \times 10^{-10}\ m/s^2}$), Galileo ($\mathbf{a = (8 \pm 3) \times 10^{-10}\ m/s^2}$), Ulysses ($\mathbf{a = (12 \pm 3) \times 10^{-10}\ m/s^2}$) is the theoretical value of acceleration $\mathbf{a_0 = Hc = 10.4922 \times 10^{-10}\ m/s^2}$. This value of acceleration corresponds to $H = 3.49981...10^{-18}\ s^{-1}$.

An additional check of the value of acceleration a_0 = Hc = 10.4922 x 10^{-10} m/s^2 on Newton's law gives the following value of the cosmological force:

$$\mathbf{F = M_U a_0 = M_U Hc = 1.21025 \times 10^{44}\ N} \qquad \mathbf{(33)}$$

This value of the force coincides completely with the value of the Planck force: $F_{Pl} = c^4/G$ = 1.21025 x 10^{44} N.

The above results (formulas (20) - (28), (33)) allow us to favor a high value of H and use the following value for the Hubble constant in the calculations (Fig. 2):

$$H = \frac{c}{r_e \bullet \alpha D_0} = 3.49981... \bullet 10^{-18} s^{-1}$$

Fig. 2. Theoretical value of Hubble constant and the formula for its calculation.

10. Large numbers from the relations of physical quantities including the Hubble constant

The value of the Hubble constant obtained from the theory (H = 3.4998 x 10^{-18} s^{-1}) gives such calculated values for the time of existence of the observable Universe and for the radius of the observable Universe:

$$T_U = H^{-1} = \frac{r_e \alpha D_0}{c} = 2.85729... \bullet 10^{17} s \qquad (34)$$

$$R_U = c / H = r_e \alpha D_0 = 0.856594... \bullet 10^{26} m \qquad (35)$$

where: r_e - classical electron radius; c - speed of light in vacuum; ħ - fine-structure constant; αD_0 - large number of scale 10^{40} (αD_0 = 3.03979... x 10^{40}).

A whole family of large numbers of different scales is associated with the constants R_U and H. Here are the formulas for their calculation and the values of the large numbers. The ratio Ru/re gives a large number of scale 10^{40}:

$$\mathbf{R_u / r_e = c / r_e H = r_e \alpha D_0 / r_e = \alpha D_0 =} \ 3.03979... \times 10^{40} \mathbf{= D_{40}} \qquad \mathbf{(36)}$$

The ratio T_U/t_{Pl} gives the large scale number 10^{60}:

$$\mathbf{T_U / t_{Pl} = \alpha D_0 \sqrt{(\alpha D_0)} = 5.2998... \times 10^{60} = D_{60}} \qquad \mathbf{(37)}$$

The ratio Ru/l_{Pl} gives a large number of scale 10^{60}:

$$\mathbf{R_u / l_{Pl} = c / l_{Pl} H = r_e \alpha D_0 / (r_e / \sqrt{(\alpha D_0)} = 5.2998... \times 10^{60} = D_{60}} \qquad \mathbf{(38)}$$

The ratio of the square of the metagalactic radius to the square of the classical electron radius gives a large number of scale 10^{80}:

$$\mathbf{R_u^2 / r_e^2 = (r_e \alpha D_0)^2 / (r_e^2) = \alpha^2 D_0^2 = 9.24033... \times 10^{80} = D_{80}} \qquad \mathbf{(39)}$$

The ratio of the square of the metagalactic radius to the square of the Planck length gives a large number of scale 10^{120}:

$$\mathbf{R_u^2 / l_{Pl}^2 = (r_e \alpha D_0)^2 / (r_e^2 / \alpha D_0) = \alpha^3 D_0^3 = 28.088... \times 10^{120} = D_{120}} \quad \mathbf{(40)}$$

The ratio of the volume of the observed universe to the volume of an electron gives a large number on the scale of 10^{120}:

$$\mathbf{R_u^3/r_e^3 = (r_e\alpha D_0)^3/(r_e^3) = \alpha^3 D_0^3 = 28.088\ldots \times 10^{120} = D_{120}} \qquad \mathbf{(41)}$$

The ratio of the volume of the observable universe to the Planck volume gives a large number of scale 10^{180}:

$$\mathbf{R_u^3/l_{Pl}^3 = (r_e\alpha D_0)^3/(r_e^3/\alpha D_0\ (\alpha D_0)^{1/2}) = \alpha^4 D_0^4\ (\alpha D_0)^{1/2} = 148,859 \times 10^{180} = D_{180}} \quad \mathbf{(42)}$$

The large numbers obtained for different scales (36) through (42) come from the relations of dimensional constants. This is only one of the methods that lead to large numbers. Next we show that there is a new method for deriving large numbers from dimensionless constants.Combining the two methods will solve a number of fundamental problems of cosmology.

11. Relation of large numbers to the fine structure constant alpha.

Teller was the first to attempt to obtain large numbers using the fine structure constant alpha. He obtained an equation in which a large number of order 10^{-60}, obtained from the relations of dimensional constants, was represented by dimensionless numbers [8, 24, 25]. In his equation, one of the dimensionless constants is the fine structure constant alpha:

$$2\frac{G\hbar H}{c^4 l_{Pl}} = 2t_{Pl}H = 2\frac{Gm_{Pl}H}{c^3} \cong \exp(-1/\alpha)$$

Although Teller's equation does not give the exact value of a large number, his idea to relate large numbers to the fine structure constant alpha is a remarkable foresight. In the following, we will show that all large numbers do indeed contain the fine structure constant alpha and have their origin in dimensionless constants.The large numbers begin at the Planck scale of 10^{20}.The first large number of the scale 10^{20} contains the fine structure constant alpha.

12. The large number $D_{20} = 1.74349 \times 10^{20}$ is the basis of all large numbers.

The Planck unit formulas (9) include the combination of the constants α and D_0 in the form:

$$D_{20} = \sqrt{\alpha D_0} \qquad (43)$$

where: D_0 is the large Weyl number ($D_0 = 4.16561... \times 10^{42}$), α is the fine structure constant.

The Newtonian constant of gravitation G (Fig. 1) and the formula for the Hubble constant H (21) include the number D_{20} to degree 2. Further we will show that the number D_{20} is included in all parameters of the observable Universe. Therefore, the number D_{20} is defined as the primary large number, and the scale 10^{20} is chosen as the base number. This base number is a generator of large numbers in a wide range from 10^{20} to 10^{180}.

The significance of the number D_{20} is:

$$D_{20} = 1.74349... \bullet 10^{20} \qquad (44)$$

The formula of the large number of scale 10^{20} is shown in Fig. 3.

$$D_{20} = \sqrt{\alpha D_0} = 1.74349 \ldots \bullet 10^{20}$$

Fig. 3. A large number of scale 10^{20}.

Equivalent formulas:

$$D_{20} = \sqrt{\alpha D_0} = \frac{r_e}{l_{Pl}} = \frac{t_0}{t_{Pl}} = \frac{\alpha\, m_{Pl}}{m_e} = 1.74349 \ldots \bullet 10^{20} \qquad \textbf{(45)}$$

13. Large number of scale 10^{40}

The ratio of the age of the Universe to the electron constant (T_U/t_0), the ratio of the radius of the Universe to the classical radius of the electron (R_U/r_e), the ratio of the electron energy to the minimum energy quantum ($m_e c^2/\alpha\hbar H$), etc., lead to the large number on the scale of 10^{40} (D_{40} = 3.03979... x 10^{40}).

The necessity of mathematical derivation of a large number of 10^{40} was pointed out by P. Davis [1]: *"It is possible that in the future explanations of some of the considered numerical coincidences will be found in the framework of theoretical physics rather than biology. In this case, the mysterious number 1040 will be derived mathematically".*

The significance of the number D_{40}:

$$D_{40} = 3.03979 \ldots \bullet 10^{40} \qquad (46)$$

The formula is shown in Fig. 4.

$$D_{40} = (D_{20})^2 = \alpha D_0 = 3.03979 \ldots \bullet 10^{40}$$

Fig. 4. Large scale number 10^{40}.

Equivalent formulas:

$$D_{40} = \frac{T_U}{t_0} = \frac{R_U}{r_e} = \frac{m_e c^2}{\alpha\hbar H} = \frac{1}{t_0 H} = \frac{r_e^2}{l_{Pl}^2} = \frac{t_0^2}{t_{Pl}^2} = \frac{\alpha^2 m_{Pl}^2}{m_e^2} = (\sqrt{\alpha D_0})^2 = \alpha D_0 = 3.03979 \ldots \bullet 10^{40} \qquad (47)$$

14. Large number of scale 10^{60}.

The large number of scale 10^{60} (D_{60} = 5.2998... x 10^{60}) is obtained from the ratio of the parameters of the observable universe to Planck units:

$$M_U / m_{Pl} = m_e D_0^2 \alpha / m_{Pl} = (\sqrt{\alpha D_0})^3 = 5.2998 \ldots \bullet 10^{60} \qquad (48)$$

$$R_U / l_{Pl} = c / l_{Pl} H = \alpha D_0 \bullet \sqrt{\alpha D_0} = 5.2998 \ldots \bullet 10^{60} \qquad (49)$$

$$T_U / t_{Pl} = 1 / t_{Pl} H = \alpha D_0 \bullet \sqrt{\alpha D_0} = 5.2998 \ldots \bullet 10^{60} \qquad (50)$$

The value of the D_{60} number is:

$$D_{60} = 5.2998... \bullet 10^{60} \qquad (51)$$

The formula is shown in Fig.5.

$$D_{60} = (D_{20})^3 = (\sqrt{\alpha D_0})^3 = 5.2998... \bullet 10^{60}$$

Fig. 5. A large number of scale 10^{60}.

Equivalent formulas:

$$D_{60} = \frac{T_U}{t_{Pl}} = \frac{R_U}{l_{Pl}} = \frac{M_U}{m_{Pl}} = \frac{c}{l_{Pl} H} = \frac{r_e^3}{l_{Pl}^3} = \frac{t_0^3}{t_{Pl}^3} = \frac{c^3}{G m_{Pl} H} = (\sqrt{\alpha D_0})^3 = 5.2998... \bullet 10^{60} \quad (52)$$

15. Large number of scale 10^{80}.

The large number of scale 10^{80} (D_{80} = 9.24033... x 10^{80}) is obtained from the ratio of the conditional area formed by the radius of the observable universe to the area formed by the radius of the electron, and from a relation of the form: $HM_u^2\alpha G/c^3m_e$, etc.

The value of the number D_{80}:

$$D_{80} = 9.24033... \bullet 10^{80} \qquad (53)$$

The formula is given in Fig. 6.

$$D_{80} = (D_{20})^4 = (\sqrt{\alpha D_0})^4 = 9.24033... \bullet 10^{80}$$

FIG.6. Large scale number 10^{80}.

Equivalent formulas:

$$D_{80} = \frac{R_U^2}{r_e^2} = \frac{HM_U^2 \alpha G}{c^3 m_e} = \frac{c^2}{r_e^2 H^2} = \frac{c r_e}{H l_{Pl}^2} = \frac{1}{r_e^2 \Lambda} = (\sqrt{\alpha D_0})^4 = 9.24033... \bullet 10^{80} \quad (54)$$

At the scale 10^{80}, the coincidence of the relations of dimensional quantities with the large number D_{80} = 9.24033... x 10^{80}, allows us to obtain the well-known formula of J.C. Carvalho [22]: $\mathbf{M_u = c^3/GH}$. This follows from the equality: $HM_u^2\alpha G/c^3m_e = R_u^2/r_e^2$. In this equation $m_eR_u^2/\alpha r_e^2 = M_u$. The result is: $HM_uG/c^3 = 1$.

16. Large number of scale 10^{100}.

The following relations lead to a large number of scale 10^{100} (D_{100} = 16.1105... x 10^{100}): $\mathbf{r_e\alpha M_u/l_{Pl}m_e}$, $\mathbf{m_ec^3/l_{Pl}\alpha\hbar H^2}$, $\mathbf{HM_u^2\alpha Gr_e/c^3m_el_{Pl}}$.

The value of the D_{100} number:

$$D_{100} = 16.1105... \bullet 10^{100} \qquad (55)$$

The formula of the D_{100} number is given in Fig.7.

$$D_{100} = (D_{20})^5 = (\sqrt{\alpha D_0})^5 = 16.1105... \bullet 10^{100}$$

Fig. 7. A large number of scale 10^{100}.

Equivalent formulas:

$$D_{100} = \frac{m_e c^3}{l_{Pl}\alpha\hbar H^2} = \frac{r_e \alpha M_U}{l_{Pl} m_e} = \frac{H M_U^2 \alpha G r_e}{c^3 m_e l_{Pl}} = \frac{R_U^2}{r_e l_{Pl}} = (\sqrt{\alpha D_0})^5 = 16.1105... \bullet 10^{100} \qquad (56)$$

From the coincidence of the ratios of the dimensional quantities of scale 10^{100}:

$$\frac{r_e \alpha M_U}{l_{Pl} m_e} = \frac{H M_U^2 \alpha G r_e}{c^3 m_e l_{Pl}} = 16.1105... \bullet 10^{100} \qquad (57)$$

it is easy to obtain the well-known formula of J. C. Carvalho [22]: **$M_u = c^3/GH$**.

17. Large number of scale 10^{120}.

At a scale of 10^{120}, very many coincidences of dimensional magnitude relations are known. The following relations lead to the large number of scale 10^{120} ($D^{120} = 28.088... \times 10^{120}$): - the ratio of the square of the ralius of the observed Universe to the square of the Planck length:

$$Ru^2/l_{Pl}^2 = (r_e \alpha D_0)^2/(r_e^2/\alpha D_0) = \alpha^3 D_0^3 = D_{120} \qquad (58)$$

- The ratio of the volume of the observed Universe to the volume of an electron:

$$Ru^3/r_e^3 = (r_e \alpha D_0)^3/(r_e^3) = \alpha^3 D_0^3 = D_{120} \qquad (59)$$

In addition, the large number $D_{120} = 28.088... \times 10^{120}$ follows from the formulas: **$GM_U^2/\hbar c$, $M_U c^2/\hbar H$, $M_u c^2 r_e \alpha^2/Gm_e^2$, $c^3/r_e^3 H^3$**.

The value of the number D_{120}:

$$D_{120} = 28.088... \bullet 10^{120} \qquad (60)$$

The formula is shown in Fig.8.

$$D_{120} = (D_{20})^6 = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120}$$

Fig.8. Large scale number 10^{120}.

Equivalent formulas:

$$D_{120} = \frac{T_U^2}{t_{Pl}^2} = \frac{R_U^2}{l_{Pl}^2} = \frac{M_U^2}{m_{Pl}^2} = \frac{c^2}{l_{Pl}^2 H^2} = \frac{R_U^3}{r_e^3} = \frac{M_U c^2}{\hbar H} = \frac{GM_U^2}{\hbar c} = \frac{c^5}{G\hbar H^2} = \frac{c^3}{G\hbar\Lambda} = \frac{1}{l_{Pl}^2 \Lambda} = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120} \qquad (61)$$

This is only a part of the equivalent formulas. To obtain new formulas, we will take advantage of the coincidence of the relations of some dimensional quantities with the large scale number $D_{120} = 28.088... \times 10^{120}$:

$$D_{120} = \frac{M_U c^2}{\hbar H} = \frac{GM_U^2}{\hbar c} = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120} \qquad (62)$$

From equation (62), when substituting from one ratio to another of mass, an additional formula follows:

$$c^5/G\hbar H^2 = \alpha^3 D_0^3 = 28{,}088 \times 10^{120} \quad (63)$$

Substituting into the formula $GM_U^2/\hbar c$ or into the formula $M_U c^2/\hbar H$ the mass value from the formula of J.C. Carvalho ($M_u = c^3/GH$) gives the same formula: $c^5/G\hbar H^2 = 28.088 \times 10^{120}$. The formula $(c^5/G\hbar H^2)^{1/2}$ for a scale of 10^{60} is obtained in [31, 32]. This formula, very important in cosmology, confirms the correctness of the choice of the high theoretical value of the Hubble constant ($H = 3.49981...10^{-18}$ s^{-1}).

18. Large scale number 10^{140}.

The following relations of dimensional constants lead to a large number of scale 10^{140} (D_{140} = 48.972... x 10^{140}):

$$D_{140} = \frac{r_e^2 m_e c^3}{l_{Pl}^3 \alpha \hbar H^2} = \frac{r_e^3 \alpha M_U}{l_{Pl}^3 m_e} = \frac{R_U^3}{l_{Pl} r_e^2} = \frac{1}{t_{Pl} t_0^2 H^3} \quad (64)$$

The value of the number D_{140}:

$$D_{140} = 48.972... \bullet 10^{140} \quad (65)$$

The formula is given in Fig. 9.

$$D_{140} = (D_{20})^7 = (\sqrt{\alpha D_0})^7 = 48.972... \bullet 10^{140}$$

Fig. 9. Large scale number 10^{140}.

Equivalent formulas:

$$D_{140} = \frac{r_e^2 m_e c^3}{l_{Pl}^3 \alpha \hbar H^2} = \frac{r_e^3 \alpha M_U}{l_{Pl}^3 m_e} = \frac{R_U^3}{l_{Pl} r_e^2} = \frac{1}{t_{Pl} t_0^2 H^3} = \frac{c}{l_{Pl} r_e^2 H \Lambda} = (\sqrt{\alpha D_0})^7 = 48.972... \bullet 10^{140} \quad (66)$$

19. Large number of scale 10^{160}.

The following relations lead to the large number of scale 10^{160} (D_{160} = 85.383... x 10^{160}): $\mathbf{M_u c^2 R_u \alpha^2/G m_e^2}$, $\mathbf{R_u M_u^2 \alpha G/c^2 r_e^2 m_e}$

The value of the number D_{160}:

$$D_{160} = 85.383... \bullet 10^{160} \quad (67)$$

The formula is given in Fig. 10.

$$D_{160} = (D_{20})^8 = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160}$$

FIG. 10. A large number of scale 10^{160}.

Equivalent formulas:

$$D_{160} = \frac{M_U R_U c^2 \alpha^2}{G m_e^2} = \frac{M_U^2 R_U G \alpha}{c^2 r_e^2 m_e} = \frac{1}{r_e^4 \Lambda^2} = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160} \quad (68)$$

On a scale of 10^{160} from Eq.

$$\frac{M_U^2 R_U G\alpha}{c^2 r_e^2 m_e} = \frac{1}{r_e^4 \Lambda^2} \quad (69)$$

a new equation follows (Fig. 11):

$$\mathbf{M_U R_U G \Lambda^2 = H^2}$$

FIG. 11. An equation that unifies the 5 most important parameters of the observable Universe.

This is a new equation that unifies the 5 most important parameters of the observed Universe. Its peculiarity is that it does not contain any constants other than cosmological parameters. Neither more nor less: "*the equation of the Universe*".

20. Large number of scale 10^{180}.

A large number of scale 10^{180} (D_{180} = 148.86... x 10^{180}) is obtained from the number D_{20} to the power of 9. The same large number (D_{180} = 148.86... x 10^{180}) gives the ratio of the volume of the universe to the Planck volume (R_U^3/l_{Pl}^3). In addition, there are other combinations of physical constants whose ratios coincide with the number D_{180} = 148.86... x 10^{180}.

The value of the number D_{180} is:

$$D_{180} = 148.86... \bullet 10^{180} \quad (70)$$

The formula for the number D_{180} is given in Fig. 12.

$$D_{180} = (D_{20})^9 = (\sqrt{\alpha D_0})^9 = 148.86... \bullet 10^{180}$$

FIG. 12. large scale number 10^{180}.

Equivalent formulas:

$$D_{180} = \frac{r_e^4 m_e c^3}{l_{Pl}^5 \alpha \hbar H^2} = \frac{r_e^5 \alpha M_U}{l_{Pl}^5 m_e} = \frac{R_U^3}{l_{Pl}^3} = \frac{c^3}{l_{Pl}^3 H^3} = \frac{c}{l_{Pl}^3 H \Lambda} = (\sqrt{\alpha D_0})^9 = 148.86... \bullet 10^{180} \quad (71)$$

21. A hierarchical ladder of large numbers generated by the primary large number D_{20} = 1.74349 x 10^{20}.

The table in FIG. 13 summarizes the large numbers that are generated by two independent methods. The first method is based on generating the large numbers by the primary large number D_{20} = 1.74349 x 10^{20} according to a power law. The second method is based on matching the ratios of the dimensional quantities.Both methods give the same values of large numbers.

	Designation, formula, value	Ratios of dimensional constants	Scale
0	$(\sqrt{\alpha D_0})^0 = 1$	$\frac{Gm_e^2}{r_e \alpha^2 \hbar H} = \frac{G\hbar}{r_e^3 H c^2} = \frac{Gm_e}{r_e^2 \alpha H c} = \frac{Gm_e^3 c}{\alpha^3 \hbar^2 H} = \frac{c^2}{M_U R_U G \Lambda} = \frac{c^3}{M_U G H} = \frac{\Lambda c^2}{H^2} = 1$	10^0
1	$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349() \bullet 10^{20}$	$D_{20} = \frac{r_e}{l_{Pl}} = \frac{t_0}{t_{Pl}} = \frac{\alpha m_{Pl}}{m_e} = \sqrt{\alpha D_0}$	10^{20}
2	$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979 \bullet 10^{40}$	$D_{40} = \frac{T_U}{t_0} = \frac{R_U}{r_e} = \frac{m_e c^2}{\alpha \hbar H} = \frac{1}{t_0 H} = \frac{r_e^2}{l_{Pl}^2} = \frac{t_0^2}{t_{Pl}^2} = \frac{\alpha^2 m_{Pl}^2}{m_e^2} = (\sqrt{\alpha D_0})^2$	10^{40}
3	$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987 \bullet 10^{60}$	$D_{60} = \frac{T_U}{t_{Pl}} = \frac{R_U}{l_{Pl}} = \frac{M_U}{m_{Pl}} = \frac{c}{l_{Pl} H} = \frac{r_e^3}{l_{Pl}^3} = \frac{t_0^3}{t_{Pl}^3} = \frac{c^3}{G m_{Pl} H} = (\sqrt{\alpha D_0})^3$	10^{60}
4	$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033... \bullet 10^{80}$	$D_{80} = \frac{R_U^2}{r_e^2} = \frac{H M_U^2 \alpha G}{c^3 m_e} = \frac{c^2}{r_e^2 H^2} = \frac{c r_e}{H l_{Pl}^2} = \frac{1}{r_e^2 \Lambda} = (\sqrt{\alpha D_0})^4$	10^{80}
5	$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105... \bullet 10^{100}$	$D_{100} = \frac{m_e c^3}{l_{Pl} \alpha \hbar H^2} = \frac{r_e \alpha M_U}{l_{Pl} m_e} = \frac{H M_U^2 \alpha G r_e}{c^3 m_e l_{Pl}} = \frac{R_U^2}{r_e l_{Pl}} = \frac{1}{r_e l_{Pl} \Lambda} = (\sqrt{\alpha D_0})^5$	10^{100}
6	$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120}$	$D_{120} = \frac{T_U^2}{t_{Pl}^2} = \frac{R_U^2}{l_{Pl}^2} = \frac{M_U^2}{m_{Pl}^2} = \frac{c^2}{l_{Pl}^2 H^2} = \frac{R_U^3}{r_e^3} = \frac{M_U c^2}{\hbar H} = \frac{G M_U^2}{\hbar c} = \frac{c^5}{G \hbar H^2} = \frac{c^3}{G \hbar \Lambda} = \frac{1}{l_{Pl}^2 \Lambda} = (\sqrt{\alpha D_0})^6$	10^{120}
7	$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972... \bullet 10^{140}$	$D_{140} = \frac{r_e^2 m_e c^3}{l_{Pl}^3 \alpha \hbar H^2} = \frac{r_e^3 \alpha M_U}{l_{Pl}^3 m_e} = \frac{R_U^3}{l_{Pl} r_e^2} = \frac{1}{t_{Pl} t_0^2 H^3} = \frac{c}{l_{Pl} r_e^2 H \Lambda} = (\sqrt{\alpha D_0})^7$	10^{140}
8	$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160}$	$D_{160} = \frac{M_U R_U c^2 \alpha^2}{G m_e^2} = \frac{M_U^2 R_U G \alpha}{c^2 r_e^2 m_e} = \frac{1}{r_e^4 \Lambda^2} = (\sqrt{\alpha D_0})^8$	10^{160}
9	$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86... \bullet 10^{180}$	$D_{180} = \frac{r_e^4 m_e c^3}{l_{Pl}^5 \alpha \hbar H^2} = \frac{r_e^5 \alpha M_U}{l_{Pl}^5 m_e} = \frac{R_U^3}{l_{Pl}^3} = \frac{c^3}{l_{Pl}^3 H^3} = \frac{c}{l_{Pl}^3 H \Lambda} = (\sqrt{\alpha D_0})^9$	10^{180}

Fig. 13: Scale ladder of large numbers and the relation of constants giving large numbers.

22. The law of scaling large numbers.

The law of scaling large numbers is shown in Fig. 14.

$$D_i = (D_{20})^i = (\sqrt{\alpha D_0})^i$$

$$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8, \pm 9$$

Fig. 14. The law of mass scaling of large numbers.

Large numbers of different scales are related to each other by a step dependence. The basis of all large numbers is the primary large number D_{20} =(αD_0)^1/2 = 1.74349...x 10^{20}, from which large numbers are generated according to a uniform law. The values of the generated large numbers coincide with high accuracy with the values of the large numbers obtained from the equation and from the relations of the dimensional parameters of the observed Universe.

The region of negative degrees -9, -8,...,-1 generates inverse values. The same numbers, give inverse ratios of the dimensional constants and parameters of the Metagalaxy. The lower boundary of the family of large numbers is the number . Identification of the upper bound in the massscaling law of large numbers requires a separate study.

The scaling law generates 10 numbers with accuracy close to the accuracy of the Newtonian constant of gravitation G. The scaling law of large numbers shows that the known family of large numbers of scales 10^{20} - 10^{122} is incomplete. The law of scaling of large numbers generates new large numbers of scales 10^{140}, 10^{160}, 10^{180}. These same new large numbers are naturally derived from the fundamental parameters of the observable Universe. The large numbers of scales 10^{140}, 10^{160}, 10^{180} obtained from the law of scaling are confirmed by the relations of physical quantities and parameters of the observable Universe.

23. A new method of calculating large numbers.

The scaling law gives a new method of obtaining large numbers from dimensionless constants and complements the known method based on the relations of dimensional physical constants and parameters of the observable Universe.

The large numbers, which are generated by the law of massscaling of large numbers, are shown in Fig. 15.

$$(\sqrt{\alpha D_0})^0 = 1$$
$$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349... \bullet 10^{20}$$
$$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979... \bullet 10^{40}$$
$$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987... \bullet 10^{60}$$
$$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033... \bullet 10^{80}$$
$$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105... \bullet 10^{100}$$
$$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120}$$
$$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972... \bullet 10^{140}$$
$$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160}$$
$$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86... \bullet 10^{180}$$

Fig. 15: Values of large numbers and formulas for their calculation.

The law of mashup of large numbers generates 10 numbers. There is only one large number on each scale from 10^0 to 10^{180}. The base number from which the large numbers are derived is the number D_{20} = 1.74349... x 10^{20}. The relations of dimensional constants, which previously could only be evaluated by order of magnitude, are now possible to compare with the exact values of the large numbers given in Fig. 14. It is possible to evaluate the coincidences of large numbers not by order of magnitude, but with an accuracy close to the accuracy of the Newtonian constant of gravitation G.

24. Combination of two methods of calculating large numbers.

As we can see, large numbers can be obtained by two methods. The first method is based on the dimensionless large number . The number D_{20} in degree 0 - 9 generates a family of large numbers on scales from 10^0 to 10^{180}. The large numbers obtained from dimensionless quantities by the scaling law have high accuracy, close to the accuracy of the Newtonian constant of gravitation G. The scaling law of large numbers shows that on each scale from 10_0 to 10^{180} there is only one large number.

The second method is based on the relations between dimensional physical quantities and parameters of the observable Universe. This method uses equations in which in one equation there are together very precise fundamental physical constants and very imprecise parameters of the observable Universe.One equation may contain values that differ in accuracy by almost 7 - 8 orders of magnitude. For a long time coincidences of large numbers were estimated approximately by order of magnitude. This led to the erroneous conclusion that there could be multiple large numbers on one particular scale.

Combining the two methods provides the key to solving many problems in physics and cosmology. The appearance of an alternative method of calculating large numbers and the combination of the two methods makes it possible to obtain by calculation those values of the parameters of the observable Universe that are difficult to obtain experimentally.

There is an opportunity to calculate "pull up" the accuracy of the parameters of the observable Universe to the accuracy of Newtonian constant of gravitation G. The possibility of quantitative comparison of the results obtained from Eq.

Fig. 16 shows the coincidence of the ratios of dimensional constants and large numbers obtained from the scaling law.

$\frac{Gm_e^2}{r_e\alpha^2\hbar H}=\frac{G\hbar}{r_e^3Hc^2}=\frac{Gm_e}{r_e^2\alpha Hc}=\frac{Gm_e^3c}{\alpha^3\hbar^2H}=\frac{c^2}{M_UR_UG\Lambda}=\frac{c^3}{M_UGH}=\frac{\Lambda c^2}{H^2}=1$	10^0
$D_{20}=\frac{r_e}{l_{Pl}}=\frac{t_0}{t_{Pl}}=\frac{\alpha m_{Pl}}{m_e}=\sqrt{\alpha D_0}$	10^{20}
$D_{40}=\frac{T_U}{t_0}=\frac{R_U}{r_e}=\frac{m_ec^2}{\alpha\hbar H}=\frac{1}{t_0H}=\frac{r_e^2}{l_{Pl}^2}=\frac{t_0^2}{t_{Pl}^2}=\frac{\alpha^2m_{Pl}^2}{m_e^2}=(\sqrt{\alpha D_0})^2$	10^{40}
$D_{60}=\frac{T_U}{t_{Pl}}=\frac{R_U}{l_{Pl}}=\frac{M_U}{m_{Pl}}=\frac{c}{l_{Pl}H}=\frac{r_e^3}{l_{Pl}^3}=\frac{t_0^3}{t_{Pl}^3}=\frac{c^3}{Gm_{Pl}H}=(\sqrt{\alpha D_0})^3$	10^{60}
$D_{80}=\frac{R_U^2}{r_e^2}=\frac{HM_U^2\alpha G}{c^3m_e}=\frac{c^2}{r_e^2H^2}=\frac{cr_e}{Hl_{Pl}^2}=\frac{1}{r_e^2\Lambda}=(\sqrt{\alpha D_0})^4$	10^{80}
$D_{100}=\frac{m_ec^3}{l_{Pl}\alpha\hbar H^2}=\frac{r_e\alpha M_U}{l_{Pl}m_e}=\frac{HM_U^2\alpha Gr_e}{c^3m_el_{Pl}}=\frac{R_U^2}{r_el_{Pl}}=\frac{1}{r_el_{Pl}\Lambda}=(\sqrt{\alpha D_0})^5$	10^{100}
$D_{120}=\frac{T_U^2}{t_{Pl}^2}=\frac{R_U^2}{l_{Pl}^2}=\frac{M_U^2}{m_{Pl}^2}=\frac{c^2}{l_{Pl}^2H^2}=\frac{R_U^3}{r_e^3}=\frac{M_Uc^2}{\hbar H}=\frac{GM_U^2}{\hbar c}=\frac{c^5}{G\hbar H^2}=\frac{c^3}{G\hbar\Lambda}=\frac{1}{l_{Pl}^2\Lambda}=(\sqrt{\alpha D_0})^6$	10^{120}
$D_{140}=\frac{r_e^2m_ec^3}{l_{Pl}^3\alpha\hbar H^2}=\frac{r_e^3\alpha M_U}{l_{Pl}^3m_e}=\frac{R_U^3}{l_{Pl}r_e^2}=\frac{1}{t_{Pl}t_0^2H^3}=\frac{c}{l_{Pl}r_e^2H\Lambda}=(\sqrt{\alpha D_0})^7$	10^{140}
$D_{160}=\frac{M_UR_Uc^2\alpha^2}{Gm_e^2}=\frac{M_U^2R_UG\alpha}{c^2r_e^2m_e}=\frac{1}{r_e^4\Lambda^2}=(\sqrt{\alpha D_0})^8$	10^{160}
$D_{180}=\frac{r_e^4m_ec^3}{l_{Pl}^5\alpha\hbar H^2}=\frac{r_e^5\alpha M_U}{l_{Pl}^5m_e}=\frac{R_U^3}{l_{Pl}^3}=\frac{c^3}{l_{Pl}^3H^3}=\frac{c}{l_{Pl}^3H\Lambda}=(\sqrt{\alpha D_0})^9$	10^{180}

Fig. 16. Coincidences of relations of dimensional constants and large numbers.

The table in Fig. 16 shows only a small part of the relations of dimensional constants, which will lead to the same large numbers as the scaling law.Realistically, there are many such relations involving the parameters of the observable Universe. This is due to the large number of equivalent formulas for calculating the constants G, H, Λ, M_U, R_U. The relations of dimensional constants demonstrate not approximate but absolute coincidences of the values of large numbers obtained by different methods. Absolute coincidences of the values of large numbers become the main criterion for determining the correct values of the fundamental parameters of the observable Universe. The law of scaling of large numbers becomes a decisive additional tool in determining the values of the fundamental parameters of the observed Universe. This is the main result, since obtaining experimental values and calculations on the scale of the observable Universe are very complicated.

25. Formula for the cosmological constant Λ from the coincidence of large numbers.

The large numbers obtained from the scaling law and their coincidence with the ratios of the dimensional physical constants on different scales allow us to obtain equations for calculating the cosmological constant Λ.

Let us use the ratio of dimensional constants on the scale 10^{40}

$$D_{40} = \frac{r_e^2}{l_{Pl}^2} \qquad (72)$$

and the ratio of dimensional constants on the scale 10^{80}:

$$D_{80} = \frac{1}{r_e^2 \Lambda} \qquad (73).$$

From the formulas (72) and (73), the equation at scale 10^{120} follows:

$$D_{120} = \frac{1}{r_e^2 \Lambda} \bullet \frac{r_e^2}{l_{Pl}^2} = \frac{1}{\Lambda l_{Pl}^2} \qquad (74)$$

Using the well-known formula for the Planck length, equation (75) leads to this equation on a scale of 10^{120}:

$$\mathbf{D_{120} = c^3/G\hbar\Lambda = \alpha^3 D_0^3} \qquad (75)$$

Thus, the exact coincidence of the ratio of the dimensional constants c, G, ħ, Λ with the large number $\alpha^3 D_0^3$ gives a useful formula containing the cosmological constant Λ. The same formula is given in [10] for The ratio between the Planck density and the cosmological density. The coincidence of the ratios of the dimensional constants and the large scale numbers 10^{120} in equation (61) leads to Eq:

$$\mathbf{c^5/G\hbar H^2 = c^3/G\hbar\Lambda = \alpha^3 D_0^3} \qquad (76)$$

From equation (76), using equation (21), we obtain a new formula for calculating the cosmological constant Λ (Fig. 17):

$$\Lambda = 1/r_e^2\alpha^2 D_0^2 = 1.36285\ldots \bullet 10^{-52} m^{-2}$$

Fig. 17. Theoretical value of the cosmological constant Λ and the formula for its calculation.

The same formula for calculating Λ ($\Lambda = 1/r_e^2\alpha^2 D_0^2$) follows from the coincidence of the ratios of dimensional quantities and large numbers on the scale 10^{160} (equation (68)) and 10^{180} (equation (71)). A very close value of Λ ($\Lambda = 1.36281(41) \times 10^{-52}$ m-2.) was obtained by L. Nottale [21, 33]. As we can see, from the theory follows the value of the cosmological constant $\Lambda = 1/R_U^2 = 1/\alpha^2 D_0^2 r_e^2 = 1.36285\ldots \times 10^{-52}$ m-2, which is equal to the inverse square of the present-day radius of the observed Universe [21, 33].

Fig. 18 shows 10 equivalent formulas for calculating the cosmological constant Λ that follow from the coincidence of large numbers.

$$\mathbf{\Lambda} = \left\{ \begin{array}{c} \dfrac{1}{r_e^2\alpha^2 D_0^2}, \quad \dfrac{m_{Pl}^2}{l_{Pl}^2 M_U^2}, \quad \dfrac{\hbar c}{l_{Pl}^2 M_U^2 G}, \\ \dfrac{H^2}{c^2}, \quad \dfrac{1}{R_U^2}, \quad \dfrac{r_e^3}{l_{Pl}^2 R_U^3}, \quad \dfrac{G\hbar H^2}{l_{Pl}^2 c^5}, \\ \dfrac{c^2}{M_U R_U G}, \quad \dfrac{H}{\sqrt{M_U R_U G}}, \quad \dfrac{cH}{M_U G}. \end{array} \right\} = \mathbf{1.36285\ldots x\ 10^{-52}\ m^{-2}}$$

Fig. 18: Equivalent formulas for calculating the cosmological constant Λ.

All formulas give the value $\Lambda = 1.36285\ldots \times 10^{-52}$ m^{-2}. And these are not all possible equivalent formulas for calculating Λ. There are many more equivalent formulas. The reason there are so many equivalent formulas for Λ has no explanation. The constant Λ is included in many relations of dimensional constants that yield large numbers. Therefore, the many formulas for calculating the cosmological constant Λ lead to many relations that give coincidences of large numbers.

The large numbers and their coincidence with the relations of the dimensional physical constants at different scales provide equations for calculating the Hubble constant. Fig. 19 shows 12 equivalent formulas for calculating the Hubble constant.

$$\mathbf{H} = \left\{ \begin{array}{l} \frac{Gm_e^2}{r_e\alpha^2\hbar}, \frac{c^3}{M_U G}, \frac{Gm_e^3 c}{\alpha^3\hbar^2}, \frac{c^3 m_e R_U^2}{M_U^2 G\alpha r_e^2}, \\ \frac{cr_e^3}{l_{Pl}^2 R_U^2}, \frac{Gm_e}{r_e^2\alpha c}, \frac{M_U c^2 r_e^3}{\hbar R_U^3}, \frac{c}{r_e\alpha D_0}, \\ \frac{c}{R_U}, \frac{c^3 l_{Pl}^3}{Gm_{Pl} r_e^3}, \frac{cm_{Pl}}{l_{Pl} M_U}, \frac{c^3 m_e}{M_U^2 G\alpha r_e^2\Lambda} \end{array} \right\} = \mathbf{3.49981 \times 10^{-18}\ s^{-1}}$$

Fig. 19: Equivalent formulas for calculating the Hubble constant H.

All formulas give a value of H = 3.49981 x 10^{-18} s^{-1}. These are by no means all possible equivalent formulas for calculating the Hubble constant. There are many more equivalent formulas. The Hubble constant is included in many relations of dimensional constants that yield large numbers. Therefore, the many equivalent formulas for calculating the Hubble constant leads to many equivalent relations that give matches of large numbers.

The reason there are so many equivalent formulas for the Newtonian constant of gravitation G, for the cosmological constant Λ, and for the Hubble constant has no explanation. Perhaps something fundamental is hidden by these multiple exact matches of equivalent formulas for H, Λ, and G.

26. Unexpected connection of the Hubble constant H with the cosmological constant Λ .

From equation (76) follows a new equation that does not exist in the LCDM model. It is an equation that reveals the relationship between the Hubble constant and the cosmological constant Λ (Fig. 20).

$$H^2 = \Lambda c^2$$

Fig. 20: Relationship between the Hubble constant and the cosmological constant Λ.

From the coincidence of the relations of the dimensional constants on a scale of 10^{120} (Fig. 16), it is easy to derive a useful equation that relates the mass constant of the observable universe to the cosmological constant Λ and the Hubble constant H:

$$\mathbf{M_U = c^2/R_U\Lambda G = cH/\Lambda G} \qquad (77)$$

27. Parameters of the observable Universe represented by the electron constants and the large numbers.

Fig. 21 summarizes the main parameters of the observed Universe represented by the electron constants and large numbers.

Name	Formula	Value
Age of the Observable Universe	$T_U = H^{-1} = \dfrac{r_e \alpha D_0}{c}$	$T_U = 2.85729... \bullet 10^{17}\, s$
Radius of the Observable Universe	$R_U = c / H = r_e \alpha D_0$	$R_U = 0.856594...\bullet 10^{26} m$
Mass of the Observable Universe	$M_U = m_e \alpha D_0^2$	$M_U = 1.15348 ... \bullet 10^{53}\, kg$
Volume of the Observed Universe	$R_U^3 = l_{Pl}^3 \bullet (\sqrt{\alpha D_0})^9$	$R_U^3 = 6.285...\bullet 10^{77} m^3$
Newtonian constant of gravitation G	$G = \dfrac{r_e^3}{t_0^2 \bullet m_e \bullet D_0} = \dfrac{c^2 r_e}{m_e D_0} = \dfrac{c^3 r_e^2}{\hbar \alpha D_0}$	$G = 6{,}6743\ldots x10^{-11}\, kg^{-1}m^3s^{-2}$
Hubble constant	$H = \dfrac{c}{r_e \bullet \alpha D_0}$	$H = 3.49981..\bullet 10^{-18} s^{-1}$
Cosmological constant Λ	$\Lambda = 1 / r_e^2 \alpha^2 D_0^2$	$\Lambda = 1.36285 ...\bullet 10^{-52} m^{-2}$
Cosmological acceleration	$a = Hc = c^2/r_e\alpha D_0$	$a = 10.492 \text{ x } 10^{-10}\, m/s^2$
Planck units	$m_{pl} = m_e \sqrt{\dfrac{D_0}{\alpha}}$, $l_{pl} = \dfrac{r_e}{\sqrt{\alpha D_0}}$, $t_{pl} = \dfrac{t_0}{\sqrt{\alpha D_0}}$.	$m_{pl} = 2.1764 \text{ x } 10^{-8}\, kg$ $l_{pl} = 1.6162 \text{ x } 10^{-35}\, m$ $t_{pl} = 5.3912 \text{ x } 10^{-44}\, s$
Stoney units	$m_S = m_e \sqrt{D_0}$, $l_S = \dfrac{r_e}{\sqrt{D_0}}$, $t_S = \dfrac{t_0}{\sqrt{D_0}}$.	$m_S = 1{,}8592{\cdot}10^{-9}\, kg$ $l_S = 1{,}3807{\cdot}10^{-36}\, m$ $t_S = 4{,}6054{\cdot}10^{-45}\, s$

Fig. 21: Basic parameters of the observed Universe.

28. Way to increase the accuracy of large numbers and parameters of the Observable Universe

The law of scaling of large numbers allows us to obtain parameters of the Observable Universe and large numbers with accuracy close to the accuracy of Newtonian constant of gravitation G. Such high accuracy for scales 10^{20} - 10^{180} opens new possibilities in cosmology and makes cosmology a more precise science. But even such accuracy is not a limit. Since the parameters of the observable Universe and the large numbers come from the electron constants, the possibility opens up to approximate their accuracy to the accuracy of the electron constants. For this purpose it is necessary to obtain a more accurate value of the Weyl number D_0. This number is obtained from the relations of dimensional physical quantities. Its accuracy is limited by the accuracy of the Newtonian constant of gravitation G. At the same time, the Weyl number D_0 has a valuable advantage. It is a dimensionless constant. This means that a large number D_0 comes from some primary dimensionless constants. We need to find a way to calculate D_0 in an alternative way without using dimensionless constants. It is possible that the large number D_0 is fundamentally computable, like the number 3.14159265, or the number 1.618033, or the number 2.7182818284. The calculation of a large number of D_0 without the use of dimensional constants will allow to "pull up" the accuracy of large numbers and parameters of the observable Universe to the accuracy of the fine structure constant alpha. Such an ambitious task is well within the power of mathematicians.

29. Conclusion

The law of scaling large numbers gives a new method for obtaining large numbers from dimensionless constants. It complements the known method based on the relations of dimensional physical quantities. The law of scaling of large numbers shows that the parameters of the Universe and the microcosm are related by scale relations in a wide range of values. It has become possible to obtain values of large numbers and parameters of the Universe with an accuracy close to the accuracy of the Newtonian constant of gravitation G. The limitation that did not allow us to estimate the values of large numbers better than by the order of magnitude has been overcome. A problem is formulated, the solution of which will allow us to calculate the parameters of the observable Universe with an accuracy close to the accuracy of the fine structure constant alpha.

The law of scaling of large numbers shows that large numbers of scales 10^{40} - 10^{122} are only a part of the complete family of large numbers. The large numbers are supplemented by new large numbers of scales 10^{140}, 10^{160}, 10^{180}. On the scales 10^{140}, 10^{160} and 10^{180} new previously unknown large numbers are obtained, which are naturally deduced from the fundamental parameters of the observable Universe. On these scales, new coincidences of the relations of dimensional constants are found. On scales 10^{160} and 10^{180}, an equation is derived from the coincidence of large numbers, which shows that the constants H and Λ are related. The origin of H and Λ from the fundamental physical constants of the electron is proved. On a scale of 10^{160}, a new equation is derived that unifies the 5 most important parameters of the observable Universe: $\mathbf{M_U R_U G \Lambda^2 = H^2}$.

The basis of all large numbers is a large number on the scale of 10_{20} (D_{20} = 1.74349... x 10^{20}), which is given the status of the primary large number. From this number all large numbers are generated. The large numbers are scale factors for recalculation of the electron constants into the parameters of the Universe.

30. Conclusions

1. The law of scaling of large numbers is derived.

2. The law of scaling of large numbers shows that large numbers of scales 10^{39}, 10^{40}, 10^{61}, 10^{122} are only a part of the family of large numbers. The large numbers are supplemented by new large numbers of scales 10^{140}, 10^{160}, 10^{180}, which are naturally derived from the fundamental parameters of the observable Universe.

3. The law of scaling of large numbers makes it possible to calculate analytically the large numbers and parameters of the observable Universe with an accuracy close to the accuracy of the Newtonian constant of gravitation G.

4. At each scale there is only one large number with which the relations of dimensional constants coincide.

5. The Planck scale number $D_{20} =(\alpha D_0)$^(1/2) = 1.74349...x 10^{20} is chosen as the primary large number. From this number the large numbers of other scales are generated according to a uniform law.

6. The parameters of the observable Universe are represented using electron constants and large numbers. The large numbers are scale factors to convert the electron constants into the parameters of the observable Universe. The parameters of the observable universe have a single origin from the electron constants.

7. An equation is derived which shows that the Hubble constant H and the cosmological constant Λ are related. The origin of H and Λ from the fundamental physical constants of the electron is proved.

8. From the coincidence of large numbers a set of equivalent formulas for analytical calculation of the Newtonian constant of gravitation G, the cosmological constant Λ and the Hubble constant is derived. All equivalent formulas give the same value of the constants. The reason for the large number of equivalent formulas for the Newtonian constant of gravitation G, for the cosmological constant Λ and for the Hubble constant has no explanation.

9. A new equation is derived, which unites 5 most important parameters of the observable Universe: $\mathbf{M_U R_U G \Lambda^2 = H^2}$.

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