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Likelihood Measures for Classifying Frequency Response Functions from Posture Control Experiments

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Abstract: The frequency response function (FRF) is an established way to describe the outcome of experiments in posture control literature. The FRF is an empirical transfer function between an input stimulus and the induced body segment sway profile, represented as a vector of complex values associated with a vector of frequencies. Having obtained an FRF from a trial with a subject, it can be useful to quantify the likelihood it belongs to a certain population, e.g., to diagnose a condition or to evaluate the human likeliness of a humanoid robot or a wearable device. In this work, a recently proposed method for FRF statistics based on confidence bands computed with Bootstrap will be summarized, and, on its basis, possible ways to quantify the likelihood of FRFs belonging to a given set will be proposed.

keywords: Frequency response function; Bootstrap; confidence bands.

MSC2020: 49-XX; 34-XX; 92-XX.

1 Introduction

The Frequency Response Function (FRF) is a common representation used in posture control experiments to describe the relationship between an input stimulus and the resulting body movement. The FRF is defined as an empirical transfer function. The FRF is a complex function of frequency, and its structure must be considered when performing statistical analysis to assess differences between groups of FRFs. An example of FRF is shown in Fig.1 with a brief explanation. For an extended description of FRFs in posture control that goes beyond the limits of this paper, see [7, 5]. The set used in the example is from [5]. Typically, statistics are performed by defining a scalar variable to be studied, such as the norm of the difference between FRFs, or by considering the components independently. However, this approach can introduce an arbitrary metric that may have little connection with the experiment. To properly consider the nature of the FRF, a method oriented to complex functions should be used. One method, based on random field theory[6], considers the two components (imaginary and real) as independent variables[3].

The intuition that an FRF, being a transfer function, can be transformed into a real-time domain signal without loss of information suggests an approach. On such real functions, the confidence bands can be defined using methods for continuous functions[1]. As the Fourier

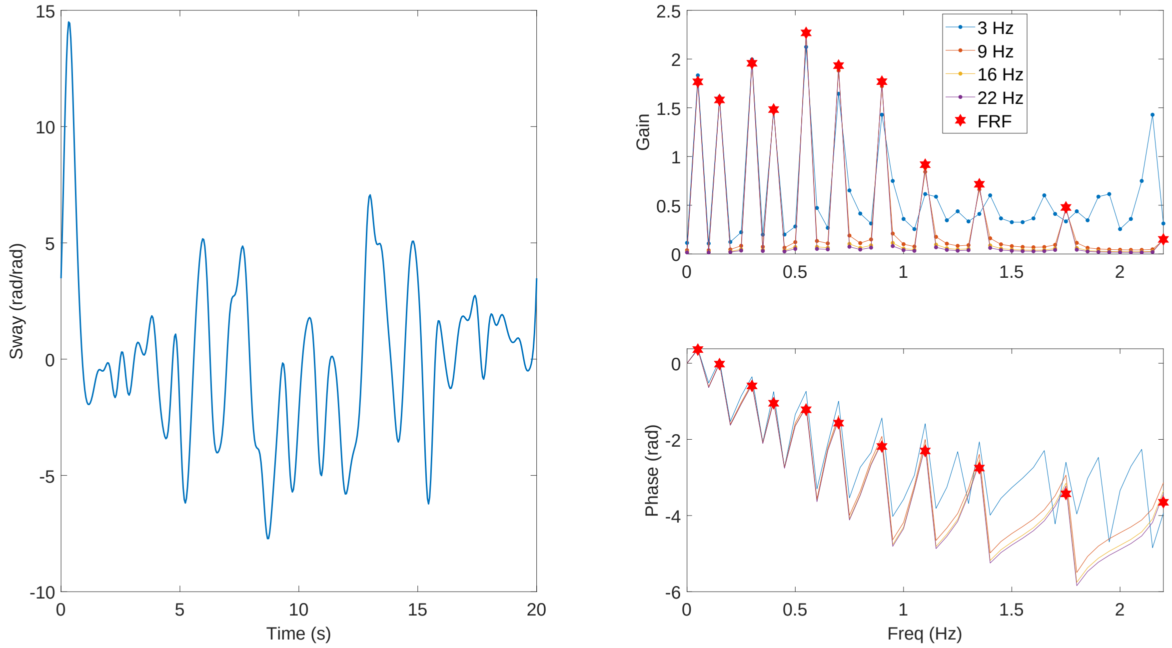

Figure 1: The PIR on the left, the FRF (red stars), and the DFT of the PIR (colored lines). The PIR is computed with eq. $x_i(t) = \sum_{k=1}^{M} \Re(H_{i,k})\cos(2\pi\varphi_k t) + \Im(H_{i,k})\sin(2\pi\varphi_k t)$. The peculiarity of the FRF as proposed in posture control analysis is that the frequencies $\varphi = [0.05, 0.15, 0.3, 0.4, 0.55, 0.7, 0.9, 1.1, 1.35, 1.75, 2.2]Hz$ are not equally spaced. The associated H is obtained as the average of the empirical transfer function over a range of frequencies. This was defined in for better plotting the signal[7]. Different sample times are tested to reconstruct the FRF through a DFT. The period of the PIR is defined as the inverse of the greatest common divisor of the frequencies in φ; the sample time used in the examples is set to ten times the highest frequency in φ, i.e., 22 Hz. Notice how the gain tends to converge to zero between the peaks.

transform of a transfer function represents the impulsive response of a system, such function is referred to as *pseudo-impulse-response*, PIR. The method to use Bootstrap to define confidence bands on PIRs to perform statistics on FRFs is described in [4], and the code is available at [2]. The average PIR

$$\bar{x}(t) = 1/N \sum_{N}^{i=1} x_i(t) \tag{1}$$

and the STD is

$$\hat{\sigma}_x(t) = \sqrt{(1/(N-1)) \sum_{N}^{i=1} |x_i(t) - \bar{x}(t)|^2} \tag{2}$$

. With these values, the prediction band can be defined for a new draw from the FRF distribution H_{n+1}, and hence for the respective PIR $x_{N+1}(t)$. With the desired confidence level $\alpha\%$, the constant C_p is defined to obtain the probability

$$P[\max_t |x_{N+1}(t) - \bar{x}(t)|/\hat{\sigma}_x(t) \leq C_p] = \alpha/100 \tag{3}$$

and the prediction band for a new FRF is

$$\bar{x}(t) \pm C_p \cdot \hat{\sigma}_x(t). \tag{4}$$

This work proposes a further application of such bands to quantify the degree to which a sample FRF belongs to a distribution.

Can likeness be defined as likeliness? The need to classify FRFs in groups arises when the posturography is used to diagnose a condition or to assess the human likeness of a behavior produced by a robot humanoid or wearable as in [5]. A modified version of the function computing the prediction band from [4] can compute the minimal band, including a given sample. See fig. 2. This works by reversing the process used to compute the prediction band, i.e., finding the confidence α given the distance from the mean: the maximum distance between the test sample and the estimated mean in the histogram produced by the Bootstrap as shown in 2. Further measures can be defined as the empirical estimation of the probability density function (pdf) and cumulative density function (cdf). Here, they are defined on the distance $D = \int (x_i(t) - \hat{x}(t))^2 dt$, but the principle can be generalized. The cdf $F(x) = P[X \leq x]$is computed empirically with a bootstrap (a mean and a STD are provided for the estimate), the pdf is computed by approximating $f(x) = dF/dx \approx \Delta F(x)/(x_2 - x_1)$ where $\Delta F(x)$ is a fixed quantity (here 1/10 of the number of samples N) and x_1 and x_2 are the values of x found in the vector ordered distances D produced by the Bootstrap moving back and forward of $N \cdot \Delta F(x)$ positions.

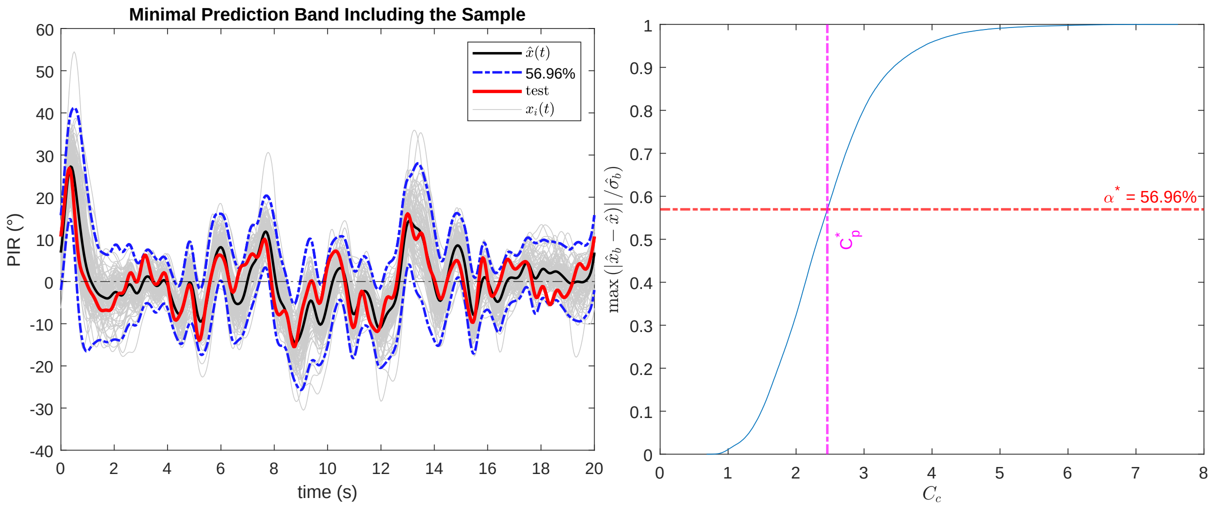

Figure 2: Minimal prediction band (left) and the cumulative histogram used to compute α (right). The histogram is produced by the Bootstrap approximating the probability as $\frac{1}{B}\sum_{b=1}^{B}\left[I\left(\max_t\left(\frac{|\hat{x}(t)-\hat{x}^b(t)|}{\hat{\sigma}_x^b(t)}\right)\right) \leq C_c\right]$. In this case, opposite to what is done in [4], the C_p is known (inferred using the tested sample), and the α is the quantity to be computed.

2 Results and Discussion and Conclusions

To show how the measures work, a sample has been removed from the set, and its minimum prediction band has been computed based on the rest of the samples, as shown in Fig. 2. The estimated probability distribution features were: cdf ($F = 0.6075$, $\sigma_F = 0.0523$) and pdf ($f = 0.0042$, $\sigma_f = 0.0016$). Future work will consist of the application of these measures to quantify the difference between groups of patients (e.g., polyneuropathy) and a control group of healthy subjects and in humanoid robotics assessment to evaluate the human likeness of the produced motion pattern as an alternative to the pseudo-statistics based on the covariance matrix that was proposed in [5] with all the advantages discussed in the introduction. The computation of cdf and pdf can be used to evaluate the likelihood of the sample. Strictly speaking, they are based on an arbitrary scalar value (the integral of the distance). This may be useful as a further test to be performed on a sample after comparing it with a prediction band with a fixed α to have a continuous score beside the binary result of such a comparison with the band. Both measures can compare the output of different identified posture control models and decide the best one representing the sample population.

References

[1] Mark W Lenhoff, Thomas J Santner, James C Otis, Margaret GE Peterson, Brian J Williams, and Sherry I Backus. Bootstrap prediction and confidence bands: a superior statistical method for analysis of gait data. *Gait & Posture*, 9(1):10–17, 1999.

[2] Vittorio Lippi. FRF statistics library. https://github.com/mcuf-idim/FRF-statistics.

[3] Vittorio Lippi. Random Field Theory for Testing Differences Between Frequency Response Functions in Posturography. In *9th International Posture Symposium*, pages 130–134. Frontiers, 2023.

[4] Vittorio Lippi. Bootstrap prediction and confidence bands for frequency response functions in posturography. forthcoming.

[5] Vittorio Lippi, Christoph Maurer, and Thomas Mergner. Human-likeness indicator for robot posture control and balance. In *International Conference on Robotics, Computer Vision and Intelligent Systems*, pages 98–113. Springer, 2020.

[6] Todd C Pataky, Mark A Robinson, Jos Vanrenterghem, Russell Savage, Karl T Bates, and Robin H Crompton. Vector field statistics for objective center-of-pressure trajectory analysis during gait, with evidence of scalar sensitivity to small coordinate system rotations. *Gait & posture*, 40(1):255–258, 2014.

[7] R. Peterka. Sensorimotor integration in human postural control. *J. Neurophysiol.*, 88(3):1097–1118, 2002.