# Calculation of Uranus' Size and Neptune's Size by Quantum Gravity Theory

Huaiyang Cui
Department of Physics, Beihang University, Beijing, 102206, China
Email: hycui@buaa.edu.cn
(July 19, 2024, submitted to viXra)

**Abstract:** Matter wave has been generalized on a planetary scale as a quantum gravity theory, and applied to the solar system. This paper shows that Uranus' size and Neptune's size are the consequence of interference of the generalized matter waves. In this calculation, Uranus' radius is determined as 2.455042e+7 (m) with a relative error of 3.2%; Neptune's radius is determined as 2.315699e+7 (m) with a relative error of 5.95%. This calculation also correctly predicts the locations of rings for Uranus and Neptune.

## 1. Introduction

The question of whether gravity has a description in terms of quantum fields is a longstanding one [1,2,3]. In recent years, matter wave has been generalized on a planetary scale as a quantum gravity theory, and applied to the solar system. The present paper continues to discuss the generalized relativistic matter wave and its extensions to the planetary science.

In analogy with the ultimate speed $c$, there is an ultimate acceleration $\beta$, nobody's acceleration can exceed this limit $\beta$ in a many-body system. In recent years, de Broglie matter wave [4,5,6] has been generalized in terms of the ultimate acceleration. Consider a particle, its generalized relativistic matter wave is given by the path integral

$$\psi = \exp[\frac{i\beta}{c^3}\int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4)] \quad . \tag{1}$$

where $u$ is the 4-velocity of the particle, $\beta$ is the ultimate acceleration determined by experiments. The constant $\beta$ has replaced the *Planck constant* in this quantum gravity theory so that *its wavelength becomes a length on planetary-scale*. The early paper [7,8] shows that celestial size is the consequence of interference of the generalized matter waves; Jupiter's radius is determined as 7.1491e+7m with a relative error of 5.05% [9]; Saturn's radius is determined as 5.80057e+7m with a relative error of 3.75% [9], as shown in Fig.1.

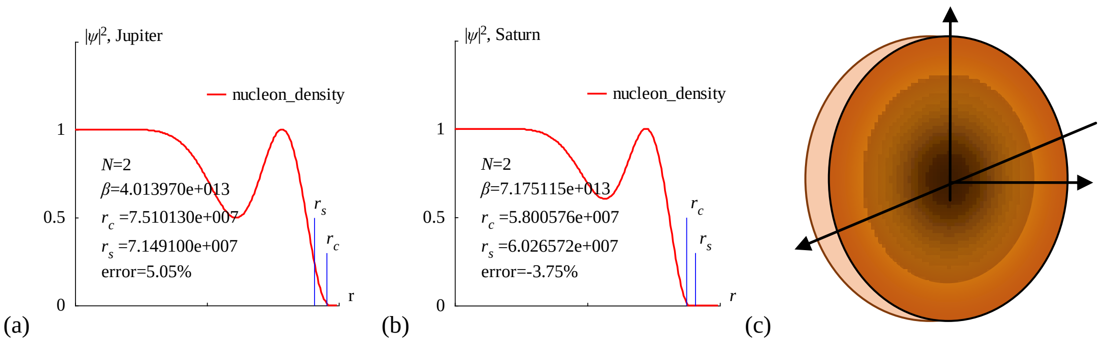



Fig.1 The nucleon distribution $|\psi|^2$ in radial direction is calculated. (a) Jupiter, (b) Saturn, (c)interior.

The present paper shows that this quantum gravity theory with the ultimate acceleration provides a mechanism to calculate the Uranus' size and Neptune's size correctly.

## 2. Extracting ultimate acceleration from a system

Similar to the Bohr model of hydrogen atom, the orbital circumference is $n$ multiple of the wavelength of the planetary-scale relativistic matter wave. According to Eq. (1), consider a satellite, we have

$$\left.\begin{aligned} \frac{\beta}{c^3}\oint_L v_l dl = 2\pi n \\ v_l = \sqrt{\frac{GM}{r}} \end{aligned}\right\} \Rightarrow \quad \sqrt{r} = \frac{c^3}{\beta\sqrt{GM}} n; \quad n = 0,1,2,\ldots \quad . \tag{2}$$

This orbital quantization rule only achieves a half success in the Uranus system and Neptune system, as shown in Fig.1. The Uranus and 12 inner satellites (Cordelia,Ophelia, etc.) satisfy the quantization equation in Fig.1(a); while other outer satellites (Mab, Miranda, etc.) fail. The Neptune and 3 inner satellites (Naiad, Thalassa, Despina) satisfy the quantization equation in Fig.1(b); while other outer satellites (Galatea, Larissa, Hippocamp, Proteus) fail. But, since we only study quantum gravity effects near the planets, so this orbital quantization rule is good enough as a foundational quantum theory. In Fig.1, the blue straight lines express a linear regression relation among the quantized orbits, so it gives Uranus' $\beta$=1.980513e+15 (m/s$^2$) and Neptune's $\beta$=2.077847e+15 (m/s$^2$) by fitting the lines. The quantum numbers $n$=40,41,42,... are assigned to the Uranus's satellites, the Uranus is assigned a quantum number $n$=0 because it is in the **central state**. The quantum numbers $n$=44,45,46,... are assigned to the Neptune's satellites, the Neptune is assigned a quantum number $n$=0 because it is in the **central state**.

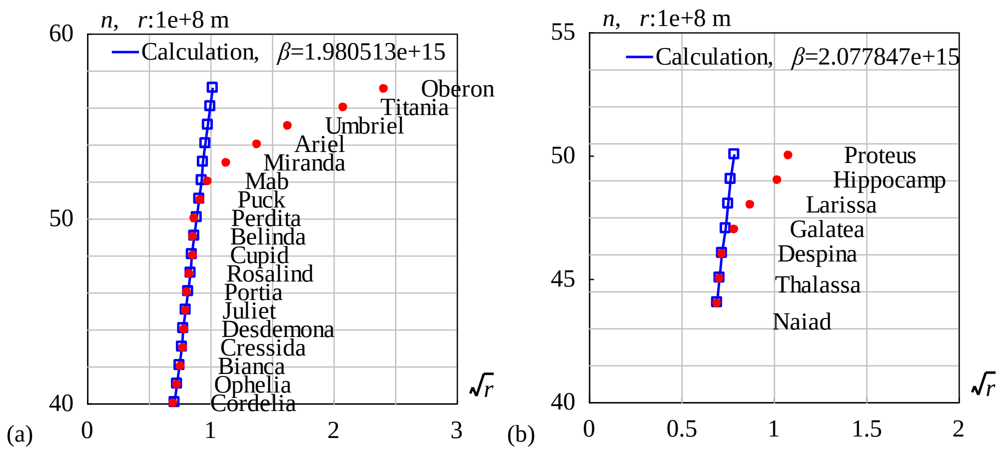



Fig.1 The inner satellites are quantized.

```
<Clet2020 Script>// C source code [10]
int i,j,k,N,N1,nP[10],Figure; double x,y,z,r,r1,M,r_unit,H,beta,pD[10],D[200],S[200];
double orbit[20]={0,0.49770,0.53790,0.59170,0.61780,0.62680,0.64350,0.66090,0.69940,0.74800,
0.75260,0.76400,0.86010,0.97700,1.29390,1.91020,2.66300,4.35910,5.83520,},a,b,A,B,r_massive,R;
double e[20]={0,0.00026,0.00992,0.00092,0.00036,0.00013,0.00066,0.00005,
0.00011,0.0013,0.00007,0.0012,0.00012,0.0025,0.0013,0.0012,0.0039,0.0011,0.0014,};
int qn[22]={0,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60}; char str[200];
char
Stars[200]={"Uranus;Cordelia;Ophelia;Bianca;Cressida;Desdemona;Juliet;Portia;Rosalind;Cupid;Belinda;Perdita;Puck;Mab;Mir
```

```
anda;Ariel;Umbriel;Titania;Oberon;"};
main(){ Figure=1; N=19;N1=13;M=14.5*5.97237E24;r_unit=1E8;
for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0; S[1]=0;
SetAxis(X_AXIS,0,0,3,"#if#rsr#t;0;1;2;3;");
SetAxis(Y_AXIS,40,40,60,"#ifn, r#t:1e+8 m, Uranus;40;50;60;");
DrawFrame(0x016a,1,0xafffaf);    k=1; N1=10000; A=-0.855; r_massive=0.89; R=sqrt(r_massive); B=-0.022;
for(i=0;i<N1;i+=1) { r=10*i/N1; r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=k) {S[k+k]=r1; S[k+k+1]=y; k+=1; } if(y>57) break;}
Format(str,"Calculation,      #ifβ#t=%e",beta);    k-=40; N1=80; SetPen(2,0x0000ff);
Polyline(k,S[N1],0.2,59,str);Plot("CARD,0,@k,XY,4,4,",S[N1]);
SetPen(2,0xff0000); Plot("OVALFILL,0,18,XY,3,3,",D[2]); nP[0]=TAKE;
for(i=1;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A

<Clet2020 Script>// C source code [10]
int i,j,k,N,N1,nP[10],Figure; double x,y,z,r,r1,M,r_unit,H,beta,pD[10],D[200],S[200],F[200];
double orbit[10]={0,0.48224,0.50074,0.52526,0.61953,0.773548,1.05283,1.17646 },a,b,A,B,r_massive,R;
double e[10]={0,0.0047,0.0018,0.0004,0.0001,0.0012,0.0005,0.0005,0,0, };
int qn[20]={0,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,}; char str[200];
char Stars[200]={" Neptune;#Y15Naiad;#Y5Thalassa;Despina;Galatea;Larissa;Hippocamp;Proteus;"};;
main(){ Figure=1; N=8; N1=3;M=17 *5.97237E24;r_unit=1E8;
for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0; S[1]=0;
SetAxis(X_AXIS,0,0,2,"#if#rsr#t;0;0.5;1;1.5;2;");
SetAxis(Y_AXIS,40,40,55,"#ifn, r#t:1e+8 m, Neptune;40;45;50;55;");
DrawFrame(0x014a,1,0xafffaf);    k=1; N1=10000; A=-0.81; r_massive=0.54; R=sqrt(r_massive); B=-0.01;
for(i=0;i<N1;i+=1) { r=4*i/N1; r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=k) {S[k+k]=r1; S[k+k+1]=y; k+=1; } if(y>50) break;}
Format(str,"Calculation,      #ifβ#t=%e",beta);    k-=44; N1=88; SetPen(2,0x0000ff);
Polyline(k,S[N1],0.2,54,str);Plot("CARD,0,@k,XY,4,4,",S[N1]);
SetPen(2,0xff0000); Plot("OVALFILL,0,7,XY,3,3,",D[2]); nP[0]=TAKE;
for(i=1;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A
```

The generalized relativistic matter wave function $\psi$ needs a further explanation. In quantum mechanics, $|\psi|^2$ equals to the probability of finding an electron due to Max Born's explanation; in astrophysics, $|\psi|^2$ equals to the probability of finding a nucleon (proton or neutron) *averagely on an astronomic scale*, we have

$$|\psi|^2 \propto \text{nucleon-density} \propto \rho \quad . \tag{3}$$

## 3. Uranus' size and Neptune's size

In the interior of Uranus or Neptune, if the coherent length of the generalized relativistic matter wave is long enough, its head may overlap with its tail when the particle moves in a closed orbit, as shown in Fig.2. Consider a point on the equatorial plane, the overlapped wave is given by

$$\begin{aligned} &\psi = \psi(r)T(t) \\ &\psi(r) = 1 + a_1 e^{i\delta} + a_2 e^{i2\delta} + \ldots + a_{N-1} e^{i(N-1)\delta} \\ &\delta(r) = \frac{\beta}{c^3}\oint_L (v_l)dl = \frac{2\pi\beta\omega r^2}{c^3} \end{aligned} \quad . \tag{4}$$

where $N$ is the overlapping number which is determined by the coherent length of the relativistic matter wave, $\delta$ is the phase difference after one orbital motion, $\omega$ is the angular speed of the planet rotation, $a_1, a_2, .., a_{N-1}$ are the amplitudes of the wavelets. The above equation is a multi-slit interference formula in optics, for a larger $N$ it becomes the Fabry-Perot interference.

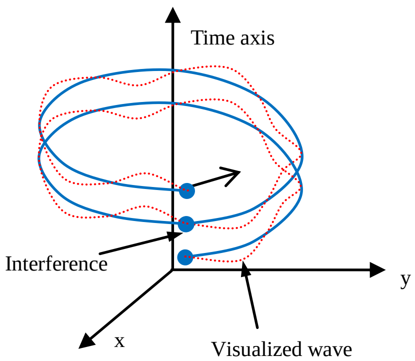



Fig.2 The head of the relativistic matter wave may overlap with its tail.

According to the study of tropic cyclones on the Pacific ocean [12,13] where gaseous clouds have $N$=2, the Uranus and Neptune also have $N$=2, because they are gaseous planets.

$$\psi(r) = 1 + a_1 e^{i\delta} = (1 + a_1 \cos\delta) + i a_1 \sin\delta \quad . \tag{5}$$

Let $r_s$ denotes the radius of planet, due to the skin effect into planetary interior, according to the two-quadric model for the height profile of Earth's troposphere refractivity [14,15], the amplitude of the first wavelet is simply assumed as

$$a_1 = \begin{cases} (\frac{r}{r_s})^4 & r \le r_s \\ 1 & r > r_s \end{cases} \quad . \tag{6}$$

Then the Uranus' size and Neptune's size can be estimated.

Uranus' angular speed at its equator is known as $\omega$=2π/(17.2399x3600) ($s^{-1}$). Its mass 8.681e+25(kg), the well-known radius 2.5362e+7(m), the mean density 1271 ($kg/m^3$), the constant $\beta$=1.980513e+15 ($m/s^2$). According to $N$=2, the matter distribution of the $|\psi|^2$ is calculated in Fig.3(a), it agrees well with the general description of Uranus interior. The radius of Uranus is determined as $r_c$ =2.455042e+7 (m) with a relative error of 3.2% in Fig.3(a), which indicates that the Uranus' radius strongly depends on its rotation.

Neptune's angular speed at its equator is known as $\omega$=2π/(16.11x3600) ($s^{-1}$). Its mass 1.0241e+26(kg), the well-known radius 2.4622e+7(m), the mean density 1638 ($kg/m^3$), the constant $\beta$=2.077847e+15 ($m/s^2$). According to $N$=2, the matter distribution of the $|\psi|^2$ is calculated in Fig.3(b), it agrees well with the general description of Neptune interior. The radius of Neptune is determined as $r_c$=2.315699e+7 (m) with a relative error of 5.95% in Fig.3(b), which indicates that the Neptune's radius strongly depends on its rotation.

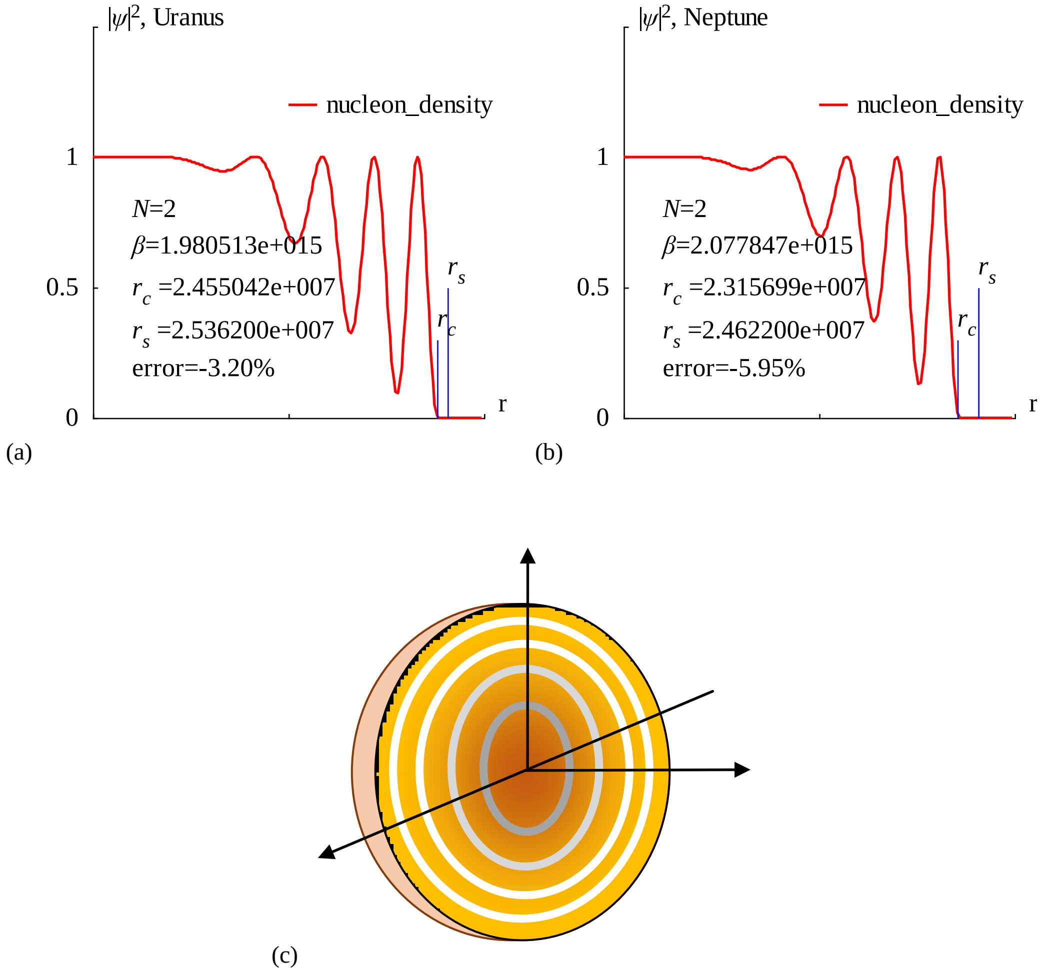



Fig.3 The nucleon distribution $|\psi|^2$ in the radial direction is calculated. (a) Uranus, (b) Neptune, (c)interior.

```
<Clet2020 Script>// C source code [10]
int i,j,k,n,N; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=200;rs=2.5362e7; r_atm =rs+rs/10; rc=r_atm; n=0; N=2;
beta=1.980513e+15;H=SPEEDC*SPEEDC*SPEEDC/beta; M=8.681e+25;
r_unit=r_atm/k; omega=2*PI/(17.2399*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs; amp=pow(x,4); d=(1+amp)*(1+amp); //skin effect
delta=2*PI*omega*r*r/H; x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d; if(r>rc) z=0;
S[n]=i;S[n+1]=z; n+=2; if(rc>=r_atm && z<0.02) rc=r; }
SetAxis(X_AXIS,0,0,k,"r; ; ; ;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Uranus;0;0.5;1; ;");
DrawFrame(FRAME_SCALE,1,0xafffaf); x=50;z=100*(rc-rs)/rs;
SetPen(2,0xff0000);Polyline(k,S,k/2,1.2," nucleon_density");SetPen(1,0xff);
x=rs/r_unit;y=0.5;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0,"#ifr#sds#t");
x=rc/r_unit;y=0.3;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0,"#ifr#sdc#t");
Format(str,"#ifN#t=%d#n#ifβ#t=%e#n#ifr#sdc#t =%e#n#ifr#sds#t =%e#nerror=%.2f%",N,beta,rc,rs,z);
TextHang(k/10,0.8,0,str);
}#v07=?>A
```

```
<Clet2020 Script>// C source code [10]
int i,j,k,n,N; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=200;rs=2.4622e7;rc=-1; r_atm =rs+rs/10; rc=r_atm;n=0; N=2;
beta=2.077847e+15;H=SPEEDC*SPEEDC*SPEEDC/beta; M=1.0241e+26;
r_unit=r_atm/k; omega=2*PI/(16.11*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs; amp=pow(x,4); d=(1+amp)*(1+amp); //skin effect
delta=2*PI*omega*r*r/H; x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d; if(r>rc) z=0;
S[n]=i;S[n+1]=z; n+=2; if(rc>=r_atm && z<0.02) rc=r; }
SetAxis(X_AXIS,0,0,k,"r; ; ; ;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Neptune;0;0.5;1; ;");
DrawFrame(FRAME_SCALE,1,0xafffaf); x=50;z=100*(rc-rs)/rs;
SetPen(2,0xff0000);Polyline(k,S,k/2,1.2," nucleon_density");SetPen(1,0xff);
x=rs/r_unit;y=0.5;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0,"#ifr#sds#t");
x=rc/r_unit;y=0.3;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0,"#ifr#sdc#t");
Format(str,"#ifN#t=%d#n#ifβ#t=%e#n#ifr#sdc#t =%e#n#ifr#sds#t =%e#nerror=%.2f%",N,beta,rc,rs,z);
TextHang(k/10,0.8,0,str);
}#v07=?>A
```

## 4. Locations of satellites and rings

The calculation of near field situations has also been carried out by the same formula Eq.(5) as shown in Fig.4 and Fig.5. For Uranus, the maxima of $|\psi(r)|^2$ point out the radii of Cordelia, Ophelia, etc., respectively; also approximately gives out the locations of Halo ring and Main ring; as shown in Fig.4, the overall relative error is about 5%.

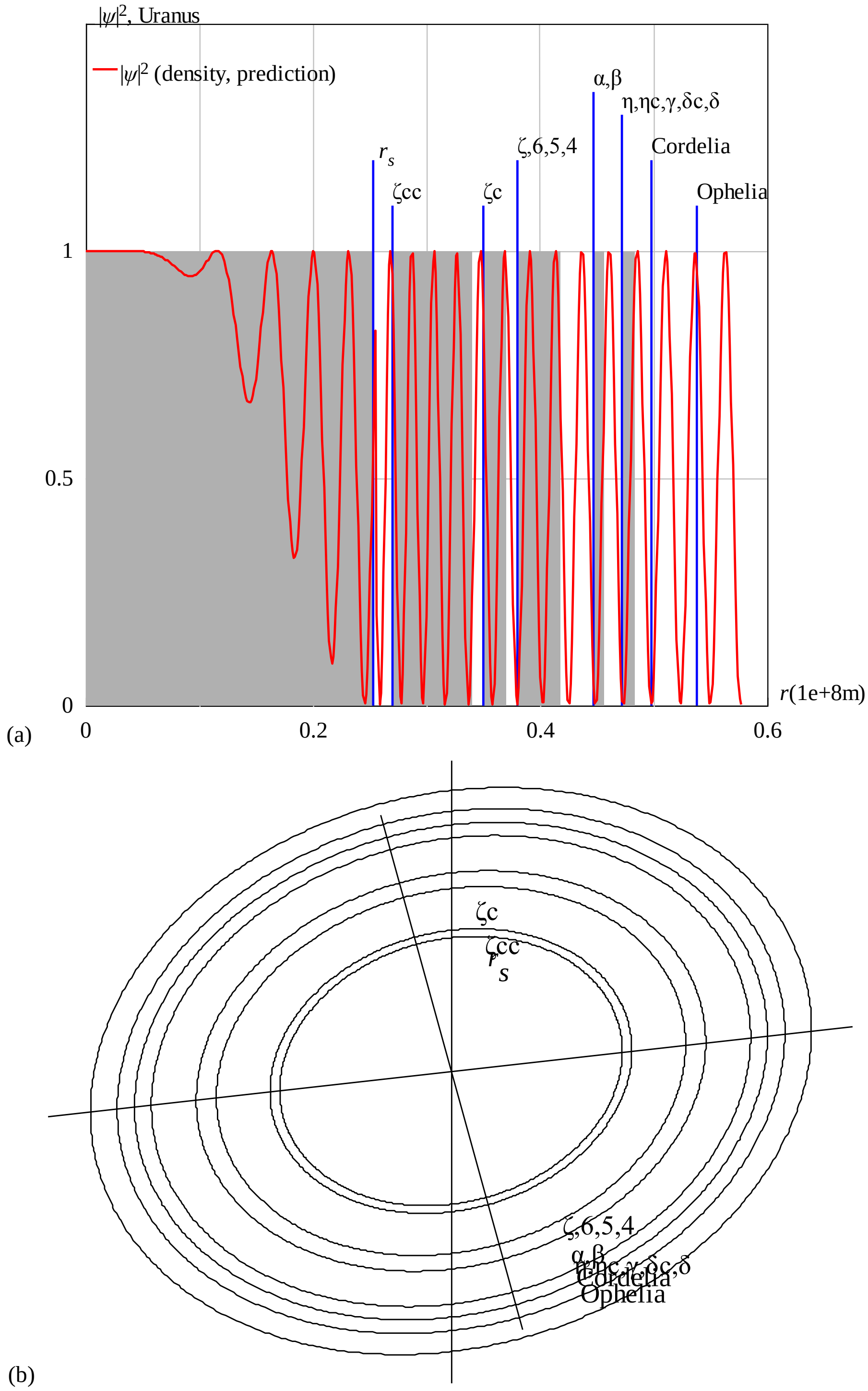


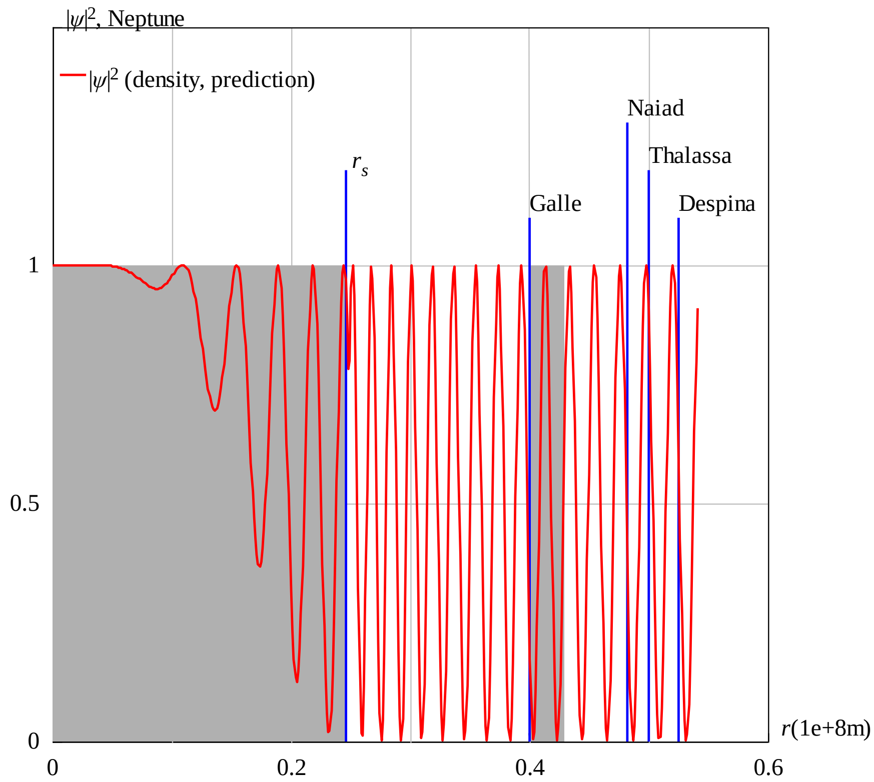

Fig.4 (a) The nucleon distribution $|\psi|^2$ is calculated in the radial direction for Uranus. (b) Orbits.

```
<Clet2020 Script>// C source code [10]
int i,j,k,n,N,nP[10]; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=500;rs=2.5362e7; r_atm =rs+rs/100; rc=r_atm; n=0; N=2;
beta=1.980513e+15;H=SPEEDC*SPEEDC*SPEEDC/beta; M=8.681e+25;
```

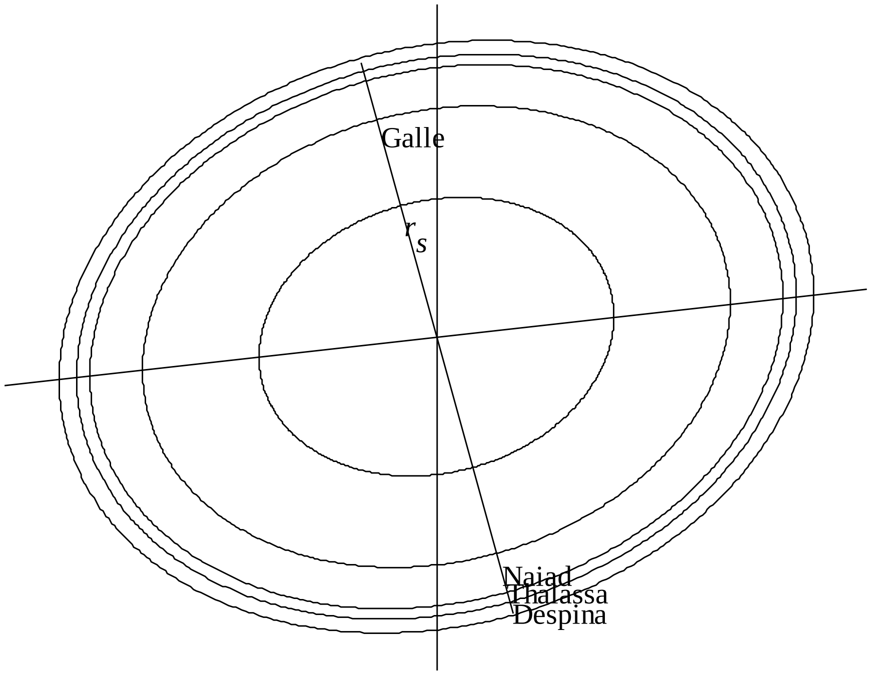
```
r_unit=0.6e+8/k; omega=2*PI/(17.2399*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs;
if(r<r_atm) {delta=2*PI*omega*r*r/H;amp=pow(x,4);}
else {delta=2*PI*sqrt(GRAVITYC*M*r)/H;amp=1;}
d=(1+amp)*(1+amp); x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d;
S[n]=i;S[n+1]=z; n+=2; if(i>480) break;}
SetAxis(X_AXIS,0,0,k,"#ifr#t(1e+8m);0;0.2;0.4;0.6;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Uranus;0;0.5;1; ;");
DrawFrame(0x0163,2,0xafffaf); SetPen(2,0xff);
x=rs/r_unit;y=1.2;D[0]=0;D[1]=0;D[2]=x;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D);D[0]=D[2]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0," #ifr#sds#t ");
y=1.45;nP[0]=TAKE;
x=0.27e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=0.34e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"ζcc");
x=0.35e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=0.37e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"ζc");
x=0.38e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=0.418e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"ζ,6,5,4");
x=0.447e8/r_unit;y=1.35;D[0]=x;D[1]=0;D[2]=0.456e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"α,β");
x=0.472e8/r_unit;y=1.3;D[0]=x;D[1]=0;D[2]=0.483e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"η,ηc,γ,δc,δ");
x=0.498e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Cordelia");
x=0.538e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Ophelia");
SetPen(2,0xff0000);Polyline(n/2,S,5,1.4,"#if|ψ|#su2#t (density, prediction)"); SetPen(1,0xff);
}#v07=?>A
```

```
<Clet2020 Script>// C source code [10]
char Stars[100]={"#ifr#sds#t;ζcc;ζc;ζ,6,5,4;α,β;η,ηc,γ,δc,δ;Cordelia;Ophelia; ;"};
double r[10]={0.2556,0.27,0.35,0.38,0.447,0.472,0.498,0.538,0,};
double a,b,x,y,dP[10],D[100],S[2000];int i,j,k,N,nP[10];char str[200];
main(){SetViewAngle(0,50,10);a=0.6;N=8;
SetAxis(X_AXIS,-a,0,a,"x;");SetAxis(Y_AXIS,-a,0,a,"y;");SetAxis(Z_AXIS,-a,0,a,"z;");
DrawFrame(FRAME_LINE,2,0x00ffff); //SetPen(1,0x808000);
for(i=0;i<N;i+=1) {b=r[i];
D[0]=0;D[1]=0;D[2]=0;D[3]=0;D[4]=1;D[5]=0;D[6]=0;D[7]=0;D[8]=1;
D[9]=600;D[10]=b;D[11]=b;
Lattice(OVAL,D,S); Plot("POLYGON,0,600,XYZ,10",S[9]);
nP[0]=TAKE;nP[1]=i;TextJob(nP,Stars,str);y=r[i];x=0.1; if(i>2) y=-y; TextHang(x,y,0,str);}
a=317.816*5.97237e24; y=30;
//for(i=0;i<N;i+=1) {x=r[i]*1e+8;b=sqrt(GRAVITYC*a/x);b=2*PI*x/b; b/=3600;
//TextAt(100,y,"i=%d      r=%e      T=%f",i,x,b); y+=30;}
}#v07=?>A
```

For Neptune, the maxima of $|\psi(r)|^2$ point out the radii of Mimas, Enceladus, Tethys and Dione, respectively; also approximately gives out the locations of ring A,B,C and D; as shown in Fig.5, the overall relative error is about 5%.




(a)




(b)

Fig.2 (a) The nucleon distribution $|\psi|^2$ is calculated in the radius direction. (b) Orbits.

```
<Clet2020 Script>//C source code [10]
int i,j,k,n,N,nP[10]; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=500;rs=2.4622e7;rc=-1; r_atm =rs+rs/100; rc=r_atm;n=0; N=2;
beta=2.077847e+15;H=SPEEDC*SPEEDC*SPEEDC/beta; M=1.0241e+26;
r_unit=0.6e8/k; omega=2*PI/(16.11*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs;
if(r<r_atm) {delta=2*PI*omega*r*r/H;amp=pow(x,4);}
else {delta=2*PI*sqrt(GRAVITYC*M*r)/H;amp=1;}
d=(1+amp)*(1+amp); x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d;
S[n]=i;S[n+1]=z; n+=2; if(i>450) break;}
SetAxis(X_AXIS,0,0,k,"#ifr#t(1e+8m);0;0.2;0.4;0.6;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Neptune;0;0.5;1; ;");
DrawFrame(0x0163,2,0xafffaf); SetPen(2,0xff);
x=rs/r_unit;y=1.2;D[0]=0;D[1]=0;D[2]=x;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D);D[0]=D[2]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0," #ifr#sds#t ");
//y=1.45;nP[0]=TAKE;
x=0.4e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=0.429e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Galle");
//x=0.530e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=0.534e8/r_unit;D[3]=1;
//Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Le Verrier");
//x=0.532e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=0.572e8/r_unit;D[3]=1;
//Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Lassell");
x=0.482e8/r_unit;y=1.3;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Naiad");
x=0.5e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Thalassa");
x=0.525e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Despina");
SetPen(2,0xff0000);Polyline(n/2,S,5,1.4,"#if|ψ|#su2#t (density, prediction)"); SetPen(1,0xff);
}#v07=?>A
```

```
<Clet2020 Script>// C source code [10]
char Stars[100]={"#ifr#sds#t;Galle;Naiad;Thalassa;Despina; ;"};
double r[10]={0.247,0.409,0.48224,0.50074,0.52526,0,};
double a,b,x,y,dP[10],D[100],S[2000];int i,j,k,N,nP[10];char str[200];
main(){SetViewAngle(0,50,10);a=0.6;N=5;
SetAxis(X_AXIS,-a,0,a,"x;");SetAxis(Y_AXIS,-a,0,a,"y;");SetAxis(Z_AXIS,-a,0,a,"z;");
DrawFrame(FRAME_LINE,2,0x00ffff); //SetPen(1,0x808000);
for(i=0;i<N;i+=1) {b=r[i];
D[0]=0;D[1]=0;D[2]=0;D[3]=0;D[4]=1;D[5]=0;D[6]=0;D[7]=0;D[8]=1;
D[9]=600;D[10]=b;D[11]=b;
Lattice(OVAL,D,S); Plot("POLYGON,0,600,XYZ,10",S[9]);
nP[0]=TAKE;nP[1]=i;TextJob(nP,Stars,str);y=r[i]+0.02*i;x=0; if(i>1) y=-y; TextHang(x,y,0,str);}
a=317.816*5.97237e24; y=30;
//for(i=0;i<N;i+=1) {x=r[i]*1e+8;b=sqrt(GRAVITYC*a/x);b=2*PI*x/b; b/=3600;
//TextAt(100,y,"i=%d      r=%e      T=%f",i,x,b); y+=30;}
}#v07=?>A
```

## 5. Improvement by the band theory

For far field situations and for improving the precision, we should invoke the band theory that is a sophisticated knowledge in semiconductors and superconductors for electronic quantum mechanics. For example, in Uranus, the first band consists of Cordelia, Ophelia, Bianca, Cressida, Desdemona, Juliet, Portia, Rosalind, Cupid, Belinda, Perdita, Puck; the second band consists of Mab, Miranda, Ariel, Umbriel, Titania, Oberon. As shown in Fig.6. There also exists **forbidden gap** in the bands, for example, Ceres is considered as locating within the forbidden gap in the solar system with other eight planets [16]. By the way, the band theory supports the dark matter concept [16,13].

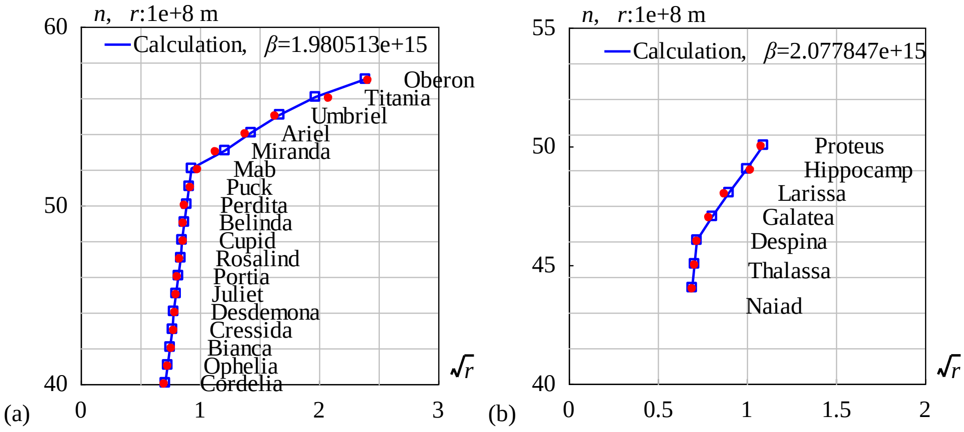



Fig.6 The satellites are quantized by the band theory.

```
<Clet2020 Script>// C source code [10]
int i,j,k,N,N1,nP[10],Figure; double x,y,z,r,r1,M,r_unit,H,beta,pD[10],D[200],S[200];
double orbit[20]={0,0.49770,0.53790,0.59170,0.61780,0.62680,0.64350,0.66090,0.69940,0.74800,
0.75260,0.76400,0.86010,0.97700,1.29390,1.91020,2.66300,4.35910,5.83520,},a,b,A,B,r_massive,R;
double e[20]={0,0.00026,0.00992,0.00092,0.00036,0.00013,0.00066,0.00005,
0.00011,0.0013,0.00007,0.0012,0.00012,0.0025,0.0013,0.0012,0.0039,0.0011,0.0014,};
int qn[22]={0,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60}; char str[200];
char
Stars[200]={"Uranus;Cordelia;Ophelia;Bianca;Cressida;Desdemona;Juliet;Portia;Rosalind;Cupid;Belinda;Perdita;Puck;Mab;Mir
anda;Ariel;Umbriel;Titania;Oberon;"};
main(){ Figure=2; N=19;N1=13;M=14.5*5.97237E24;r_unit=1E8;
for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0; S[1]=0;
SetAxis(X_AXIS,0,0,3,"#if#rsr#t;0;1;2;3;");
SetAxis(Y_AXIS,40,40,60,"#ifn, r#t:1e+8 m, Uranus;40;50;60;");
DrawFrame(0x016a,1,0xafffaf);    k=1; N1=10000; A=-0.855; r_massive=0.89; R=sqrt(r_massive); B=-0.022;
for(i=0;i<N1;i+=1) { r=10*i/N1; r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=k) {S[k+k]=r1; S[k+k+1]=y; k+=1; } if(y>57) break;}
Format(str,"Calculation,     #ifβ#t=%e",beta);    k-=40; N1=80; SetPen(2,0x0000ff);
Polyline(k,S[N1],0.2,59,str);Plot("CARD,0,@k,XY,4,4,",S[N1]);
SetPen(2,0xff0000); Plot("OVALFILL,0,18,XY,3,3,",D[2]); nP[0]=TAKE;
for(i=1;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A#t
```

```
<Clet2020 Script>// C source code [10]
int i,j,k,N,N1,nP[10],Figure; double x,y,z,r,r1,M,r_unit,H,beta,pD[10],D[200],S[200],F[200];
double orbit[10]={0,0.48224,0.50074,0.52526,0.61953,0.773548,1.05283,1.17646 },a,b,A,B,r_massive,R;
double e[10]={0,0.0047,0.0018,0.0004,0.0001,0.0012,0.0005,0.0005,0,0, };
int qn[20]={0,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,}; char str[200];
char Stars[200]={" Neptune;#Y15Naiad;#Y5Thalassa;Despina;Galatea;Larissa;Hippocamp;Proteus;"};;
main(){ Figure=2; N=8; N1=3;M=17 *5.97237E24;r_unit=1E8;
for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0; S[1]=0;
SetAxis(X_AXIS,0,0,2,"#if#rsr#t;0;0.5;1;1.5;2;");
SetAxis(Y_AXIS,40,40,55,"#ifn, r#t:1e+8 m, Neptune;40;45;50;55;");
DrawFrame(0x014a,1,0xafffaf);    k=1; N1=10000; A=-0.81; r_massive=0.54; R=sqrt(r_massive); B=-0.01;
for(i=0;i<N1;i+=1) { r=4*i/N1; r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=k) {S[k+k]=r1; S[k+k+1]=y; k+=1; } if(y>50) break;}
Format(str,"Calculation,     #ifβ#t=%e",beta);    k-=44; N1=88; SetPen(2,0x0000ff);
```

```
Polyline(k,S[N1],0.2,54,str);Plot("CARD,0,@k,XY,4,4,",S[N1]);
SetPen(2,0xff0000); Plot("OVALFILL,0,7,XY,3,3,",D[2]); nP[0]=TAKE;
for(i=1;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A#t
```

## 6. Conclusions

Matter wave has been generalized on a planetary scale as a quantum gravity theory, and applied to the solar system. This paper shows that Uranus' size and Neptune's size are the consequence of interference of the generalized matter waves. In this calculation, Uranus' radius is determined as 2.455042e+7 (m) with a relative error of 3.2%; Neptune's radius is determined as 2.315699e+7 (m) with a relative error of 5.95%. This calculation also correctly predicts the locations of rings for Uranus and Neptune.

References


[1]C. Marletto, and V. Vedral, Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity, Phys. Rev. Lett., 119, 240402 (2017)
[2]T. Guerreiro, Quantum effects in gravity waves, Classical and Quantum Gravity, 37 (2020) 155001 (13pp).
[3]S. Carlip, D. Chiou, W. Ni, R. Woodard, Quantum Gravity: A Brief History of Ideas and Some Prospects, International Journal of Modern Physics D, V,24,14,2015,1530028. DOI:10.1142/S0218271815300281.
[4]de Broglie, L., CRAS,175(1922):811-813, translated in 2012 by H. C. Shen in Selected works of de Broglie.
[5]de Broglie, Waves and quanta, Nature, 112, 2815(1923): 540.
[6]de Broglie, Recherches sur la théorie des Quanta, translated in 2004 by A. F. Kracklauer as De Broglie, Louis, On the Theory of Quanta. 1925.
[7]Huaiyang Cui, Determination of Solar Radius and Earth's Radius by Relativistic Matter Wave, Journal of Applied Mathematics and Physics, (2023)11:1, DOI: 10.4236/jamp.2023.111006.
[8]Huaiyang Cui, Biological Clock of Relativistic Matter Wave and Calculation of Human Mean Lifespan 84 Years. Modern Physics, (2023)13(2): 28-41. DOI:10.12677/mp.2023.132005; Huaiyang Cui, Modelling of Human Lifespan -- Based on Quantum Gravity Theory with Ultimate Acceleration, viXra:2302.0121. https://vixra.org/abs/2302.0121
[9]Huaiyang Cui, (2024) Calculation of Jupiter's Size, Saturn's Size, Tides and Zonal Winds by Quantum Gravity Theory, viXra: 2310.0148. https://vixra.org/abs/ 2310.0148.
[10]Clet Lab, Clet: a C compiler, download at https://drive.google.com/file/d/1OjKqANcgZ-9V5 6rgcoMtOu9w4rP49sgN/view?usp=sharing
[11]NASA, https://solarscience.msfc.nasa.gov/interior.shtml.
[12]Huaiyang Cui, (2022) Calculation of Tropic Cyclone Inner Structure Using Quantum Theory with Ultimate Acceleration, viXra: 2209.0139. https://vixra.org/abs/ 2209.0139.
[13]Huaiyang Cui, Relativistic Matter Wave and Quantum Computer, Amazon Kindle eBook, 2021.
[14]H.S.Hopfied,Two-Quartic Tropospheric Refractivity Profile for Correcting Satellite Data, Journal of Geophysics Research, 1969,Vol.74,No.18, 4487-4499.
[15]V.L.Pisacane, The space Environment and Its Effects on Space Systems, American Institute of Aeronautics and Astronautics, Inc. 2016, ISBN:978-1-62410-353-7, USA.
[16]Huaiyang Cui, (2023) Ceres Extinction due to Quantum Gravity Effect, viXra: 2306.0078. https://vixra.org/abs/ 2306.0078