A Recasting of MOND, Modified Newtonian Dynamics

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Abstract

Summary

MOND, Modified Newtonian Dynamics, provides an alternative to dark matter and is successful at explaining many astronomical scenarios including the flat rotation curves of disk galaxies. MOND operates at very low accelerations (less than 10^{-10} m/s^2) and can be interpreted either as a modification of Newton's law of inertia, or as a modification of Newton's law of gravity. In this paper we recast MOND in terms of our conjecture of a weighting function that determines the dynamical mass from the baryonic mass. This enables us to preserve both Newton's law of inertia and Newton's law of gravity.

1 Introduction

MOND, modified Newtonian dynamics, (Milgrom, 1983) is an alternative to dark matter and provides an explanation for many astronomical scenarios where dark matter is invoked. The best known scenario is the flat rotation curves of spiral galaxies. These are expected to show a $1/\sqrt{r}$ decline in rotational velocity but show a near constant velocity instead (Sanders, 2010). One way of solving the problem is by the addition of around 5 times the mass in the form of dark matter. MOND solves the problem by modifying the acceleration in Newton's second law of motion so that no dark matter is needed.

MOND provides explanations for many other scenarios, not just spiral galaxies. These include globular clusters, elliptical galaxies, galaxy clusters, cosmology and the formation of structure (Sanders & McGaugh, 2002). And, unlike dark matter, MOND provides a natural explanation for the observed Tully-Fisher relation (Milgrom, 1983) and RAR the radial acceleration relation (McGaugh et al, 2016). Milgrom maintains a complete description of MOND, including new research results, on Scholarpedia (Milgrom, 2024).

There is a lot of resistance in the astronomical community to consider MOND, and the vast majority of astronomers remain convinced that some form of dark matter must exist. Most papers on dark matter do not mention MOND at all. There are some genuine concerns with MOND, such as being a non-relativistic theory, that it breaks Newton's law of inertia, and that it fails to explain some features of the cosmic microwave background (Sanders, 2010). However, unlike dark matter, MOND has made numerous predictions that have been substantiated by observations later on (Milgrom, 2014).

In previous papers (JoKe 2024; JoKe 2023) we have put forward our conjecture that there is no dark matter and that a weighting function exists that determines the dynamical mass from the baryonic mass. This conjecture has the potential to explain all those astronomical scenarios where dark matter is invoked. In this paper we show that MOND can be recast in terms of our weighting function. When we do this we no longer need to modify either Newton's law of inertia or Newton's law of gravity.

2 Mass

In this section, we take a quick look at the different types of mass as used in Newton's second law and Newton's law of gravitation.

Newton's second law of motion, the law of inertia, is

$$\boldsymbol{F} = \boldsymbol{m_I}\, \boldsymbol{a} \tag{1}$$

where $\boldsymbol{F}$ is the force; $\boldsymbol{m_I}$ the inertial mass; $\boldsymbol{a}$ the acceleration. So, in Newton's second law it is the "inertial mass" that makes an appearance.

Newton's law of gravitation is

$$\boldsymbol{F} = \frac{\boldsymbol{G}\, \boldsymbol{M_A}\, \boldsymbol{m_P}}{\boldsymbol{r}^2} \tag{2}$$

where $\boldsymbol{G}$ is the gravitational constant; $\boldsymbol{M_A}$ is the active mass; $\boldsymbol{m_P}$, the passive mass; $\boldsymbol{r}$ the separation. This means $\boldsymbol{F}$ is the force acting on mass $\boldsymbol{m_P}$ by mass $\boldsymbol{M_A}$. So in Newton's law of gravitation it is both an "active gravitational mass" and a "passive gravitational mass" that appear.

Theoretically the inertial mass, the active gravitational mass, and the passive gravitational mass can all be different. In practice, experiments and observations show that these labels are superfluous and that, for any object, all three masses are exactly the same. This is the usual position adopted by the physical sciences.

When we look at MOND, we see that it can be interpreted as modifying the inertial mass.

3 MOND and inertia

MOND, modified Newtonian dynamics, replaces Newton's second law, equation (1), with (Sanders & McGaugh, 2002)

$$\boldsymbol{F} = \boldsymbol{m}\,\boldsymbol{a}\;\mu\left\{\frac{\boldsymbol{a}}{\boldsymbol{a_o}}\right\} \tag{3}$$

where μ is a function of ***a/a***$_o$; $\boldsymbol{a_o}$ is a universal constant acceleration with a value of around

$$\boldsymbol{a_o} \approx \mathbf{1.2} \times \mathbf{10^{-10}}\ \textbf{m/s}^2 \tag{4}$$

The function $\boldsymbol{\mu}$ satisfies the asymptotic conditions

$$\mu\left\{\frac{\boldsymbol{a}}{\boldsymbol{a_o}}\right\} = \mathbf{1} \qquad \textbf{for}\ \boldsymbol{a} \gg \boldsymbol{a_o} \tag{5}$$

and

$$\mu\left\{\frac{\boldsymbol{a}}{\boldsymbol{a_o}}\right\} = \frac{\boldsymbol{a}}{\boldsymbol{a_o}} \qquad \textbf{for}\ \boldsymbol{a} \ll \boldsymbol{a_o} \tag{6}$$

In high acceleration situations, $\boldsymbol{a} \gg \boldsymbol{a_o}$, equation (5) holds, equation (3) reverts back to equation (1), and we are back with Newton's second law. These situations cover the Earth, the solar system, and the central regions of galaxies; in fact most regions until we get out to the flat part of galaxy rotation curves where MOND provides a better explanation.

In low acceleration situations, $\boldsymbol{a} \ll \boldsymbol{a_o}$,equation (6) holds and equation (3) becomes

$$\boldsymbol{F} = \boldsymbol{m}\left(\frac{\boldsymbol{a}^2}{\boldsymbol{a_o}}\right) \tag{7}$$

This represents a change to law of inertia. It means that for a given force, ***F*** , the resultant acceleration, $\boldsymbol{a}$, is larger than that given by Newton's second law, equation (3),

$$\boldsymbol{a} = \frac{\boldsymbol{F}}{\boldsymbol{m}}\left(\frac{\boldsymbol{a_o}}{\boldsymbol{a}}\right) > \frac{\boldsymbol{F}}{\boldsymbol{m}} \tag{8}$$

MOND equation (7) holds in the outer regions of disk galaxies where it provides a good explanation for the flatness of the rotation curves. Essentially the larger acceleration supports a larger rotational velocity. Other situations where MOND is successful include galaxy clusters where equation (7) can explain the higher than expected velocities of cluster members.

4 MOND and gravity

Newton's law of gravity gives the gravitational force, $\boldsymbol{F}$, of mass $\boldsymbol{M}$ acting on mass $\boldsymbol{m}$ as

$$\boldsymbol{F} = -\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r^2}} \tag{9}$$

If we apply Newton's 2nd law, equation (1), then we obtain the Newtonian acceleration as

$$\boldsymbol{F} = \boldsymbol{m\,g_N} = -\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r^2}} \tag{10}$$

or

$$\boldsymbol{g_N} = -\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}} \tag{11}$$

where $\boldsymbol{g_N}$ is the Newtonian acceleration. So, the acceleration depends on the inverse square of the distance.

If, instead, we apply MOND equation (7), we obtain (Sanders & McGaugh, 2002)

$$\boldsymbol{F} = \boldsymbol{m}\left(\frac{\boldsymbol{a_M^2}}{\boldsymbol{a_o}}\right) = -\frac{\boldsymbol{G\,M\,m}}{\boldsymbol{r^2}} \tag{12}$$

or

$$\boldsymbol{a_M} = \sqrt{\frac{\boldsymbol{G\,M}}{\boldsymbol{r^2}}}\,\sqrt{\boldsymbol{a_o}} = \sqrt{\boldsymbol{g_N}}\,\sqrt{\boldsymbol{a_o}} \tag{13}$$

where $\boldsymbol{a_M}$ is the MOND acceleration. So the acceleration depends on the inverse distance, and not the inverse square of the distance.

The primary justification for MOND is that it explains successfully the flat rotation curves of disk galaxies. It does this with the introduction of the single universal constant, a_o, that applies to all rotation curves. This can be compared to dark matter where a different dark matter profile has to be applied to every galaxy. Given the baryonic mass distribution, MOND can predict the rotation curve. Dark matter cannot predict the rotation curve, it can only determine the amount of dark matter required once the baryonic matter distribution and the rotation curve are known.

5 MOND and galaxy rotation curves

We now look at the rotation curves of disk galaxies. We consider a star of mass, $\boldsymbol{m}$, in circular motion around the centre of a galaxy of mass $\boldsymbol{M}$. The centripetal acceleration, $\boldsymbol{a}$, is given by

$$\boldsymbol{a} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}} \tag{14}$$

where $\boldsymbol{v}$ is the rotational speed; $\boldsymbol{r}$ is the distance.

For Newtonian gravity, where the acceleration is given by equation (11), we have

$$\frac{\boldsymbol{v}^2}{\boldsymbol{r}} = \frac{\boldsymbol{G\,M}}{\boldsymbol{r}^2} \tag{15}$$

or

$$\boldsymbol{v} = \sqrt{\frac{\boldsymbol{G\,M}}{\boldsymbol{r}}} \tag{16}$$

So, Newtonian gravity predicts that the rotational velocity of a disk galaxy should fall off with a $1/\sqrt{r}$ dependence. This goes against observations where the rotational velocity is observed to remain approximately constant.

For MOND gravity, where the acceleration is given by equation (13)

$$\frac{\boldsymbol{v}^2}{\boldsymbol{r}} = \sqrt{\frac{\boldsymbol{G\,M}}{\boldsymbol{r}^2}}\,\sqrt{\boldsymbol{a_o}} \tag{17}$$

So, MOND predicts that the rotational velocity of disk galaxies should be constant, or that the rotation curves should be flat. This, of course, only applies to the outer regions where the acceleration is small.
So, for disk galaxies, MOND successfully explains the observed rotation curves, and Newtonian gravitation fails to do so.

Equation (17) can be rewritten as

$$\boldsymbol{v}^4 = \boldsymbol{G\,M\,a_o} = \textbf{constant} \tag{18}$$

So, MOND also predicts that the fourth power of the rotational velocity should be proportional to the galaxy mass. This is an observational fact, known as the Tully-Fisher relation (Milgrom, 2014).

6 Our conjecture

We have put forward an alternative explanation for the rotation curves of disk galaxies and other scenarios; this is different from both dark matter and MOND. Our conjecture is that there exists a weighting function that determines the dynamical mass of an object from its baryonic mass. This conjecture has been set out in a number of papers (JoKe, 2020; JoKe, 2023; JoKe, 2024).

We keep Newton's 2nd law, equation (1), but we replace Newton's law of gravity with

$$\boldsymbol{F} = -\frac{\boldsymbol{G\,m\,M_{dyn}}}{\boldsymbol{r^2}} \tag{19}$$

where M_{dyn} is the dynamical mass defined by

$$\boldsymbol{M_{dyn}} = \boldsymbol{M}\left(\frac{\boldsymbol{\xi_o}}{\boldsymbol{\xi(r)}}\right) \tag{20}$$

where $\boldsymbol{\xi}$ is our weighting function; $\boldsymbol{\xi_o}$ is the value of the weighting function at mass $\boldsymbol{M}$; $\boldsymbol{\xi(r)}$, is the value of the weighting function at mass $\boldsymbol{m}$.

The rotational velocity of a disk galaxy, equation (15), then becomes

$$\boldsymbol{v^2} = \frac{\boldsymbol{G\,M}}{\boldsymbol{r}}\left(\frac{\boldsymbol{\xi_o}}{\boldsymbol{\xi(r)}}\right) \tag{21}$$

It is clear that if our $\boldsymbol{\xi(r)}$ weighting function has a roughly $\boldsymbol{1/r}$ dependence, then equation (21) leads to a constant rotational velocity and a flat rotation curve.

Observations (JoKe, 2020) can be used to define our $\boldsymbol{\xi(r)}$ weighting function, and these lead to

$$\boldsymbol{\xi(r)} = \boldsymbol{r^{-\alpha}} \tag{22}$$

where the exponent, $\boldsymbol{\alpha}$, follows

$$\boldsymbol{0.5 < \alpha < 1.8} \tag{23}$$

So, there is good observational evidence for the existence of our weighting function and that equations (19) & (20) hold.

7 A recasting of MOND

For our conjecture, the acceleration, $\boldsymbol{g_c}$, as defined by equations (19) & (20) is

$$\boldsymbol{g_c}\,(\boldsymbol{r}) \;=\; -\,\frac{\boldsymbol{G}}{\boldsymbol{r}^2}\,\boldsymbol{M}\left(\frac{\boldsymbol{\xi_o}}{\boldsymbol{\xi}(\boldsymbol{r})}\right) \tag{24}$$

For MOND, we can rewrite equation (13) to give the acceleration, $\boldsymbol{a_M}$, as

$$\boldsymbol{a_M}(\boldsymbol{r}) \;=\; -\,\frac{\boldsymbol{G}}{\boldsymbol{r}^2}\,\boldsymbol{M}\left(\frac{\boldsymbol{a_o}}{\boldsymbol{a_M}(\boldsymbol{r})}\right) \tag{25}$$

Equations (24) and (25) now have exactly the same form. So, we have recast MOND to match our conjecture. Clearly, in any real situation, the accelerations given by equations (24) & (25) must be the same. This means

$$\frac{\boldsymbol{\xi_o}}{\boldsymbol{\xi}(\boldsymbol{r})} \;=\; \frac{\boldsymbol{a_o}}{\boldsymbol{a_M}(\boldsymbol{r})} \tag{26}$$

So, our weighting function and the MOND acceleration should have the same functional form. We have always taken it that our weighting function, $\boldsymbol{\xi}$, is dimensionless. But the MOND acceleration clearly has the dimensions of an acceleration, [L T^{-2}]. However, equation (26) has a ratio of quantities on both sides. So, the actual units do not matter as they cancel out.

If we go back to force, rather than acceleration, then we can rewrite equation (24) for our conjecture for the force on mass $\boldsymbol{m}$ as

$$\boldsymbol{F_c} \;=\; -\,\frac{\boldsymbol{G\,m\,M}}{\boldsymbol{r}^2}\left(\frac{\boldsymbol{\xi_o}}{\boldsymbol{\xi}(\boldsymbol{r})}\right) \;=\; \boldsymbol{m}\;\boldsymbol{a} \tag{27}$$

And for MOND, we can rewrite equation (25) as

$$\boldsymbol{F_M} \;=\; -\,\frac{\boldsymbol{G\,m\,M}}{\boldsymbol{r}^2}\left(\frac{\boldsymbol{a_o}}{\boldsymbol{a_M}(\boldsymbol{r})}\right) \;=\; \boldsymbol{m\,a} \tag{28}$$

It is clear, that in both cases, we have preserved Newton's second law, his law of inertia, equation (1). The forces are the same, and the accelerations are the same.

8 Comparing the approaches

Having recast MOND to match our weighting function, we can make a number of comparisons. Both MOND and our conjecture agree that dark matter does not exist, and that the only matter in the Universe is the so-called baryonic matter of galaxies, stars, gas and dust.

Conceptually, MOND and our conjecture are completely different.
MOND sees the dark matter problem as a problem with acceleration, and seeks to solve the problem by modifying Newton's second law. This is clear from equations (3), (7) and (25).
Our conjecture sees the dark matter problem as a problem with the dynamical mass and seeks to solve this by introducing a weighting function that determines the dynamical mass from the baryonic mass. This is clear from equation (24).

MOND breaks Newton's second law, the law of inertia. This is a strong reason why many scientists are reluctant to embrace MOND. MOND is sometimes described as modifying inertia rather than modifying gravity.
Our conjecture preserves Newton's second law. Our weighting function preserves the inertial mass and, instead, modifies the active gravitational mass.

MOND naturally explains the Tully-Fisher relation, whereby the mass of a disk galaxy is proportional to the fourth power of its rotational velocity, equation (18).
Our conjecture provides no straightforward explanation of Tully-Fisher.

MOND introduces a new universal constant, $\boldsymbol{a_o}$, with the units of acceleration. This single value is sufficient to explain the rotation curves of disk galaxies.
Our conjecture has no such constant and, instead, requires a different value of the exponent of the power law, equation (22), for each galaxy.

MOND's acceleration constant, $\boldsymbol{a_o}$, means MOND has one gravitational law for high accelerations, equation (11), and another gravitational law for low accelerations, equation (13).
With our conjecture there is no such switchover and our weighting function, $\boldsymbol{\xi(r)}$, varies continuously across objects irrespective of the magnitude of the acceleration.

9 Discussion

MOND is a leading contender to explain missing mass problems in astronomy without invoking some form of dark matter. MOND and our conjecture are completely different theories. MOND sees the dark matter problem as a problem with acceleration. Our conjecture sees the dark matter problem as a problem with the dynamical mass.

In previous papers we have explained our conjecture as a variation of the energy scale (JoKe 2020), and this is still our preferred option. This is a somewhat difficult concept to understand, which is the main reason we have introduced the simpler idea of a weighting function.

In this paper we have managed to recast MOND and our conjecture so that they resemble one another in the way they define acceleration. This is made clear in equations (24) & (25). Although equations (24) & (25) make MOND and our conjecture appear to be similar, the two theories are completely distinct and are in no way equivalent. However, by recasting MOND, as in equation (25) for acceleration and equation (28) for force, we have managed to preserve Newton's law of inertia.

MOND and our conjecture are two different ideas for replacing dark matter. In this paper we are making no judgement that MOND is wrong and our conjecture is right. Both have strengths and weaknesses. The more theories that are put forward to explain away dark matter, the more likely we are to find the answer.

No artificial intelligence (AI) has been used in this work.

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