Theoretical justification of the relationship between electric charge and mass

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August 12, 2024

Abstract

This paper proposes a novel theoretical framework that introduces a fifth spatial dimension, termed "space density," as a fundamental aspect influencing both gravitational and electric fields. While the properties of electromagnetic and gravitational interactions are well-studied empirically, the underlying nature of these forces, their interrelation, and the physical substance that constitutes them remain elusive. This research explores the concept of "space density" within a five-dimensional space, hypothesizing that variations in this dimension can lead to phenomena analogous to gravitational and electric fields. Through a series of mathematical models, we demonstrate how space density distribution behaves around spherical objects and discuss the implications for classical field theories. Our findings suggest a need to reconsider traditional views on space as merely a metric-bound entity, proposing instead that space itself possesses inherent properties and degrees of freedom. This hypothesis opens new avenues for understanding fundamental interactions in the universe from both physical and philosophical perspectives.

I Introduction

The electromagnetic and gravitational forces are among the most fundamental interactions known to physics. These forces govern the behavior of matter and energy on scales ranging from subatomic particles to the cosmos. Despite the extensive empirical data and theoretical models that describe the behavior of these forces, their true nature, and the material essence from which they arise, remain subjects of deep inquiry.

From a physical standpoint, we understand how these forces operate and can predict their effects with great accuracy. However, questions persist: What exactly are these forces? How are they interrelated? And most importantly, what is the proto-matter, the fundamental substance, from which these forces emerge? These questions touch not only on physical principles but also on philosophical considerations about the nature of reality.

In this paper, we propose a theoretical model that introduces a fifth spatial dimension, referred to as "space density." We hypothesize that this dimension plays a critical role in the formation of gravitational and electric fields. Our model suggests that the conventional three-dimensional space, coupled with time, is insufficient to fully explain the origin of these forces. Instead, space itself may possess intrinsic properties that contribute to the formation of these fields. By extending our understanding of space to include an additional dimension, we explore the potential for new interpretations of gravitational and electromagnetic interactions.

II Hypothesis

We hypothesize that the electromagnetic and gravitational fields are manifestations of a more fundamental property of space, which we refer to as ”space density.” This property is defined within a five-dimensional framework, where the fifth dimension is orthogonal to the conventional three spatial dimensions and one temporal dimension.

In this model, ”space density” represents a measure of how space itself can be compressed or expanded independently of its metric. This density is not analogous to the material density we are familiar with in three-dimensional space but instead reflects a fundamental characteristic of space that influences the formation of gravitational and electric fields.

Our hypothesis is built upon several key postulates:

- **Space Density and Field Formation:** The distribution of space density within a given region directly influences the formation of gravitational and electric fields. High concentrations of space density can lead to effects analogous to gravitational fields, while variations in space density can manifest as electric fields.

- **Spherical Symmetry in Perturbations:** The perturbations in space density caused by external influences (e.g., massive objects) exhibit spherical symmetry. This symmetry is a fundamental characteristic of how space density responds to perturbations, leading to predictable patterns in field formation.

- **Conservation of Space Density:** The total amount of space density within a closed system is conserved. This implies that when space density is altered in one region, there must be a corresponding change elsewhere to maintain overall balance.

- **Entropy Minimization:** Space tends to evolve towards states of minimal entropy with respect to space density distribution. This principle drives the natural tendency of space to return to a uniform density distribution following perturbations, analogous to the thermodynamic principles governing physical systems.

By exploring these postulates within the framework of five-dimensional space, we aim to provide a deeper understanding of the origins of gravitational and electromagnetic fields. This model challenges the traditional view that these fields are independent and instead suggests they are interconnected through the intrinsic properties of space itself.

III Methodology

3.1 Derivation of Space Density for a Compressed Spherical Region

To investigate the behavior of space density within a five-dimensional framework, we begin by considering a hypothetical spherical region of space that undergoes compression. Let this spherical region be denoted by $S(R_1)$, with R_1 representing the initial radius of the sphere before compression. After compression, the radius of the sphere is reduced to R_1'.

3.1.1 Space Density Distribution Inside the Compressed Sphere

The space density inside the compressed sphere, ρ_{inside}, can be determined by considering the conservation of the space density quantity within the sphere. Initially, the uncompressed sphere has a uniform space density ρ_0. After compression, the density inside the sphere increases due to the reduced volume, and is given by:

$$\rho_{\text{inside}} = \rho_0 + \rho_1$$

where ρ_1 represents the additional density due to compression.

Using the conservation of space density, the relationship between the densities before and after compression can be expressed as:

$$\rho_0 V(R_1) = \rho_{\text{inside}} V(R_1')$$

Substituting the volumes of the spheres:

$$\rho_0 \cdot \frac{4}{3}\pi R_1^3 = (\rho_0 + \rho_1) \cdot \frac{4}{3}\pi R_1'^3$$

Simplifying, we obtain:

$$\rho_0 R_1^3 = (\rho_0 + \rho_1) R_1'^3$$

Solving for ρ_1, the added density inside the sphere:

$$\rho_1 = \rho_0 \left(\frac{R_1^3}{R_1'^3} - 1 \right)$$

3.1.2 Space Density Distribution Outside the Compressed Sphere

Outside the compressed sphere, we assume that the reduction in space density, $\Delta\rho_{\text{decrease}}(r)$, follows an inverse fourth power law with respect to the distance r from the center of the sphere:

$$\Delta\rho_{\text{decrease}}(r) = \frac{A}{r^4}$$

where A is a normalization coefficient that ensures the total decrease in space density outside the sphere matches the increase inside the sphere.

3.1.3 Normalization Coefficient

To find the normalization coefficient A, we equate the total added density inside the sphere to the total reduced density outside it. The total added density inside the sphere is:

$$\rho_1 V(R_1') = \rho_1 \cdot \frac{4}{3}\pi R_1'^3$$

The total reduced density outside the sphere is given by:

$$\int_{R_1'}^{\infty} \Delta\rho_{\text{decrease}}(r) \cdot 4\pi r^2 \, dr = \int_{R_1'}^{\infty} \frac{A}{r^4} \cdot 4\pi r^2 \, dr$$

Evaluating the integral:

$$4\pi A \int_{R_1'}^{\infty} \frac{1}{r^2} \, dr = 4\pi A \left[-\frac{1}{r} \right]_{R_1'}^{\infty} = \frac{4\pi A}{R_1'}$$

Equating the added and reduced densities:

$$\rho_1 \cdot \frac{4}{3}\pi R_1'^3 = \frac{4\pi A}{R_1'}$$

Solving for A, we obtain:

$$A = \rho_1 \cdot \frac{R_1'^4}{3}$$

3.1.4 Final Formula for Space Density Decrease Outside the Sphere

Substituting the expression for A, the final formula for the decrease in space density outside the compressed sphere is:

$$\Delta\rho_{\text{decrease}}(r) = \frac{\rho_1 \cdot R_1' \cdot V(R_1')}{4\pi r^4} = \frac{\rho_1 \cdot R_1' \cdot \frac{4}{3}\pi R_1'^3}{4\pi r^4} = \frac{\rho_1 \cdot R_1'^4}{3r^4}$$

Thus, the space density distribution outside the compressed sphere decreases as $1/r^4$, consistent with the conservation of space density across the system.

IV Interaction of Two Compressed Space Spheres

In this section, we explore the interaction between two compressed spherical regions of space. By examining the space density distribution around these spheres, we derive the impact of one sphere on the density distribution of the other. This analysis is crucial for understanding the nature of forces and interactions that arise from the variations in space density.

4.1 Illustration of Space Density Distribution

Before diving into the mathematical derivations, we present a graphical representation of the space density distribution around two compressed spheres. This figure provides a visual understanding of how the density distribution is affected as the spheres are brought closer together.

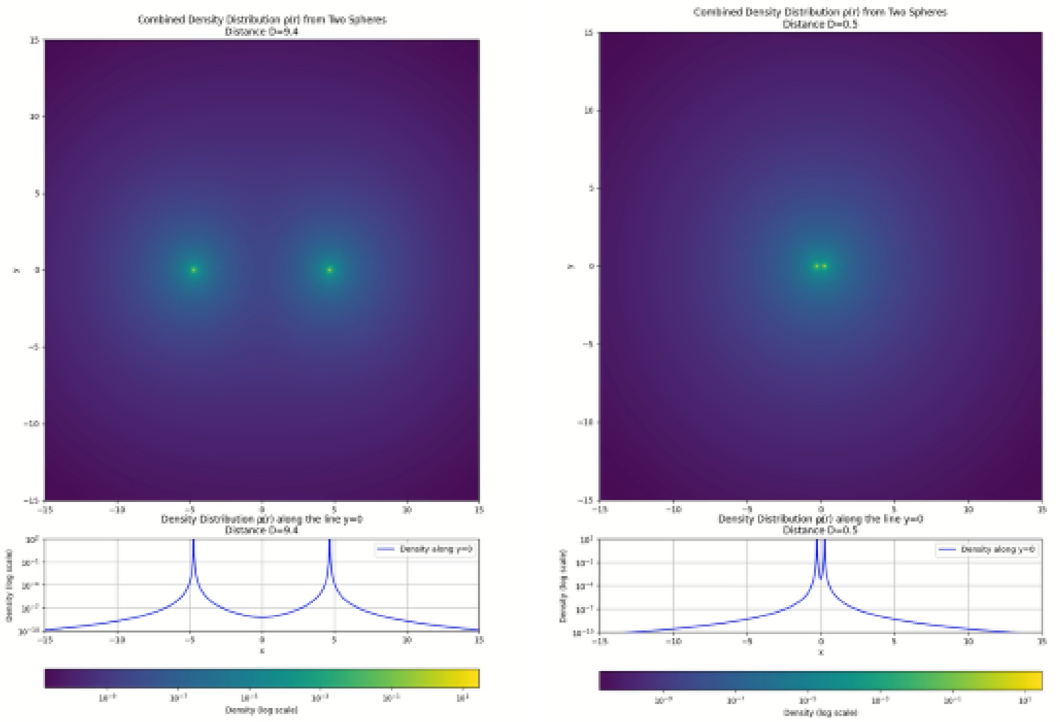

Figure 1: Space density distribution around two compressed spheres. The graph illustrates how the space density varies along the line connecting the centers of the spheres as they are brought closer together.

4.2 Integral of the Density Gradient for One Sphere

Consider the function $\Delta\rho_{\text{decrease}}(r'_1)$, which represents the density distribution for a single sphere, and is spherically symmetric with respect to the coordinate system r'_1. The function is given by:

$$\Delta\rho_{\text{decrease}}(r'_1) = \frac{R'_1\rho_1 V(R'_1)}{4\pi r_1'^4},$$

where R'_1 is the radius of the sphere, ρ_1 is the density at the radius R'_1, and $V(R'_1)$ is a volume-dependent function.

4.2.1 Gradient of the Density Function in r'_1 Coordinate System

We first compute the gradient of the function $\Delta\rho_{\text{decrease}}(r'_1)$ with respect to the radial coordinate r'_1. The gradient operator in spherical coordinates for a radially symmetric function is given by:

$$\nabla_{r'_1}\Delta\rho_{\text{decrease}}(r'_1) = \frac{d}{dr_{1'}}\left(\frac{R'_1\rho_1 V(R'_1)}{4\pi r_1'^4}\right)\hat{r}_{1'},$$

where $\hat{r}_{1'}$ is the unit vector in the radial direction.

Taking the derivative with respect to r'_1, we obtain:

$$\frac{d}{dr_{1'}}\left(\frac{R'_1\rho_1 V(R'_1)}{4\pi r_1'^4}\right) = -\frac{4R'_1\rho_1 V(R'_1)}{4\pi r_1'^5}.$$

Thus, the gradient of the density function in the r'_1 coordinate system is:

$$\nabla_{r'_1}\Delta\rho_{\text{decrease}}(r'_1) = -\frac{R'_1\rho_1 V(R'_1)}{\pi r_1'^5}\hat{r}_{1'}.$$

4.2.2 Integral of the Gradient from R'_1 to Infinity

Next, we integrate the gradient of the density function from R'_1 to infinity:

$$\int_{R'_1}^{\infty} \nabla_{r'_1}\Delta\rho_{\text{decrease}}(r'_1)\, dV_{r'_1},$$

where $dV_{r'_1} = 4\pi r_1'^2\, dr'_1$ is the volume element in spherical coordinates. Substituting the gradient we found earlier:

$$\int_{R'_1}^{\infty}\left(-\frac{R'_1\rho_1 V(R'_1)}{\pi r_1'^5}\right)4\pi r_1'^2\, dr'_1 = -4R'_1\rho_1 V(R'_1)\int_{R'_1}^{\infty}\frac{1}{r_1'^3}\, dr'_1.$$

To solve the integral:

$$\int \frac{1}{r_1'^3}\, dr'_1 = -\frac{1}{2r_1'^2}.$$

Evaluating this from R'_1 to infinity gives:

$$\left[-\frac{1}{2r_1'^2}\right]_{R'_1}^{\infty} = 0 - \left(-\frac{1}{2R_1'^2}\right) = \frac{1}{2R_1'^2}.$$

Finally, substituting this into the integral:

$$\int_{R_1'}^{\infty} \nabla_{r_1'} \Delta\rho_{\text{decrease}}(r_1')\, dV_{r_1'} = -4R_1'\rho_1 V(R_1') \cdot \frac{1}{2R_1'^2} = -2\frac{R_1'\rho_1 V(R_1')}{R_1'^2}.$$

Simplifying the expression:

$$\int_{R_1'}^{\infty} \nabla_{r_1'} \Delta\rho_{\text{decrease}}(r_1')\, dV_{r_1'} = -2\frac{\rho_1 V(R_1')}{R_1'}.$$

4.3 Integral of the Density Gradient for the Second Sphere

Consider the density distribution for the second sphere $\Delta\rho_{\text{decrease}}(r_2')$, which is spherically symmetric with respect to the coordinate system r_2'. The function is given by:

$$\Delta\rho_{\text{decrease}}(r_2') = \frac{R_2'\rho_2 V(R_2')}{4\pi r_2'^4},$$

where $r_2' = r_1' - D$, and D is the fixed distance between the origins of the coordinate systems r_1' and r_2'.

4.3.1 Gradient of the Density Function in r_2' Coordinate System

To compute the gradient of $\Delta\rho_{\text{decrease}}(r_2')$ with respect to r_1', we must apply the chain rule. The gradient of the function $\Delta\rho_{\text{decrease}}(r_2')$ with respect to r_2' is given by:

$$\nabla_{r_2'} \Delta\rho_{\text{decrease}}(r_2') = \frac{d}{dr_{2'}} \left(\frac{R_2'\rho_2 V(R_2')}{4\pi r_2'^4} \right) \hat{r}_{2'}.$$

Using the chain rule, we have:

$$\nabla_{r_1'} \Delta\rho_{\text{decrease}}(r_2') = \frac{d\Delta\rho_{\text{decrease}}(r_2')}{dr_{2'}} \cdot \frac{dr_{2'}}{dr_{1'}} \hat{r}_{1'},$$

where $\frac{dr_{2'}}{dr_{1'}} = \frac{d}{dr_{1'}}(r_1' - D) = 1$, since D is a constant.
Thus, the gradient in terms of r_1' is:

$$\nabla_{r_1'} \Delta\rho_{\text{decrease}}(r_2') = \frac{d}{dr_{2'}} \left(\frac{R_2'\rho_2 V(R_2')}{4\pi (r_1' - D)^4} \right) \hat{r}_{1'}.$$

Taking the derivative with respect to r_2':

$$\frac{d}{dr_{2'}} \left(\frac{R_2'\rho_2 V(R_2')}{4\pi r_2'^4} \right) = -\frac{4R_2'\rho_2 V(R_2')}{4\pi r_2'^5}.$$

Thus, the gradient with respect to r_1' becomes:

$$\nabla_{r_1'} \Delta\rho_{\text{decrease}}(r_2') = -\frac{R_2'\rho_2 V(R_2')}{\pi (r_1' - D)^5} \hat{r}_{1'}.$$

4.3.2 Application of the Substitution Theorem

To perform the integration, we apply the substitution theorem. The substitution is $r_2' = r_1' - D$, which implies $r_1' = r_2' + D$.

Verification of the Chain Rule Application The chain rule can be applied because the function $\Delta\rho_{\text{decrease}}(r'_2)$ is continuously differentiable with respect to r'_2. Moreover, the relationship between r'_1 and r'_2 is linear, ensuring that the derivative $\frac{dr'_1}{dr'_2} = 1$ holds.

Verification of the Substitution Theorem Application For the substitution theorem, we must check the following:

1. **Continuity of the Transformation:** The transformation $r'_2 = r'_1 - D$ is continuous and differentiable.

2. **Computation of the Jacobian:** The Jacobian of the transformation $r'_1 = r'_2 + D$ is given by $\frac{dr'_1}{dr'_2} = 1$.

3. **Transformation of the Integration Limits:** The integration limits transform as follows: - Lower limit: $r'_1 = R'_1$ corresponds to $r'_2 = R'_1 - D$. - Upper limit: $r'_1 = \infty$ corresponds to $r'_2 = \infty$.

Thus, the substitution theorem is applicable, and the integral in the r'_2 coordinate system is:

$$\int_{R'_1}^{\infty} \nabla_{r'_1} \Delta\rho_{\text{decrease}}(r'_2)\, dV_{r'_1} = \int_{R'_1-D}^{\infty} \nabla_{r'_2} \Delta\rho_{\text{decrease}}(r'_2)\, dV_{r'_2}.$$

4.3.3 Integral of the Gradient in the r'_2 Coordinate System

Now, we integrate the gradient of the density function $\Delta\rho_{\text{decrease}}(r'_2)$ over the volume element $dV_{r'_2} = 4\pi r_2'^2\, dr'_2$:

$$\int_{R'_1-D}^{\infty} \left(-\frac{R'_2\rho_2 V(R'_2)}{\pi r_2'^5}\right) 4\pi r_2'^2\, dr'_2 = -4R'_2\rho_2 V(R'_2) \int_{R'_1-D}^{\infty} \frac{1}{r_2'^3}\, dr'_2.$$

The integral simplifies to:

$$-4R'_2\rho_2 V(R'_2) \int_{R'_1-D}^{\infty} \frac{1}{r_2'^3}\, dr'_2.$$

Solving the integral:

$$\int \frac{1}{r_2'^3}\, dr'_2 = -\frac{1}{2r_2'^2}.$$

Evaluating at the limits:

$$\left[-\frac{1}{2r_2'^2}\right]_{R'_1-D}^{\infty} = \frac{1}{2(R'_1-D)^2}.$$

Thus, the integral is:

$$\int_{R'_1}^{\infty} \nabla_{r'_1} \Delta\rho_{\text{decrease}}(r'_2)\, dV_{r'_1} = -2\frac{R'_2\rho_2 V(R'_2)}{(R'_1-D)^2}.$$

4.4 Perturbation of the Density Distribution of the First Sphere in the Presence of the Second Sphere

4.4.1 Definition of Perturbation Quantity

The perturbation of the density distribution of the first sphere in the presence of the second sphere, located at a distance D, is defined as the difference between the integral of the gradient of the total density distribution for both spheres and the integral of the gradient of the density distribution for a single sphere, both computed with respect to the coordinate system r_1'. This perturbation represents the influence of the second sphere on the density distribution of the first sphere.

Mathematically, the perturbation quantity $\Delta W_{r_1'}(D)$ is given by:

$$\Delta W_{r_1'}(D) = \left(\int_{R_1'}^{\infty} \nabla_{r_1'} \Delta \rho_{\text{total}}(r) \, dV_{r_1'} \right) - \left(\int_{R_1'}^{\infty} \nabla_{r_1'} \Delta \rho_{\text{decrease}}(r_1') \, dV_{r_1'} \right),$$

where:

$$\int_{R_1'}^{\infty} \nabla_{r_1'} \Delta \rho_{\text{total}}(r) \, dV_{r_1'} = -2 \frac{\rho_1 V(R_1')}{R_1'} - 2 \frac{R_2' \rho_2 V(R_2')}{(R_1' - D)^2},$$

is the total density distribution considering both spheres, and

$$\int_{R_1'}^{\infty} \nabla_{r_1'} \Delta \rho_{\text{decrease}}(r_1') \, dV_{r_1'} = -2 \frac{\rho_1 V(R_1')}{R_1'},$$

is the density distribution of the first sphere.

$$\Delta \rho_{\text{decrease}}(r_2') = \frac{R_2' \rho_2 V(R_2')}{4\pi r_2'^4}, \quad \Delta \rho_{\text{decrease}}(r_1') = \frac{R_1' \rho_1 V(R_1')}{4\pi r_1'^4},$$

This definition captures the additional effect of the second sphere on the density distribution of the first sphere, effectively quantifying the perturbation introduced by the presence of the second sphere.

The perturbation quantity $\Delta W_{r_1'}(D)$ can now be computed as the difference between these two integrals:

$$\Delta W_{r_1'}(D) = \left(-2 \frac{\rho_1 V(R_1')}{R_1'} - 2 \frac{R_2' \rho_2 V(R_2')}{(R_1' - D)^2} \right) - \left(-2 \frac{\rho_1 V(R_1')}{R_1'} \right).$$

Simplifying the expression:

$$\Delta W_{r_1'}(D) = -2 \frac{R_2' \rho_2 V(R_2')}{(R_1' - D)^2}.$$

4.4.2 Approximation for Large Distances $D \gg R_1'$

In the approximation where $D \gg R_1'$, the perturbation formula simplifies and closely resembles the expression for the electric field intensity generated by a point charge. Specifically, the term $\rho_2 \cdot V(R_1')$ can be interpreted as the equivalent of an electric charge Q of the second sphere, representing the added spatial density of the second sphere within the volume $V(R_1')$. The radius R_1' in the numerator acts as a normalization constant, while D represents the distance between the centers of the spheres $S(R_1')$ and $S(R_2')$, where the additional spatial density is concentrated and can be analogized to electric charges.

Given this analogy, the perturbation quantity $\Delta W_{r_1'}(D)$ for large distances can be expressed as:

$$\boxed{\Delta W_{r_1'}(D) \approx 2\frac{R_2' Q}{D^2}}, \tag{1}$$

where:

$$Q = \rho_2 V(R_2')$$

can be interpreted as the "electric charge" of the second sphere in this analogy, and D is the distance between the centers of the spheres.

This formula highlights the inverse-square relationship between the perturbation and the distance D, which is characteristic of fields such as the electric field generated by point charges.

4.4.3 Physical Interpretation

Given the resemblance of this perturbation formula to the electric field intensity formula derived from Coulomb's law, which was originally obtained based on empirical data, we can suggest with a certain degree of confidence that our assumptions regarding the properties of spatial density in the context of real physical phenomena, such as the electric field, are valid. This analogy provides a conceptual bridge between the abstract mathematical formulation of spatial density perturbation and the well-established physical laws governing electric fields.

Thus, the presence of the second sphere at a distance D leads to a perturbation in the spatial density distribution of the first sphere, analogous to the influence of a point charge on the electric field at a distance. This connection not only reinforces the validity of our theoretical approach but also provides a deeper understanding of the interplay between spatial density distributions and their physical interpretations.

4.5 Results and Further Discussion

The results obtained from the analysis of the interaction between two compressed spheres suggest a profound connection between the concepts of space density and classical field theories. The derived formula for the interaction between space density disturbances bears a striking resemblance to Coulomb's law for electric fields, implying that what we understand as electric charge may be deeply rooted in the fundamental properties of space.

This resemblance opens up new perspectives for interpreting the nature of electric and gravitational fields, suggesting that these fields are not merely byproducts of the presence of matter but are intrinsic to the fabric of space itself. The hypothesis that space can possess a "density" that influences field formation challenges the traditional views that treat space as a passive backdrop for physical phenomena.

The mathematical model presented herein provides a new framework for understanding the forces that govern the universe. By introducing the concept of a fifth dimension, we gain a new lens through which to view the interaction of forces, potentially leading to a unified theory that encompasses both gravitational and electromagnetic interactions.

The potential implications of this model are vast. If the connection between space density and field formation is confirmed through further theoretical and experimental work, it could lead to a reevaluation of fundamental concepts in physics. This model could provide new insights into the unification of forces, the nature of dark matter and dark energy, and the role of additional dimensions in the structure of the universe.

Future research will need to explore the broader applicability of this model, including its implications for quantum field theory, cosmology, and high-energy physics. Additionally, experimental verification of the predicted space density distributions and their effects on observable phenomena will be crucial in validating this theory. The introduction of the concept of space density as a fundamental property of space opens new avenues for both theoretical exploration and experimental investigation.

V Solution of the Integral of the Gradient Over the Entire Volume for the Equation of Space Density Distribution of One Sphere

In this section, we solve the integral of the gradient over the entire volume for the space density distribution equation of a single sphere. The approach utilizes the Heaviside function, which allows us to effectively describe the boundary conditions and the sharp transitions in space density distribution. This detailed derivation ensures that the conservation laws are respected and provides insight into the nature of space density disturbances.

Let's write our distribution under boundary conditions using the Heaviside function and take the integral of the gradient of this space density distribution over the entire volume. The idea is that I assume that mass in the classical understanding of mass is also related to space density. Curvature of space together with its metric (second-order curvature) and curvature of space by Curvature of space relative to its measurement such as space density is inevitably related to boundary conditions! Based on the postulates of our space, inside the compressed sphere, the space density will always be uniform, and thus, to comply with the law of conservation of space, a sharp density transition boundary inevitably arises, which can be described by the Heaviside function, and it is this boundary — as a strong disturbance of space density — that causes the curvature of space relative to its metric. Here is an illustration showing the space density distribution along any radius vector from the center of disturbance to infinity:

5.1 Representation of Space Density Distribution Using the Heaviside Function

The space density distribution, $\rho(r)$, for a single sphere can be expressed using the Heaviside function $H(x)$ to accurately represent the density inside and outside the compressed sphere. The basic density distribution is defined as:

$$\rho(r) = \begin{cases} \rho_0 + \rho_1, & \text{if } r \leq R_1' \\ \rho_0 - \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4}, & \text{if } r > R_1' \end{cases}$$

The increase in density $\Delta\rho_{\text{increase}}(r)$ inside the compressed region can be expressed as:

$$\Delta\rho_{\text{increase}}(r) = \begin{cases} \rho_1, & \text{if } r \leq R_1' \\ 0, & \text{if } r > R_1' \end{cases}$$

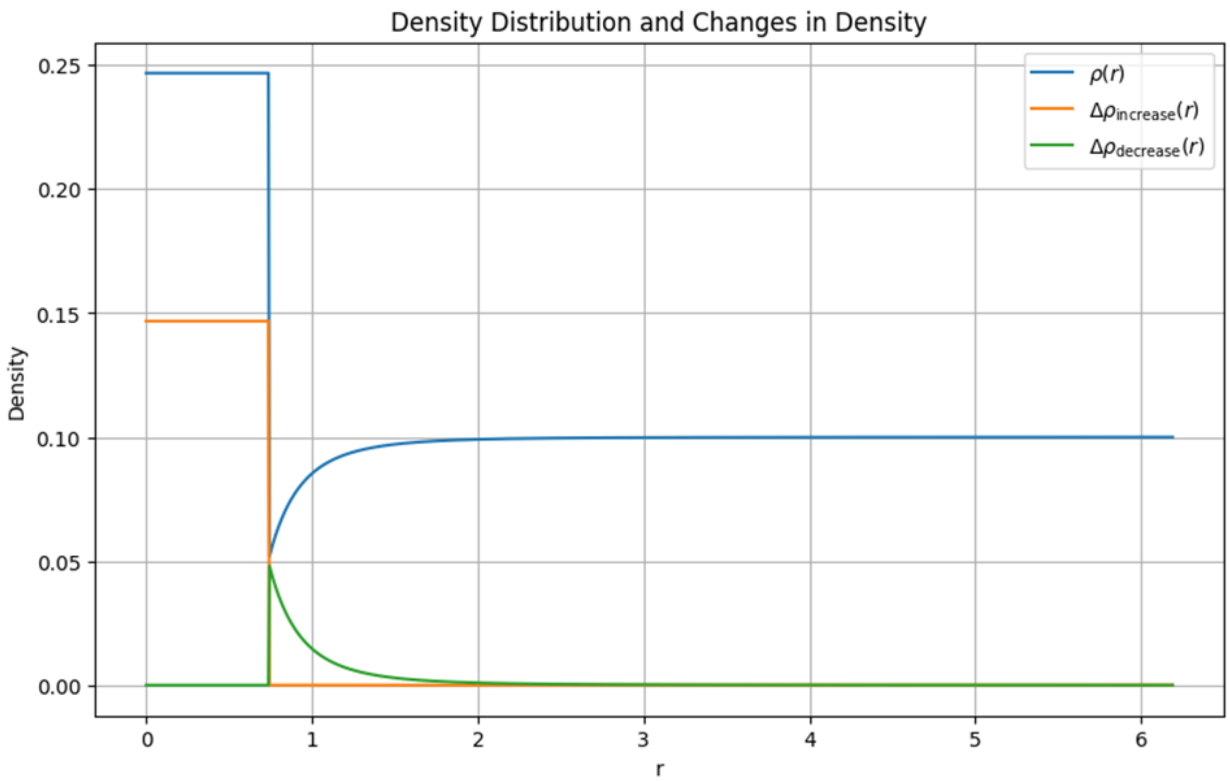

Figure 2: Graphs of the distribution of space density on a straight line passing through the center of a compressed sphere

Similarly, the decrease in density $\Delta\rho_{\text{decrease}}(r)$ outside the sphere is:

$$\Delta\rho_{\text{decrease}}(r) = \begin{cases} 0, & \text{if } r \leq R_1' \\ \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4}, & \text{if } r > R_1' \end{cases}$$

We can now rewrite these expressions in terms of the Heaviside function $H(x)$:

$$\Delta\rho_{\text{increase}}(r) = \rho_1 H(R_1' - r)$$

$$\Delta\rho_{\text{decrease}}(r) = \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} H(r - R_1')$$

Thus, the total change in density $\Delta\rho(r)$ is:

$$\Delta\rho(r) = \rho_1 H(R_1' - r) - \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} H(r - R_1')$$

5.2 Verification of Boundary Conditions Using Heaviside Function

To verify that this expression satisfies the boundary conditions, we analyze it at the boundary $r = R_1'$:

1. For $r \leq R_1'$:

$$\Delta\rho(r) = \rho_1 H(R_1' - r) - \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} H(r - R_1')$$

Since $H(R_1' - r) = 1$ and $H(r - R_1') = 0$, we obtain:

$$\Delta\rho(r) = \rho_1$$

2. For $r > R_1'$:

$$\Delta\rho(r) = \rho_1 H(R_1' - r) - \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} H(r - R_1')$$

Since $H(R_1' - r) = 0$ and $H(r - R_1') = 1$, we obtain:

$$\Delta\rho(r) = -\frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4}$$

Thus, the function $\Delta\rho(r)$ correctly describes the transition in space density at the boundary $r = R_1'$.

5.3 Integration Over the Entire Volume to Verify Density Conservation

Next, we verify that the total space density is conserved by integrating $\Delta\rho(r)$ over the entire volume:

$$\int_0^\infty \Delta\rho(r) \cdot 4\pi r^2 \, dr$$

This integral can be split into two parts, corresponding to $\Delta\rho_{\text{increase}}(r)$ and $\Delta\rho_{\text{decrease}}(r)$.

1. For $\Delta\rho_{\text{increase}}(r)$:

$$\int_0^{R_1'} \rho_1 \cdot 4\pi r^2 \, dr = 4\pi\rho_1 \int_0^{R_1'} r^2 \, dr = 4\pi\rho_1 \left[\frac{r^3}{3}\right]_0^{R_1'} = 4\pi\rho_1 \cdot \frac{R_1'^3}{3}$$

2. For $\Delta\rho_{\text{decrease}}(r)$:

$$\int_{R_1'}^\infty -\frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} \cdot 4\pi r^2 \, dr = -R_1' \cdot \rho_1 \cdot V(R_1') \int_{R_1'}^\infty \frac{1}{r^2} \, dr$$

Evaluating the integral:

$$-R_1' \cdot \rho_1 \cdot V(R_1') \cdot \left[-\frac{1}{r}\right]_{R_1'}^\infty = -R_1' \cdot \rho_1 \cdot V(R_1') \cdot \frac{1}{R_1'}$$

Simplifying gives:

$$-\rho_1 \cdot V(R_1')$$

Adding both parts, we confirm that the total density is conserved:

$$4\pi\rho_1 \cdot \frac{R_1'^3}{3} - \rho_1 \cdot V(R_1') = 0$$

This result verifies that the chosen density distribution satisfies the conservation of space density.

5.4 Gradient Integral Calculation and Equilibrium Check

In this section, we calculate the integral of the gradient of the space density distribution over the entire volume to determine whether the space is in a perturbed or equilibrium state. The space density distribution $\Delta\rho(r)$ was previously defined using the Heaviside function. We now focus on the integration of the gradient $\nabla\Delta\rho(r)$.

5.4.1 Space Density Gradient

The gradient $\nabla \Delta\rho(r)$ for the space density distribution $\Delta\rho(r)$, as defined earlier, is computed as follows:

1. **Derivative of the increase in density $\Delta\rho_{\text{increase}}(r)$:**

$$\frac{\partial}{\partial r}\left(\rho_1 H(R_1' - r)\right) = -\rho_1 \delta(r - R_1')$$

2. **Derivative of the decrease in density $\Delta\rho_{\text{decrease}}(r)$:**

$$\frac{\partial}{\partial r}\left(\frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} H(r - R_1')\right) = -\frac{4 \cdot R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^5} H(r - R_1') + \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} \delta(r - R_1')$$

Thus, the partial derivative of the total space density is:

$$\frac{\partial \Delta\rho(r)}{\partial r} = -\rho_1 \delta(r - R_1') - \frac{4 \cdot R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^5} H(r - R_1') + \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} \delta(r - R_1')$$

5.4.2 Integration Over the Entire Volume

We now integrate the gradient $\nabla \Delta\rho(r)$ over the entire volume to determine the total disturbance:

$$\int_0^\infty \nabla \Delta\rho(r) \cdot 4\pi r^2 \, dr$$

This integral is split into three parts:

1. **Integral of $-\rho_1 \delta(r - R_1')$:**

$$\int_0^\infty -\rho_1 \delta(r - R_1') \cdot 4\pi r^2 \, dr = -\rho_1 \cdot 4\pi (R_1')^2$$

2. **Integral of $-\frac{4 \cdot R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^5} H(r - R_1')$:**

$$\int_{R_1'}^\infty -\frac{4 \cdot R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^5} \cdot 4\pi r^2 \, dr = -4 \cdot R_1' \cdot \rho_1 \cdot V(R_1') \int_{R_1'}^\infty \frac{1}{r^3} \, dr$$

Evaluating the integral:

$$-4 \cdot R_1' \cdot \rho_1 \cdot V(R_1') \left[-\frac{1}{2r^2}\right]_{R_1'}^\infty = 2 \cdot R_1' \cdot \rho_1 \cdot V(R_1') \cdot \frac{1}{(R_1')^2} = 2 \cdot \rho_1 \cdot V(R_1') \cdot \frac{1}{R_1'}$$

3. **Integral of $\frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} \delta(r - R_1')$:**

$$\int_0^\infty \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} \delta(r - R_1') \cdot 4\pi r^2 \, dr = \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi} \cdot 4\pi \cdot \frac{1}{(R_1')^2} = \frac{\rho_1 \cdot V(R_1')}{R_1'}$$

5.4.3 Final Result of the Gradient Integral

Combining all the parts, the total integral over the entire volume is:

$$\int_0^\infty \nabla \Delta\rho(r) \cdot 4\pi r^2 \, dr = -\rho_1 \cdot 4\pi (R_1')^2 + 2 \cdot \rho_1 \cdot V(R_1') \cdot \frac{1}{R_1'} - \frac{\rho_1 \cdot V(R_1')}{R_1'}$$

Simplifying further, we have:

$$\int_0^\infty \nabla\Delta\rho(r) \cdot 4\pi r^2 \, dr = -\rho_1 \cdot 4\pi(R_1')^2 + 2 \cdot \frac{\rho_1 \cdot \frac{4}{3}\pi(R_1')^3}{R_1'} - \frac{\rho_1 \cdot \frac{4}{3}\pi(R_1')^3}{R_1'}$$

This simplifies to:

$$\int_0^\infty \nabla\Delta\rho(r) \cdot 4\pi r^2 \, dr = -4\pi\rho_1(R_1')^2 + \frac{8\pi\rho_1(R_1')^2}{3} - \frac{4\pi\rho_1(R_1')^2}{3}$$

Finally, combining these terms:

$$\int_0^\infty \nabla\Delta\rho(r) \cdot 4\pi r^2 \, dr = -\frac{4\pi\rho_1(R_1')^2}{3}$$

This result demonstrates that the product of three-dimensional density and the surface area of the sphere expresses the magnitude of the spatial density perturbation. The negative sign indicates that the system is in a perturbed state and that to restore equilibrium, space must undergo further adjustments, such as curvature.

5.5 Conclusions of the Gradient Integral

The integration of the space density gradient over the entire volume has yielded a significant result that reinforces the hypothesis that space density plays a crucial role in the formation of gravitational and electromagnetic fields. The non-zero result of the gradient integral, with its negative sign, indicates that the system is in a perturbed state, necessitating further adjustments to reach equilibrium.

This perturbation can be interpreted as the curvature of space, which directly relates to the changes in space density distribution caused by the compression of a spherical region. The result suggests that the curvature of space and the disturbances in space density are intimately connected, providing a new perspective on the nature of gravitational interactions.

Furthermore, the derived expression for the total disturbance highlights the relationship between three-dimensional surface area, space density, and the resulting perturbation. This relationship hints at a deeper connection between space density and the forces that govern the behavior of matter and energy in the universe.

The introduction of the concept of space density as a fundamental property of space itself, capable of influencing the distribution and interaction of fields, opens new avenues for understanding the fundamental forces of nature. This theoretical framework offers the potential to unify gravitational and electromagnetic phenomena under a common conceptual umbrella, leading to new insights into the nature of matter, energy, and the structure of the universe.

5.5.1 Reduction to Gravitational Charge Form

1. We define the total space charge Q as the difference in volumes before and after compression, multiplied by the initial space density ρ_0:

$$Q = (V(R_1) - V(R_1')) \cdot \rho_0$$

where $V(R_1)$ and $V(R_1')$ are the volumes of spheres with radii R_1 and R_1', respectively.

2. The space density after compression ρ_1 is defined as:

$$\rho_1 = \frac{Q}{V(R_1')}$$

where $V(R_1')$ is the volume of the compressed sphere.

We use the formula for the space density obtained from the compression of a single sphere:

$$\int_0^\infty \nabla\Delta\rho(r) \cdot dV = -\frac{2\rho_1 S(R_1')}{3}$$

where $S(R_1')$ is the surface area of the sphere with radius R_1', and ρ_1 is the space density after compression.

To reduce the formula to a form resembling the gravitational charge, we multiply both the numerator and denominator by R_1':

$$\frac{\text{Numerator} \cdot R_1'}{\text{Denominator} \cdot R_1'} = \frac{Q \cdot V(R_1')}{V(R_1') \cdot R_1'}$$

Now let's expand the formula for the volume of a sphere:

$$V(R_1') = \frac{4}{3}\pi(R_1')^3$$

Substituting this value:

$$\frac{Q}{\frac{4}{3}\pi(R_1')^3}$$

We use the formula for the volume of a four-dimensional sphere:

$$V_4(R_1') = \frac{\pi^2(R_1')^4}{2}$$

Thus:

$$\frac{Q}{V_4(R_1')} = \frac{Q}{\frac{\pi^2(R_1')^4}{2}} = \frac{2Q}{\pi^2(R_1')^4}$$

Now we can write our formula for the space density perturbation:

$$\frac{(3\pi Q) \cdot V_3(R_1')}{4 \cdot V_4(R_1')}$$

If we take the four-dimensional density $\rho_1' = \frac{Q}{V_4(R_1')}$, then our formula for the space density perturbation becomes:

$$\int_0^\infty \nabla\Delta\rho(r) \cdot dV = \frac{3\pi\rho_1' V_3(R_1')}{4}$$

where $V_3(R_1')$ is the three-dimensional volume of a sphere with radius R_1', and $\rho_1' = \frac{Q}{V_4(R_1')}$ is the four-dimensional density.

This formula resembles the expression for gravitational charge, where the product of four-dimensional density and three-dimensional volume expresses the magnitude of the space charge.

5.6 Transition to Gravitational Equations

Based on the concept of gravitational charge that we introduced, we can develop conclusions that lay the groundwork for deriving gravitational equations. The gravitational charge, as we interpreted it, describes the perturbation in space density and can be expressed through the curvature coefficient $K(r)$, which represents the deviation of the spatial metric due to this charge.

The curvature of the spatial metric $K(r)$, similar to the density distribution outside the compressed sphere, will be proportional to $1/r^4$, with an appropriate normalization factor. This coefficient $K(r)$ describes the distribution of spacetime curvature caused by the density and allows us to establish an equation for this distribution.

The next step involves considering the perturbation created by a second gravitational charge on the distribution of the $K(r)$ coefficient, which describes the curvature of the spacetime metric due to the first charge. This perturbation and its impact on the curvature lead us to equations analogous to gravitational equations.

However, it is essential to note that the obtained formula, analogous to the equations for electric charges, will describe the repulsion of gravitational charges. In classical theory, it is known that massive bodies, associated with spacetime curvature, attract each other. This apparent paradox can be explained by considering that the curvature of space density, created by the second object, will interact directly with the "lump of density," representing both gravitational and electric charges.

It should be remembered that the gravitational charge is the product of the sphere's surface area and the three-dimensional density, which we interpreted as a gravitational charge—the product of four-dimensional density and the volume of the compressed sphere, to bring the formula to a form analogous to that of an electric charge. Space, striving to minimize entropy, will "expel" the space lump (the first gravitational charge) in the form of the first sphere from regions of lower curvature $K(r)$ to areas with greater curvature, that is, towards the region where the second space lump (the second gravitational charge) is located. Undoubtedly, these are superficial and preliminary considerations; the result obtained requires, in my opinion, deeper collective analysis, which is the goal of my paper.

Thus, a region will emerge where the forces of attraction and repulsion are balanced. Beyond this region, objects causing spacetime curvature will begin to repel each other, which is observed in cosmology as the accelerated recession of cosmic objects. This approach opens up prospects for a deeper understanding of not only gravity but also the general mechanisms of matter interaction with spacetime curvature, especially in the context of the expanding universe.

These considerations emphasize the importance of the relationship between space density and metric curvature, which could lead to a reevaluation of current gravitational theories and offer a new perspective on the nature of fundamental forces.

5.7 Gravitational Charge and Curvature of Space

The gravitational charge associated with the mass of the compressed sphere can be used to calculate the curvature of space generated by this mass. This curvature, in accordance with general relativity, determines the gravitational field created by the sphere. Thus, the mass of the sphere, derived from the compression energy, plays a fundamental role in shaping the surrounding spacetime.

This relationship between spatial density, compression energy, and gravitational mass offers a deeper understanding of the nature of mass and its connection to the geometry of space. It provides a framework for exploring the fundamental interactions in the universe and bridges the gap between classical mechanics and relativistic physics.

VI The Relationship Between Spatial Density and the Mass of a Compressed Sphere

In this section, we derive the relationship between the energy required to compress a sphere from its initial radius R_1 to a final radius R_1', and the mass of the compressed sphere. This relationship is crucial for understanding how the energy contained within the compressed sphere determines the curvature of space, and thus the gravitational field generated by the sphere.

6.1 Energy Required for Compressing a Sphere from R_1 to R_1'

6.1.1 Initial Equation

We have:

$$\int_0^\infty \nabla \Delta\rho(r) \cdot 4\pi r^2 \, dr = -\frac{4\pi\rho_1 (R_1')^2}{3}$$

where $\rho_1 = \frac{Q}{V(R_1')}$ and $Q = (V(R_1) - V(R_1')) \cdot \rho_0$.

6.1.2 Simplification of the Integrand

Multiply the numerator by $\rho_0 \cdot (R_1')^2$ and expand the brackets:

$$-\frac{4\pi}{3} \cdot \frac{\rho_0 (V(R_1) - V(R_1')) \cdot (R_1')^2}{V(R_1')} = -\frac{4\pi\rho_0}{3} \cdot \left(\frac{V(R_1) \cdot (R_1')^2}{V(R_1')} - \frac{V(R_1') \cdot (R_1')^2}{V(R_1')} \right)$$

6.1.3 Simplification of Each Term

First term:

$$\frac{V(R_1) \cdot (R_1')^2}{V(R_1')}$$

Second term:

$$\frac{V(R_1') \cdot (R_1')^2}{V(R_1')} = (R_1')^2$$

6.1.4 Substituting Volume Values

Substituting the volume values:

$$V(R_1) = \frac{4}{3}\pi R_1^3, \quad V(R_1') = \frac{4}{3}\pi (R_1')^3$$

we get:

$$-\frac{4\pi\rho_0}{3} \cdot \left(\frac{\frac{4}{3}\pi R_1^3 \cdot (R_1')^2}{\frac{4}{3}\pi (R_1')^3} - (R_1')^2 \right) = -\frac{4\pi\rho_0}{3} \cdot \left(\frac{R_1^3 \cdot (R_1')^2}{(R_1')^3} - (R_1')^2 \right)$$

6.1.5 Simplifying Volumes in the Second Term

$$-\frac{4\pi\rho_0}{3} \cdot \left(\frac{R_1^3}{(R_1')} - (R_1')^2 \right)$$

6.1.6 Integration of Each Expression

First integral:

$$-\rho_0 \cdot \frac{4\pi}{3} \int_{R_1}^{R_1'} \frac{R_1^3}{R_1'} \, dR_1'$$

Second integral:

$$\rho_0 \cdot \frac{4\pi}{3} \int_{R_1}^{R_1'} (R_1')^2 \, dR_1'$$

6.1.7 Integrating the First Term

Integration of the first term:

$$-\frac{4\pi\rho_0 R_1^3}{3} \int_{R_1}^{R_1'} \frac{1}{R_1'} \, dR_1' = -\frac{4\pi\rho_0 R_1^3}{3} \ln\left(\frac{R_1'}{R_1}\right)$$

6.1.8 Integrating the Second Term

Integration of the second term:

$$\frac{4\pi\rho_0}{3} \int_{R_1}^{R_1'} (R_1')^2 \, dR_1' = \frac{4\pi\rho_0}{3} \left[\frac{(R_1')^3}{3} \right]_{R_1}^{R_1'} = \frac{4\pi\rho_0}{9} \left[(R_1')^3 - R_1^3 \right]$$

6.1.9 Total Energy E

Combining the results:

$$E = -\frac{4\pi\rho_0 R_1^3}{3} \ln\left(\frac{R_1'}{R_1}\right) + \frac{4\pi\rho_0}{9} \left[(R_1')^3 - R_1^3 \right]$$

This expression represents the energy required to compress the sphere from R_1 to R_1'. This energy is equivalent to the amount of energy contained in the compressed sphere, which causes space to curve along with its metric, thereby determining the mass of the sphere.

Let's simplify the resulting expression for energy by factoring out ρ_0:

$$E = \frac{4\pi\rho_0}{3} \left[-R_1^3 \ln\left(\frac{R_1'}{R_1}\right) + \frac{1}{3} \left((R_1')^3 - R_1^3 \right) \right]$$

6.2 Mass of the Compressed Sphere

6.2.1 Using Einstein's famous equation $E = mc^2$, we can find the mass m of the compressed sphere:

$$m = \frac{E}{c^2}$$

6.2.2 Substitute the expression for E:

$$m = \frac{4\pi\rho_0}{3c^2}\left[-R_1^3 \ln\left(\frac{R_1'}{R_1}\right) + \frac{1}{3}\left((R_1')^3 - R_1^3\right)\right]$$

This expression defines the mass of the compressed sphere based on the energy required for its compression. This result illustrates how the energy associated with compressing the sphere translates into equivalent mass, which curves space and creates a gravitational field.

6.3 Physical Meaning

This expression shows that the mass of the sphere, and hence its gravitational influence, is related to the energy required to compress the space inside it. Thus, the energy contained in the compressed sphere determines the degree of space curvature, which is equivalent to the concept of mass in general relativity.

VII Conclusions and Final Remarks

In this paper, an attempt was made to theoretically derive Coulomb's law, which currently serves as an empirical generalization of experimental data. The main objective of the research was to explain fundamental interactions in nature through the introduction of a new concept — spatial density, which can quantitatively describe the warping of space without altering its metric. This approach allowed us to propose a model that links the concepts of gravity, electric fields, and the mass of matter on a deeper level than existing theories.

7.1 Key Findings

1. Derivation of Coulomb's Law:

By utilizing simple mathematics and fundamental physical concepts, a formula analogous to Coulomb's law was derived through the introduction of spatial density. This was achieved by recognizing that spatial density plays a crucial role in the formation of fields and forces similar to those in electromagnetism.

2. Concept of Spatial Density:

Spatial density was introduced as a new physical quantity capable of describing the warping of space. Unlike the traditional notion of warping through the space-time metric, this concept offers an alternative view of the interaction between matter and fields.

Spatial density was also linked to the concept of a "gravitational charge," interpreted as the force that keeps space compressed. In this context, the gravitational charge explains how spatial density contributes to the warping of space and the creation of gravitational effects.

3. Integration of the Compression Process:

It is proposed that as the compression of a sphere $S(R_1)$ to $S(R_1')$ increases, where R_1 is the sphere in an uncompressed state with a density ρ_0, the gravitational charge corresponds to the force keeping the space compressed. By integrating the compression process of spatial density from R_1 to R_1' (where $R_1 > R_1'$), one can obtain the energy required to compress the sphere. This is crucial for understanding how energy is related to the warping of space created by mass.

4. Connection with Existing Theories:

The paper demonstrates that the proposed model does not contradict the laws of electrostatics and quantum field theory but rather complements them by providing a deeper explanation of concepts such as matter, force, and energy. In particular, the model offers new insights into the origin of mass in material objects.

5. Comparison with Existing Theories:

The introduction of spatial density as a mechanism responsible for warping provides a way to integrate it with established theories such as general relativity and quantum field theory. Spatial density can be associated with the gravitational charge that causes the warping of the space metric, consistent with the ideas of the Higgs boson and its role in imparting mass to matter.

6. Philosophical Aspects of Physics:

The work touches not only on the technical aspects of deriving physical laws but also on philosophical questions about the nature of matter and energy. This makes the theory a universal platform for further research in fundamental physics.

7.2 Final Remarks

The concept of spatial density proposed in this paper represents an important step towards the construction of a unified Theory of Everything, which combines gravitational and electromagnetic interactions and explains the nature of mass in matter. The derivation of a formula analogous to Coulomb's law and the understanding of the gravitational charge as the force holding space in a compressed state provide a new foundation for studying physical processes.

By integrating the process of compressing spatial density from the uncompressed state to the compressed state, we can obtain the energy required to compress the sphere. This energy is related to the gravitational charge that causes the warping of space. This connects gravity and the mass of matter with the fundamental concept of spatial density.

However, many questions remain open. At present, spatial density remains a hypothetical quantity, and the mechanisms that cause it to behave in a certain way require further study. Future research should focus on experimentally confirming the proposed ideas and gaining a deeper theoretical understanding of the mechanism of interaction between spatial density and matter and fields.

Thus, this work is a first step towards a deeper and more comprehensive theory that requires collective efforts and further development. This new approach has the potential to lead to fundamental discoveries and revolutionize our understanding of the nature of forces, fields, and matter.