On the physical foundations of Gravitation with a dual space

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Abstract

Starting from first principles, we point out an apparent contradiction in the behavior of light in the metric space of a stationary frame in gravity. We show that the issue is resolved provided an independent moving 'dual space' exists along side the metric space of general relativity (GR). In this dual space clocks run at equal rates to first order but coincide with the time dilation of metric space, due rather to different light path lengths. We find its mathematical properties coincide with the Gullstrand–Painleve coordinates, however the interpretation dictated by this model requires some different concepts to that of the standard river model. Objects fall at equal rate in this space not because of equality of gravitational and inertial mass but because they are stationary in the dual space which itself is falling. The space behaves more Galilean than one might expect. We explore how escape velocity is modelled in this moving spacetime and address how the dual space re-interprets gravitational redshift to unify it with doppler and Hubble and as well as perhaps dark energy. natural implications for mass variation and radiating charges stationery in gravity are also presented as a logical result of adopting the framework. We then touch on how the dual space can be modelled as a vector field in geometric algebra.

1 Introduction

The concept of spacetime encapsulated in Einstein's theory of General Relativity (GR), underpins our current understanding of the universe. In this theory, spacetime is a four-dimensional deformable continuum in which events occur.

The original classical view of space and time begins from Aristotle, followed by Newton and Kant [Kan08]. Space and time were conceptualized as separate, absolute, and unchanging entities. However, following the advent of Special Relativity (SR), which introduced the properties of length contraction and time dilation, Minkowski proposed in 1909 that space and time be combined into a unified four-dimensional continuum, deformed by the Lorentz transformations [Min09]. This actually followed an earlier idea by Hamilton of combining space and time within the four-dimensional quaternions, followed by Poincaré in 1906 who noted that the Lorentz transformations imply a four-dimensional spacetime. Then, GR developed by Einstein in 1915, extended Minkowski's concept of spacetime by treating it as a curved manifold influenced by the presence of matter and energy. In this theory, gravity is not a Newtonian force but a manifestation of the curvature of spacetime, in which particles follow geodesics [Ein03]. This revolutionary concept in GR of spacetime as a flexible, dynamic entity distorted by the presence of mass-energy-pressure has been confirmed by numerous experiments [Gre04].

More recently, the advent of quantum mechanics has led to the proposal of a quantum theory of gravity, in which spacetime is no longer a smooth continuum but a foam-like structure at the smallest scales. This quantum foam is believed to be composed of tiny, Planck-scale fluctuations, which give rise to the discrete nature of space and time [Rov14].

GR is a coordinate independent theory, however several different coordinate systems have been found to improve understanding of the physics. The first solution found to Einstein's field equations was the Schwarzschild coordinates. However, these coordinates have a discontinuity across the event horizon, and so alternate coordinates systems were investigated. A few years later the Gullstrand–Painlevé coordinates were found, and they are not only regular across the horizon, but they describe a free-fall inertial observer within a flat space. The coordinates also have a clear physical interpretation as the inflow of space at the local escape velocity. Hence, they are often considered as the most intuitive, particularly for radial timelike geodesics [Mac19]. Later, other coordinates were developed, with various benefits, such as the Eddington–Finkelstein coordinates in 1924, Lemaître coordinates in 1932 and Kruskal–Szekeres coordinates in 1960.

This paper builds on a completely different approach to the above approaches. it builds on a simple physical argument by a reexamination of the behavior of light and the equivalence principle. The result coincides with the special properties of the Gullstrand–Painlevé coordinates and provide a natural intuitive understanding of gravity, as the inflow of spacetime. From the vantage point of this free-fall observer, we show some the standard results of GR as well as some novel insights and results.

2 Foundation arguments for a gravitational Dual Space

2.1 Gravitational time dilation using acceleration and the equivalence principle

We begin by showing how time dilation can be predicted to occur in gravity using a fairly straight forward argue using the equivalence principle. We first use a standard definition of length L, as the time t light c takes to travel between two separated points as

$$L = ct. \tag{1}$$

This was first adopted in 1983 by the International Commitee for weights and measures (ICWM) "Resolution 1 of the 17th The International system of units (SI) Conference Generale des Poids et Mesures (CGPM))

Let us consider an inertial observer O located in flat space far from any gravitational sources, close to a transparent rocket also at rest with respect to O. The rocket's length is $L' = ct'$, so the time it takes light to propagate the rockets length is $t' = L'/c$ to an observer in the rocket and for O it is $t = L'/c$, so $t' = t$. Let the rocket now accelerate at a constant acceleration a with respect to O, where its velocity is $v = at << c$. Let a ray of light be sent from the top of the rocket to the bottom. Now with respect to O the light will propagate from the top of the rocket at c, and so at any instant of time, it travels a distance $L = ct$. However by virtue of the rocket's floor accelerating at a upwards with respect to O to meet the light at instant velocity $-at$, the floor covers a distance $d = -\frac{1}{2}at^2$. Hence O will observe the light to reach the floor of the rocket at the shorter distance

$$ct = ct' - \frac{1}{2}at^2. \tag{2}$$

Therefore O will see the light reach the floor not at time $t = L'/c$ but the shorter time

$$t = L'/c - d/at. \tag{3}$$

O therefore observes the instant relative velocity between light and the floor to be

$$v = (c + at) > c. \tag{4}$$

For O observing upward light originating from the floor of the rocket, we have, due to the ceiling of the rocket accelerating away from the light

O will see the longer light length

$$ct = ct' + \frac{1}{2}at^2. \tag{5}$$

The longer time

$$t = L'/c + d/at. \tag{6}$$

The instant relative velocity between light and the rocket ceiling

$$v = (c + at) < c. \tag{7}$$

From this type of analysis and using his equivalence principle, Einstein deduced that due to the agreement of different clock readings for the events of light arrival between both the inside and outside observer's, that the inside observer must interpret the result as time dilation for the top and bottom clocks. Another key reason for this conclusion of the inside observer's is that the length of the light paths inside their frame in the rocket is of fixed equal length in both directions. The outside observer simply understands the different clock times as being due to the different lengths for the light paths $L = ct$.

We now wish to consider three questions.

1- Does there exist in gravity an equivalent 'outside' inertial observer to O that can view a rocket stationary on the surface of a planet in gravity?

2- If such a frame exists will it also measure the same results for the different lengths of light?

3- For this inertial observer, are there any further implications for gravity?

2.2 An apparent contradiction with light path lengths in gravity

If we consider a rocket again stationary on the surface of a large mass in gravity then to first order, ignoring any relativistic effects gravity has on length measures, we can consider the length again to be L'

Now on quick reflection it is easy to see that an equivalent frame to the inertial observer O in flat space in our acceleration case, there is the local free fall frame in gravity, which we can name O'.

With respect to question two, we find in accordance with the acceleration case, O' will also observe the downward light, to move with speed c and again with respect to the floor, with speed $c + at$.

Now suppose O' holds a long ruler of length R in the vertical. Then suppose that O' fires a light beam from the top of the ruler at the instant O' falls with the ruler, so that the event of the light reaching the ruler's end coincides with the event of ruler's end arriving at the floor of the rocket. Then by our definition of length, Eq. (1), light would only have travelled the distance $R = ct$ in O's frame.

Let us rename R to $R = L_d$ to reflect that it is the length of light that O' records downward in the rocket.

Now for an arbitrary frame S stationary in the rocket the light length is $L_s = ct$. So we have for the two lengths the relation

$$L_d = L_s - \frac{1}{2}at^2. \tag{8}$$

Hence $L_d < L_s$ Which is equivalent to Eq. (2)

By the same analysis, the upward light will be for the relative speed between the ceiling and light $(c - at)$ and for this upward distance we have

$$L_u = L_r + \frac{1}{2}at^2. \tag{9}$$

We must point out that for at least part of the journey O' is not able the see the full length of the upward light path unless O' falls below the level of the rocket floor. However we still find that for any length $L_u > L_r$ always holds. We have answered question two above in the affirmative, both the inertial frame O and O' are able to see different lengths of light in their respective observed frames.

We now ask a pivotal question: Given L_r is a fixed length and the rocket is also stationary on the surface of a gravitational source mass, how does light in this frame have $L_d < L_u$?

That is, how is $L_u > L_r > L_d$? Clearly these different lengths of light L_d and L_u raises an apparent impossibility in the currently understood metric space of GR. It suggests that light's dual behavior is possible only if it moves concurrent through another space, not of the same properties, but dual to that of the metric space of GR.

What properties might such a space have ? We will attempt to answer this and the above question now.

2.3 Resolving the apparent contradiction: The flow of spacetime

If we are to resolve the issue above we should define at least two fundamental differences between the two apparent spaces.

First: The length L of the light paths in the metric space is constant in both directions, while in the dual space the length is longer upwards and shorter downwards.

Second: In the metric space time dilation occurs for the fixed vertical height. In the dual space, to a first approximation, the clocks are constant vertically.

In a first step to resolving this considering the observer O of the acceleration again. Since the rocket is moving through space with respect to O with increasing velocity, O views different light lengths upward and downward as an obvious consequence of this acceleration. Second, since the rocket is accelerating, through a flat space it is therefore moving forward towards many fixed and distant points at different positions in spacetime.

However, as noted and is the core of our problem, no such acceleration or movement through space occurs for the fixed rocket in gravity. Hence the most reasonable explanation that fits the anomalous behavior of light, is that what we mean by spacetime and everything in it above the rocket, including all mass and energy as well as any fields and the quantum vacuum, move toward the stationary rocket.

This is in contrast the the metric space of GR which does not move in this way. Hence this is a separate space to the metric space. It is a locally flat spacetime and it moves with respect to the curved spacetime of GR. It moves towards the source mass from a higher position in the metric field.

We will now refer to this spacetime as a dual space. One property it has is that objects falling in the metric space are actually stationary in this dual space. In GR, geodesic motion is usually thought of as the straightest possible path through a curved spacetime that an object moves. However in dual space, there is a constant inflow of spacetime and any objects affected by it move to some extent in its direction, depending on the history of their own motion.

A not so obvious property of this spacetime, is that the time light take to move, a given length, maps to the time in the metric space. We will explore this property further below.

We note further that while this dual space is locally flat at each point, it moves at different accelerations at these points. This is due to non-uniformity of free fall acceleration at each point in gravity. In the frame of dual spacetime, these different accelerations manifest as the well known tidal forces, that characterize gravity.

It should be clear from this model that objects fall at the same rate not because their inertial and gravitational masses are equal and therefore cancel. Rather all masses are stationary in the dual space and so with respect to each other.

3 Some implications and characteristics of the space

Now that we have established dual space as an entity that exists independently of metric space, we want to examine some consequences of this .

3.0.1 Escape velocity and the true distance through 'space'

Let us see what the escape velocity $v_{esc} = \sqrt{\frac{2MG}{r}}$ means in light of this model. It can be considered as the instant velocity required by any mass m to overcome the equal and opposite downward velocity v_d, of dual space S_d at the position r with a_d acceleration. The equation is the same as the escape velocity but with the simple subscript $_d$ replacing the escape velocity subscript $_{esc}$ so we have $v_d = \sqrt{\frac{2MG}{r}}$

Now according to this view of dual space, v_d began its fall from the mass M, at a position where space is nearly flat. However, for more practical purposes, if we take the earth as M and use the common escape velocity of $11.2km/s$, then we can take this as the speed v_d of our dual space at the earth's surface at the fixed position r. So we have that the Space S_d begin its journey at a tiny starting acceleration a distance l from the earth surface. Now for the mass m to leave the earth at the escape velocity, it must also leave according to the usual escape velocity of $v = \sqrt{\frac{2MG}{r}}$ also equal to $11.2km/s$. Now m and S_d must take the same time for these opposite routes. The mass therefore has to move the distance l as well as the distance through S_d which is also equal to l. The total distance m travelled is therefore $2l$.

3.1 Clarifying the Velocity field and the Acceleration field of Dual space

Ignoring the velocity dependence of gravity [BCA20] and tidal forces to first order the acceleration field is independent of the velocity field.

Clearly with this view of space any object falling in gravity is moving with the free fall of space. However clearly this cannot mean at the same speed of space. Interpreting escape velocity: In order for an object to overcome the traditional escape velocity, in this model it need to equal or overcome the escape velocity.

3.1.1 Mass and length changes as a function of free fall.

Suppose we have some observer of mass m standing on the surface of any sizable mass M, where $M >> m$. Let this observer record the instant velocity $v = at$ of a mass m_o falling past them and let m_0 originate from infinity. Since (by eqn $v_d = \sqrt{\frac{2MG}{r}}$), in this theory $v = v_d$ this is the velocity space is moving past this person. Furthermore by $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^c}}}$ we have that mass increases with velocity. This velocity is the usual velocity of motion v through flat space with respect to some object or frame of reference. But in this theory we have that $v = v_d$, so this is the velocity of the dual space at any instant $v = at$ for $m = \frac{m_0}{\sqrt{1-\frac{v_d^2}{c^2}}}$. Hence for a mass m sitting on the surface of any sizable mass, we only need its instant local velocity v to ascertain that m possesses a greater mass than any stationary mass m_0 at infinity. Where the originating mass comes from infinity.

Also from eqn ($v_d = \sqrt{\frac{2MG}{r}}$), this can be written as $m = \frac{m_0}{\sqrt{1-\frac{2MG}{rc^2}}}$. So that m increases as r decreases or M increases. Since this theory posits that spacetime itself is falling, then we would also expect that any quantum fluctuations and short lived sub atomic particles produced in stationary flat space, will remain stationary with the dual space. Hence, since not all subatomic particles pass through matter, then we would expect an energy density increase at the interface of the source mass via this mechanism. Whether this fits the above equation needs experimental evidence.

Now regarding length we have by the Schwarzschild metric we have for length $L = \frac{L_0}{\sqrt{1-\frac{2MG}{rc^2}}}$. Hence for an observer stationary on the surface of Jupiter, we have by a trivial substitution $L = \frac{L_0}{\sqrt{1-\frac{v_d^2}{c^2}}}$. Length increase can be found by the same local measurement process applied by the mass increase observer.

3.1.2 Gravitational time dilation moving space

In this theory the movement of space in gravity at any velocity gt is equivalent also to any velocity between objects in ordinary Galilean space so that for two objects in relative velocity that experience a non relativisitc doppler for light we have

Now, in special relativity (SR), the time dilation, based on a relative velocity v in flat space, is given by

$$\frac{t}{t_0} = \frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}, \tag{10}$$

where γ is the relativistic gamma factor. Again $v = v_d = g_d t$ and so substituting in the escape velocity from Eq. (16), describing the inflow velocity of space, we find

$$\frac{t}{t_0} = \sqrt{1 - \frac{2GM}{rc^2}}, \tag{11}$$

where t_0 is the time of the observer at infinity, this is the equation for the gravitational time dilation. Hence, we can view gravitational time dilation in this model as being due to light losing energy against the inflowing space. Thus Eq. (11) also describes the gravitational redshift from a massive body.

3.1.3 Redshift in the Dual Gravitational space as Doppler

We can easily derive the first order 'gravitational redshift', $\omega = \omega_0(1 - \frac{gL}{c^2})$ for inside the rocket and therefore for gravity, from Doppler the effect, $\omega = \omega_0(1 - \frac{v}{c})$, by letting $v = at$, where $v << c$, and $t = \frac{L}{c}$ where L is the length of the rocket. However in the dual space of gravity, at is the velocity of dual space moving against the background Schwarzschild space. Hence light trying to moving against this space moves according to usual Doppler (equation $\omega = \omega_0(1 - \frac{gL}{c^2})$) and so is doppler in nature with no time dilation. Therefore we can consider emitted gravitational light in dual space as being of the same mechanism as Doppler. Hubble redshift or redshift with distance in the universe at large associated with so called dark energy is better thought of as tidal force, which in dual space are accelerations of dual space at different positions.

4 Applications

4.1 The dual spaces of gravity and free-fall

We now illustrate these two spaces, with two different experiments: 1) a ball thrown up in an accelerating rocket and 2) a ball is thrown upwards from the surface of the Earth at approximate escape velocity. In order to keep the comparison clear, we will assume that the gravity field is constant.

4.1.1 Two frames for a rocket

We consider a rocket accelerating at a constant acceleration $a = g$. Then, an external inertial observer, will see the rocket moving a distance $s = \frac{1}{2}gt^2$, assuming we begin observations at time $t = 0$ when the observer is momentarily comoving with the rocket. If we now consider the ball thrown vertically upwards in the rocket at $t = 0$, with an initial velocity v_0, then once the ball is released and moving independently it will be moving inertially, just like the external inertial observer frame. However, there will be a relative velocity of v_0 with respect to the outside observer, and so the external observer will see an inertial trajectory for the ball of $s = v_0 t$. However, the observer in the rocket, who is accelerating upwards towards the ball with an acceleration g will see the ball following an inverted parabola with an equation

$$s = v_0 t - \frac{1}{2}gt^2. \tag{12}$$

We can easily calculate that the ball will indeed return to the launch point ($s = 0$) at $t = \frac{2v_0}{g}$. The accelerating observer will thus see a parabola, the same as an observer on the Earth in gravity. This is to be expected from the equivalence principle, as we are permitted to equate uniform gravity to an accelerating frame.

4.1.2 The equivalence to two frames in gravity

Now, the equivalent reference frame to the external inertial observer viewing the rocket and ball, is a free-fall observer in gravity. As an example, the free-fall observer could correspond to a person stepping off a building, at the instant the ball is thrown up. This free-fall observer will follow an accelerating trajectory downwards of $s = \frac{1}{2}gt^2$, as seen from the surface. Also, an observer standing on the Earth will see the ball following an inverted parabola, with an equation identical to Eq. (12). Therefore, to find the trajectory of the ball, as seen by the free-fall observer, we need to subtract these two expressions, finding $s = v_0 t$. As expected, the free-fall observer in gravity sees exactly the same trajectory as the inertial observer viewing the ball motion in the rocket.

We note that for an observer on the Earth, they are actually in an accelerating frame, with clocks running at different rates with altitude. However, for the free-fall observer, as the ball is also now in free-fall, there will be no relative acceleration. That is, as expected, for two free-fall objects, there can only be relative velocity, and we are now in flat space, where all clocks run at the same rate. That is, the free-fall frame is the dual space to an observer stationary in gravity, and is a flat space. Significantly, what we saw as a parabolic curve in gravity, now in the dual space, manifests as a straight line.

4.1.3 Free-fall vector field

4.1.4 Charges in a gravitational field

A free fall observer in gravity obeys the laws of special relativity and inertial flat space. These are exactly the laws for an observer stationary to the dual space. Hence from the perspective of geometric algebra Maxwell's equations in dual space take the simple form $\partial F = J$. It follows therefore that if a charge radiates in acceleration then a charge at rest on the surface of the Earth must, from the perspective of an observer stationary in the free fall dual space, also radiate. The equation of radiation is identical to the known equations of an accelerating electron in flat space since the dual space occupies a flat metric. What is more clear with the dual space model is that now we can see that a stationary electron in a gravitational field has a space moving past it and is therefore entirely equivalent to the electron accelerating through space. We do not at this stage propose to know with any precision the nature of this space however due to its existence and symmetric behavior to ordinary acceleration we make this prediction with confidence. We have therefore in Geometric algebra based on Larmor's nonrelativistic formulation which describes the electromagnetic radiation from an accelerating charge. In geometric algebra, this can be written as:

$$F = (q/4\pi c)(a \wedge v)/r \tag{13}$$

where: F is the electromagnetic field, q is the charge, a is the acceleration, v is the velocity, r is the distance from the charge and c is the speed of light . $\wedge$ represents the wedge product.

However, to describe the radiating electromagnetic waves, we need to consider the electromagnetic field in terms of the electromagnetic potential, A.

$$A = (q/4\pi c)(v/r) + (q/4\pi c)(\partial/\partial t)(-a/r) \tag{14}$$

Using the geometric algebraic identity: $\nabla \wedge A = F$. Hence we can derive the electromagnetic field F from the potential A.

4.1.5 A unified field ?

The inflowing space can be modeled as a radial velocity vector field $\boldsymbol{v}$, at each point. For the Kerr metric, describing a rotating black hole, surprisingly, we simply need to add a local twist at each point [HL08], which can be represented by a bivector $j\boldsymbol{w}$. We can thus write for the flow field $F = \boldsymbol{v} + j\boldsymbol{w}$, a six dimensional field. This is closely analogous to the electromagnetic field $F = \boldsymbol{E} + j\boldsymbol{B}$. Hence, this provides a possible unification of gravity and electromagnetism, where the same flow field F creates electromagnetic forces in a local reference frame, but creates gravity when the field is flowing. The flow field obeys the Galilean transforms whereas motion within the field obeys the Lorentz transformations.

Hence, in this view, the universe would be approximately static, with the redshift arising from the outwardly flowing spacetime.

4.2 The Gullstrand–Painlevé coordinates and the dual space

The solution of Einstein's field equations around a stationary, non-rotating mass is the Schwarzschild metric

$$ds^2 = \left(1-\beta^2\right)c^2dt^2 - \left(1-\beta^2\right)^{-1}dr^2 - r^2d\Omega^2, \tag{15}$$

where

$$\beta = \frac{v}{c} = \sqrt{\frac{r_s}{r}} \tag{16}$$

is the escape velocity at a distance r, where $r_s = \sqrt{\frac{2GM}{c^2}}$ is the Schwarzschild radius. Now, make the coordinate transformation $dt = dT - \frac{\beta}{1-\beta^2}$, the line element becomes

$$\begin{aligned} ds^2 &= \left(1-\beta^2\right)c^2dT^2 - 2\beta c dT dr - dr^2 - r^2d\Omega^2 \\ &= c^2dT^2 - (dr + \beta c dT)^2 - r^2d\Omega^2, \end{aligned} \tag{17}$$

which are the Gullstrand–Painlevé coordinates. The spatial component $dr + \beta cdT = dr + v_e dT$, describes a flow of spacetime towards the gravitating mass at the escape velocity β. The local time coordinate T is now equal to the proper time of a free-falling observer from infinity, and at constant time with $dT = 0$, the space is flat, as reflected in a unit coefficient on dr^2. The line element is also regular across the event horizon, with the inflow rate reaching the speed of light at the event horizon, and increasing steadily towards the singularity $r = 0$.

Now, electromagnetic radiation (EM) satisfies $ds^2 = 0$, so that Eq. (17) produces the factorised equation $(cdT + (dr + \beta cdT))(cdT - (dr + \beta cdT)) = 0$, indicating two solutions. Dividing through by dT, we find

$$v_{\mathrm{EM}} = \frac{dr}{dT} = c\,(\pm 1 - \beta) = \pm c - v_{esc} = c\left(\pm 1 - \sqrt{\frac{r_s}{r}}\right). \tag{18}$$

Hence, at the event horizon, with $r = r_s$, we can see that the outbound light velocity is zero, as it is balanced by the similar inflow velocity of space. However, the inbound light has a velocity of $2c$, in these coordinates. Hence, the inflowing space viewpoint provides a natural explanation for the event horizon.

The Gullstrand–Painlevé coordinates, describes the velocity of inflowing space at a given radius r and thus also corresponds to the free-fall observer in gravity, within a flat space. For an observer falling from infinity, they are at rest with respect to the this space as it moves towards the centre of the mass. Hence, moving in from infinity they have effectively travelled zero distance. Reversing their direction, they will travel further than the distance dr in this space, as there is an extra distance travelled through the dual space.

What could be considered as simply a coordinate transformation, we could view the Gullstrand–Painlevé coordinates as correctly describing the flow of space from infinity, against a backdrop of the static Schwarzschild coordinates.

4.2.1 Origins of inertia ?

If there is such a phenomena as 'moving space' as this theory claims as dual space separete to the metric space of GR, then appealing to symmetry, then it is difficult to argue that acceleration through space of a mass to cause inertia according to Newton, is not the same as the acceleration of space with respect to a mass. In the case of applying $F = ma$ to a mass when there is no reference inertial mass to refer the acceleration to, then the differences are even more difficult to distinguish. These identical results are already encapsulated in the equivalence principle, So in this sense this is not new. What is different is whether this phenomena is actually caused by a 'physical entity' formerly considered to be empty. This theory claims this to be the case.

In so far as inertia is defined as resistance to acceleration through space and this dual space is an accelerating field of space moving past an object, then we conclude that it is indistinguishable from inertia and therefore is a candidate that brings us closer to understanding what the phenomena is.

5 Discussion

There have been other models of gravity as a river of space falling freely. Most derive their arguments from the Gullstrand–Painlev´e coordinates and have varying interpretations of what the coordinates mean.

'Eg Hamilton and Lisle4 who developed a river model of non-rotating black holes by expressing the Schwarzschild metric in the Gullstrand-Painlev´e coordinates. In that model space itself is pictured as a river flowing through a flat background while objects moving in the river move according to the rules of special relativity'. We dont deal with any background space to the dual space, rather we see that it flows with respect to metric space.

Our starting point however is different from the Gullstand-Painlev'e coordinates coordinates. We have built the argument directly off the physical behavior of light in gravity in the chosen frame.

This leads us a little deeper in addressing what do we mean by 'dual space moves'? The problem is that this dual space is normal Euclidean 3-D space comprising points and lines orthogonal to each other, out of which we build up planes and then volumes. It also encompasses the 4-D flat space of special relativity. So then if we claim that these actual spaces move when a mass is embedded within them, then in what manner ? This is a question for future research, we hope we have given enough of a starting point, especially with its separate independence to the metric space g_{uv} that GR is comprised.

However once we defined length as $L = ct$, and we found that in dual space, this produces different vertical lengths in the up down directions for a fixed length metric length L, then this further complicates what we mean by distance and therefore space itself. Furthermore, the different lengths of light in dual space, play the role of different rates of time in the metric space. Not withstanding that we can in the frame of the dual space observer O, falling and watching light move towards the ground at a speed different to O say, that there is another space and distance through which the dual space itself is observed to move.

6 Conclusion

By the resolution of an apparent physical contradiction in the behavior of light paths in a stationary frame in gravity, we have shown the most reasonable resolution is the existance a second independently moving dual space to the metric space of GR. This space contrasts with the metric space of GR in that in it clocks run at the same rate locally over the vertical distance and that the different light length times coincide with the clock times in the metric frame. Falling objects are stationary to the dual space, while stationary objects in the metric space move with through it. This has implications for interpreting the escape velocity where the distance to escape is twice the distance travelled the the apparent distance. We have explored other physical implications in this dual space such as localised methods of determining length increase and gravitational time dilation is doppler shift and predictions of mass increase. This framework then allows for a unified description of gravitational time dilation, gravitational redshift, and cosmologicalredshift.

We presented the Gullstrand–Painlevé coordinates which also can be interpreted as describing an inflowing spacetime at the local escape velocity, hence from an independent analysis we arrive at similar descriptions.

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