$\tan(\pi/2) = 0$ from Jacobi's Method in Diagonalization of Matrices

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Abstract: In this note, we would like to show the simple result $\tan(\pi/2) = 0$ from Jacobi's formula in diagonalization of matrices.

Key Words: Division by zero, division by zero calculus, $1/0 = 0/0 = 0$, $\tan(\pi/2) = 0$, Jacobi's formula in diagonalization of matrices.

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1 Result

We assume the elementary properties of the division by zero and division by zero calculus. See the basic references. However, here, we would like to point out the simple result

$$\tan\frac{\pi}{2} = 0,$$

in order to see its naturalness from Jacobi's formula in diagonalization of matrices.

For a real symmetric matrix

$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix},$$

we consider the matrix

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

Then we have, for

$${}^tP = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$${}^tPAP = \begin{pmatrix} a\cos^2\theta + 2b\sin\theta\cos\theta + c\sin^2\theta & b\cos 2\theta + (c-a)\sin\theta\cos\theta \\ b\cos 2\theta + (c-a)\sin\theta\cos\theta & c\cos^2\theta - 2b\sin\theta\cos\theta + a\sin^2\theta \end{pmatrix}.$$

Therefore, it is a diagonal matrix if and only if

$$b\cos 2\theta + (c-a)\sin\theta\cos\theta = 0;$$

that is,

$$\tan 2\theta = \frac{2b}{a-c}.$$

Here, we see that for $a - c = 0$, $\cos 2\theta = 0$; that is, $\theta = \pi/4$. We thus have the desired result

$$\tan\frac{\pi}{2} = \frac{1}{0} = 0$$

from the fact

$$\frac{1}{0} = 0.$$

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References

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