Planck length

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Abstract

The paper reveals the essence of the Planck length and its relevance to quantum gravity.

1 Introduction

The **Planck length** (denoted by ℓ_P) is a quantity of the dimension of length, composed of fundamental constants — the speed of light, the Planck constant, and the gravitational constant:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}$$

where:
$\hbar$ is the Dirac constant $\frac{h}{2\pi}$, where h is the Planck constant
G is the gravitational constant,
c is the speed of light in a vacuum.

Up to a numerical factor, such a combination is unique, so it is considered a natural unit of length. It is part of the Planck system of units.

Numerically, the Planck length is[1]

$$\ell_P = 1.616255(18) \cdot 10^{-35}\text{m}$$

The last two digits in brackets denote the uncertainty (standard deviation) of the last two digits.

The Planck length (and the Planck time associated with it) define the scales at which modern physical theories stop working: the geometry of spacetime predicted by general relativity loses its meaning at distances of the order of the Planck length due to quantum effects. Natural phenomena at these scales should be adequately described by a theory that combines general relativity and quantum mechanics — quantum gravity.

2 Theoretical Relevance

The Planck length is the length scale at which quantum gravity becomes relevant. The Planck length is approximately the size of a black hole, where quantum and gravitational effects are on the same scale: the Compton wavelength and the Schwarzschild radius are the same.

The uncertainty principle $\Delta r_s \Delta r \geq \ell_P^2$, where r_s is the gravitational radius, r is the radial coordinate, and ℓ_P is the Planck length, will have to play a major role in quantum gravity.[2] This uncertainty principle is another form of the Heisenberg uncertainty principle between momentum and position applied to the Planck scale. Indeed, this relation can be written as follows: $\Delta(2Gm/c^2)\Delta r \geq G\hbar/c^3$, where G is the gravitational constant, m is the mass of the body, c is the speed of light, $\hbar$ is the reduced Planck constant. By canceling the same constants on both sides, we obtain the Heisenberg uncertainty principle $\Delta(mc)\,\Delta r \geq \hbar/2$. In relativistic physics, in a frame of reference at rest relative to a micro-object, there is a minimum measurement error of its coordinates $\Delta r \sim \hbar/mc$. This error corresponds to the uncertainty of the momentum $\Delta p \sim mc$, corresponding to the minimum threshold energy for the formation of a particle-antiparticle pair, as a result of which the measurement process itself loses its meaning.

The uncertainty principle $\Delta r_s \Delta r \geq \ell_P^2$ predicts the appearance of virtual black holes and wormholes (quantum foam) at the Planck scale.[2][3][4]

Proof: the equation for the invariant interval dS in the Schwarzschild solution is

$$dS^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - r_s/r} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2)$$

We substitute, according to the uncertainty relation $r_s \approx \ell_P^2/r$. We get[2]

$$dS^2 \approx \left(1 - \frac{\ell_P^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \ell_P^2/r^2} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2)$$

It is seen that on the Planck scale $r = \ell_P$ the invariant interval dS in special and general relativity is limited from below by the Planck length (division by zero appears), which means the formation of real and virtual Planck black holes.

The space-time metric $g_{00} = 1 - \Delta g \approx 1 - \ell_P^2/(\Delta r)^2$ fluctuates and generates quantum foam. These fluctuations $\Delta g \sim \ell_P^2/(\Delta r)^2$ in the macrocosm and in the world of atoms are very small compared to 1 and become noticeable only at the Planck scale. Lorentz invariance is violated at the Planck scale. The formula for fluctuations of the gravitational potential $\Delta g \sim \ell_P^2/(\Delta r)^2$ agrees with the Bohr-Rosenfeld uncertainty relation $\Delta g\,(\Delta r)^2 \gtrsim \ell_P^2$.[5][6] Because of the smallness of $\ell_P^2/(\Delta r)^2$, the formula for the invariant interval dS in special relativity is always written in the Galilean metric $(+1, -1, -1, -1)$, which is not actually true. The correct formula must take into account the fluctuations of the space-time metric Δg and the presence of virtual black holes and wormholes (quantum foam) at Planck-scale distances. Ignoring this fact leads to ultraviolet divergences in quantum field theory.[7][8] Quantum fluctuations in geometry are superimposed on the large-scale slowly varying curvature predicted by classical deterministic general relativity. Classical curvature and quantum fluctuations coexist with each other.[3]

Any attempt to probe the possible existence of shorter distances by collisions at higher energies will inevitably lead to the formation of black holes. Collisions at higher energies will not break matter into smaller pieces, but will simply produce larger black holes. [9] Decreasing Δr will lead to increasing Δr_s and vice versa. Further increasing the energy will lead to the appearance of larger black holes with worse, not better, resolution. Therefore, the Planck length is the minimum distance that can be probed.[13]

Planck black holes with a mass of 10^{-5}g may not "evaporate", but be stable formations - maximons ($\Delta r_s > 0$).[8] The entire mass of the black hole will "evaporate"[10] except for that part of it that is associated with the energy of zero-point, quantum oscillations of the black hole matter. Such oscillations do not increase the temperature of the object and their energy cannot be radiated.[11] An alternative to this process could be the "evaporation" of macroscopic black holes to Planck sizes, and then their disappearance into a sea of virtual black holes.[12]

The Planck length imposes practical limits on current physics. To measure Planck-length distances, one would need a particle with a Planck energy about four quadrillion times greater than the Large Hadron Collider can give it.[14]

3 Relationship between Compton wavelength and Schwarzschild radius

A particle of mass m has a reduced Compton wavelength

$$\bar{\lambda}_{\mathrm{C}} = \frac{\lambda_{\mathrm{C}}}{2\pi} = \frac{\hbar}{mc}$$

On the other hand, the Schwarzschild radius of the same particle is

$$r_s = \frac{2Gm}{c^2} = 2\,\frac{G}{c^3}\,mc$$

The product of these quantities is

$$r_s\bar{\lambda}_{\mathrm{C}} = 2\,\frac{G}{c^3}\,\hbar = 2\ell_P^2$$

4 Planck Length and Euclidean Geometry

The gravitational field undergoes zero-point oscillations, and the geometry associated with it also oscillates. The ratio of the circumference to the radius oscillates around the Euclidean value: the smaller the scale, the greater the deviations from Euclidean geometry. Let us estimate the order of the wavelength of zero-point gravitational oscillations at which the geometry becomes completely unlike Euclidean.[15] The degree of deviation of *zeta* geometry from Euclidean in the gravitational field is determined by the ratio of the gravitational potential φ and the square of the speed of light c : $\zeta = \varphi/c^2$. When $\zeta \ll 1$, the geometry is close to Euclidean; at $\zeta \sim 1$ all similarity disappears. The energy of oscillation of scale l is equal to $E = \hbar\nu \sim \hbar c/l$ (c/l is the order of the oscillation frequency). The gravitational potential created by mass m at such a length is $\varphi = Gm/l$, where G is the constant of universal gravitation. Instead of m we should substitute the mass, which, according to Einstein's formula, corresponds to the energy E ($m = E/c^2$). We get $\varphi = GE/l\,c^2 = G\hbar/l^2c$. Dividing this expression by c^2, we obtain the deviation value $\zeta = G\hbar/c^3l^2 = \ell_P^2/l^2$. Equating $\zeta = 1$, we find the length at which Euclidean geometry is completely distorted. It is equal to the Planck length $\ell_P = \sqrt{G\hbar/c^3} \approx 10^{-35}m$.

As noted by Regge (1958), "for a space-time region of size l, the uncertainty of the Christoffel symbols $\Delta\Gamma$ must be of the order of ℓ_P^2/l^3, and that of the metric tensor Δg of the order of ℓ_P^2/l^2. If l is a macroscopic length, the quantum constraints are fantastically small and can be neglected even on atomic scales. If the value of l is comparable to

ℓ_P, then the content of the former (ordinary) concept of space becomes more and more difficult and the influence of microcurvature becomes obvious [16] Hypothetically, this could mean that space-time becomes quantum foam at the Planck scale.[17]

5 Planck Length and Einstein's equation

The uncertainty relation between the gravitational radius and the Compton wavelength of a particle is a special case of the general Heisenberg uncertainty relation on the Planck scale

$$\Delta R_\mu \Delta x_\mu \geq \ell_P^2 \tag{5.1}$$

where R_μ is a component of the radius of curvature of a small region of spacetime; x_μ is the conjugate coordinate of the small region.

In fact, the indicated uncertainty relations can be obtained based on Einstein's equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{5.2}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}$ is the Einstein tensor, which combines the Ricci tensor, scalar curvature and metric tensor, $R_{\mu\nu}$ - Ricci tensor, obtained from the spacetime curvature tensor R_{abcd} by convolving it over a pair of indices, R is the scalar curvature, that is, the convoluted Ricci tensor, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant, and $T_{\mu\nu}$ is the energy-momentum tensor of matter, π is pi, c is speed of light in vacuum, G is Newton's gravitational constant.

In this form, the essence of the right side of Einstein's equations (5.2) is greatly obscured. It is advisable to rewrite these equations by grouping the constants into separate factors that have a specific meaning

$$\left(\frac{1}{4\,\pi}\right)(G_{\mu\nu} + \Lambda g_{\mu\nu}) = 2\left(\frac{G}{c^3}\right)\left(\frac{1}{c}\;T_{\mu\nu}\right) \tag{5.3}$$

A simple rearrangement of the factors allows us to gain deeper insight into the physical nature of the phenomenon. It is known that the factor $(1/c)\,T_{\mu\nu}$ is associated with the density and flow of energy-momentum of matter,[20] and with the help of the factor (G/c^3) you can make the transition to the Planck scale, since the same factor is present in the expression for the Planck length $\ell_P = \sqrt{(G/c^3)\,\hbar}$.

When deriving his equations, Einstein assumed that physical space-time is Riemannian, that is, curved. A small region of Riemannian space is close to flat space.

Example: if you cut out a small enough area from a sphere, the geometry will be imitated by Euclidean geometry. A similar technique—isolating the simplest from a more complex geometry (in this case, Euclidean geometry) by isolating a small part of the total space (here a sphere)—is a very common technique. Using the example of a sphere, it becomes clear that with a decrease in curvature or an increase in size, the surface locally approaches Euclidean space. Locally - in the small - the sphere can be approximated by part of the plane; globally - as a whole - impossible. This approximation is also realized in a more general case, when all curvature components decrease.[18]

For any tensor field $N_{\mu\nu\ldots}$ the value $N_{\mu\nu\ldots}\sqrt{-g}$ can be called the tensor density, where g is the determinant of the metric tensor $g_{\mu\nu}$. When the region of integration is small, $\int N_{\mu\nu\ldots}\sqrt{-g}\,d^4x$ is a tensor. If the region of integration is not small, then this integral will not be a tensor, since it represents the sum of tensors given at different points and, therefore, is not transformed according to any simple law when transforming coordinates.[19] Only small areas are considered here. The above is also true when integrating over the three-dimensional hypersurface S^ν.

Thus, Einstein's equations for a small region of pseudo-Riemannian spacetime can be integrated over the three-dimensional hypersurface S^ν.

$$\frac{1}{4\pi}\int (G_{\mu\nu} + \Lambda g_{\mu\nu})\sqrt{-g}\,dS^\nu = 2\left(\frac{G}{c^3}\right)\frac{1}{c}\int T_{\mu\nu}\sqrt{-g}\,dS^\nu \tag{5.4}$$

Since the integrable region of spacetime is small, that is, it is practically flat, from (5.4) we obtain the tensor equation

$$R_\mu = \frac{2G}{c^3}P_\mu \tag{5.5}$$

where $P_\mu = \frac{1}{c}\int T_{\mu\nu}\sqrt{-g}\,dS^\nu$ is the 4-pulse component matter; $R_\mu = \frac{1}{4\pi}\int (G_{\mu\nu} + \Lambda g_{\mu\nu})\sqrt{-g}\,dS^\nu$ is a component of the radius of curvature of a small region of space-time.

The resulting tensor equation (5.5) can be rewritten in another form. Since $P_\mu = mc\,U_\mu$ then

$$R_\mu = \frac{2G}{c^3}\,P_\mu = \frac{2G}{c^3}mc\,U_\mu = r_s\,U_\mu \tag{5.6}$$

where r_s is the Schwarzschild radius (invariant of the radius of curvature), U_μ is the 4-speed, m is the gravitational mass. This entry reveals the physical meaning of the quantity R_μ, as the μ-component of the Schwarzschild radius. Note that here $R_\mu R^\mu = r_s^2$ (compare, for example, with $dx_\mu dx^\mu = dS^2$).

Here the expression for the gravitational radius $r_s = 2\,(G/c^3)mc$ is a more convenient form of notation than the form $r_s = 2\,(G/c^2)m$. In this case, continuity is visible between the resulting tensor equation (5.5) and the expression for the gravitational radius of a massive body. This happens due to the presence of the (G/c^3) multiplier.

For a static spherically symmetric field and a static matter distribution we have $U_0 = 1, U_i = 0\,(i = 1, 2, 3)$. In this case we get

$$R_0 = \frac{2G}{c^3}mc\,U_0 = \frac{2G}{c^3}\,mc = r_s \tag{5.7}$$

In a small region, space-time is practically flat and the tensor equation (5.5) can be written in operator form

$$\hat{R}_\mu = \frac{2G}{c^3}\hat{P}_\mu = \frac{2G}{c^3}(-i\hbar)\frac{\partial}{\partial x^\mu} = -2i\,\ell_P^2\frac{\partial}{\partial x^\mu} \tag{5.8}$$

where $\hbar$ is the Dirac constant. Then the commutator of the operators $\hat{R}_\mu$ and $\hat{x}_\mu$ is equal to

$$[\hat{R}_\mu, \hat{x}_\mu] = -2i\ell_P^2 \tag{5.9}$$

This implies the above uncertainty relations

$$\Delta R_\mu \Delta x_\mu \geq \ell_P^2 \tag{5.10}$$

Substituting into (5.10) the values $R_\mu = \frac{2G}{c^3}P_\mu$ and $\ell_P^2 = \frac{\hbar G}{c^3}$ and canceling the same constants on the right and left, we arrive at the Heisenberg uncertainty relations.

$$\Delta P_\mu \Delta x_\mu \geq \frac{\hbar}{2} \tag{5.11}$$

Note that now, according to the equation $R_\mu = (2G/c^3)P_\mu$, along with the expressions for energy-momentum quanta $P_\mu = \hbar\, k_\mu$ the expressions for the quantity $R_\mu = 2\ell_P^2\, k_\mu$ are valid (but not spacetime quanta), where k_μ is the wave 4-vector. That is, the quantity R_μ (component of the Schwarzschild radius) is quantized, but the quantization step is extremely small.

For a static spherically symmetric field and a static distribution of matter, the found uncertainty relation takes the form

$$\Delta R_0 \Delta x_0 = \Delta r_s \Delta r \geq \ell_P^2 \tag{5.12}$$

where r_s is the Schwarzschild radius, r is the radial coordinate. Here $R_0 = r_s$, and $x_0 = c\,t = r$, since at the Planck level matter moves at the speed of light.

For vacuum at the Planck level, the last uncertainty relation $\Delta r_s \Delta r \geq \ell_P^2$ will be characteristic, since a state of motion or a velocity vector cannot be assigned to vacuum. In Minkowski space, due to its high symmetry, vacuum is the same state for all inertial frames of reference; in any frame of reference it will appear to be at rest (static). Therefore, the Planck vacuum, according to the specified uncertainty relation, will generate wormholes and tiny virtual black holes (quantum foam).

From equations (5.5) and (5.8) it is clear that the basic equation of the quantum theory of gravity (Klimets equation)[1] should have the following form (similar to the Schrodinger equation)[2][20][21]

$$\boxed{-2i\ell_P^2 \frac{\partial}{\partial x^\mu} |\Psi(x_\mu)\rangle = \hat{R}_\mu |\Psi(x_\mu)\rangle} \tag{5.13}$$

In equation (5.13), spatial and temporal coordinates have equal rights. The $\hat{R}_\mu$ operator acts as a generator of infinitesimal displacements of quantum states. Its form depends on the specific situation. To be continued...

6 Quantization of space and the Planck length

In the mid-20th century, the hypothesis of quantization of space-time[22] on the way to unifying quantum mechanics and general relativity led to the assumption that there are space-time cells with a minimum possible length equal to the fundamental length.[23] According to this hypothesis, the degree of influence of space quantization on transmitted light depends on the size of the cell. For research, intense radiation that has traveled as great a distance as possible is needed. The flow of electromagnetic radiation (photons) from point objects (stars, galaxies), before reaching the observer, must repeatedly "overcome" the scale of Planck time, as a result of which its speed will change slightly, so that the image of the object will be distorted. And the further the object is located, the more such distortions, caused by the "cellular" nature of space and time, will accumulate by the time its light reaches the observer on Earth. This effect will lead to a "smearing" of the image of the object. Now a group of scientists has used data from a survey of the gamma-ray burst GRB 041219A carried out by the European space telescope INTEGRAL (observatory)—Integral. The gamma-ray burst GRB 041219A was among the one percent of the brightest gamma-ray bursts over the entire observation period, and the distance to its source is at least 300 million light years. Integral's observations made it possible to limit the cell size from above by several orders of magnitude more accurately than all previous experiments of this kind. Data analysis showed that if spatial granularity exists at all, it should be at the level of 10^{-48} meters or less.[24] It turned out that no "smearing" of the images of objects could be detected at all. The images of objects turned out to be completely sharp. This contradicts the hypothesis of the quantum nature of space-time on a microscale. Perhaps there should be no fuzzy images of distant objects at all. It is, of course, too early to talk about the complete discrediting of the theory of quantization of space and time. Theorists have at least two options for explaining this strange fact. The first option is based on the fact that at the micro level - on the Planck scale - space and time vary simultaneously with each other, so that the speed of photon propagation does not change. The second explanation assumes that the velocity inhomogeneities are determined not by the Planck length, but by its square (of the order of $10^{-66}cm^2$, so that these inhomogeneities become immeasurably small.[25] The second option is consistent with Sections 1-3 of this article. Indeed, in a gravitational field, the coordinate speed of light changes, as a result of which light rays are curved. If we denote by c the physical speed of light at the origin, then the coordinate speed of light c_c at some place with gravitational potential φ will be equal to $c_c \approx c(1 + \varphi/c^2)$. But then, as was shown above, on the Planck scale $c_c \approx c(1 - \ell_P^2/l^2)$. That is, the fluctuations of the speed of light $\Delta c \approx c\ell_P^2/l^2$ are determined not by the Planck length, but by the square of the Planck length and are therefore immeasurably small. For example, if the wavelength of visible light is $\lambda \approx 10^{-5}cm$, then in this case the ratio $\ell_P^2/\lambda^2 = 10^{-66}/10^{-10} = 10^{-56}$ will be less than the ratio $\ell_P/\lambda = 10^{-33}/10^{-5} = 10^{-28}$ [?]y 28 orders of magnitude. Therefore, images of distant stars and galaxies are perfectly sharp even at metagalactic distances.

From the picture of space-time foam presented by Wheeler,[26] it follows that for photons with a wavelength λ propagating in the foam, the travel time T from the source to the detector must be indefinite in accordance with the law , which can only depend on the distance traveled x, the wavelength of the particle λ and the Planck scale ℓ_P with a shape of type $\delta T \sim x^n \ell_P^{1+m-n}/\lambda^m$, where m and n are model-dependent powers, and $1 + m - n > 0$. The phenomenology of quantum gravity currently focuses mainly on effects suppressed at the first power of the Planck scale, since stronger suppression leads to even weaker effects.[1] Therefore, the picture that experimenters are now focusing on corresponds to the following choice: $n = m = 1$, that is, $\delta T \sim x\,\ell_P/\lambda$.

From a modern point of view, the hypothesis[37] of the quantization of spacetime is unsatisfactory. In fact, from Einstein's equations, as has been shown, the quantization of the curvature of spacetime (quantization of the Schwarzschild radius) follows. In accordance with this, the dispersion of light rays from distant galaxies is determined not by the Planck length, but by its square, $n = 1; m = 2$ and $\delta T \sim x\,\ell_P^2/\lambda^2$, therefore, fluctuations in the speed of light will be immeasurably small and images of distant sources will be sharp even at metagalactic distances.[24]

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