The Significant Gaps and Errors in the Interpretations of Special Relativity

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Abstract

This article critically examines the commonly overlooked gaps and errors in the interpretations of the Special Theory of Relativity (STR), particularly in the treatment of the external and internal observers, the role of the Lorentz transformation, and the misapplication of moving reference frames. The core argument revolves around the objective versus subjective observations within STR, emphasizing the pivotal role of an external observer who perceives spacetime as a whole, in contrast to the limited perspective of internal observers. The discussion further extends to mathematical misinterpretations, such as the dimensional alignment of time and space, the improper handling of inertial frames, and inconsistencies in the application of Newtonian mechanics within relativistic contexts. These gaps underscore the need for a refined conceptual framework in interpreting STR, paving the way for a more rigorous understanding of relativistic dynamics and its implications for modern physics.

A very serious shortcoming in various works on the special theory of relativity (STR) is the lack of a definition for an external observer, although his "services" (e.g., his observations) are frequently utilized. In STR, the definition of an external observer is as follows: **The external observer is situated outside spacetime and "sees" the entire spacetime instantaneously, meaning they are not subject to the limitation of the finite speed at which information about the state of spacetime is obtained**. (*Note: In the case of quantum entanglement, we are also dealing with the immediate transmission of information outside space-time*).

Only an external observer has the ability to perform a Lorentz transformation that transforms the entire spacetime into another spacetime, because only they have access to the entire spacetime. For example, when we draw a model of two-dimensional spacetime on a sheet of paper and perform various operations on it, including a Lorentz transformation, we act as external observers. On the other hand, an internal observer is an observer located inside the spacetime. Thus, in our model of two-dimensional spacetime, the internal observer is a resident of the sheet of paper. Meanwhile, we, as inhabitants of the Universe, are internal observers of the four-dimensional spacetime in which we live.

The external observer is an objective observer and is the appropriate person to discover objective effects related to STR. It must be understood that science, especially physics, deals with uncovering the objective laws of Nature. Subjective (local) observations often lead to false conclusions. A good example of this is the geocentric model of our Solar System, which was the result of subjective observations. It was only Copernicus who managed to "detach" himself from Earth and look at the Solar System from the outside, in an objective way. He sketched it on a piece of paper (on parchment) just as we analyze the model of two-dimensional spacetime on a sheet of paper.

In popular works on STR, Einstein's thought experiments involving a train traveling at relativistic speed are often presented. Based on these experiments, he concluded that in a moving frame of reference, time flows more slowly than in a stationary frame. (*Note! The correct way to phrase this is: A moving clock measures time more slowly than a stationary clock. Further on, there will be an explanation of why the former phrasing is incorrect*). This slowing down is consistent with the Lorentz factor. These conclusions from the thought experiment were the result of observations made by an external observer. However, Einstein himself and other popularizers of STR claim that these are observations made from within spacetime. Not at all! This is a clear misrepresentation. The diagrams they draw, where they illustrate the path of light rays, are drawn and analyzed from the perspective of an external (objective) observer. They are outside the spacetime model, outside the sheet of paper. And in such an analysis, we are indeed dealing with the objective slowing down of a moving clock in accordance with the Lorentz factor.

Let's consider what the internal observer standing on the platform, for example, sees when observing the clock placed on the roof of a train moving at relativistic speed. Let's analyze the case of a train moving at, say, 0.6c (where the Lorentz factor is 1.25). Assume we are using atomic (optical) clocks, which are synchronized with a certain frequency of light. Thus, the ticking of the clocks occurs according to that frequency. When the train approaches the platform, the observer on the platform will notice that the clock on the train is ticking twice as fast as the clock on the platform, because from the Doppler formula for light, they will see the ticking frequency of that clock as twice the frequency of the platform clock. However, when the train begins to move away, they will observe that the same clock is ticking twice as slowly as their clock. The observer on the train will notice a similar effect. As they approach the platform, they will see that the clock on the platform is running twice as fast as their own clock, but when they start moving away, they will observe it ticking twice as slowly. This is entirely different from what the Lorentz factor suggests. The slowing down of the moving clock according to the Lorentz factor is an objective phenomenon, and only an external (objective) observer is capable of making such observations.

Let's now focus on the two-dimensional model of spacetime, which we will draw on a sheet of paper. We draw a Cartesian coordinate system, where we place the time axis horizontally and label it with the letter $\boldsymbol{T}$. Here, an important note immediately comes to mind. In all interpretations of STR, the time axis is usually drawn as a vertical axis. This is puzzling because in all graphs representing the progression of various phenomena and physical parameters over time, the time axis is always shown as horizontal. Everyone is accustomed to this, and the sudden change of the time axis from horizontal to vertical may cause some discomfort and difficulties in approaching STR intuitively.

On the other hand, we will draw the spatial axis in our two-dimensional spacetime model as vertical and label it with the letter $\boldsymbol{Y}$, following convention. (We will denote the spatial axis in the three-dimensional spacetime model as $\boldsymbol{X}$, representing the axis running away from us, perpendicular to both the $\boldsymbol{Y}$ and $\boldsymbol{T}$ axes). It is often assumed in works on STR that the speed of light is dimensionless and equals 1. Consequently, the velocities of massive point-like objects must also be dimensionless and fall within the right-open interval $\boldsymbol{\langle 0\,,1)}$. In this case, the time axis must have the same units as the spatial axis. If we assume that the unit of the $\boldsymbol{Y}$ axis is meters, then we must also scale the $\boldsymbol{T}$ axis in meters. (Under these conditions, one second is a distance measure equal to $299\,792\,458\,m$). In this way, we obtain a homogeneous, flat metric space. Material points trace worldlines in our model, which are described by functions in the form $\boldsymbol{y = f(t)}$, and the velocity $\boldsymbol{v}$ of a given point at time $\boldsymbol{t}$ is defined as the derivative:

$$\boldsymbol{v(t) = \frac{df(t)}{dt}} \tag{1}$$

At this point, let's make a digression about time. Time is a very important parameter in physics, especially in STR. Therefore, it is surprising that such a crucial parameter has not been precisely defined. This is yet another significant gap in the interpretation of STR. In STR, we deal with the concept of time, which is attributed to two different categories:

1. Time as one of the spacetime coordinates, with special significance. In this

approach, time as a coordinate has spatial properties and is relative, because the Lorentz transformation changes the value of the time coordinate of individual points, just as it changes the values of the spatial coordinates. The time axis in spacetime is usually denoted by the capital letter $\boldsymbol{T}$

2. Time as a proper (individual) parameter of material points moving in spacetime. Proper time has no spatial properties, it is a scalar that continually increases and is invariant under Lorentz transformations. In this approach, time is not relative, although it may pass differently for each massive point.

When we assign two fundamentally different (opposing) categories to the same concept (time), it's no surprise that difficulties arise in defining it. On one hand, time is assumed to have spatial properties (meaning it behaves like a vector that can be both positive and negative) and is relative, as the Lorentz transformation rescales the time coordinate. On the other hand, proper time is a scalar that continuously increases and is invariant under Lorentz transformations (in all reference frames, it uniquely defines an object's position on its worldline).

Due to these differences in the two approaches, it is justified to denote the time axis in our model of metric spacetime with a capital letter $\boldsymbol{U}$. Thus, the time coordinate of points will be denoted by a lowercase letter $\boldsymbol{u}$, while the proper time of material points will still be denoted by a lowercase letter $\boldsymbol{t}$. Through this maneuver, spacetime as a combination of two different types of dimensions disappears, and we obtain a two-dimensional, flat, homogeneous, metric space, so there is no need to call it spacetime. Instead, it can be called U-space, due to the unique significance of the dimension $\boldsymbol{u}$. Our model of U-space is presented in Figure 1 below. With these changes, the velocity of massive point objects will be defined by the following formula:

$$\boldsymbol{v(u) = \frac{df(u)}{du}} \tag{2}$$

Nowhere in the interpretations of STR has it been emphasized that by adopting the speed of light as one, we obtain a flat, homogeneous metric space. Therefore, using the concept of spacetime in this case is a mistake.

If the concept of "time" is reserved exclusively for the proper time of massive point objects, we can provide a very simple mathematical formula that will define time:

$$\boldsymbol{t(u) = \int_{u_0}^{u} \sqrt{1 - |v(u)|^2}\, du} \tag{3}$$

Where $\boldsymbol{u_0}$ is the point where the worldline of a given massive point object begins. It should be noted that, according to this definition, time has the same

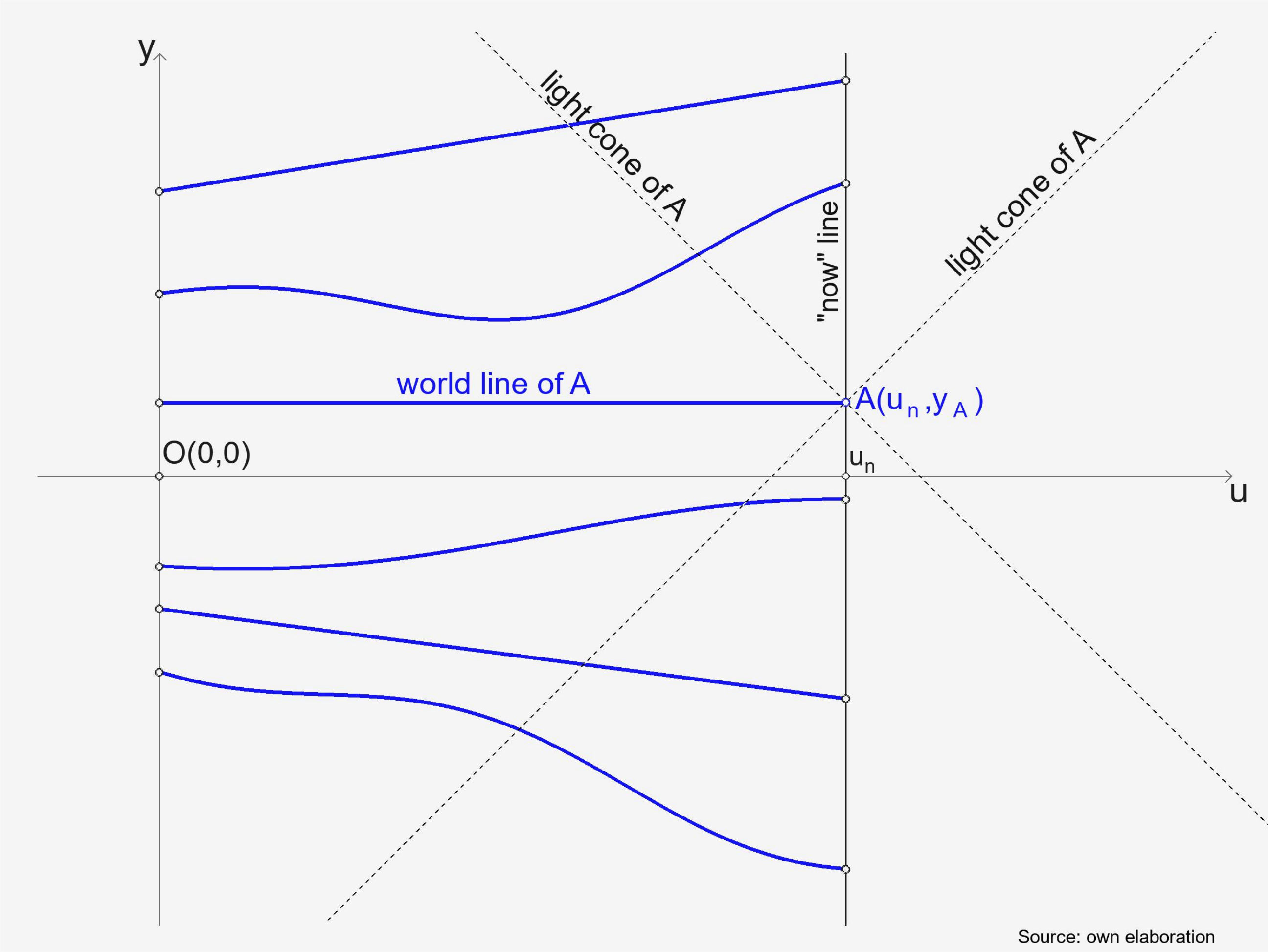

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Figure 1

dimension as the coordinate $\boldsymbol{u}$, meaning the unit of time is a unit of distance, such as meters. As inhabitants of U-space, we do not have direct access to the $\boldsymbol{u}$ dimension and cannot measure it with distance measuring instruments, but we can measure it with clocks. One second corresponds to a distance of 299 792 458 meters.

> *Note: As internal observers, we essentially deal exclusively with proper time, which continually increases as we move along the U-axis. Considering our understanding of time and distance, we can assume that each observer in their (private) reference frame always moves along the U-axis at the same speed of light. Here, we encounter a kind of Copernican problem: light (light rays) is stationary, which is why time does not pass for light, while we move at the speed of light, even though it seems to us that we are at rest. (A light ray is stationary in the sense that in all reference frames it is always at the same 45-degree angle to the U-axis and is fixed at the point from which it was emitted).*

Let us now focus on the Lorentz transformation, which is the foundation of STR. A serious oversight by physicists (including Einstein) is the lack of a clear definition of the Lorentz transformation vector. This vector should have its own special notation; for example, let's denote it by the symbol $\vec{\boldsymbol{w}}$, in contrast to the symbol $\vec{\boldsymbol{v}}$, which we use to denote the velocities of massive point objects, and the symbol $\vec{\boldsymbol{a}}$ for their acceleration. The vector $\vec{\boldsymbol{w}}$ represents the velocity of a hypothetical object that, after applying the Lorentz transformation with this vector, will be at rest. Therefore, the Lorentz transformation does not have to apply to any specific physical object with velocity $\vec{\boldsymbol{v}}$.

Below are the Lorentz transformation formulas for the two-dimensional U-space. They differ slightly from the formulas typically given for spacetime because, due to the assumption that the speed of light equals 1, the parameter $\boldsymbol{c}$ does not appear, making the formulas more straightforward. (*Note: In our two-dimensional U-space model, we do not need to use vector notation with an arrow, as all vectors in our model are one-dimensional*).

$$\boldsymbol{u'} = \frac{\boldsymbol{u - yw}}{\sqrt{\boldsymbol{1 - w^2}}} \tag{4}$$

$$\boldsymbol{y'} = \frac{\boldsymbol{y - uw}}{\sqrt{\boldsymbol{1 - w^2}}} \tag{5}$$

$$\boldsymbol{v'} = \frac{\boldsymbol{v - w}}{\boldsymbol{1 - vw}} \tag{6}$$

$$\boldsymbol{a'} = \boldsymbol{a}\left(\frac{\sqrt{\boldsymbol{1 - w^2}}}{\boldsymbol{1 - vw}}\right)^{\boldsymbol{3}} \tag{7}$$

In STR, the Lorentz transformation is the only permissible transformation for switching between reference frames. In STR, the Galilean transformation cannot be applied! However, physicists openly ignore this prohibition by using the concept of moving reference frames (inertial and non-inertial) in works related to STR. Moving frames are defined in the Galilean-Newtonian space, where absolute time exists, and switching between moving reference frames is only possible through the Galilean transformation. Of course, in situations where non-relativistic speeds are involved, physicists use the Galilean transformation to define moving reference frames (inertial and non-inertial), such as those associated with an elevator, a train carriage, a car, or a space station. In these cases, the error introduced by using the Galilean transformation is insignificant.

In contrast, in STR, there is no possibility of defining moving reference frames (inertial and non-inertial) because the Lorentz transformation does not change the position of the coordinate origin in the sense that both reference frames are "anchored" at the same point in spacetime (U-space), meaning their coordinate origins coincide. Therefore, reference frames in STR are "bound" to each other at this point, and it is not possible to think of them as moving separately, as in the case of the Galilean transformation. Thus, saying that STR involves moving reference frames is a significant error. In STR, we are only dealing with moving objects (observers). Each moving observer has their own rest frame, which they can share only with objects that are also at rest in their rest frame.

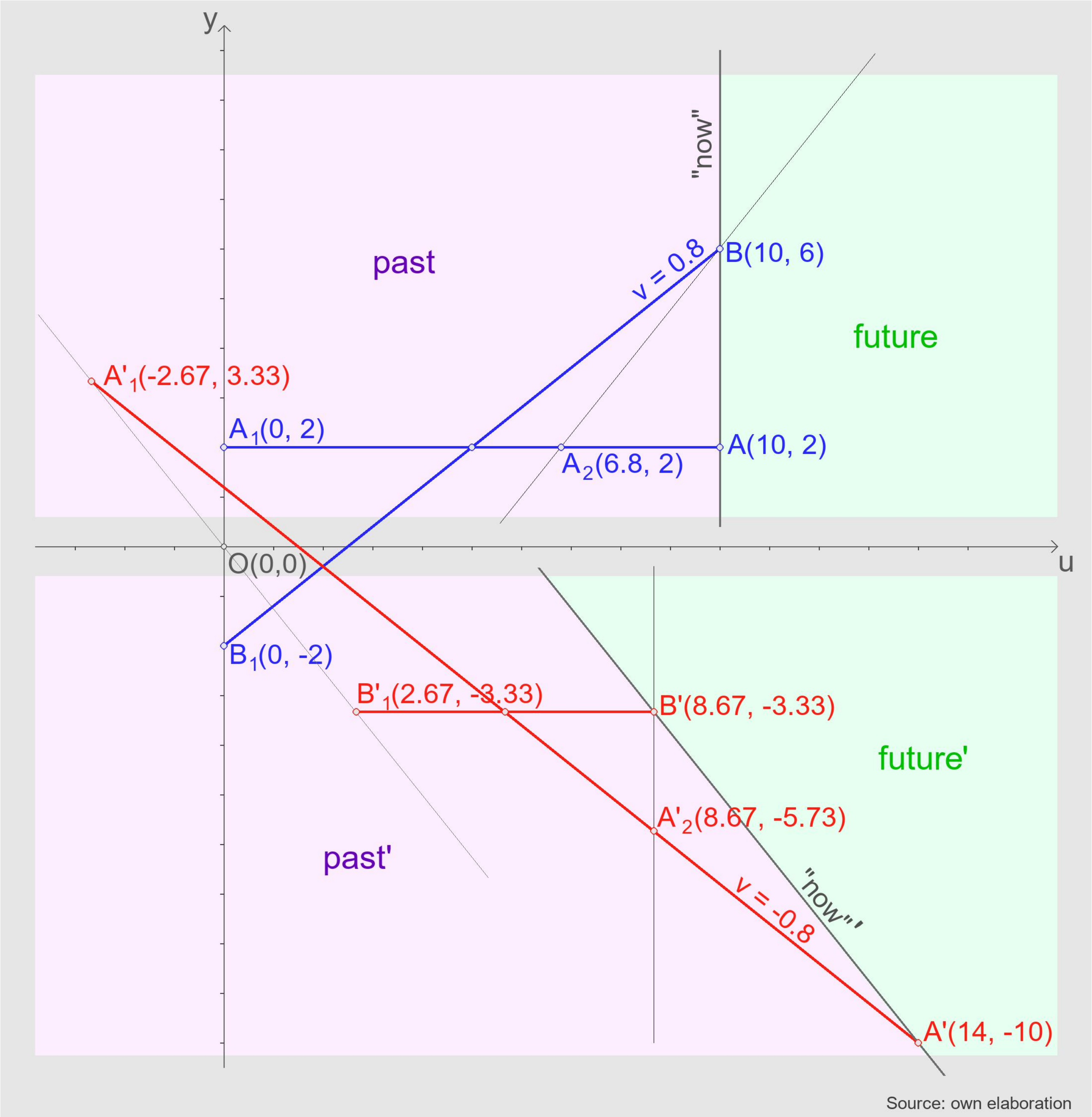
In all moving reference frames (originating from the absolute space-time of Galileo-Newton), the relative velocity between two objects is the same. However, in STR, the relative velocity varies in different Lorentz reference frames and is limited to $\boldsymbol{2c}$. Only in the rest frames of these two objects does it have the same absolute value and is limited to $\boldsymbol{c}$. This is yet another reason that makes it impossible to define moving reference frames in STR.

It is also necessary to define what an inertial and a non-inertial object is. An inertial object (observer) is one that moves along a straight segment of its worldline (an object at rest is also an inertial object moving along a straight worldline). On the other hand, a non-inertial object (observer) is one that moves along a curved segment of its worldline. The straight (curved) worldline of an object is straight (curved) in all other reference frames.

Another serious omission by popularizers of STR is the lack of a definition for the "now" space (Einstein also did not provide such a definition). Instead, for each reference frame, a so-called hyperplane of simultaneity is defined, where all points on this hyperplane share the same time coordinate (in our U-space model, they have the same $\boldsymbol{u}$ coordinate). According to this definition, the hyperplane of simultaneity in a given reference frame is perpendicular to the time axis (to the U-axis). However, the only logical and intuitive definition of the "now" space is as follows: The "now" space is the hypersurface that divides spacetime (U-space) into the past (history) and the future.

One must realize that only an external observer has access to the "now" space, while internal observers have only a single-point contact with this space, as all objects and the surrounding reality interact solely through the light cone directed into the past. Therefore, they have no contact with their surrounding "present." In our two-dimensional U-space model (Figure 1), which represents a specific reference frame, we can define the "now" space as a line perpendicular to the U-axis at the point $\boldsymbol{u_n}$ (u-now).

In order for time to pass in our two-dimensional U-space model, the "now" space must move towards increasing values of the $\boldsymbol{u}$ coordinate. Therefore, we, as the creators of this model, must manually shift the "now" line to the right. We cannot move it back even slightly (to the left), as that would violate the cause-and-effect principle. However, the most important point is that the source of time (movement) in our model lies outside U-space (outside spacetime). (*Note: Let’s set aside for now the question of who or what moves our "present" toward increasing values of the* $\boldsymbol{u}$ *coordinate in our universe*).



Source: own elaboration

Figure 2

Let’s perform a Lorentz transformation on our model using any arbitrary vector w. After the transformation, it turns out that the "now" line, which separates the past (history) from the future, has been tilted at a certain angle to the U-axis (see Figure 2). This indicates that the "present" does not necessarily have to be a hyperplane where all points share the same $\boldsymbol{u}$ coordinate. The "present" can be a continuous, curved hypersurface (curved line) that meets one condition: the hyperplane (straight line) tangent to the "present" at any point must be at an angle greater than 45° with respect to the U-axis, because no parts of the surrounding "present" can be within any observer's light cone.

In our U-space model, the movement of the "present" by us, as the creators of the model, determines the passage of time for the internal observers. Thus, we can, for example, stop time in certain regions of the "present" by not moving it forward, while in other areas, we can continue making such shifts. We can even stop time across the entire U-space and resume the animation of our model the next day. Stopping the "present" in certain regions or across the entire U-space is not noticeable to the internal observers, as their experience of time depends solely on the movement of this space by us and progresses according to the increase in the $\boldsymbol{u}$ coordinate.

What is "the present" in our model of the two-dimensional U-space for us as the animators of this model? It is a line (not necessarily straight) on which, at a given moment of our "external" time, all the massive objects contained in U-space are located simultaneously. In this context, one could say that only objectively simultaneous events, which do not necessarily have the same value of the u-coordinate, are found in "the present." Therefore, the definition of simultaneity of events makes sense only from the point of view of an external (objective) observer, and we do not deal with the relativity of simultaneity here. However, from the perspective of internal observers, who do not have access to the surrounding "present," it makes no sense to speak of simultaneous events. Each internal observer has their own "private" light cone, through which they only see the past. (For example, when looking in a mirror, we see our past.) The past seen by the internal observer depends on the position they occupy in space. The light cones of two internal observers located, for instance, in neighboring rooms or on opposite sides of the world differ fundamentally.

In the context of Special Relativity, there is another myth. It is believed that spacetime is a static, four-dimensional structure in which all events (past, present, and future) are, in a sense, already "fixed." The movement of the "now" moment is not considered. For example, the block universe concept assumes that spacetime does not require animation—past, present, and future exist simultaneously, and our perception of movement through time is a subjective experience resulting from how our consciousnesses interpret our own time. This approach clearly defines the universe as deterministic, meaning that our belief in free will, according to this concept, is a fiction, and saying that "we are the masters of our own fate" is pointless. Nothing could be further from the truth!

The truth is this: The "now" space separates the past (the history) from the future. In the past, we deal with events that have already occurred. We cannot change anything in the past, just like in a recorded film. In the future, however, we deal with probable events that have not yet happened. The so-called superposition of quantum states is a set of probable events that may occur in the future. The wave function of particles contains these probable future states. In the "now" space, a selection (the so-called measurement) occurs. From the probable states of the future, individual events are chosen through a random selection process and placed in the past. Events that participated in the selection but were not chosen disappear irreversibly (time cannot be reversed). This is the so-called collapse of the wave function. Nature itself, in the "now" space, performs countless measurements (random selections). No observer is needed for this. One could put it this way: events that have not yet happened are represented by diffuse states of probability, and the "now" moment acts as a selection mechanism in which certain events become reality while others disappear. The future is not predetermined, but dynamically chosen.

We must realize that the "now" hypersurface exists objectively, even though we, as internal observers, have no idea what shape it takes. However, each of us has a physical "single-point" contact with it, and each of us physically feels its movement, that is, the passage of time. People try to predict the future with some degree of probability, but in reality, we don't know what will happen to us in a minute, a day, a month, a year, etc. The "present" is a tangible entity for us because we exist only in the "present."

In all scientific and popular-scientific works on STR, we encounter yet another serious omission. Little is mentioned, or sometimes nothing at all, about the consequences of STR for Newton's second and third laws of motion. Let's start with the third law of motion, which states that two bodies exert equal and simultaneous forces on each other. It turns out that the third law does not function in the case of remote interactions. (In nature, all interactions are remote.) When STR was announced, it became clear that interactions are one-sided due to the limited speed at which they propagate. For example, the Earth feels the gravitational force from the point where it "sees" the Sun. However, the Earth in its current position will pull on the Sun only in the future (in 8 minutes) with a force that will generally not equal the force with which the Sun is pulling on it at this moment. This is because, over the 16 minutes it takes for gravitational interaction to travel the distance from the Sun to the Earth and back, the distance between the Earth and the Sun changes. Additionally, there is the problem of in which frame of reference this distance should be measured. Should it be in the Sun's frame, the Earth's frame, or some other reference frame altogether? If, in the first case (the Sun acting on the Earth), Nature uses the Sun's reference frame to measure the distance, then in the second case (the Earth acting on the Sun), the Earth's reference frame must consistently be used. It's immediately clear what consequences this has, for example, for electromagnetic interactions. Looking at a proton-electron system when they move relative to each other at relativistic speeds, the mutual distances in these two reference frames can differ from each other by even several times.

The fact that the speed of interactions is limited is responsible, among other things, for the emergence of the so-called magnetic field in electric interactions. Given the analogy between electric and gravitational interactions, we should assume that in the case of the gravitational field, we should also observe effects that we can call "gravitomagnetic," and there is evidence for this. For example, in electric interactions, there is a phenomenon known as synchrotron radiation, which occurs in synchrotrons when electric charges orbit at relativistic speeds. We should expect a similar radiation in the gravitational version when two massive objects (e.g., black holes) orbit each other. When two black holes "collide" (in a spiral), just before merging into one black hole, they reach relativistic orbital velocities, and at that moment, these so-called gravitational waves, which are essentially the gravitational version of synchrotron radiation, become so intense that they are detected by observatories like LIGO and VIRGO, even over vast distances.

An experiment was also conducted that clearly confirmed the existence of Earth's gravitomagnetic field, which results from its rotational motion. On March 21, 2006, at the European Space Research and Technology Centre in the Netherlands, the European Space Agency announced the results of an experiment in which a superconducting disk was spun up to 6,500 revolutions per minute, and precise measurements showed an increase in the weight of the disk. The conclusion of this experiment was as follows: "The experiment demonstrated that a superconducting gyroscope is capable of generating a powerful gravitomagnetic field, and is therefore the gravitational equivalent of a magnetic coil. Although this is one hundred millionth of the acceleration caused by Earth's gravitational field, the measured field is astonishingly one hundred billion, billion times greater than predicted by Einstein's general theory of relativity."

Unfortunately, the experiment was not fully completed, as the disk was not spun in the opposite direction. In that case, the measurements should have shown that the weight of the disk decreased slightly by the same amount it had increased with the previous rotations. This experiment was conducted in the Netherlands (around **51°** north latitude), where the angle between the Earth's axis and the disk was about **39°**. The largest difference in weight between the stationary and rotating disk would have been recorded at the pole, where both gravitomagnetic coils, i.e., the disk and the Earth, would be coaxial. However, it is important to remember that in electrical interactions, like charges repel, whereas in gravitational interactions, they attract. Therefore, if such a disk were spun at the pole in the same direction as the Earth's rotation, an additional repulsive force would occur, and if spun in the opposite direction, an additional attractive force would arise.

Gravitomagnetic forces are also responsible for the formation of Saturn's extremely thin and wide rings, as well as those of other planets. Simply put, these gravitomagnetic forces press the material of the rings towards the plane of the equator. Of course, these rings must rotate in the opposite direction to the rotation of the parent planet, based on what we discussed earlier about the direction of forces acting on a rotating disk. So far, there have been several space missions in the region of Saturn, but no observations have been made that definitively identify the rotational direction of the rings beyond any doubt.

Let's now move on to Newton's second law of motion. The second law of motion states that if a force $\boldsymbol{\vec{F}}$ acts on a body of mass $\boldsymbol{m}$, the body moves with an acceleration $\boldsymbol{\vec{a}}$, which is directly proportional to the force and inversely proportional to the mass of the body. This law is expressed as follows:

$$\boldsymbol{\vec{a}} = \frac{\boldsymbol{1}}{\boldsymbol{m}}\boldsymbol{\vec{F}} \tag{8}$$

Physicists up until Einstein's time were puzzled by the fact that the mass from equation (8), also known as inertial mass, is equal to the gravitational mass from the universal law of gravitation. Thus, even in Newton's era, we were dealing with the equivalence principle. However, after the announcement of STR, the relativistic version of the second law of dynamics took the following form:

$$\boldsymbol{\vec{a}} = \frac{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}}{\boldsymbol{m_r}}\left[\boldsymbol{\vec{F}} - \frac{\boldsymbol{1}}{\boldsymbol{c^2}}\left(\boldsymbol{\vec{F}} \cdot \boldsymbol{\vec{v}}\right)\boldsymbol{\vec{v}}\right] \tag{9}$$

Where $\boldsymbol{m_r}$ denotes the rest mass of the body. Equation (9) can be rearranged and expressed as a relationship between the force vector, acceleration, and velocity:

$$\boldsymbol{\vec{F}} = \frac{\boldsymbol{m_r}}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}}\left(\boldsymbol{\vec{a}} + \frac{\boldsymbol{\vec{a}} \cdot \boldsymbol{\vec{v}}}{\boldsymbol{c^2} - |\boldsymbol{\vec{v}}|^2}\boldsymbol{\vec{v}}\right) \tag{10}$$

Equations (9) and (10) represent the relativistic form of the second law of dynamics.

Note: The expression $\frac{m_r}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}}$ *from equation (10) was called relativistic mass. According to some, this term was meant to replace the name "inertial mass" from Newton's era.*

Equation (9) can also be written as follows:

$$\frac{\vec{a}}{c^2} = \frac{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}}{m_r\, c^2}\left[\vec{F} - \left(\vec{F}\cdot\frac{\vec{v}}{c}\right)\frac{\vec{v}}{c}\right] \tag{11}$$

If we now denote the total energy of the body as $E_t = \frac{m_s c^2}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}}$ and apply the second law of dynamics to U-space, where we have assumed that the speed of light equals one, the velocity vector $\vec{v}$, defined by equation (2), is dimensionless and confined within a sphere of radius one, and the acceleration $\vec{a}(u) = \frac{d\vec{v}(u)}{du}$ has the dimension of $\left[\frac{1}{m}\right]$, then equation (11) will take the form:

$$\vec{a} = \frac{1}{E_t}\left[\vec{F} - \left(\vec{F}\cdot\vec{v}\right)\vec{v}\right] \tag{12}$$

Note: To convert acceleration expressed in $\left[\frac{m}{s^2}\right]$*, which is defined for spacetime, into acceleration expressed in* $\left[\frac{1}{m}\right]$ *in U-space, it must be divided by* c^2*.*

Equation (12) can be appropriately transformed to obtain the vector formula for force:

$$\vec{F} = E_t\left(\vec{a} + \frac{\vec{a}\cdot\vec{v}}{1-|\vec{v}|^2}\vec{v}\right) \tag{13}$$

Equations (12) and (13) represent the relativistic form of the second law of dynamics for U-space, assuming that the rest mass is a constant parameter.

A very important point that the proponents of STR often forget is the omission of the fact that, with the announcement of STR, the concept of "inertial mass" as the proportionality factor between the force vector and acceleration disappeared. In the relativistic formulas (9) and (10) of the second law of dynamics, such a proportionality factor cannot be identified. The concept of inertial mass has become blurred. We are dealing with a complex relationship involving the directions of the vectors: force, acceleration, and velocity. In general, the direction of the acceleration vector does not align with the direction of the force vector. However, there are two cases where their directions coincide. One, when the force and velocity vectors have the same direction. Two, when they are perpendicular to each other. In these two situations, equation (9) takes the following forms:

$$\vec{a} = \frac{\left(\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}\right)^3}{m_r} \vec{F} \tag{14}$$

$$\vec{a} = \frac{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}}{m_r} \vec{F} \tag{15}$$

Based on these formulas, two peculiar terms were coined: longitudinal mass, which is the inverse of the coefficient before $\vec{F}$ in formula (14), and transverse mass from formula (15), which in this case equals the expression referred to as relativistic mass. In this context, the concept of inertia of a body should be used, as this parameter is not identical to the concept of mass. (Instead of longitudinal mass and transverse mass, one should say longitudinal inertia and transverse inertia).

It is clearly evident that the inertia of a body, as a parameter expressing the ratio of force to acceleration, depends on the angle between the velocity vector and the force vector. The inertia is smallest when the force vector is perpendicular to the velocity vector and increases as this angle decreases, reaching its maximum value when the directions of these vectors coincide. Moreover, the inertia of a body changes with the change of the reference frame, since in different reference frames the body has different velocities.

Therefore, Einstein's confirmation of the equivalence principle (the equality of inertial and gravitational mass), originating from Newton's era, was a mistake. If gravitational mass were equal to inertial mass, then, for example, the Moon, when it has an orbital velocity directed toward the Sun, would, according to the equivalence principle, have a different gravitational mass (gravitational charge) for the Sun and a different one for the Earth. The difference between these gravitational masses would amount to about 850 million tons. Furthermore, the gravitational mass of the Moon for the Sun would constantly change due to the continual change in the direction of the Moon's velocity relative to the direction of the Sun's gravitational force. This is absurd. Moreover, the gravitational mass of bodies would differ in different reference frames, whereas, for example, electric charge is a Lorentz invariant and has the same value in all reference frames. Therefore, it is the rest mass of bodies that is equal to gravitational mass, because rest mass is a Lorentz invariant. Why would Nature (God), which created such a brilliant work as the Universe, lack consistency and establish radically different rules for interactions that are similar to each other (i.e., gravitational and electric)?

And returning to the equivalence principle. Someone might say that during free fall, the force of gravity is balanced by inertial forces. Yes, that's true, but this balance of forces results from Newton's third law of motion. However, the acceleration of a falling body depends not only on its rest mass (gravitational mass) but also on its velocity. When two bodies fall side by side at different velocities (e.g., from different heights), at the moment they pass each other, they are not moving with the same acceleration. The faster body moves with less acceleration, as shown in equation (9). (*This difference in acceleration becomes greater the larger the relative velocity of the bodies*). Thus, these two bodies move relative to each other with acceleration. However, in the absence of a gravitational field, the relative motion of freely moving bodies on the same trajectory is uniform. It is clear that the statement, "all the laws of motion for bodies in free fall are the same as in an inertial frame," is false. It must be remembered that physics is concerned with discovering the objective laws of Nature; therefore, it cannot be based on the subjective (local) perceptions of the observer. The fact that a person in free fall (e.g., in an elevator) feels as though there is no gravitational field around them is just an "illusion" because nothing changes the objective fact that they are in a gravitational field, moving along a curved worldline, and their kinetic energy is constantly increasing. This is the fundamental (objective) difference between free fall, where we are dealing exclusively with curved worldlines, and the so-called inertial frame, where the worldlines of free objects are straight.

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