Turbo Multiuser Detection with Unknown Interferers

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Abstract

We consider the problem of turbo multiuser detection for synchronous and asynchronous CDMA in the presence of unknown users. Turbo multiuser detectors, as previously developed, typically require knowledge of the signature waveforms of all of the users in the system and ignore users whose signature sequences are unknown, e.g., users outside the cell. In this paper we develop turbo multiuser detection for CDMA uplink systems and other environments in which the receiver has knowledge of the signature waveforms of only $\breve{K} \leq K$ users. Subspace techniques are used to estimate the interference from the unknown users and the interference estimate is subtracted from the received signal. We will see that the new receiver significantly outperforms the conventional turbo multiuser for moderate and high signal-to-noise ratios. It is also seen that the traditional turbo receiver provides little gain through iteration when unknown users are present.

Keywords: turbo multiuser detection, instantaneous MMSE filtering, soft interference cancellation, wireless communications

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1 Introduction

Most of the early work on multiuser detection for code-division multiple-access (CDMA) focused on uncoded systems. Since most practical CDMA systems use error control coding, more recent work has addressed coded systems. Optimal joint decoding and symbol detection for coded asynchronous CDMA, for example, was investigated in [1]. However, the computational complexity resulting from the combined trellises of the convolutional code and the multiuser detector is $O(2^{K\nu})$ where K is the number of users and ν is the constraint length of the code. Some suboptimal techniques that separate the functions of symbol detection and channel decoding are studied in [2].

More recently, iterative (Turbo) receivers for coded CDMA have received much attention. The inspiration behind these receivers is the decoding of turbo codes [3, 4], in which the transmitted signal contains two-dimensional redundancy in the form of two recursive, systematic convolutional codes separated by an interleaver. Decoding is accomplished via an iterative process in which extrinsic likelihood information is fed back and forth between two soft-input soft-output channel decoders. Turbo receivers for CDMA typically use only a single convolutional code. The second form of redundancy is induced by the channel (in the form of ISI, multipath, etc.) or by the structure of the transmitted signal. Examples of turbo equalization for single-user convolutionally coded transmission over intersymbol interference (ISI) channels include [5, 6]. Turbo multiuser detection methods based on different interference cancellation schemes are proposed in [7, 8, 9, 10, 11]. In [12], Moher develops an optimal iterative multiuser detector for synchronous coded CDMA based on cross-entropy minimization. Reed *et al.* [13] developed an iterative receiver that has some similarities to [5] but whose application is to *turbo*-coded CDMA. These receivers can achieve near-optimal performance, but complexity is still exponential in the number of users (for the multiuser detection applications) or the channel ISI length (for single user ISI applications) unless significant suboptimal modifications are made.

Honig *et al.* [14] developed a turbo receiver for synchronous CDMA that reduces complexity by using a suboptimal linear filtering operation for multiuser detection. Later, Wang and Poor developed a low-complexity turbo receiver for coded asynchronous CDMA in fading-multipath channels that relies on a MMSE-based multiuser detector working in conjunction with a MAP channel decoder [15]. Complexity was reduced to $O(N^2\iota^2 + 2^\nu)$ where N is the processing gain and ι is the maximum delay in symbol intervals. In a separate work, Wang and Host-Madsen [16] developed (non-iterative) multiuser detectors for CDMA uplink environments in which the receiver has knowledge of the signature sequences of all of the users within the cell, but no knowledge of the sequences of users outside the cell. They termed these receivers "group-blind". In this paper we merge Wang and Poor's low-complexity turbo receiver structure with the concept of group-blind multiuser detection to develop a turbo group-blind receiver for synchronous and asynchronous CDMA in which some of the users have spreading codes that are unknown. We will compare the performance of the new turbo group-blind receiver to the traditional turbo receiver developed in [15]. We will see that the group-blind turbo receiver provides a significant performance gain over a non-iterative receiver in this environment, while the traditional turbo receiver provides little performance gain through iteration when unknown users are present.

The remainder of this paper is organized as follows. In Section 2, the system under study is described. In Section 3 we develop the soft-input soft-output (SISO) MMSE group-blind multiuser detector that is used in our system. Simulation results are presented in Section 4, and Section 5 concludes.

2 System Description

We consider a coded synchronous CDMA system with K users, employing normalized modulation waveforms $\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K$, and signaling through an AWGN channel. A block diagram of the transmitter/receiver structure appears in Figure 1. The binary information bits for user k, $\{d_k(n)\}$, are encoded using a convolutional code, resulting in a coded bit stream $\{b_k(m)\}$. A code-bit interleaver is used to reduce the probability of error bursts and to remove correlation in the coded bit stream. The coded, interleaved bits are then mapped to BPSK symbols, yielding a symbol stream $\{b_k(i)\}$. Each symbol is then modulated by a spreading waveform $\boldsymbol{s}_k$, and transmitted through the channel. The received signal is the superposition of the K users' transmitted signals plus the ambient noise, given by

$$\begin{aligned} \boldsymbol{r}(i) &= \sum_{k=1}^{K} A_k b_k(i) \boldsymbol{s}_k + \boldsymbol{n}(i) && (1) \\ &= \boldsymbol{S}\boldsymbol{A}\boldsymbol{b}(i) + \boldsymbol{n}(i), \quad i = 0, \ldots, M-1. && (2) \end{aligned}$$

In (2), M is the number of data symbols per user per frame; $\boldsymbol{A} \triangleq \mathrm{diag}(A_1, \ldots, A_K)$ where A_k is the received amplitude of the k-th user; $\boldsymbol{b}(i) \triangleq [b_1(i) \cdots b_K(i)]^T$ where $b_k(i)$ denotes the i-th symbol of the k-th user; $\boldsymbol{S} \triangleq [\boldsymbol{s}_1 \cdots \boldsymbol{s}_K]$ where $\boldsymbol{s}_k$ is the normalized spreading waveform of the k-th user; $\boldsymbol{n}(i)$ is a zero mean i.i.d. Gaussian noise vector with variance σ^2 that is independent of the symbol sequences. The spreading waveform is of the form

$$\boldsymbol{s}_k \triangleq \frac{1}{\sqrt{N}} [\beta_0^k \ \ \beta_1^k \cdots \beta_{N-1}^k]^T, \quad \beta_j^k \in \{+1, -1\}, \tag{3}$$

where N is the processing gain.

In the group-blind multiuser detection scenario, we assume we have knowledge of the first $\breve{K}(\breve{K} \leq K)$ users' spreading sequences (and received amplitudes), whereas the rest of the users are unknown to the receiver. Denote $\breve{\boldsymbol{S}}$ as the matrix consisting of the first $\breve{K}$ columns of $\boldsymbol{S}$. Denote the remaining $\bar{K} = K - \breve{K}$ columns of $\boldsymbol{S}$ by $\bar{\boldsymbol{S}}$. These signature sequences are unknown to the receiver. Let $\breve{\boldsymbol{b}}(i)$ be the $\breve{K}$-vector containing the first $\breve{K}$ bits of $\boldsymbol{b}(i)$ and let $\bar{\boldsymbol{b}}(i)$ contain the remaining $\bar{K}$ bits. Similarly, denote $\breve{\boldsymbol{A}} \stackrel{\triangle}{=} \mathrm{diag}(A_1, \ldots, A_{\breve{K}})$ and $\bar{\boldsymbol{A}} \stackrel{\triangle}{=} \mathrm{diag}(A_{\breve{K}+1}, \ldots, A_K)$. Then we may write (2) as

$$\boldsymbol{r}(i) = \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\breve{\boldsymbol{b}}(i) + \bar{\boldsymbol{S}}\bar{\boldsymbol{A}}\bar{\boldsymbol{b}}(i) + \boldsymbol{n}(i). \tag{4}$$

Since we do not have knowledge of $\bar{\boldsymbol{S}}$ we cannot hope to demodulate $\bar{\boldsymbol{b}}(i)$. We therefore write (4) as

$$\boldsymbol{r}(i) = \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\breve{\boldsymbol{b}}(i) + \boldsymbol{I}(i) + \boldsymbol{n}(i), \tag{5}$$

where $\boldsymbol{I}(i) \stackrel{\triangle}{=} \bar{\boldsymbol{S}}\bar{\boldsymbol{A}}\bar{\boldsymbol{b}}(i)$ is regarded as an interference term that is to be estimated and removed by our multiuser detector before it computes the *a posteriori* log-likelihood ratios (LLRs) for the bits in $\breve{\boldsymbol{b}}(i)$.

In Figure 1, the MAP channel decoder accepts, as inputs, the set of extrinsic LLRs of all the code bits in the frame. It delivers, as output, updated LLRs of the code bits and, at the last iteration, the LLRs of the information bits. A forward-backward recursion of the type in [17] was developed in [15] and was used here. For brevity, the details are omitted. We note in passing that although some related work has been completed [18], the issue of passing extrinsic information versus full likelihood ratio information is not fully resolved. It may be true that for some system loads, passing the full likelihood ratio information may improve performance. Since a full analysis is beyond the scope of this letter, we will use the standard approach of passing extrinsic information.

3 SISO Group-Blind Multiuser Detectors

The heart of the turbo group-blind receiver is the soft-input soft-output (SISO) group-blind multiuser detector. The detector accepts, as inputs, the *a priori* LLRs for the code bits of the known users delivered by the SISO MAP channel decoder and produces, as outputs, updated LLRs for these code bits. This is accomplished by soft interference cancellation and MMSE filtering. Specifically, using the *a priori* LLRs and knowledge of the signature sequences and received amplitudes of the known users, the detector performs a soft-interference cancellation for each user, in which estimates of the multiuser interference from the other known users *and* an estimate for the interference caused by the *unknown* users are subtracted from the received signal. This is in contrast to previously developed turbo multiuser detectors which ignore the interference from unknown users.

Residual interference is suppressed by passing the resulting signal through an MMSE filter. The *a posteriori* LLR can be computed from the MMSE filter output. In this section we develop SISO group-blind detectors for synchronous and asynchronous CDMA.

3.1 SISO Group-Blind Detector for Synchronous CDMA

The detector first forms soft estimates of the user code bits as $\tilde{b}_k(i) \stackrel{\triangle}{=} E\{b_k(i)\} = \tanh\left(\frac{1}{2}\lambda_2[b_k(i)]\right)$ [15] where $\lambda_2[b_k(i)]$ is the *a priori* LLR of the k-th bit during the i-th time slot delivered by the MAP decoder and is given by

$$\lambda_2[b_k(i)] = \log\frac{\Pr[b_k(i) = +1]}{\Pr[b_k(i) = -1]}, \qquad 1 \leq k \leq \breve{K}, 0 \leq i \leq M-1. \tag{6}$$

We denote hard estimates of the code bits as $\hat{b}_k(i) \stackrel{\triangle}{=} \mathrm{sign}[\tilde{b}_k(i)]$ and define $\hat{\boldsymbol{b}}(i) \stackrel{\triangle}{=} [\hat{b}_1(i) \;\; \hat{b}_2(i) \cdots \hat{b}_{\breve{K}}(i)]^T$.

In the next step we form an estimate of interference of the unknown users, $\boldsymbol{I}(i)$, which we denote by $\hat{\boldsymbol{I}}(i)$. We begin by forming a preliminary estimate

$$\begin{aligned}
\boldsymbol{\gamma}(i) &\stackrel{\triangle}{=} \boldsymbol{r}(i) - \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\hat{\boldsymbol{b}}(i) &(7)\\
&= \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\breve{\boldsymbol{b}}(i) + \bar{\boldsymbol{S}}\bar{\boldsymbol{A}}\bar{\boldsymbol{b}}(i) + \boldsymbol{n}(i) - \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\hat{\boldsymbol{b}}(i) &(8)\\
&= \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}[\breve{\boldsymbol{b}}(i) - \hat{\boldsymbol{b}}(i)] + \bar{\boldsymbol{S}}\bar{\boldsymbol{A}}\bar{\boldsymbol{b}}(i) + \boldsymbol{n}(i) &(9)\\
&= \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\boldsymbol{d}(i) + \bar{\boldsymbol{S}}\bar{\boldsymbol{A}}\bar{\boldsymbol{b}}(i) + \boldsymbol{n}(i), &(10)
\end{aligned}$$

where $\boldsymbol{d}(i) \stackrel{\triangle}{=} [d_1(i) \;\; d_2(i) \cdots d_{\breve{K}}(i)]^T$ and $d_j(i), 1 \leq j \leq \breve{K}$, is a random variable defined by

$$d_j(i) \stackrel{\triangle}{=} b_j(i) - \hat{b}_j(i). \tag{11}$$

We will see that our ability to form a soft estimate for $d_j(i)$ will allow us to perform the soft interference cancellation mentioned above. Clearly, $d_j(i)$ can take on one of three values, $\{-2, 0, 2\}$, i.e. 0 or $2b_j(i)$. The probability that $d_j(i)$ is equal to zero is the probability that our hard estimate is correct and is given by

$$\Pr[d_j(i) = 0] = \Pr\left[b_j(i) = \mathrm{sign}\left\{\tanh\left(\frac{\lambda_2[b_j(i)]}{2}\right)\right\}\right]. \tag{12}$$

It is shown in [15] that for $b \in \{-1, 1\}$, the probability that $b_j(i) = b$ is given by

$$\Pr[b_j(i) = b] = \frac{1}{2} + \frac{b}{2}\tanh\left(\frac{\lambda_2[b_j(i)]}{2}\right) \tag{13}$$

We substitute $\mathrm{sign}\left\{\tanh\left(\frac{\lambda_2[b_j(i)]}{2}\right)\right\}$ for b in (13) and we find that

$$\begin{aligned}
\Pr[d_j(i) = 0] &= \frac{1}{2}\left[1 + \mathrm{sign}\left\{\tanh\left(\frac{\lambda_2[b_j(i)]}{2}\right)\right\}\tanh\left(\frac{\lambda_2[b_j(i)]}{2}\right)\right] &(14)\\
&= \frac{1}{2}\left[1 + \tanh\left(\frac{|\lambda_2[b_j(i)]|}{2}\right)\right] &(15)
\end{aligned}$$

Therefore, $d_j(i)$ is a random variable that can be described as

$$d_j(i) = \begin{cases} 0 & \text{with probability} \quad \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{|\lambda_2[b_j(i)]|}{2}\right) \\ 2b_j(i) & \text{with probability} \quad \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{|\lambda_2[b_j(i)]|}{2}\right). \end{cases} \tag{16}$$

Denote by $\boldsymbol{U}_u$ the $\bar{K}$ largest eigenvectors of the eigendecomposition of $E\{\boldsymbol{\gamma}(i)\boldsymbol{\gamma}(i)^T\}$. When perfect prior information is available, $\boldsymbol{d}(i) = \boldsymbol{0}$ and $\boldsymbol{U}_u$ represents the (exact) signal subspace of the unknown users, i.e. the interference subspace. In order to refine our estimate of $\boldsymbol{I}(i)$ we project $\boldsymbol{\gamma}(i)$ onto $\boldsymbol{U}_u$. The result is

$$\hat{\boldsymbol{I}}(i) = \boldsymbol{U}_u\boldsymbol{U}_u^T\left\{\breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\boldsymbol{d}(i) + \bar{\boldsymbol{S}}\bar{\boldsymbol{A}}\bar{\boldsymbol{b}}(i) + \boldsymbol{n}(i)\right\}. \tag{17}$$

If we denote $\tilde{\boldsymbol{S}} \triangleq \boldsymbol{U}_u\boldsymbol{U}_u^T\breve{\boldsymbol{S}}$ and $\boldsymbol{v}(i) \triangleq \boldsymbol{U}_u\boldsymbol{U}_u^T\boldsymbol{n}(i)$, we have

$$\hat{\boldsymbol{I}}(i) = \tilde{\boldsymbol{S}}\breve{\boldsymbol{A}}\boldsymbol{d}(i) + \bar{\boldsymbol{S}}\bar{\boldsymbol{A}}\bar{\boldsymbol{b}}(i) + \boldsymbol{v}(i). \tag{18}$$

Now we subtract the interference estimate from the received signal and form a new vector

$$\begin{aligned} \boldsymbol{\zeta}(i) &\triangleq \boldsymbol{r}(i) - \hat{\boldsymbol{I}}(i) && (19)\\ &= \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\breve{\boldsymbol{b}}(i) - \tilde{\boldsymbol{S}}\breve{\boldsymbol{A}}\boldsymbol{d}(i) + \boldsymbol{w}(i), && (20) \end{aligned}$$

where $\boldsymbol{w}(i) \triangleq \boldsymbol{n}(i) - \boldsymbol{v}(i)$.

For each known user we perform a soft interference cancellation on $\boldsymbol{\zeta}(i)$ to obtain

$$\boldsymbol{r}_k(i) \triangleq \boldsymbol{\zeta}(i) - \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\tilde{\boldsymbol{b}}_k(i) + \tilde{\boldsymbol{S}}\breve{\boldsymbol{A}}\tilde{\boldsymbol{d}}(i), \quad 1 \le k \le \breve{K}, \tag{21}$$

where $\tilde{\boldsymbol{b}}_k(i) \triangleq [\tilde{b}_1(i)\cdots\tilde{b}_{k-1}(i)\ 0\ \tilde{b}_{k+1}(i)\cdots\tilde{b}_{\breve{K}}(i)]$ and $\tilde{\boldsymbol{d}}(i) \triangleq [\tilde{d}_1(i)\ \ \tilde{d}_2(i)\cdots\tilde{d}_{\breve{K}}(i)]^T$ where $\tilde{d}_j(i)$ is a soft estimate for $d_j(i)$ and is given by

$$\begin{aligned} \tilde{d}_j(i) &\triangleq E\{d_j(i)\} && (22)\\ &= E\{E\{d_j(i)\,|\,b_j(i)\}\} && (23)\\ &= \tilde{b}_j(i)\left[1 - \tanh\left(\frac{|\lambda_2[b_j(i)]|}{2}\right)\right], \quad 1 \le j \le \breve{K}. && (24) \end{aligned}$$

Substituting (20) into (21) we obtain

$$\begin{aligned} \boldsymbol{r}_k(i) &= \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}[\breve{\boldsymbol{b}}(i) - \tilde{\boldsymbol{b}}_k(i)] - \tilde{\boldsymbol{S}}\breve{\boldsymbol{A}}[\boldsymbol{d}(i) - \tilde{\boldsymbol{d}}(i)] + \boldsymbol{w}(i) && (25)\\ &= \breve{\boldsymbol{H}}[\breve{\boldsymbol{b}}(i) - \tilde{\boldsymbol{b}}_k(i)] - \tilde{\boldsymbol{H}}[\boldsymbol{d}(i) - \tilde{\boldsymbol{d}}(i)] + \boldsymbol{w}(i), && (26) \end{aligned}$$

where $\breve{\boldsymbol{H}} \triangleq \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}$ and $\tilde{\boldsymbol{H}} \triangleq \tilde{\boldsymbol{S}}\breve{\boldsymbol{A}}$.

An instantaneous linear MMSE filter is then applied to $\boldsymbol{r}_k(i)$ to obtain

$$z_k(i) \stackrel{\triangle}{=} \boldsymbol{x}_k(i)^T \boldsymbol{r}_k(i). \tag{27}$$

The filter $\boldsymbol{x}_k(i) \in \mathbb{R}^N$ is chosen to minimize the mean-squared error between the code bit $b_k(i)$ and the filter output $z_k(i)$, i.e.,

$$\boldsymbol{x}_k(i) \stackrel{\triangle}{=} \arg \min_{\boldsymbol{x} \in \mathbb{R}^N} E\{[b_k(i) - \boldsymbol{x}^T \boldsymbol{r}_k(i)]^2\}, \tag{28}$$

where the expectation is with respect to the ambient noise and the interfering users. The solution to (28) is given by

$$\boldsymbol{x}_k(i) = E\{\boldsymbol{r}_k(i)\boldsymbol{r}_k(i)^T\}^{-1} E\{b_k(i)\boldsymbol{r}_k(i)\}. \tag{29}$$

It is easy to show that

$$\begin{aligned}
E\{\boldsymbol{r}_k(i)\boldsymbol{r}_k(i)^T\} &= E\left\{[\breve{\boldsymbol{H}} \;\; \tilde{\boldsymbol{H}}] \begin{bmatrix} -(\breve{\boldsymbol{b}}(i) - \tilde{\boldsymbol{b}}_k(i)) \\ \boldsymbol{d}(i) - \tilde{\boldsymbol{d}}(i) \end{bmatrix} \begin{bmatrix} -(\breve{\boldsymbol{b}}(i) - \tilde{\boldsymbol{b}}_k(i)) \\ \boldsymbol{d}(i) - \tilde{\boldsymbol{d}}(i) \end{bmatrix}^T \begin{bmatrix} \breve{\boldsymbol{H}}^T \\ \tilde{\boldsymbol{H}}^T \end{bmatrix} \right\} + & (30) \\
& \quad \sigma^2 \left[\boldsymbol{I} - \boldsymbol{U}_u \boldsymbol{U}_u^T\right] & (31) \\
&= \boldsymbol{\mathcal{H}} \underbrace{\mathrm{Cov}\left\{ \begin{bmatrix} \tilde{\boldsymbol{b}}_k(i) - \breve{\boldsymbol{b}}(i) \\ \boldsymbol{d}(i) - \tilde{\boldsymbol{d}}(i) \end{bmatrix} \right\}}_{\boldsymbol{\Delta}_k(i)} \boldsymbol{\mathcal{H}}^T + \sigma^2 \left[\boldsymbol{I} - \boldsymbol{U}_u \boldsymbol{U}_u^T\right], & (32)
\end{aligned}$$

where $\boldsymbol{\mathcal{H}} \stackrel{\triangle}{=} [\breve{\boldsymbol{H}} \;\; \tilde{\boldsymbol{H}}]$. $\boldsymbol{\Delta}_k(i)$ has size $2\breve{K} \times 2\breve{K}$ and may be partitioned into four diagonal $\breve{K} \times \breve{K}$ blocks as

$$\boldsymbol{\Delta}_k(i) = \begin{bmatrix} \boldsymbol{\Delta}_{11}(i) & \boldsymbol{\Delta}_{12}(i) \\ \boldsymbol{\Delta}_{21}(i) & \boldsymbol{\Delta}_{22}(i) \end{bmatrix}, \tag{33}$$

where, for convenience, we have dropped the user index k from the submatrix notation. The diagonal elements of $\boldsymbol{\Delta}_{11}(i)$ are given by

$$\begin{aligned}
[\boldsymbol{\Delta}_{11}(i)]_{jj} &= \mathrm{Var}\{b_j(i)\} & (34) \\
&= \begin{cases} 1 - \tilde{b}_j^2(i) & 1 \le j \le \breve{K}, j \ne k \\ 1 & j = k \end{cases} & (35)
\end{aligned}$$

Similarly, the diagonal elements of $\boldsymbol{\Delta}_{22}(i)$ are given by

$$\begin{aligned}
[\boldsymbol{\Delta}_{22}(i)]_{jj} &= \mathrm{Var}\{d_j(i)\} & (36) \\
&= 2\alpha_j(i) - \tilde{b}_j^2(i)\alpha_j^2(i), \qquad 1 \le j \le \breve{K}, & (37)
\end{aligned}$$

where

$$\alpha_j(i) \stackrel{\triangle}{=} 1 - \tanh\left(\frac{|\lambda_2[b_j(i)]|}{2}\right). \tag{38}$$

The diagonal elements of $\boldsymbol{\Delta}_{12}(i)$ and $\boldsymbol{\Delta}_{21}(i)$ are identical and are given by

$$\begin{aligned} [\boldsymbol{\Delta}_{12}(i)]_{jj} &= -\text{Cov}\{b_j(i), d_j(i)\} && (39) \\ &= \alpha_j(i)[\tilde{b}_j^2(i) - 1], \qquad 1 \leq j \leq \breve{K}. && (40) \end{aligned}$$

It is also easy to see that

$$E\{b_k(i)\boldsymbol{r}_k(i)\} = \breve{\boldsymbol{H}}\boldsymbol{e}_k - \alpha_k(i)\tilde{\boldsymbol{H}}\boldsymbol{e}_k, \tag{41}$$

where $\boldsymbol{e}_k$ is a $\breve{K}$-vector whose elements are all zero except for the k-th element which is 1. Using (32) and (41) in (29), we may write the MMSE filter for user k as

$$\boldsymbol{x}_k(i) = \left[\boldsymbol{\mathcal{H}}\boldsymbol{\Delta}_k(i)\boldsymbol{\mathcal{H}}^T + \sigma^2\left[\boldsymbol{I} - \boldsymbol{U}_u\boldsymbol{U}_u^T\right]\right]^{-1}\left[\breve{\boldsymbol{H}}\boldsymbol{e}_k - \alpha_k(i)\tilde{\boldsymbol{H}}\boldsymbol{e}_k\right]. \tag{42}$$

If we make the common assumption that the MMSE filter output is Gaussian [19], we may write

$$\begin{aligned} z_k(i) &\stackrel{\triangle}{=} \boldsymbol{x}_k^T(i)\boldsymbol{r}_k(i) && (43) \\ &= \mu_k(i)b_k(i) + \eta_k(i), && (44) \end{aligned}$$

where $\mu_k(i)$ is the equivalent amplitude of the k-th user's signal at the filter output, and $\eta_k(i) \sim \mathcal{N}(0, \nu_k^2(i))$ is a Gaussian noise sample. We may compute the parameter $\mu_k(i)$ as

$$\begin{aligned} \mu_k(i) &= E\{z_k(i)b_k(i)\} && (45) \\ &= \boldsymbol{x}_k^T E\{b_k(i)\boldsymbol{r}_k(i)\} && (46) \\ &= \left[\breve{\boldsymbol{H}}^T\boldsymbol{\Theta}_k^{-1}(i)\breve{\boldsymbol{H}}\right]_{kk} + \alpha_k^2(i)\left[\tilde{\boldsymbol{H}}^T\boldsymbol{\Theta}_k^{-1}(i)\tilde{\boldsymbol{H}}\right]_{kk} - 2\alpha_k(i)\left[\breve{\boldsymbol{H}}^T\boldsymbol{\Theta}_k^{-1}(i)\tilde{\boldsymbol{H}}\right]_{kk}, && (47) \end{aligned}$$

where $\boldsymbol{\Theta}_k(i) \stackrel{\triangle}{=} \left[\boldsymbol{\mathcal{H}}\boldsymbol{\Delta}_k(i)\boldsymbol{\mathcal{H}}^T + \sigma^2\left[\boldsymbol{I} - \boldsymbol{U}_u\boldsymbol{U}_u^T\right]\right]$.

Finally, the extrinsic information, $\lambda_1[b_k(i)]$, delivered by the SISO multiuser detector is given by

$$\begin{aligned} \lambda_1[b_k(i)] &\stackrel{\triangle}{=} \log\frac{p[z_k(i)|b_k(i) = +1]}{p[z_k(i)|b_k(i) = -1]} && (48) \\ &= -\frac{[z_k(i) - \mu_k(i)]^2}{2\nu_k^2(i)} + \frac{[z_k(i) + \mu_k(i)]^2}{2\nu_k^2(i)} && (49) \\ &= \frac{2z_k(i)}{1 - \mu_k(i)}. && (50) \end{aligned}$$

This algorithm is summarized in Table 1. Each step in the algorithm is annotated with its approximate complexity in floating point operations *per user per symbol*. The major computation involved in generating the MMSE filter output is the inversion of the matrix $\boldsymbol{\Theta}_k(i)$ in Step (5). Notice, however, that $\boldsymbol{\mathcal{H}}\boldsymbol{\Delta}_k(i)\boldsymbol{\mathcal{H}}^T = \breve{\boldsymbol{H}}\boldsymbol{\Delta}_{11}(i)\breve{\boldsymbol{H}}^T + \tilde{\boldsymbol{H}}\boldsymbol{\Delta}_{21}(i)\breve{\boldsymbol{H}}^T + \breve{\boldsymbol{H}}\boldsymbol{\Delta}_{12}(i)\tilde{\boldsymbol{H}}^T + \tilde{\boldsymbol{H}}\boldsymbol{\Delta}_{22}(i)\tilde{\boldsymbol{H}}^T$ can be written as the sum of $8\breve{K}$ vector outer products (vector length N) and $\boldsymbol{U}_u \boldsymbol{U}_u^T$ can be written as the sum of $\bar{K}$ vector outer products (vector length N) and is independent of k. Hence, the matrix inversion can be computed iteratively using the matrix inversion lemma as in [15], resulting in a complexity of $O\left(N^2 + \frac{\bar{K}N^2}{M\breve{K}}\right)$ rather than $O(N^3)$.

3.2 Sliding Window Group-Blind Detector for Asynchronous CDMA

It is not difficult to extend the results of the previous subsection to asynchronous CDMA. The received signal due to user $k(1 \leq k \leq K)$ is given by

$$y_k(t) = A_k \sum_{i=0}^{M-1} b_k[i] \sum_{j=0}^{N-1} c_k[j] \psi(t - jT_c - iT - d_k), \tag{51}$$

where d_k is the delay of the k-th user' signal, $\{c_k[j]\}_{j=0}^{N-1}$ is a signature sequence of ± 1's assigned to the k-th user and $\psi(t)$ is a normalized chip waveform of duration $T_c = T/N$. The total received signal, given by

$$r(t) = \sum_{k=1}^{K} y_k(t) + v(t), \tag{52}$$

is matched filtered to the chip waveform and sampled at the chip rate, The n-th matched filter output during the i-th symbol interval is

$$\begin{aligned} r[i,n] &\triangleq \int_{iT+nT_c}^{iT+(n+1)T_c} r(t)\psi(t - iT - nT_c)\mathrm{d}t \\ &= \sum_{k=1}^{K} \underbrace{\left\{ \int_{iT+nT_c}^{iT+(n+1)T_c} \psi(t - iT - nT_c) y_k(t) \mathrm{d}t \right\}}_{y_k[i,n]} + \underbrace{\int_{iT+nT_c}^{iT_s+(n+1)T_c} v(t)\psi(t - iT - nT_c)\mathrm{d}t}_{v[i,n]} . \end{aligned} \tag{53}$$

Substituting (51) into (53) we obtain

$$\begin{aligned} y_k[i,n] &= A_k \sum_{p=0}^{M-1} b_k[p] \sum_{j=0}^{N-1} c_k[j] \int_{iT+nT_c}^{iT+(n+1)T_c} \psi(t - iT - nT_c)\psi(t - jT_c - pT - d_k)\mathrm{d}t && (54) \\ &= \sum_{p=0}^{\iota_k - 1} b_k[i-p] \underbrace{A_k \sum_{j=0}^{N-1} c_k[j] \int_0^{T_c} \psi(t)\psi(t - jT_c + nT_c + pT - d_k)\mathrm{d}t}_{h_k[p,n]} && (55) \end{aligned}$$

where $\iota_k \triangleq 1 + \lceil (d_k + T_c)/T \rceil$. Then

$$r[i,n] = h_k[0,n] b_k[i] + \underbrace{\sum_{j=1}^{\iota_k - 1} h_k[j,n] b_k[i-j]}_{\text{ISI}} + \underbrace{\sum_{k' \neq k} y_{k'}[i,n]}_{\text{MAI}} + v[i,n]. \tag{56}$$

Denote

$$\underline{r}[i] \stackrel{\triangle}{=} \begin{bmatrix} r[i,0] \\ \vdots \\ r[i,N-1] \end{bmatrix}, \quad \underline{v}[i] \stackrel{\triangle}{=} \begin{bmatrix} v[i,0] \\ \vdots \\ v[i,N-1] \end{bmatrix}, \quad \underline{b}[i] \stackrel{\triangle}{=} \begin{bmatrix} b_1[i] \\ \vdots \\ b_K[i] \end{bmatrix}, \tag{57}$$

and, for $j = 0, 1, \ldots, \iota_k - 1$,

$$\underline{H}[j] \stackrel{\triangle}{=} \begin{bmatrix} h_1[j,0] & \cdots & h_{\breve{K}}[j,0] & \cdots & h_K[j,0] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_1[j,N-1] & \cdots & h_{\breve{K}}[j,N-1] & \cdots & h_K[j,N-1] \end{bmatrix}. \tag{58}$$

Then

$$\underline{r}[i] = \underline{H}[i] \star \underline{b}[i] + \underline{v}[i]. \tag{59}$$

By stacking $\iota \stackrel{\triangle}{=} \max_k \iota_k$ successive received sample vectors, we define

$$\underbrace{\underline{\boldsymbol{r}}[i]}_{N\iota\times 1} \stackrel{\triangle}{=} \begin{bmatrix} \underline{r}[i] \\ \vdots \\ \underline{r}[i+\iota-1] \end{bmatrix}, \quad \underbrace{\underline{\boldsymbol{v}}[i]}_{N\iota\times 1} \stackrel{\triangle}{=} \begin{bmatrix} \underline{v}[i] \\ \vdots \\ \underline{v}[i+\iota-1] \end{bmatrix}, \quad \underbrace{\underline{\boldsymbol{b}}[i]}_{r\times 1} \stackrel{\triangle}{=} \begin{bmatrix} \underline{b}[i-\iota+1] \\ \vdots \\ \underline{b}[i+\iota-1] \end{bmatrix}, \tag{60}$$

and

$$\underbrace{\underline{\boldsymbol{H}}}_{N\iota\times r} \stackrel{\triangle}{=} \begin{bmatrix} \underline{H}[\iota-1] & \cdots & \underline{H}[0] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \underline{H}[\iota-1] & \cdots & \underline{H}[0] \end{bmatrix}, \tag{61}$$

where $r \stackrel{\triangle}{=} K(2\iota - 1)$. Then we can write the received signal in matrix form as

$$\underline{\boldsymbol{r}}[i] = \underline{\boldsymbol{H}}\underline{\boldsymbol{b}}[i] + \underline{\boldsymbol{v}}[i]. \tag{62}$$

Define the set of matrices $\{\breve{\underline{\boldsymbol{H}}}_j\}_{j=0}^{2\iota-2}$ such that $\breve{\underline{\boldsymbol{H}}}_j$ is the $N\iota \times \breve{K}$ matrix composed of columns $jK+1$ through $jK+\breve{K}$ of the matrix $\underline{\boldsymbol{H}}$. We define the matrix $\breve{\underline{\boldsymbol{H}}} \stackrel{\triangle}{=} [\breve{\underline{\boldsymbol{H}}}_0 \;\; \breve{\underline{\boldsymbol{H}}}_1 \cdots \breve{\underline{\boldsymbol{H}}}_{2\iota-2}]$. The size of $\breve{\underline{\boldsymbol{H}}}$ is $N\iota \times \breve{K}(2\iota-1)$. We denote by $\bar{\underline{\boldsymbol{H}}}$ the matrix that contains the remaining $\bar{K}(2\iota-1)$ columns of $\underline{\boldsymbol{H}}$. We define $\breve{\underline{\boldsymbol{b}}}[i]$ and $\bar{\underline{\boldsymbol{b}}}[i]$ by performing a similar separation of the elements of $\underline{\boldsymbol{b}}[i]$. Then we may write (62) as

$$\underline{\boldsymbol{r}}[i] = \breve{\underline{\boldsymbol{H}}}\breve{\underline{\boldsymbol{b}}}[i] + \bar{\underline{\boldsymbol{H}}}\bar{\underline{\boldsymbol{b}}}[i] + \underline{\boldsymbol{v}}[i]. \tag{63}$$

This equation is the asynchronous analog to (4). We can obtain estimates of $b_1[i], b_2[i], \ldots, b_{\breve{K}}[i]$ with straightforward modifications to the algorithm in Table 1.

4 Simulation Results

In this section we present simulation results to demonstrate the performance of the proposed turbo group-blind multiuser receiver for asynchronous CDMA. The processing gain of the system is 7 and the total number of users is 7. The number of known users is either 2 or 5, as noted on the figures. The spreading sequences are random and the same sequences are used for all simulations. All users employ the same rate $\frac{1}{2}$, constraint length 3 convolutional code (with generators $g_1 = [110]$ and $g_2 = [111]$). Each user uses a different random interleaver, and the same interleavers are used in all simulations. The block size of information bits for each user is 128. The maximum delay, in symbol intervals is 1. All users use the same transmitted power and the chip pulse waveform is a raised cosine with roll-off factor .5.

Figure 2 illustrates the average bit-error-rate performance of the known users for the group-blind turbo receiver and the conventional turbo receiver [15] for the first 4 iterations. The number of known users is 5. For the sake of comparison, we have also included plots for the conventional turbo receiver when *all* of the users are known. The three sets of plots in this figure are denoted in the legend by "GBMUD", "TMUD", and "ALL KNOWN" respectively. Note that the curves for the first iteration are identical for GBMUD and TMUD. Hence we have suppress the plot of the first iteration for TMUD to improve clarity. Notice that iteration does not significantly improve the performance of the conventional turbo receiver while the group-blind receiver provides significant gains through iteration at moderate and high signal-to-noise ratios. We can also see that the use of more than three iterations does not provide significant benefits.

In Figure 3, the number of known users has been changed to 2. As we would expect, there is a performance degradation for both the conventional and group-blind turbo receivers. In fact, the conventional receiver gains nothing through iteration for this scenario because there are now 5 users whose interference is simply ignored. It is also apparent that the group-blind turbo receiver will not be able to mitigate all of the interference of the unknown users, even for a large number of iterations. This is due, in part, to the use of an imperfect interference subspace estimate in the SISO group-blind multiuser detector.

5 Conclusions

In this paper we have developed a new turbo multiuser receiver for CDMA systems that is capable of suppressing interference not only from known users, but also from users whose signature sequences and received amplitudes are unknown. This technique differs from previously developed turbo multiuser receivers that ignore unknown interferers. We have seen that this so-called *group-*

blind turbo multiuser receiver provides a significant performance improvement over a non-iterative receiver, whose performance is illustrated in Figures 2 and 3 by the first iteration. We have also seen that the traditional turbo receiver fails to provide a useful performance gain over a non-iterative receiver when unknown users are present.

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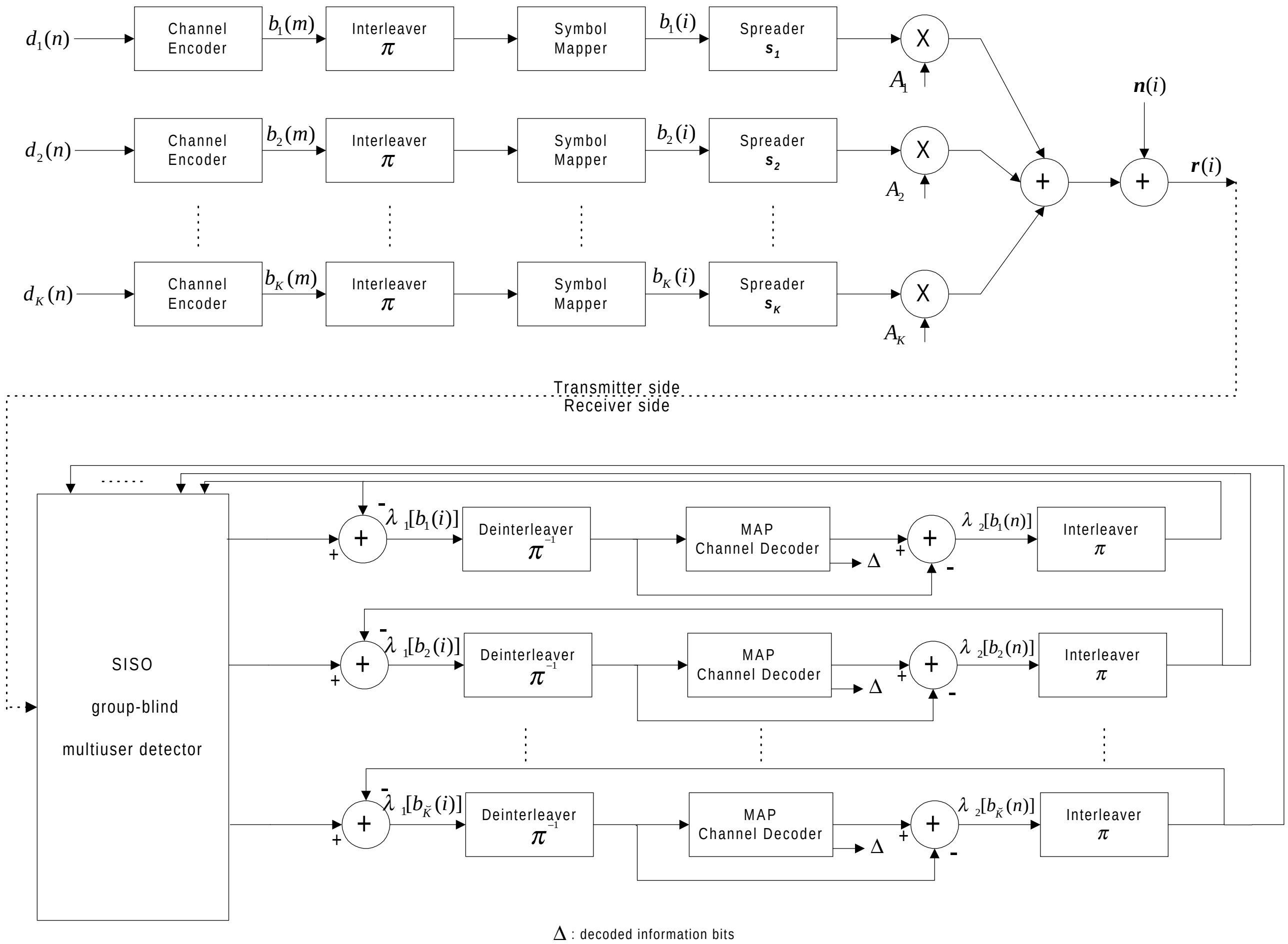

Figure 1: Turbo group-blind multiuser detector transmitter/receiver structure.

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Given: $\boldsymbol{r}(l), \lambda_2[b_j(l)],\ 0 \le l \le M-1, 1 \le j \le \breve{K}$.
1. $[O(1)]$ For $1 \le k \le \breve{K}$ and for $0 \le j \le M-1$, form soft and hard estimates of code bits: $\tilde{b}_k(j) = \tanh\left[\lambda_2[b_k(j)]/2\right]$, $\hat{b}_k(j) = \mathrm{sign}[\tilde{b}_k(j)]$. Define the vectors $\hat{\boldsymbol{b}}(j) \stackrel{\triangle}{=} [\hat{b}_1(j)\ \ \hat{b}_2(j) \cdots \hat{b}_{\breve{K}}(j)]^T$ and $\tilde{\boldsymbol{b}}_k(j) \stackrel{\triangle}{=} [\tilde{b}_1(j) \cdots \tilde{b}_{k-1}(j)\ 0\ \tilde{b}_{k+1}(j) \cdots \tilde{b}_{\breve{K}}(j)]$. 2. Form $\hat{\boldsymbol{U}}_u$, an estimate of $\boldsymbol{U}_u$, by using the $\bar{K}$ largest eigenvectors of $\frac{1}{M}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^T$, where $\boldsymbol{\Gamma} \stackrel{\triangle}{=} [\boldsymbol{\gamma}(0) \cdots \boldsymbol{\gamma}(M-1)]$ and $\boldsymbol{\gamma}(j) \stackrel{\triangle}{=} \boldsymbol{r}(j) - \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\hat{\boldsymbol{b}}(j)$. 3. for $i = 0, 1, \ldots, M-1$ (1). $\left[O\left(\frac{N^2}{\breve{K}}\right)\right]$ Refine the estimate from step 2 by projection: $\hat{\boldsymbol{I}}(i) = \hat{\boldsymbol{U}}_u \hat{\boldsymbol{U}}_u^T \boldsymbol{\gamma}(i)$. (2). $[O\,(1)]$ Define $\tilde{\boldsymbol{d}}(i) \stackrel{\triangle}{=} [\tilde{d}_1(i)\ \ \tilde{d}_2(i) \cdots \tilde{d}_{\breve{K}}(i)]^T$ and compute $\tilde{d}_j(i)$ according to: $\tilde{d}_j(i) = \tilde{b}_j(i)\alpha_j(i)$, where $\alpha_j(i)$ is defined in (38). (3). $\left[O\left(\frac{N^2}{\breve{K}}\right)\right]$ Subtract $\hat{\boldsymbol{I}}(i)$ from $\boldsymbol{r}(i)$ and for $1 \le k \le \breve{K}$ perform soft interference cancellation: $\boldsymbol{r}_k(i) = \boldsymbol{r}(i) - \hat{\boldsymbol{I}}(i) - \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}\tilde{\boldsymbol{b}}_k(i) + \tilde{\boldsymbol{S}}\breve{\boldsymbol{A}}\tilde{\boldsymbol{d}}(i)$. (4). $[O\,(1)]$ Calculate $\boldsymbol{\Delta}_k(i)$ according to (34)-(40). (5). $O\left[N^2 + \frac{\bar{K}N^2}{M\breve{K}}\right]$ Calculate and apply the MMSE filter: $\boldsymbol{x}_k(i) = \left[\boldsymbol{\mathcal{H}}\boldsymbol{\Delta}_k(i)\boldsymbol{\mathcal{H}}^T + \sigma^2\left[\boldsymbol{I} - \hat{\boldsymbol{U}}_u\hat{\boldsymbol{U}}_u^T\right]\right]^{-1}\left[\breve{\boldsymbol{H}}\boldsymbol{e}_k - \alpha_k(i)\tilde{\boldsymbol{H}}\boldsymbol{e}_k\right]$, $z_k(i) = \boldsymbol{x}_k(i)^T\boldsymbol{r}_k(i)$. where $\boldsymbol{\mathcal{H}} \stackrel{\triangle}{=} [\breve{\boldsymbol{H}}\ \ \tilde{\boldsymbol{H}}]$ and where $\breve{\boldsymbol{H}} \stackrel{\triangle}{=} \breve{\boldsymbol{S}}\breve{\boldsymbol{A}}$ and $\tilde{\boldsymbol{H}} \stackrel{\triangle}{=} \tilde{\boldsymbol{S}}\breve{\boldsymbol{A}}$. (6). $[O\,(1)]$ Compute $\mu_k(i)$ according to (47). (7). $[O\,(1)]$ For $1 \le k \le \breve{K}$ compute the *a posteriori* LLR for code bit $b_k(i)$: $\lambda_1[b_k(i)] = 2z_k(i)/[1-\mu_k(i)]$. end

Table 1: SISO group-blind multiuser detection algorithm

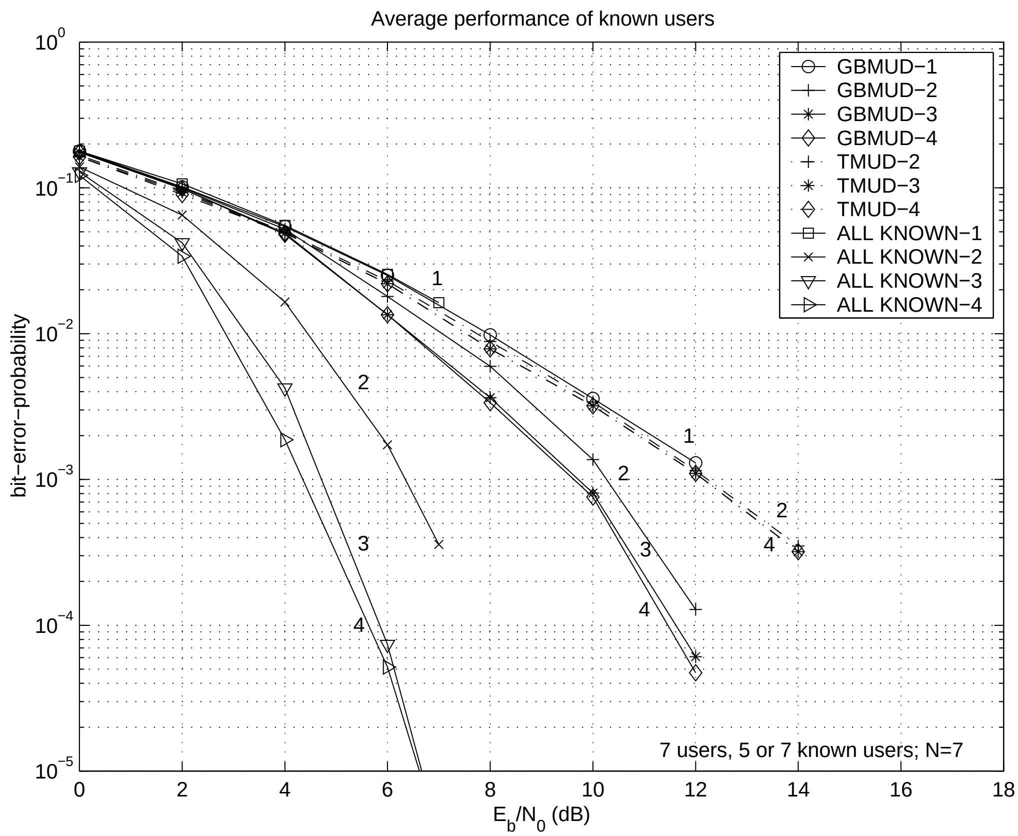

Figure 2: Performance of the group-blind iterative multiuser receiver with 5 known users. Curves denoted GB-TMUD are produced using the turbo group-blind multiuser receiver and those denoted TMUD are produced using the traditional turbo multiuser receiver. Also included are plots for TMUD when *all* users are known.

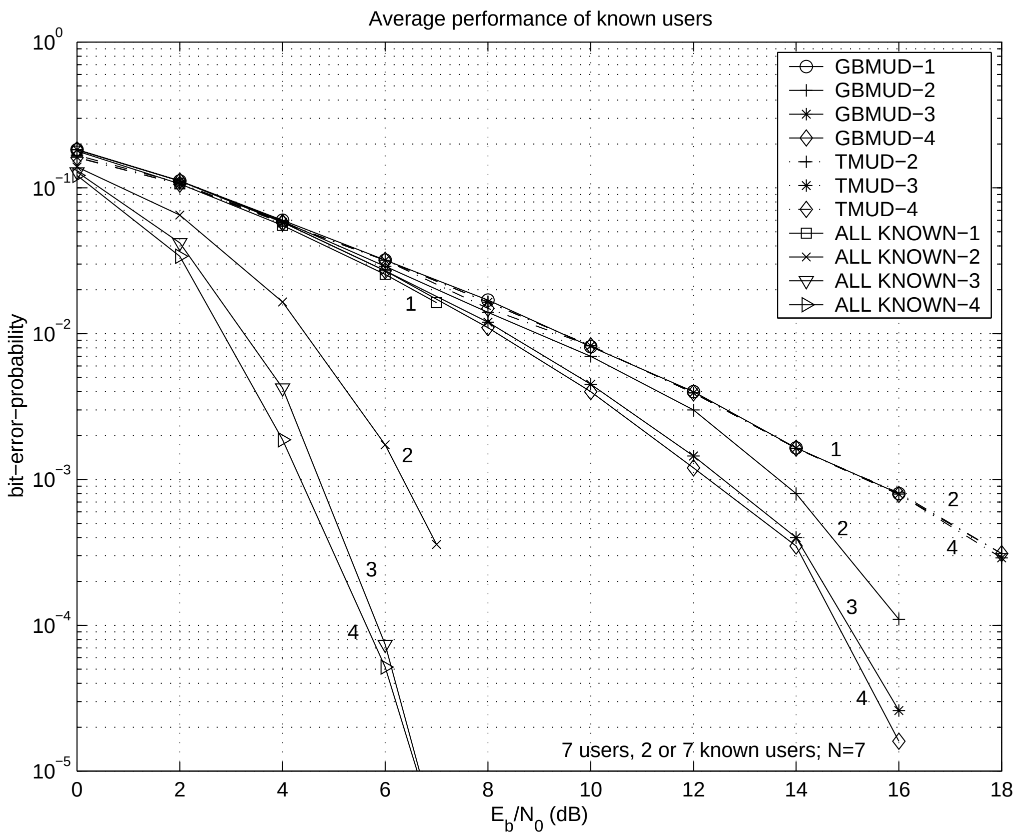

Figure 3: Performance of the group-blind iterative multiuser receiver with 2 known users. Curves denoted GB-TMUD are produced using the turbo group-blind multiuser receiver and those denoted TMUD are produced using the traditional turbo multiuser receiver. Also included are plots for TMUD when *all* users are known.