An Iterative Algorithm for Asynchronous Coded Multiuser Detection

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***Abstract*—A multiuser detection algorithm, applicable to asynchronous users having the same signaling waveform and power levels, is presented. Users are assumed to employ forward error correction coding but with different pseudorandom interleaving. The algorithm is derived from iterative techniques for cross-entropy minimization, similar to turbo decoding. Simulations show that the detector is limited by theoretical channel capacity at low signal-to-noise ratio (SNR) and asymptotically achieves single user performance at high SNR.**

***Index Terms*— Cross-entropy, multiuser decoding, multiuser detection, relative entropy.**

IN THIS LETTER, the asynchronous multiple access problem is addressed. Contrary to most current multiuser detection literature, we assume users are not spread but rather have identical modulating waveforms with minimal excess bandwidth. In related work, the maximum-likelihood multiuser detector for coded code-division multiple access (CDMA) signals is described in [1]. The synchronous multiple access problem for nonspread signals is described in [2], and techniques for solving it are described in [3] and [4]. The approach described here relies upon the forward error correction (FEC) coding to separate the users, and some ideas are borrowed from turbo coding/decoding to achieve this [5].

To describe the problem, we use notation similar to the literature on multiuser detectors for CDMA [6] although nonspread systems are being considered. In particular, we consider multiuser systems that have the following equivalent discrete time representation [6], with the appropriate boundary conditions

$$\boldsymbol{y}(i) = H(-1)\boldsymbol{b}(i-1) + H(0)\boldsymbol{b}(i) + H(1)\boldsymbol{b}(i+1) + \boldsymbol{n}(i) \qquad i = 1, \cdots, N \tag{1}$$

where $\boldsymbol{y}(i) \in \Re^K$ is the vector of matched filter outputs for symbol period i (optimal sampling for each user); the vector of the K-user bits at time i is represented by $\boldsymbol{b}(i) = (b_1(i), \cdots b_K(i))^T \in \{-1, +1\}^K$; the cross-correlation matrices $H(i) \in \Re^{K \times K}$ between the modulating waveforms $s_k(t)$ are given by

$$(H(i))_{kl} = \int_{-\infty}^{\infty} \mathrm{Re}[s_k(t - iT - \tau_k)s_l^*(t - \tau_l)]\, dt \tag{2}$$

where T is the symbol period, $\{\tau_k\}$ are the relative delays and $\boldsymbol{n}(i) \in \Re^K$ is the vector of noise samples at the matched filter output with correlation $E[\boldsymbol{n}(i)\boldsymbol{n}(i+j)^T] = H(j)\sigma^2$. For simplicity of discussion, we assume that $s_k(t) = s_l(t)$ for all k and l and are zero outside the symbol interval $[0, T]$. It is assumed that the delays are known and user sequences include FEC coding but with different random interleaving. The proposed algorithm can be easily extended to a more general situation including differing modulating waveforms, power levels, codes, symbol alphabets, phase and frequency offsets. The intent here is to illustrate the basic ideas of the algorithm.

The algorithm is derived from iterative techniques for minimizing cross-entropy [3], [4]. With this approach the FEC codes of the different users and the intersymbol interference (ISI) relationships (1) are viewed as constraints on the probability distributions of the symbols $\{b_k(i)\colon i = 1 \cdots N, k = 1 \cdots K\}$ and implicitly on the underlying information sequences [3], [4]. In particular, given an initial symbol distribution, the object is to determine a resultant distribution consistent with the constraints and having minimum cross-entropy (MCE) with respect to the initial distribution. The advantage of the iterative MCE approach is that one can consider each of the constraints independently as long as one continues to iterate through all of the constraints until convergence occurs. The MCE approach determines the a posteriori distribution of the symbols; the related maximum *a posteriori* (MAP) detector determines which symbol in this distribution has the maximum probability (although it may not explicitly calculate the probability distribution over all symbols).

The detector based on the iterative cross-entropy minimization algorithm is shown in Fig. 1. The initial distribution for each iteration is the combination of the *a priori* information regarding the bits and the extrinsic information for each user provided by the previous decoding, if any. The algorithm first considers the ISI constraint (1) at each symbol period. Recalling the relationship of MCE detection to MAP detection, the MCE distribution on the mth iteration for the ith ISI constraint is given by

$$\begin{aligned} r_m[\boldsymbol{b}(i)] &= p[\boldsymbol{b}(i)|\boldsymbol{y}(i)] \\ &= \sum_{\boldsymbol{b}(i-1),\boldsymbol{b}(i+1)} p[\boldsymbol{b}(i)|\boldsymbol{y}(i), \boldsymbol{b}(i-1), \boldsymbol{b}(i+1)] \\ &\quad \times q_m[\boldsymbol{b}(i-1), \boldsymbol{b}(i+1)] \end{aligned} \tag{3}$$

where $q_m[\cdots]$ is the *a priori* distribution for the mth iteration. In the following example, the *a priori* distribution $q_0[\cdots]$ is an independent uniform distribution over all binary symbol vectors. In (3), the conditioning is restricted to the adjacent

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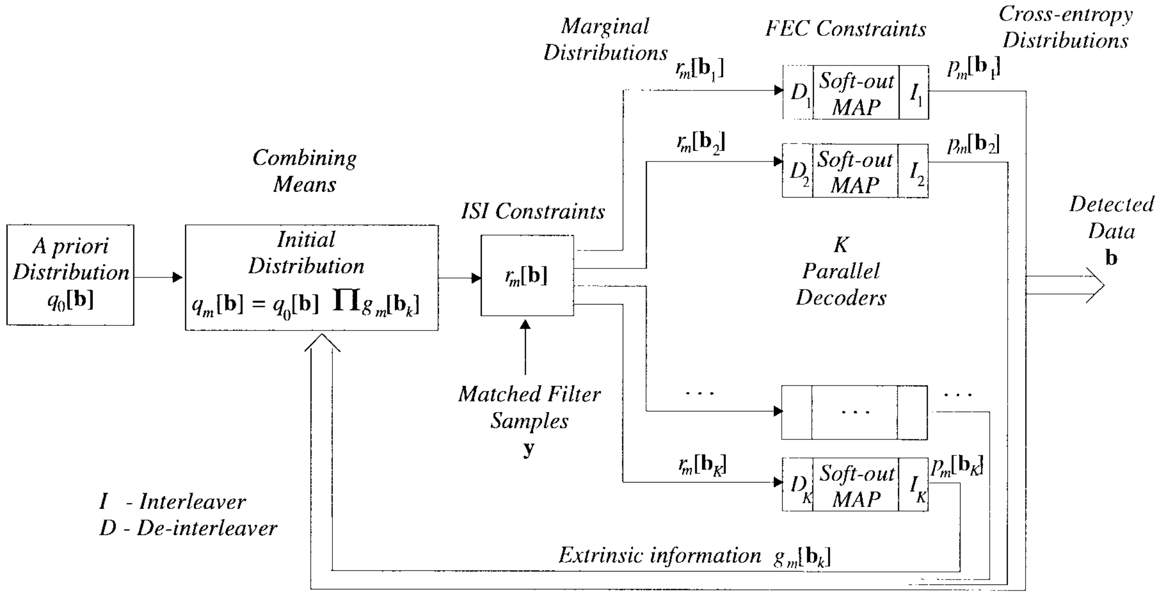

Fig. 1. Illustration of iterative multiuser detection algorithm.

symbol vectors because this results in a simple form for the distribution. The conditional distribution under the summation in (3) can be rearranged to give

$$\begin{aligned} p[\boldsymbol{b}(i)|\boldsymbol{y}(i), \boldsymbol{b}(i-1), \boldsymbol{b}(i+1)] \\ = p[\boldsymbol{y}(i)|\boldsymbol{b}(i-1), \boldsymbol{b}(i), \boldsymbol{b}(i+1)]p[\boldsymbol{b}(i)]/\nu_m \end{aligned} \tag{4}$$

where ν_m is a constant the normalizes the probability mass over all $\boldsymbol{b}(i)$. The relationship (4) assumes that the initial probabilities, $q_m[\cdots]$, of the symbol vectors $\boldsymbol{b}(i-1), \boldsymbol{b}(i)$, and $\boldsymbol{b}(i+1)$, are independent. The conditional distribution on the right-hand side of (4) can be evaluated by letting $\boldsymbol{s}(i) = \boldsymbol{y}(i) - H(-1)\boldsymbol{b}(i-1) - H(0)\boldsymbol{b}(i) - H(1)\boldsymbol{b}(i+1)$ and then, for an additive Gaussian noise channel, one has

$$\begin{aligned} p[\boldsymbol{y}(i)|\boldsymbol{b}(i-1), \boldsymbol{b}(i), \boldsymbol{b}(i+1)] \\ = \frac{1}{(2\pi\sigma)^{K/2}\det(H(0))^{1/2}} \exp\{-\boldsymbol{s}(i)^T H(0)^{-1}\boldsymbol{s}(i)/2\sigma^2\}. \end{aligned} \tag{5}$$

The marginals of the output distribution from the ISI constraint $r_m[b_k(i)]$ are then fed to the K parallel branches embedded in the iterative structure. Each branch considers only the bits of one particular user. The branch structure is somewhat analogous to the Turbo-decoding of parallel codes [5]. Each branch consists of a deinterleaver (D_k), a soft-output MAP symbol detector [7], and a corresponding interleaver (I_k). The D and I pair correspond to the random interleaving assumed at the transmitter. The soft-output MAP decoder is equivalent to determining the MCE distribution under the corresponding FEC code constraints with the assumption that the bits are independent [3], [4].

The output of each interleaver is the cross-entropy distribution for each user's symbols and implicitly determines the estimated information sequence. The extrinsic portion of this distribution [5] (the information learned from the decoding) is then fed back to form the starting distribution for the next iteration.

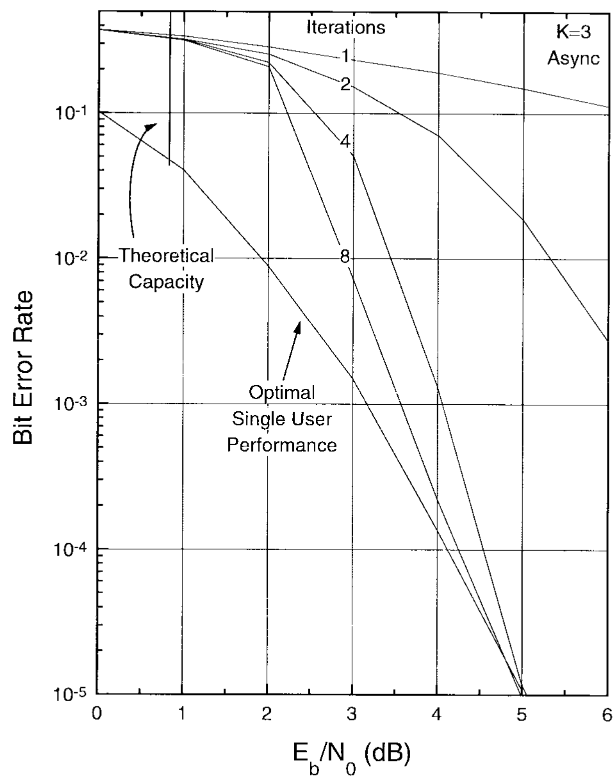

Fig. 2. Comparison of BER performance of iterative detector for three asynchronous users with rectangular signaling to theoretical capacity and optimal single user performance (1, 2, 4, and 8 iterations).

The simulated bit-error rate performance of this detector is illustrated in Fig. 2 for the case for $K = 3$ equal power users, each using a rectangular signaling pulse with the relative delays $\tau_k = (k-1)T/3$. Each user has a block size of 500 information bits and uses the same rate $r = 1/2$ constraint length 5 convolutional code but with different random interleaving. In Fig. 2, the bit error rate performance of the iterative detector on an additive white Gaussian noise (AWGN) channel is shown as a function of the single user E_b/N_o for 1, 2, 4, and 8 iterations of the algorithm. Performance converges rapidly to single user performance at higher E_b/N_o ratios. At low E_b/N_o ratios, there is a performance threshold. This threshold is related to the theoretical capacity of the channel [4]. In Fig. 2, we have indicated the theoretical AWGN channel capacity, i.e., the minimum E_b/N_o for transmitting a total of $K_r = 1.5$ bits per channel use (when the channels uses are independent).

In conclusion, an algorithm has been presented for iterative multiuser detection for nonspread asynchronous users sharing the same channel. It can be shown that this algorithm is closely related to the optimal minimum cross-entropy detector subject to an independence assumption. As a result, when one uses FEC coding with random interleaving, one can achieve the maximum asymptotic multiuser efficiency [6], as the simulation result presented here indicates. Further results, when users have unequal powers, indicate that the approach is near–far resistant [6]. A further observation is that this algorithm collapses to an interesting approach for dealing with ISI in the single user channel.

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