Multiuser Decoding for Multibeam Systems

Michael L. Moher, *Member, IEEE*

***Abstract*—An iterative multiuser decoding algorithm for co-channel BPSK/QPSK users in a multibeam system is presented. The approach can be applied to the return link of multibeam satellites and to terrestrial systems with sectored base-station antennas. It allows the reuse of the same spectrum in each beam. The algorithm is based on the extension of turbo-decoding techniques to the iterative decoding of parallel users. Simulation results show one can asymptotically achieve single user performance in a high multiuser interference environment; often this includes some diversity gain. The complexity of the algorithm is approximately $O(2^K + 2^\kappa)$ operations per bit per iteration where K is the number of co-channel users and κ is the constraint length of the forward error correction code.**

***Index Terms*—Antenna arrays, channel coding, decoding, diversity methods, iterative methods, multiaccess communications, multibeam antennas, multiuser channels.**

I. Introduction

FREQUENCY reuse of the radio spectrum is one key constraint limiting the spectral efficiency of many communications systems. Code division multiple access (CDMA) has enjoyed recent success as a strategy for maximizing the spectral efficiency of both terrestrial cellular systems and multibeam satellite systems, partly due to its potential to reuse the same spectrum in each cell or beam. However, these systems are still far from the theoretical Shannon capacity of multiuser systems [1]. To further increase the capacity of CDMA there has been much research into multiuser detection and, recently, multiuser decoding [2]–[9].

In [4], it is demonstrated that the optimum multiuser decoder for an asynchronous CDMA system employing forward error correction coding combines the trellises of both the asynchronous detector and the forward error correction (FEC) code. This decoder is a variation of the Viterbi detector that has complexity of approximately $O(2^{\kappa K})$ operations per bit where κ is the code constraint length and K is the number of users. This exponential complexity makes the system impractical for most CDMA systems. Consequently, there has been considerable research into simpler, suboptimum decoding strategies. As described in [8], these range from partitioned strategies, e.g., [9], where the detection is separated from the decoding function, to integrated strategies, e.g., [5] and [8], that often include feedback and iteration between the decoding and detection stages.

As described in [11], a limitation of all these strategies is the degradation of performance as the correlation between users increases. They rely on a spread modulation such as direct sequence spread spectrum where the correlation between users is intentionally kept low. Consequently, they are not applicable for nonspread BPSK or QPSK modulation. In [11], an iterative multiuser decoding algorithm is presented addressing this problem with highly correlated users sharing the same channel. This algorithm has independently been discovered by Reed *et al.* [10], and related work can be also be found in [20]. However, the focus of these latter investigators has remained on CDMA applications. In this paper, we describe how the iterative algorithm of [11] can be applied to co-channel BPSK/QPSK users in two particular scenarios—the multibeam satellite scenario and the sectored base-station antenna scenario—and allow complete reuse of the spectrum in every beam. This paper also extends the work of Miller and Schwartz who considered the spatial-temporal processing required for multiuser detection of uncoded CDMA users [12].

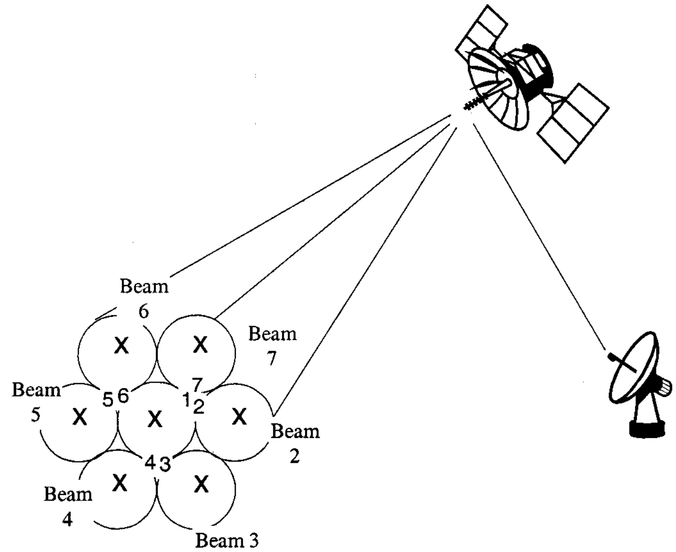

Fig. 1. Illustration of co-channel interference with multi-beam satellites. (The $\times$'s and numbers indicate user positions for different simulation scenarios.)

In Section II, the return-link multibeam satellite and sectored antenna scenarios are described with the corresponding mathematical model. This is followed in Section III by a description of the algorithm for multiuser decoding. In Section IV, we provide simulation results showing the system's performance, and conclude the paper in Section V. For simplicity of presentation, we shall assume synchronous BPSK users throughout the development. The algorithm can be extended to handle asynchronous users [19] and other modulations but these require greater complexity.

II. Multiuser Detection Model

The first scenario that we will consider is the return link of a multibeam satellite, as illustrated in Fig. 1. In this scenario, one

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M. L. Moher is with the Communications Research Centre, Station "H," Ottawa K2H 8S2, ON, Canada.
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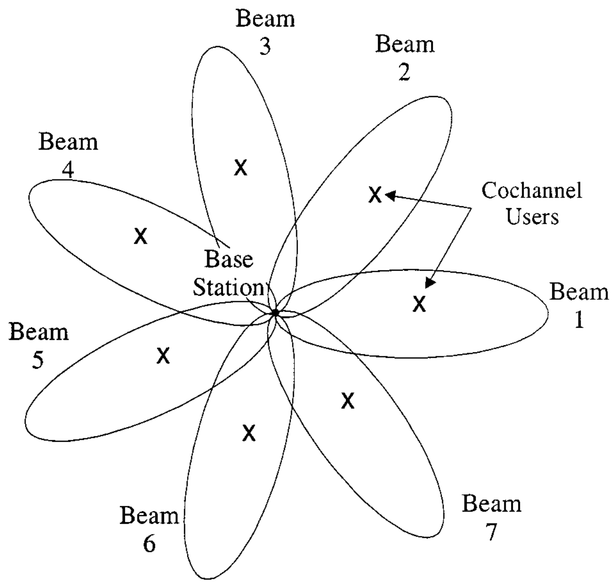

Fig. 2. Illustration of multiple co-channel users with base station using sectored antenna. (The $\times$s indicate positions of users sharing the same channel.)

usually has access to the received signals of each spotbeam at the same earth station. This makes joint processing of the different beams possible. Traditionally, satellites use an FDMA strategy for multiplexing users and, when this is extended to multiple beam satellites, frequencies are generally not reused in adjacent beams but separated by one or two beamwidths to provide greater isolation. Here, we consider the case where frequencies are reused in every beam and each user's signal will be received in each spotbeam, albeit with varying degrees of attenuation. For example, with a multibeam satellite, the spotbeam edges are often only 3 dB lower than the center of the beam. This amounts to a multiuser detection problem with diversity.

The second scenario occurs in a terrestrial setting when one has a base station using a sectored antenna, as illustrated in Fig. 2. This may occur in a mobile telephony service, packet radio service, or in proposed Local Multipoint Communications Systems (LMCS). If an FDMA access strategy is chosen, then the typical approach is to separate frequency re-uses by at least one beamwidth or more, at the expense of system capacity; quite analogous to the multibeam satellite example. Thus, this case can be considered in the same framework as the multibeam satellite scenario.

Multiuser detection techniques with spatial diversity were first investigated in [12] for uncoded CDMA systems. Here, we extend this investigation to include forward error correction and nonspread systems.

Assume K BPSK users, each transmitting a block of N bits in a bit-synchronous fashion using the following notation. For user k at bit period i, the corresponding transmitted bit will be represented by $b_k(i)$. The complete sequence of the kth user's bits will be represented by the vector $\mathbf{b}_k = (b_k(1), \ldots, b_k(N)) \in \{-1, +1\}^N$ and the vector of the K-user bits at time i is represented by $\mathbf{b}(i) = (b_1(i), \ldots, b_K(i))^T$. The matrix of all user bits is represented by $b \in \{-1, +1\}^{K \times N}$. This same notation is extended to other quantities.

These K users simultaneously access the communications channel each using a modulating waveform $s_k(t)$ with a symbol period T. Nominally, one can assume each user is using rectangular pulses. In general, there can be p receivers for K users, i.e., p antennas, and each user will suffer a different attenuation and phase rotation with respect to each antenna, as indicated in Fig. 3. In the simulations, we assume that $p = K$. The case $p = 1$ is described in [11]. With a minor alteration in the notation of [12], the noise-free portion of the received signal of the kth user signal received at a nominal antenna can be represented as

$$s_k(\mathbf{b}_k, t) = \sum_{i=1}^{N} b_k(i) \sqrt{w_k} s_k(t - iT - \tau_k). \tag{1}$$

The parameter τ_k represents the relative transmission delay of the kth users. For the synchronous case, $\tau_1 = \cdots = \tau_K = 0$. The modulating waveforms, $s_k(t)$, are assumed to be normalized to unit energy, and the relative received amplitude levels of the different users are characterized by the positive parameters $\{\sqrt{w_k}\}$. The channel amplitude $\sqrt{w_k}$ in (1) represents individual user gain effects such as propagation losses, power control, and fading. The vector of p antenna outputs for user k can be represented as

$$\mathbf{s}_k(\mathbf{b}_k, t) = \mathbf{a}_k s_k(\mathbf{b}_k, t). \tag{2}$$

The vector $\mathbf{a}_k$ in (2) represents the complex gain of the p different receivers (sensors) for user k. For the kth user, $\mathbf{a}_k$ typically has the form

$$\mathbf{a}_k = \begin{bmatrix} 1 \\ a_{k2} e^{j\theta_{k2}} \\ \vdots \\ a_{kp} e^{j\theta_{kp}} \end{bmatrix}. \tag{3}$$

This assumes only amplitude and phase differences between the different users. In practice, there could be frequency differences as well but if these are relatively small relative to the symbol rate, they could be modeled as a time-varying $\mathbf{a}_k(i)$.

The noise-free received signal from all K users with spatial diversity can then be expressed as

$$\mathbf{S}(\mathbf{b}, t) = \mathbf{A}\mathbf{s}(\mathbf{b}, t) \tag{4}$$

where $\mathbf{S}$ is the p-dimensional vector representing the output of the p different receivers, $\mathbf{s}(\mathbf{b}, t) = (s_1(\mathbf{b}_1, t), \ldots, s_K(\mathbf{b}_K, t))^T$, and $\mathbf{A} = (\mathbf{a}_1 \cdots \mathbf{a}_K)$ is the $p \times K$ directions matrix.

When combined with additive Gaussian noise, the p-vector of received signals is

$$\mathbf{r}(t) = \mathbf{S}(\mathbf{b}, t) + \mathbf{n}(t) \tag{5}$$

where $\mathbf{n}(t)$ is a p-vector of Gaussian noise processes. For independent antennas, we model the noise covariance as

$$E[\mathbf{n}(t)\mathbf{n}(s)^H] = \sigma^2 \mathbf{I} \delta(t - s) \tag{6}$$

where $\mathbf{I}$ is the $p \times p$ identity matrix and the superscript H represents conjugate transpose. If the different beams are formed by

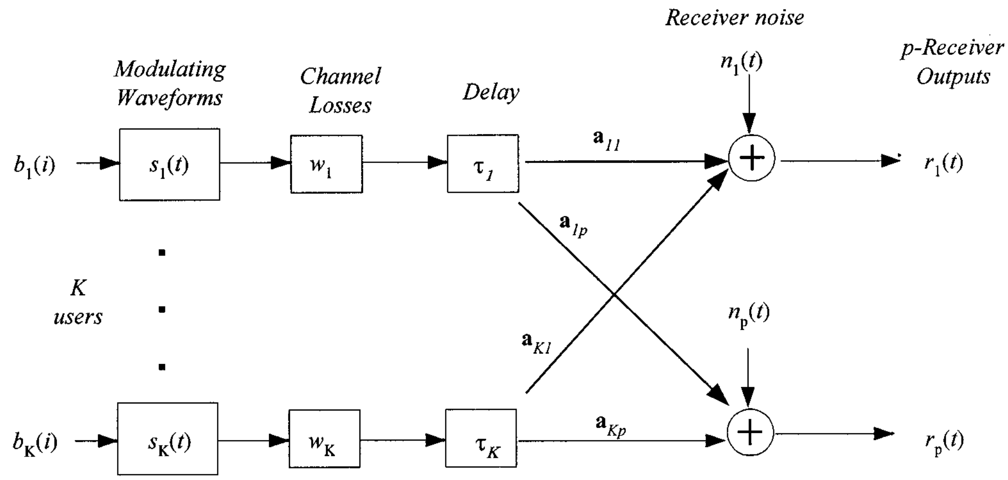

Fig. 3. Illustration of multiuser communication system model.

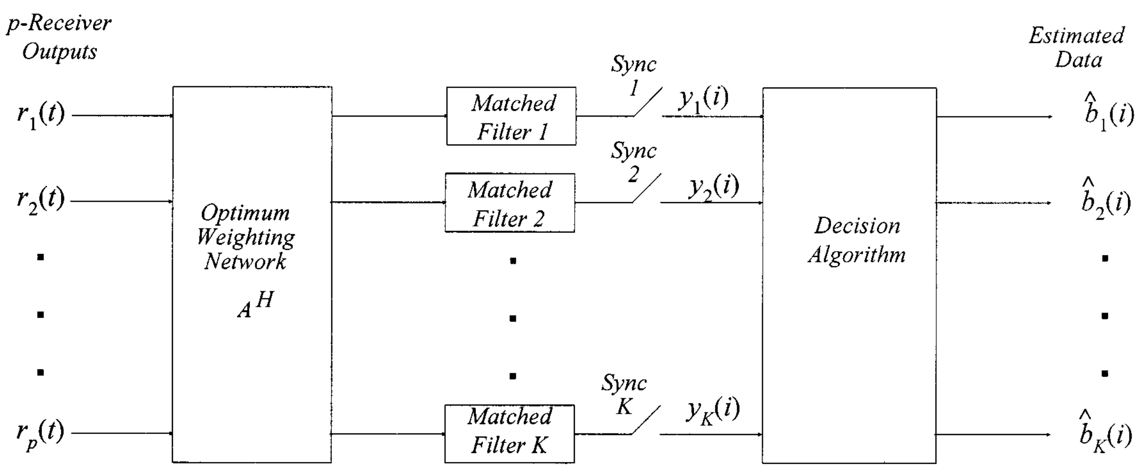

Fig. 4. Illustration of multi-user array processing algorithm.

a phased array, then one would expect correlation between the different elements of $\mathbf{n}(t)$. The effect of such correlation will be discussed later.

It is assumed that the modulating waveform of each user is known at the receiver, and that the K-user coherent receiver locks to the signaling interval and phase of each active user. This is a nontrivial problem but we do not consider the design of such a multiuser synchronization circuit in this paper.

Maximum-likelihood detection requires maximizing the corresponding likelihood function. In [12], it is shown that a sufficient statistic for evaluating the log-likelihood function in the multibeam scenario is

$$y_k(i) = \mathrm{Re}\left[\mathbf{a}_k^H \int_{-\infty}^{\infty} \mathbf{r}(t) s_k^*(t - iT - \tau_k)\, dt\right]. \tag{7}$$

This implies that the sufficient statistic is obtained by passing the received noisy signal vector through a K-output optimal weighting network represented by matrix A^H followed by a bank of K matched filters. The weighting matrix A^H realizes beams directed at each of the users maximizing the signal to noise ratio for user k at the output of the weighting (beamforming) network. This is illustrated in Fig. 4.

With this approach the $K \times K$ spatial-temporal cross-correlation matrix for the different users in the synchronous case is given by

$$R_{ij} = \mathrm{Re}\left[\mathbf{a}_i^H \mathbf{a}_j \int_{-\infty}^{\infty} s_i(t - \tau_i) s_j^*(t - \tau_j)\, dt\right]. \tag{8}$$

Since the modulating waveform is normalized, the integral term in (8) has a maximum magnitude of one. However, the contribution of the antenna network may often exceed one, and consequently, the system has effectively some diversity gain. Analogous to [12], the corresponding equivalent discrete time model for generating the matched filter outputs $\mathbf{y}(i)$ in the synchronous case is given by

$$\mathbf{y}(i) = RW^{1/2}\mathbf{b}(i) + \mathbf{z}(i), \qquad i = 1, \ldots, N \tag{9}$$

where $\mathbf{z}(i)$ is the sampled complex noise K-vector. The matched filter outputs represent sufficient statistics for obtaining the optimum solution and W is the diagonal matrix with nonzero elements $\{w_k\}$. The receiver of Fig. 4 maximizes the signal to noise ratio on the samples $\mathbf{y}(i)$. Although the

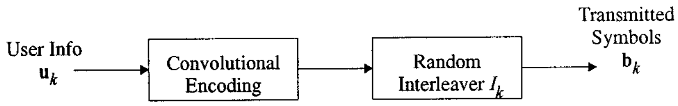

Fig. 5. Illustration of encoding algorithm.

input noise is assumed white, after the weighting network the sampled output noise process is nonwhite. In particular,

$$E[\mathbf{z}(i)\mathbf{z}(j)^T] = \sigma^2 R\delta(i-j) \tag{10}$$

which is analogous to the nondiversity case [11]. If the input noise of (6) is not white then the noise correlation matrix in (10) is modified accordingly.

III. The Multiuser Decoding Algorithm

A. Encoding Algorithm

For the transmission/detection strategy described in the previous section, we include the following FEC encoding strategy. We assume each user uses the same convolutional code for FEC. This is not a requirement but it simplifies the presentation and system design. In addition, we assume each user shuffles the encoded bits with a different pseudorandom interleaver, as illustrated in Fig. 5. The interleaving effectively gives each user a different code and this has some important consequences that are described in the following.

In the multiuser detection literature, efficiency is a measure used to compare the performance of detectors. The efficiency of a multiuser detector, η_k, is a function of the user k. It represents the power efficiency of the kth user with multiple users present, relative to the power efficiency of the kth user by itself, in an additive white Gaussian channel at high signal-to-noise ratio (SNR) [13]. In a diversity situation such as being considered here, however, we shall use as the reference point: the performance in an additive white Gaussian noise channel with equivalent diversity. For all uncoded systems and many coded systems, the detector efficiency decreases as the correlation between user waveforms increases. This is true even with the optimum detector for these systems. This is why many proposed multiuser detectors are limited to CDMA-like applications where the waveform correlations are low.

When FEC encoding is employed with the random interleaving strategy described above, the efficiency is a random variable that depends upon the choice of the random interleavers for the K different users. We let $\eta_{k,N}$ denote this random variable where N is the interleaver length. From [11] we have the following result that applies directly to the model given in (9).

Theorem 1. (Optimal asymptotic efficiency with coding and random interleaving [11]): For a synchronous Gaussian channel with K equal power users and K randomly selected interleavers of length N

$$\lim_{N\to\infty} \Pr[\eta_{k,N} = 1] = 1, \qquad k = 1, \ldots, K \tag{11}$$

with an optimum decoder when the cross-correlation matrix R is positive definite. ∎

This states that for randomly chosen interleavers, the probability of a multiuser system achieving single user performance approaches one as the block length N increases. While the above result relies on the use of an optimum decoder, the importance of the result is that it does not require the user cross-correlations to be small. In fact, even the requirement of a positive-definite R does not seem necessary in practice. This implies that it can be applied to modulation techniques such as BPSK and QPSK.

B. Decoding Algorithm

The following iterative decoding structure can be derived either intuitively or theoretically from a minimum cross-entropy framework [14]. In either case, it is best understood by using some of the concepts developed to explain Turbo decoding [15], [16]. The important concepts are those of intrinsic and extrinsic information [17]. Intrinsic information refers to that information inherent in a bit as received over the channel, i.e., the soft decision. Extrinsic information is that information provided about a bit from the other received bits due to the constraints imposed by the FEC code. For a single user, the optimum soft-output decoder produces the sum of the intrinsic and extrinsic information, often expressed as a log-likelihood ratio.

The two major components to the multiuser decoding algorithm shown in Fig. 6 are a combining block and a parallel decoding block. The input to each decoder is the combination of the intrinsic information obtained from the channel with the extrinsic information for each user provided by the previous decoding. It is an iterative scheme where the extrinsic output of the soft-output decoders are fed back for combining with the original intrinsic input.

The parallel decoding block consists of K parallel single-user decoders each matched to the interleaver and FEC code of a specific user and only considers the bits of that user. These are soft-output decoders that produce a probability estimate (soft-output) for each symbol. The soft decoding algorithm used in our simulations can be found in [18]. Following [17], this probability estimate (soft-output) for a symbol p_{out}, can be separated into two parts, the input (intrinsic) probability estimate, p_{int}, and the additional (extrinsic) information added via the decoding, p_{ext}. In particular

$$p_{\text{out}} = \frac{p_{\text{int}} p_{\text{ext}}}{p_{\text{int}} p_{\text{ext}} + (1-p_{\text{int}})(1-p_{\text{ext}})} \tag{12}$$

and the extrinsic information can be extracted from the output via the expression

$$p_{\text{ext}} = \frac{p_{\text{out}}/p_{\text{int}}}{p_{\text{out}}/p_{\text{int}} + (1-p_{\text{out}})/(1-p_{\text{int}})}. \tag{13}$$

In the following description of the algorithm, let $p_m[b_k(i)]$ represent the soft-output of the decoder for the ith symbol of the kth user after the mth iteration. The corresponding extrinsic information, $g_m[b_k(i)]$, is defined, from (13), as

$$g_m[b_k(i)] = \frac{\dfrac{p_m[b_k(i)]}{q_{m-1}[b_k(i)]}}{\dfrac{p_m[b_k(i)]}{q_{m-1}[b_k(i)]} + \dfrac{1-p_m[b_k(i)]}{1-q_{m-1}[b_k(i)]}} \tag{14}$$

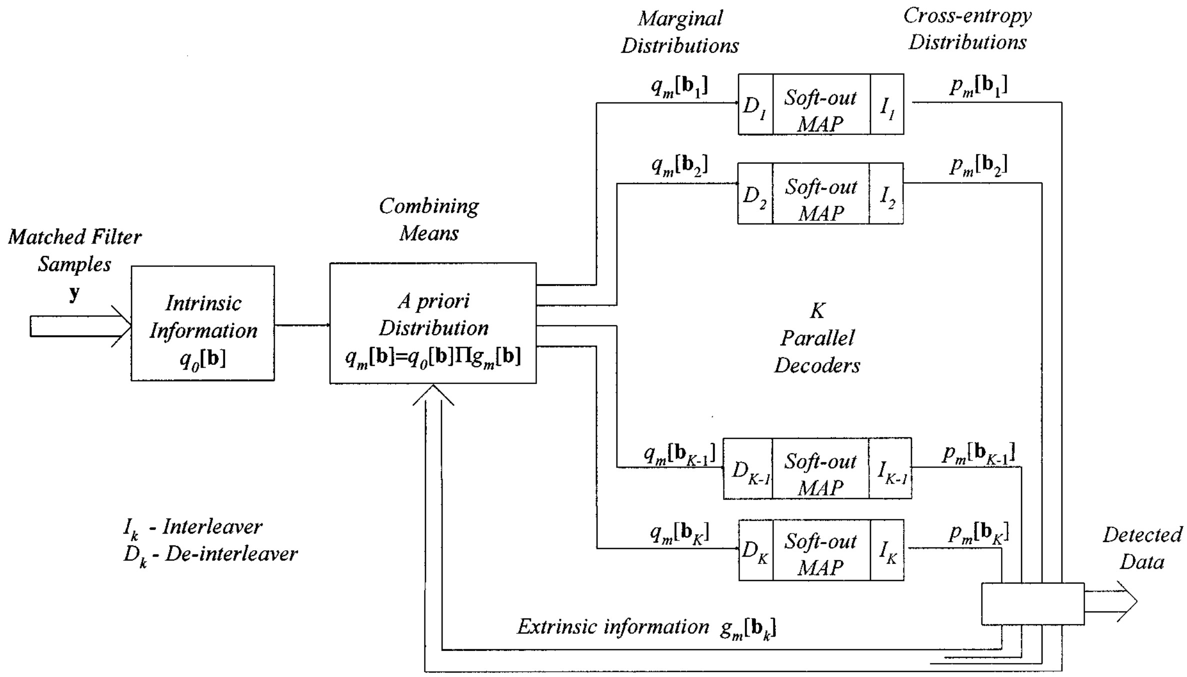

Fig. 6. Illustration of iterative multiuser detection algorithm.

where $q_m[b_k(i)]$ is defined in the following. It is the set $\{g_m[b_k(i)]\text{: } k = 1, \ldots, K,\ i = 1, \ldots, N\}$, of all extrinsic information that is fed back to the combining algorithm. For this algorithm, we do not assume the FEC code is systematic as is the case with Turbo codes.

The intrinsic distribution of the input corresponds to the distribution of the matched filter outputs. For a memoryless Gaussian channel with matched filter outputs $\{y_k(i)\text{: } k = 1, \ldots, K, i = 1, \ldots, N\}$, the *intrinsic* distribution for the ith symbol period is given by [from (9)]

$$q_o[\mathbf{b}(i)] = \frac{\det(R)^{-1/2}}{(2\pi\sigma^2)^{K/2}} \exp\left\{-\frac{1}{2\sigma^2} \mathbf{f}(i)^T R^{-1}\mathbf{f}(i)\right\} \equiv q_o[b_1, \ldots, b_K] \tag{15}$$

where $\mathbf{f}(i) = \mathbf{y}(i) - RW^{1/2}\mathbf{b}(i)$ and the noise has covariance $\sigma^2 R$.

With these definitions, the multiuser decoding algorithm has the following steps.

1) Compute the marginal distributions $q_o[b_k(i)]$ corresponding to (15)

$$q_o[b_k(i)] = \sum_{b_1, \ldots, b_{k-1}, b_{k+1}, \ldots, b_K} q[\mathbf{b}(i)]. \tag{16}$$

2) Decode each user's bits separately based on $\{q_m[b_k(i)]\text{: } i = 1, \ldots, N\}$ and compute the corresponding extrinsic information $\{g_m[b_k(i)]\text{: } i = 1, \ldots, N\}$ at the output of the decoder. Note that this decoding includes the random interleaving and de-interleaving corresponding to the user of interest. At any time, one can use the output of the soft-decoder to make a hard decision on the bit, as in [18].

3) Combine the extrinsic information with the original intrinsic information, and compute the marginal distributions for each of the k decoders in the next iteration

$$q_m[b_k(i)] = \sum_{b_1, \ldots, b_{k-1}, b_{k+1}, \ldots, b_K} q_o[\mathbf{b}(i)] g_m[b_1(i)] \cdots g_m[b_{k-1}(i)] g_m[b_{k+1}(i)] \cdots g_m[b_K(i)]. \tag{17}$$

4) Repeat steps 2 and 3 a fixed number of times or until convergence.

This algorithm relies on the independence of the intrinsic and extrinsic information for the bit sequence of each user. This is provided in large part by the use of random interleavers. Note that we also exclude the extrinsic information for the bit of interest in (17). This was found to improve performance and is justified in [11] and [14]. The majority of the performance improvement is obtained with eight or fewer iterations. One can iterate for a fixed number of iterations or other stopping criteria can be used [17].

This provides a manageable algorithm that is suitable for multiuser decoding. From a complexity viewpoint there are K decoders (or a single decoder reused K times) that is matched to the code of a single user. This results in a complexity of approximately $O(2^\kappa)$ per bit where κ is the constraint length of the binary code being used. With the combining algorithm, one must compute the probabilities of all 2^K K-user bit patterns in any symbol interval, and then the corresponding marginals. This has a complexity proportional to $O(2^K)$. Combining these two results, the overall complexity is approximately $O(2^K + 2^\kappa)$ operations per bit per iteration.

IV. SIMULATION RESULTS

In this section, we present the simulation results for the proposed iterative decoding algorithm. All simulations use a block size of 500 information bits for each user. Each user uses the same rate 1/2 constraint length 5 convolutional code with generators [10 011] and [11 101]. Each user uses a different pseudorandom interleaver and the same set of interleavers is used for all simulation runs. The interleavers were chosen at random and no attempt was made to optimize them. Each simulation point is tested for the minimum of 1600 errors or four million bits.

A. Multibeam Satellite Examples

In this first example, we consider a satellite with seven spotbeams arranged with user terminals located in the center of each beam, as indicated by $\times$s in Fig. 1. To characterize the spotbeam gain pattern, we assume that the antenna gain is 0 dB at beam center, -6 dB at a distance of one beamwidth, and negligible at a distance of two beamwidths or more. These are nominal values chosen to illustrate the approach; in practice, the antenna gain patterns will be somewhat less uniform. For this nominal example the antenna gain matrix A is given by

$$A = \begin{bmatrix} 1 & \alpha & \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & 1 & \alpha & 0 & 0 & 0 & \alpha \\ \alpha & \alpha & 1 & \alpha & 0 & 0 & 0 \\ \alpha & 0 & \alpha & 1 & \alpha & 0 & 0 \\ \alpha & 0 & 0 & \alpha & 1 & \alpha & 0 \\ \alpha & 0 & 0 & 0 & \alpha & 1 & \alpha \\ \alpha & \alpha & 0 & 0 & 0 & \alpha & 1 \end{bmatrix} \tag{18}$$

where $\alpha = 0.5$ corresponding to a 6-dB attenuation, and the crosscorrelation matrix is $R = A^H A$. This assumes that all users are synchronous and use the same modulating waveform, e.g., BPSK with rectangular signaling. Since all entries in A are real, it implicitly implies worst case phasing of the users, i.e., all users have identical phase and maximize the mutual interference. In Fig. 7, the performance of the iterative detector for the six outer users (the outer circle of $\times$s in Fig. 1) is shown for one, two, four, and eight iterations. Also shown for comparison is ideal single user performance. As the number of iterations increases, performance exceeds single user performance. The reason for this is the diversity gain present in this channel. The diagonal elements of R corresponding to the six outer users are $1 + 3\alpha^2$ corresponding to a diversity gain of 2.4 dB with $\alpha = 0.5$. In fact, as Fig. 7 indicates asymptotic performance approaches that of a single user with the same level of diversity, as the number of iterations increases.

The performance of the user in the center beam is shown in Fig. 8. The performance of the center user is initially worse than the six outer user because of the greater multiple access interference but becomes better as the number of iterations increases due of increased diversity gain. The diversity factor for this beam is $1 + 6\alpha^2$ (3.9 dB). The simulation results indicate that this will be achieved asymptotically but requires more iterations of the decoder.

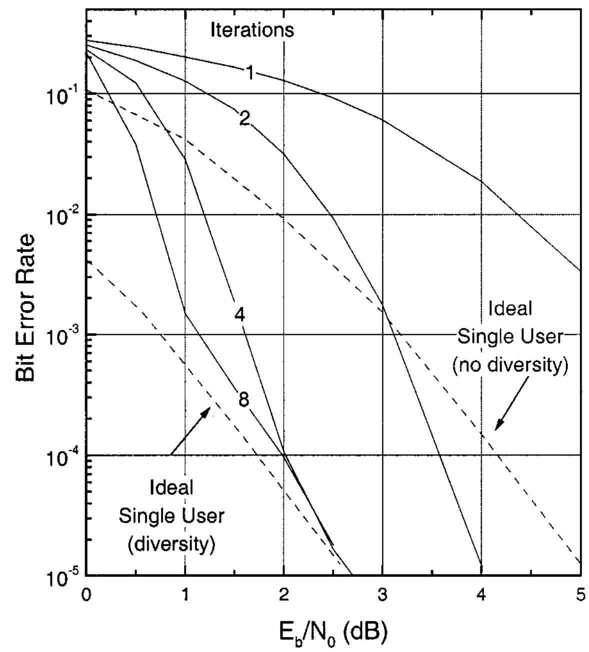

Fig. 7. Comparison of BER performance of six outer users ($\times$'s) in Fig. 1 with the iterative decoder (one, two, four, and eight iterations) to ideal single user performance.

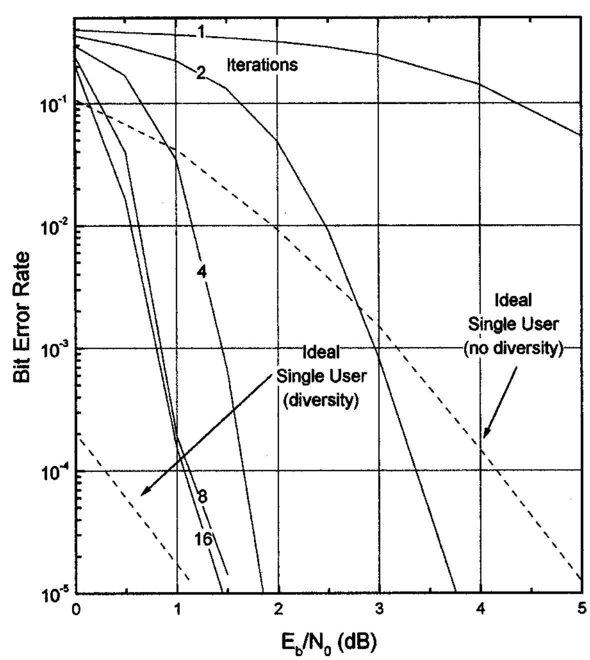

Fig. 8. Comparison of BER performance of the center beam user ($\times$) in Fig. 1 with the iterative decoder (one, two, four, eight, and 16 iterations) to ideal single user performance.

As a second satellite example, we use what might be considered a worst case arrangement of users with the seven spotbeams. In this case, there is again one user per beam but all users are located at the beam edge touching the center beam—the

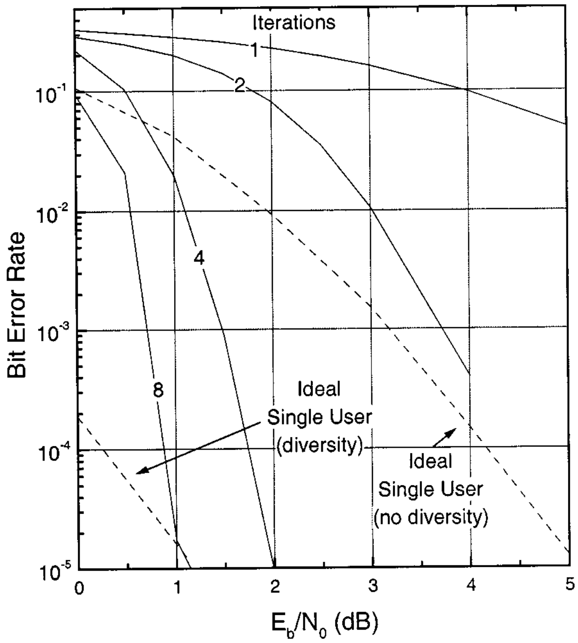

Fig. 9. Comparison of BER performance of users 4 and 5 in Fig. 1 with the iterative decoder (one, two, four, and eight iterations) to ideal single user performance.

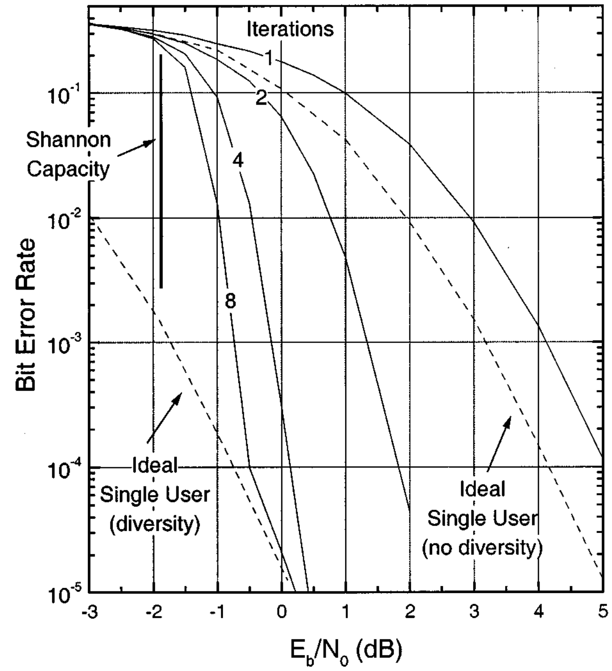

Fig. 10. Comparison of BER performance with seven users in the sectored antenna scenario with the iterative decoder (one, two, four, and eight iterations) to ideal single user performance.

small numbers in Fig. 1. The user in the center beam is also at the beam edge. In this case the antenna gain matrix is given by ($k = 1$ corresponds to the center beam user)

$$A = \begin{bmatrix} 1 & \beta & \beta & \beta & \beta & \beta & \beta \\ \beta & 1 & \alpha & \alpha & 0 & 0 & \beta \\ \alpha & \alpha & 1 & \beta & 0 & 0 & \alpha \\ 0 & 0 & \beta & 1 & \alpha & \alpha & 0 \\ 0 & 0 & \alpha & \alpha & 1 & \beta & 0 \\ \alpha & \alpha & 0 & 0 & \beta & 1 & \alpha \\ \beta & \beta & 0 & 0 & \alpha & \alpha & 1 \end{bmatrix}. \tag{19}$$

We assume all users are transmitting unity power. In this case, β represents the isolation with respect to a user that is just outside the beam service area. In the simulation model of the channel we let $\beta = 1$, implying no isolation. However, in the receiver we assume $\beta = 0.99$ to permit the calculation of R^{-1} in (15). The parameter α represents the isolation of a user separated by one beamwidth. As before, we let $\alpha = 0.50$. Not all users are equivalent in this scenario but there are a number of symmetries. In particular, users 4 and 5 should have identical performance. In Fig. 9 we show the average bit error rate (BER) performance of users 4 and 5. The diversity factor for these two users (the diagonal element of $R = A^H A$) is $1 + \beta^2 + 2\alpha^2$ which corresponds to 3.9 dB. This gain is achieved after eight iterations as shown in Fig. 9. Note that for the two users, 4 and 5, in this example, the signal to multiple access interference ratio is -5.8 dB. (From a system viewpoint, the results of Fig. 9 are optimistic if compared directly to those of Figs. 7 and 8. There is a 3 dB loss in antenna gain in the Fig. 9 scenario that does not appear in the BER versus E_b/N_o plots.)

B. Sectored Antenna Example

In this case, we consider a base station with a sectored beam, as illustrated in Fig. 2. We assume frequency reuse in each beam and that the users, as indicated, have a gain matrix of

$$A = \begin{bmatrix} 1 & \alpha & \alpha^2 & 0 & 0 & \alpha^2 & \alpha \\ \alpha & 1 & \alpha & \alpha^2 & 0 & 0 & \alpha^2 \\ \alpha^2 & \alpha & 1 & \alpha & \alpha^2 & 0 & 0 \\ 0 & \alpha^2 & \alpha & 1 & \alpha & \alpha^2 & 0 \\ 0 & 0 & \alpha^2 & \alpha & 1 & \alpha & \alpha^2 \\ \alpha^2 & 0 & 0 & \alpha^2 & \alpha & 1 & \alpha \\ \alpha & \alpha^2 & 0 & 0 & \alpha^2 & \alpha & 1 \end{bmatrix} \tag{20}$$

where $\alpha = 0.80$ corresponding to approximately 2 dB isolation between adjacent beam users. In this case, the performance of all users is equivalent and the average performance is shown in Fig. 10. In this case, the diversity gain is $D_g = 1 + 2\alpha^2 + 2\alpha^4$, which corresponds to 4.9 dB. This corresponds exactly to the gain observed relative to single user performance at a BER of 10^{-5}.

C. Other Issues

1) Interleaver Dependence: The interleavers used in these simulations were chosen at random. The same interleavers were used for all simulations having the same block size. No attempt was made to optimize these interleavers. The tests were repeated with a number of different interleavers, all of which were chosen at random. The simulated performance results with the different interleavers were virtually identical. There are, however, bad interleavers. For example, no interleaving results in degraded performance and the amount of degradation can be predicted by theory in some cases [11]. However, as Theorem 1 indicates these bad interleavers should be rare as the block size increases.

The interleaver size does not affect the asymptotic results but it does play a role at low SNR [11]. As seen in Fig. 10, many of the BER results indicate a threshold below which the system is not usable. Increasing the interleaver size increases the steepness of the approach to the single user performance above this threshold; it does not, however, affect the threshold.

Results in [11] indicate that the thresholds are related to Shannon capacity constraints rather than the FEC code. The Shannon capacity indicated in Fig. 10 is the theoretical E_b/N_o required to transmit 1/2 bit per user per channel use error free, for the channel described by (20). It assumes binary signaling for the individual users. Once again, this demonstrates that the threshold phenomenom is due to theoretical limits.

2) Code Dependence: The dependence of performance on the forward correction code is also of interest. For comparison with the constraint length 5 rate 1/2 convolutional code used to this point, rate 1/2 convolutional codes with constraint lengths of 4 and 7 were also tested. The results are qualitatively similar to the constraint length 5 code in all cases.

3) Practical Considerations: In a multiple access environment, synchronizing all users is an added overhead. However, successful field trials of pseudo-synchronous CDMA satellite return links certainly suggests that it should be feasible for low-rate non-CDMA systems. The multiuser decoding techniques described here can also be extended to asynchronous users [19]. With asynchronous users, there is an increase in the computational requirements of the combining means used in the algorithm, and the synchronization recovery for the different users is challenging as well.

There is a question of the sensitivity of performance to errors in determining the cross-correlation matrices R and the channel gains W. Our limited investigation has indicated no strong sensitivity to small perturbations of R and W relative to their true values. In fact, when R is singular, we intentionally perturb the R used in the receiver, in order to produce an invertible matrix. This does not have any significant effect on performance. The approach also works for unequal power users.

V. Conclusion

In this paper, we have shown that multiuser detection of BPSK users in multibeam systems can result in improved single user performance and greatly improved system capacity. In fact, the approach allows reuse of the same spectrum in each beam. To achieve these gains, users must be forward error correction coded but a simple convolutional code with a distinct random interleaver for each user is sufficient. In fact, users in these systems not only asymptotically achieve single user performance but usually exceed it due to the diversity gains inherent in a multibeam system. The existence of diversity gains depends upon the antenna implementation. With a phased array implementation that introduces noise correlation between the beams, the diversity gains may be smaller or nonexistent.

The described approach can be applied to other scenarios, such as interference between satellites and adjacent cells in a cellular environment. However, in these extensions, the problem of getting the information to one central point for joint processing may limit the practicality of the technique.

While the multiuser decoding algorithm is not trivial, it is manageable. It has a complexity of approximately $O(2^K + 2^\kappa)$ operations per bit per iteration where K is the number of users being detected and κ is the constraint length of the code. What is important in the consideration of complexity, is that large gains in spectral efficiency can be made for small values of K and κ, and with relatively short block lengths.

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Michael L. Moher (M'84) received the B.A.Sc. degree in engineering science and the M.A.Sc. degree in electrical engineering from the University of Toronto, Toronto, Canada. He subsequently received the Ph.D. degree in systems and computer engineering from Carleton University, Ottawa, Canada.

From 1980 to 1982 , he was with Telesat Canada where he was responsible for the analysis of new and upgraded thin route satellite links, and was involved in the field trials of a new 90 Mbit/s Ku-band data service. From 1982 to 1988, he led various projects and studies at Miller Communications, Kanata, that analyzed, simulated, and implemented hardware prototypes of advanced communications systems for military, broadcast, and mobile communications. Since 1988, he has been with the Communications Research Centre, Ottawa, Canada, where he has led various proof-of-concept modem developments for mobile satellite applications, participated in international standards activities, and acted as an industrial consultant for several new wireless and mobile satellite systems. His current research interests are in the area of digital signal processing and detection, and multiuser decoding.