Turbo Greedy Multiuser Detection

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Abstract—Recently, a novel scheme for iterative multiuser detection and turbo decoding was proposed by Damnjanovic and Vojcic. In this scheme, multiuser detection and single-user turbo decoding are tightly coupled to maximize the overall gain. The extrinsic probabilities for the coded bits of the interfering users, obtained after each turbo decoding iteration, are used as *a priori* probabilities in the following multiuser iteration and the extrinsic information for the systematic bits of the desired user is used as a priori information in the next single-user turbo decoding iteration. Turbo decoding of parallel concatenated convolutional codes is carried out in parallel fashion. It has been shown that the proposed detector approaches the multiuser capacity limit within 1 dB in the low signal-to-noise ratio region. However, the main drawback of the scheme is its exponential complexity in the number of users, which is due to the complexity of the maximum *a posteriori* probability (MAP) multiuser detector. In this paper, we show that the complexity of the scheme can be significantly reduced by replacing the MAP multiuser detector with an iterative detector derived from the greedy multiuser detector proposed by AlRustamani and Vojcic. In this paper, we demonstrate that, for both the additive white Gaussian noise and the frequency-nonselective Rayleigh fading, the substantial reduction in complexity of the iterative scheme proposed by Damnjanovic and Vojcic when the greedy detector is utilized introduces a slight degradation in performance; the loss in performance is ≤ 0.5 dB compared to MAP detector for systems with number of users double the spreading gain.

Index Terms—Code division multiaccess, concatenated coding, iterative methods, multiuser channels.

I. INTRODUCTION

RECENTLY, researchers have focused their attention on iterative decoding [6]–[12] and their application to the coded multiuser systems [1], [2], [13]–[20]. Iterative multiuser detection/decoding offers a way of combining the process of multiuser detection and decoding. In such schemes, the problem of multiuser detection and channel decoding are partitioned into two parts. However, the multiuser detector and the decoder utilize soft-input, soft-output (SISO) algorithms and operate in an iterative feedback mode as in turbo decoding [6], [7], [18]. The

soft-output of a bank of single-user decoders is fed back to the multiuser detector and used as *a priori* information for the next multiuser detection iteration.

The optimal iterative multiuser detector for convolutionally coded synchronous multiuser systems, based on the iterative techniques for cross-entropy minimization, is derived [13]. A suboptimum implementation with exponential complexity in the number of users is also presented. The scheme was examined for systems characterized with equal and high crosscorrelation between users. The results show that performance asymptotically approaching the single-user bound is achievable in the high signal-to-noise ratio (SNR) region [13]. The main drawback of that scheme is a relatively poor single-user performance of a convolutional code at low SNR. It is reasonable to believe that this limitation can be mitigated with more powerful turbo codes. In [13], the authors investigated the use of log-MAP [10], [21] algorithm in their iterative scheme for both multiuser detection and decoding for a convolutionally coded asynchronous multiuser system over the additive white Gaussian channel (AWGN) and the flat Rayleigh-fading channel. The results for both channels show that the performance is close to the single-user bound [19]. However, the main disadvantage of the iterative schemes in [13], [19] is that the complexity is exponential in the number of users and, thus, not practical for real systems. Therefore, researchers have focused on developing iterative schemes with substantially lower complexity.

In [14], an iterative scheme is developed for turbo coded synchronous multiuser system. Parallel concatenated convolutional codes (PCCCs) are used. After several iterations, the single-user turbo decoding is stopped, and *a posteriori* probabilities are used as *a priori* ones in the next multiuser iteration. The procedure is repeated iteratively. The MAP multiuser detector is examined and the M-algorithm [11] is introduced as a complexity reduction technique for multiuser detection. The simulation results suggest that iterations improve performance, but the reported results are still far from the multiuser capacity limit. The loss in performance when the M algorithm is employed is about 1 dB compared to the MAP detector for systems with $K = 5$ and $N_c = 7$ and random spreading sequences, where K is the number of users and N_c is the spreading gain. An iterative scheme for the asynchronous channel is studied in [16], where the CDMA channel is viewed as a time-varying convolutional code. Thus, at the receiver, the single-user encoder and the multiuser channel are considered as a concatenation of two convolutional codes and are decoded in a serial turbo code fashion. The M algorithm is also investigated to reduce the complexity of the iterative scheme [16] and the loss in performance, compared to the full complexity scheme, is about 1 dB for systems with $K = 5$ and $N_c = 7$. In [15] a low complexity SISO

multiuser detection for asynchronous multipath channel is developed based on soft interference cancellation and minimum mean square error (MMSE) filtering. The scheme was examined for systems with equal and high cross-correlation. The loss in performance due to the complexity reduction is negligible; however, the performance is far from the single-user bound at low SNR. An iterative scheme for multiuser detection/decoding based on the MMSE criterion and convolutional coding was developed in [17]. The performance of the scheme is 1 dB away from the single-user bound for a system with $K = N_c = 7$ and random spreading sequences.

A novel iterative multiuser detector/decoder has been proposed in [1], [2]. In this scheme, multiuser detection and single-user turbo decoding are tightly coupled to maximize the overall gain. Unlike in [14], multiuser iterations are embedded into single user turbo decoding iterations. The extrinsic probabilities for the coded bits of the interfering users, obtained after each turbo decoding iteration, are used as *a priori* probabilities in the following multiuser iteration and the extrinsic information for the systematic bits of the desired user is used as *a priori* information in the next single-user turbo decoding iteration. Turbo decoding of PCCC is done in parallel fashion [12]. The simulation results indicate that, at low SNR, the iterative receiver with binary signaling can operate close to the multiuser capacity limit [1], [2]. However, since the scheme employs MAP multiuser detection, the complexity is exponential in the number of users.

In this paper, we introduce a complexity reduction technique for the scheme presented in [1] and [2] that substantially reduces the computational load, yet insignificantly degrades performance. In this technique, the MAP detector is replaced with an iterative detector that is based on the greedy principle proposed in [3] and [4]. The complexity of the greedy algorithm is approximately $K^2 \log K$ operations per bit interval, where K is the number of users, and it has been shown that its performance is close to that of an isolated single user, in low SNR region. The results in [3], [4] show that in the presence of moderate-to-high noise, near–far effect, fading, and asynchronous transmission, near-optimum performance is achieved by the greedy detection for a number of users that is close to double the processing gain. Furthermore, it has been shown in [3] and [4] that the greedy multiuser detector considerably outperforms the most popular alternatives studied in the literature, such as: conventional, successive interference cancellation, decorrelator, sequential and multistage detectors, especially for moderate and high loads in low and moderate SNR; exactly where practical coded systems would operate. The performance of the proposed scheme over the AWGN was investigated in [20]; in this paper, we extend the results to the frequency-nonselective Rayleigh-fading channel. The results show that the loss in performance is ≤ 0.5 dB for a system with the number of users double the spreading gain $(K = 2N_c)$.

The paper is organized as follows. In Section II, the system model is presented. Section III provides a brief description of the iterative multiuser detector proposed in [1], [2]. A description of the greedy multiuser detector and its utilization in the proposed complexity reduction technique is given in Section IV. In Section V, we present the numerical results while our concluding remarks are given in Section VI.

II. SYSTEM MODEL

Consider a PCCC coded synchronous baseband multiuser system with K users, employing unit energy time limited spreading waveforms $s_k(t) \in \Re(0, T)$, $k = 1, 2, \ldots, K$. A block of message bits, $\mathbf{u}_k$, is encoded by a constituent encoder C_1 to create a parity sequence, $\mathbf{x}_{k,1}$, and interleaved and encoded by a constituent encoder C_2 to create the second parity sequence, $\mathbf{x}_{k,2}$. The systematic bits, $\mathbf{u}_k$, and the parity bits, $\mathbf{x}_{k,1}$ and $\mathbf{x}_{k,2}$, are then multiplexed to create a coded output sequence $\mathbf{b}_k = [\mathbf{u}_k, \mathbf{x}_{k,1}, \mathbf{x}_{k,2}]^{\mathrm{T}}$, resulting in the rate 1/3 code. The coded bits at the output of the PCCC encoder are modulated by the spreading waveform $s_k(t)$ and then transmitted over a multiuser channel. For a frequency-nonselective fading channel, the signal bandwidth, W, is significantly smaller than the coherence bandwidth of the channel, $(\Delta f)_c$, and the multipath components are not resolvable [23]. In this case, the received signal is the transmitted signal multiplied by a complex-valued random process representing the time-variant characteristics of the channel. Furthermore, we assume that the symbol duration is smaller than the coherence time of the channel such that attenuation and phase shift are essentially constant for the duration of at least one symbol interval [23]. Therefore, the received signal can be written as

$$r(t) = \sum_{k=1}^{K} \sum_{i=0}^{N-1} A_k(i) b_k(i) s_k(t - iT) + n(t) \qquad (1)$$

where $A_k(i)$ is a zero-mean complex-valued Gaussian variable, such that, $E\{A_p{}^*(i) A_k(i)\} = 0 \ \forall \ p \neq k$, and $E\{A_k{}^*(i) A_k(i)\} = \Phi_k((j - i)T)$ is the space–time correlation function of the channel associated with kth user. $*$ denotes complex conjugate and $E\{\cdot\}$ is the expected value of the k-th user. $b_k(i) \in \{+1, -1\}$ is the value of the kth user's ith coded bit, T is the symbol duration, $n(t)$ is a complex zero-mean Gaussian random process and N is the coded packet length. The AWGN channel can be viewed as a special case for which $A_k(i) = 1$ for $k = 1, \ldots, K$ and $i = 0, \ldots, N-1$ and $n(t)$ is real Gaussian random process. We assume that at the receiver the values $A_k(i)$, $k = 1, \ldots, K$ and $i = 0, \ldots, N-1$, are known, i.e., perfect knowledge of channel state information. The sufficient statistic for demodulation of K coded bits in the i-th interval is given by the vector $\mathbf{y}(i) = [y_1(i), \ldots, y_K(i)]$, whose kth component is the output of a filter matched to $s_k(t)$

$$y(i) = \int_{iT}^{(i+1)\mathrm{T}} s_k{}^*(t - iT) r(t) dt, \quad k = 1, 2, \ldots, K. \quad (2)$$

The vector of sufficient statistic can be written as

$$\mathbf{y}(i) = \mathbf{R} \mathbf{A}(i) \mathbf{b}(i) + \mathbf{n}(i) \qquad (3)$$

where

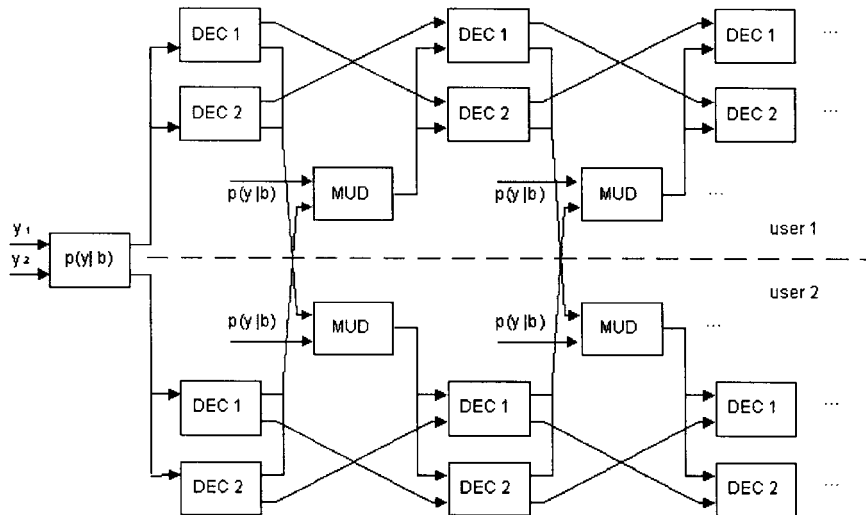|	
$\mathbf{R}$	normalized cross-correlation matrix of spreading waveforms;
$\mathbf{R}_{k,l}$	$\rho_{kl} = \int_0^{\mathrm{T}} s_k{}^*(t) s_l(t) dt$;
$\mathbf{A}(i)$	$[A_1(i), \ldots, A_K(i)]$;
$\mathbf{b}(i)$	$[b_1(i), \ldots, b_K(i)]$;
$\mathbf{n}(i)$	$[n_1(i), \ldots, n_K(i)]$, where $n_k = \int_0^{\mathrm{T}} s_k{}^*(t) n(t) dt$.

Fig. 1.   Principal block diagram of the proposed iterative multiuser receiver, two-user case.

III. ITERATIVE RECEIVER

In the first iteration, multiuser detection is activated disjointly of single-user turbo decoding. The purpose of multiuser detection is to provide the single-user turbo decoders with the soft input metrics. In the ith interval, at the beginning of the iterative process, the MUD computes the likelihood vector $p(\mathbf{y}(i)|\mathbf{b}(i))$, which is a multivariate conditional Gaussian distribution. $p(\mathbf{y}(i)|\mathbf{b}(i))$ is computed only once, and it is used in subsequent multiuser iterations.

The marginal likelihood $p(\mathbf{y}(i)|b_k(i))$ is given by

$$p(\mathbf{y}(i)|b_k(i)) = \frac{\sum_{\substack{b_j(i) \\ j \neq k}} p(\mathbf{y}(i)|\mathbf{b}(i)) \prod_{j=1}^{K} p(b_j(i))}{p(b_k(i))} \qquad (4)$$

where we assume statistical independence among the coded bits $b_j(i)$, $j = 1, \ldots, K$. The soft input metric for the single-user turbo decoder of user k is *a posteriori* distribution

$$p_{1,k}(b_k(i)) = \alpha p(\mathbf{y}(i)|b_k(i)) \qquad (5)$$

where the first subscript "1" in $p_{1,k}(b_k(i))$ refers to the first iteration and α is a constant that normalizes the probability mass. In the first iteration, *a priori* probabilities for coded bits are all equal, i.e., $p(b_j(i) = \pm 1) = 1/2$.

The decoders of the constituent codes are activated simultaneously. The goal is to ensure that the output of the turbo decoder equally weights the contribution from both constituent decoders and to minimize the decoding delay through parallel processing. The single-user MAP decoder is modified to produce *a posteriori* probabilities of coded bits. After one parallel iteration, *a posteriori* probabilities of all coded bits are obtained, and the extrinsic information of the systematic bits becomes *a priori* information for the next single-user parallel turbo decoding iteration. The extrinsic probabilities of the systematic bits at the output of decoder 1 are fed as *a priori* ones to decoder 2, and the extrinsic probabilities of the systematic bits at the output of decoder 2 are fed as *a priori* ones to decoder 1. Notice, however, that there is some additional extrinsic information that may be

exploited. The extrinsic probabilities for coded bits of the interfering users can be factored out from *a posteriori* probabilities, which are generated by the single-user turbo decoders. For user $j \neq k$, after the n-th iteration, the extrinsic distribution is denoted as $p_{n,j}^{E}(b_j(i))$, and can be fed back as *a priori* distribution for next multiuser iteration. After the $(n+1)$th multiuser iteration, updated marginal *a posteriori* distribution of the coded bits is equal to

$$p_{n+l,k}(b_k(i)) = \alpha \sum_{\substack{b_j(i) \\ j \neq k}} p(\mathbf{y}(i)|\mathbf{b}(i)) \prod_{\substack{j=1 \\ j \neq k}}^{K} p_{n,j}^{E}(b_j(i)) \qquad (6)$$

and it is fed as the soft input metric into the single-user turbo decoder of user k. The procedure is iteratively repeated until some predetermined stopping condition is met. The final bit decisions are obtained based on *a posteriori* probabilities of the systematic bits at the output of single-user MAP decoders. The principal block diagram of the proposed iterative receiver, for a two-user example, is presented in Fig. 1. Note that the probabilities $p(\mathbf{y}/\mathbf{b})$ is calculated only once in the first iteration, and they are used in subsequent MUD iterations which are presented as inputs to the MUD blocks in Fig. 1.

IV. REDUCTION IN COMPLEXITY

The complexity of the original scheme [1], [2] discussed above is $O(K2^K + 2^{\nu+1})$ per bit per user per iteration, since $O(K2^K)$ computations are needed to compute (6), and for the constituent convolutional codes of constraint length ν, the decoding complexity is $O(2^{\nu+1})$. Therefore, the main computational burden is imposed by the MAP multiuser detector. The complexity of the scheme can be profoundly reduced if we utilize a suboptimum multiuser detector that examines a subset of the possible vectors $\mathbf{b}(i)$ corresponding to large values of likelihoods $p(\mathbf{y}(i)|\mathbf{b}(i))$ in (4). Instead of the MAP detector, we propose the use of the greedy algorithm for multiuser detection presented in [3] and [4].

To employ the greedy algorithm in iterative decoding, the metric used in the algorithm is modified to incorporate the updated *a priori* distribution from the single-user turbo decoders

$$\Psi = \ln\left(p(\mathbf{y}(i)|\mathbf{b}(i))\prod_{j=1}^{K} p_{n,j}^{E}(b_j(i))\right). \tag{7}$$

It can be easily shown that after eliminating all the terms that are equal for all the vectors $\mathbf{b}(i)$, we can write the above metric as

$$\Psi = \ln\left(\prod_{j=1}^{K} p_{n,j}^{E}(b_j(i))\right)$$
$$+ \frac{1}{N_o}\left(\sum_{k=1}^{K} C_k b_k(i) + \sum_{k=1}^{K-1}\sum_{j=k+1}^{K} D_{kj} b_k(i) b_j(i)\right) \tag{8}$$

where

$$C_k = \text{Re}\{A_k^*(i)y_k(i)\} \text{ and } D_{kj} = \text{Re}\{-A_k^*(i)A_j(i)\rho_{kj}\}$$

$\text{Re}\{x\}$ is real part of x and N_o is the variance of the noise $n(t)$.

Based on the greedy principle, the greedy multiuser detector views the absolute values of the coefficients $\{C_k, D_{kj}, \forall k, j \in \{1,\ldots,K\} \text{ and } j > k\}$ as weights that indicate the order in which bits can be estimated. The dominating coefficients, the large values, will have more effect on deciding the bit value than smaller values, i.e., we take into consideration the order of the coefficients contribution to the metric value, ψ. Initially the absolute values of the coefficients are sorted in a descending order. The subscripts of these coefficients indicate in which order the bits might be estimated. Therefore, the algorithm consists of $M_s \leq K(K+1)/2$ stages, which is the total number of the coefficients C_k and D_{kj}. The mth stage corresponds to the mth coefficient after sorting. In some cases, the number of stages may be less than $K(K+1)/2$ for reasons that will be evident later.

Let $\mathbf{b}^{(m)} = \left[b_1^{(m)}\ldots b_K^{(m)}\right]$ be the estimate of the vector $\mathbf{b}(i)$ in the mth stage and $\mathbf{b}^{(0)} = [0\ldots0]$. In the mth stage, if the corresponding coefficient is C_k, then the kth bit in $\mathbf{b}^{(m-1)}$ is examined and updated. This is done by calculating the metric ψ in (8) twice for the two possible vectors obtained by substituting the values of $b_k^{(m-1)}(i) = \pm 1$ in $\mathbf{b}^{(m-1)}$

- $\mathbf{b}_1^{(m-1)} = \left[b_1^{(m-1)}(i)\ldots b_k(i) = -1\ldots b_K^{(m-1)}(i)\right]$ and
- $\mathbf{b}_2^{(m-1)} = \left[b_1^{(m-1)}(i)\ldots b_k(i) = +1\ldots b_K^{(m-1)}(i)\right]$.

The vector that results in the largest metric value is chosen as the new estimate of the vector $\mathbf{b}(i)$ in the mth stage

$$\mathbf{b}^{(m)} = \mathbf{b}_p^{(m-1)} \text{ if and only if } \Psi\left(\mathbf{b}_p^{(m-1)}\right)$$
$$\geq \Psi\left(\mathbf{b}_p^{(m-1)}\right) \forall q \neq p \text{ and } p, q = 1, 2.$$

On the other hand, if the coefficient is of the form D_{kj}, then the kth and jth bits in $\mathbf{b}^{(m-1)}$ are examined and updated. This is done by calculating the metric Ψ in (8) four times for all the four possible vectors obtained by substituting $b_k^{(m-1)}(i) = \pm 1$ and $b_j^{(m-1)}(i) = \pm 1$ in $\mathbf{b}^{(m-1)}$.

- $\mathbf{b}_1^{(m-1)} = \left[b_1^{(m-1)}(i)\ldots b_k(i) = -1\ldots b_j(i) = -1\ldots b_K^{(m-1)}(i)\right]$;
- $\mathbf{b}_2^{(m-1)} = \left[b_1^{(m-1)}(i)\ldots b_k(i) = -1\ldots b_j(i) = +1\ldots b_K^{(m-1)}(i)\right]$;
- $\mathbf{b}_3^{(m-1)} = \left[b_1^{(m-1)}(i)\ldots b_k(i) = +1\ldots b_j(i) = -1\ldots b_K^{(m-1)}(i)\right]$;
- $\mathbf{b}_4^{(m-1)} = \left[b_1^{(m-1)}(i)\ldots b_k(i) = +1\ldots b_j(i) = +1\ldots b_K^{(m-1)}(i)\right]$.

The vector that results in the largest metric value is chosen as the new estimate of the vector $\mathbf{b}^{(i)}$ in the mth stage

$$\mathbf{b}^{(m)} = \mathbf{b}_p^{(m-1)} \text{ if and only if } \Psi\left(\mathbf{b}_p^{(m-1)}\right)$$
$$\geq \Psi\left(\mathbf{b}_p^{(m-1)}\right) \forall q \neq p \text{ and } p, q = 1, 2, 3, 4.$$

Therefore, at the mth stage, one or two bits in $\mathbf{b}^{(m-1)}$, depending on the coefficient corresponding to the mth stage, are examined and updated based on the estimates of the bits made in the previous stages, corresponding to the largest increment to the likelihood metric.

As mentioned earlier, the number of stages might be less than $K(K+1)/2$ in some cases. The cases in which the number of stages can be reduced are summarized in Lemma 1.

Lemma 1: In the greedy algorithm, if two consecutive stages examine $\{C_k, D_{kl}\}$ or $\{D_{kl}, C_k\}$, or if three consecutive stages examine $\{C_k, C_l, D_{kl}\}$ or all the other five possible permutations of C_k, C_l and D_{kl}, then deleting the stages corresponding to C_k (and C_l) will not affect the result.

Proof: The vectors examined in the consecutive stages $\{C_k, D_{kl}\}$ or $\{C_k, C_l, D_{kl}\}$, or all other possible permutations, have the same elements in all other positions besides k and l, i.e., same symbol estimates for all other users except users k and l. Moreover, the 2^M examined vectors in the stages C_k (and C_l) are subsets of the 2^{2M} examined vectors in the stage D_{kl}. Therefore, deleting stages C_k (and $\hat{A}_l$) will not have an effect on the results.

One possibility to improve the performance of the greedy algorithm is to forward the L largest metric values and the corresponding bit vectors from one stage to the next one. Therefore, the input to a stage m will be the largest L metric values $(\Psi(\mathbf{b}^{(m-1,1)}),\ldots,\Psi(\mathbf{b}^{(m-1,L)})$ and the corresponding estimates of $\mathbf{b}\left(\mathbf{b}^{(m-1,1)},\ldots,\mathbf{b}^{(m-1,L)}\right)$. In stage m, for every $\mathbf{b}^{(m-1,i)}$ $i = 1,\ldots,L$, the metric values are calculated for the $2L$ or $4L$ vectors obtained by examining the bits indicated by the mth coefficient and the largest L values and the corresponding bit vectors are chosen as the output of the stage. The improvement in performance by increasing the value of L is at the expense of a slight increase in the computational complexity. A detailed description of the algorithm is given in[3]–[5].

Since $\mathbf{b}^{(0)} = [0\ldots0]$, estimates of all bits might be available at least until stage $\lfloor K/2\rfloor$ or at most until stage $K(K-1)/2$, where $\lfloor x\rfloor$ is the smallest integer greater than or equal to x. Once
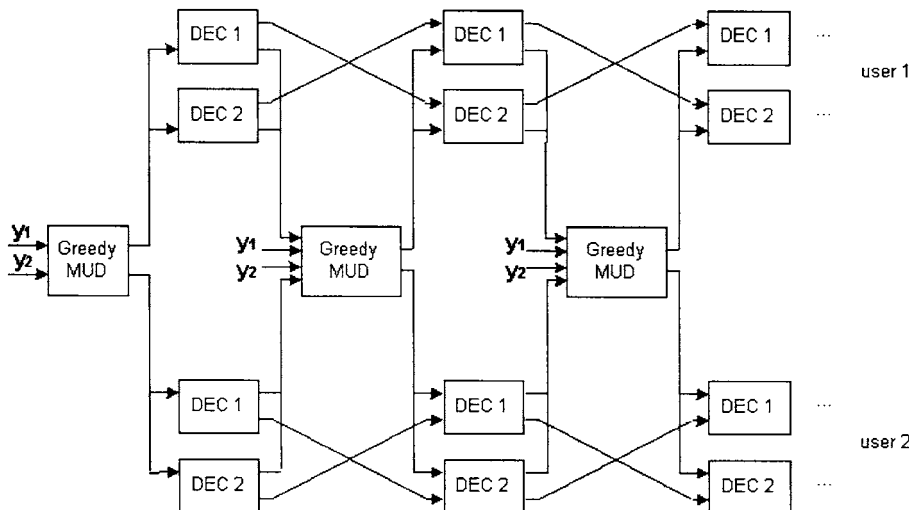
Fig. 2. Principal block diagram of the proposed reduced complexity iterative multiuser receiver, two-user case.

estimates of all bits are available, the greedy detector saves the new bit vectors that are examined in every stage and the corresponding metric value, ψ. Therefore, instead of calculating all 2^K metric values, the number of the values the greedy detector computes, which is a random variable that depends on the order of detection and the speed of convergence of the algorithm to a maximum, is greater than $M_{\min} = 2K$ and less than $M_{\max} = L(2 + 3K(K-1)/2)$. The detector assumes that the rest of the values are negligibly small and can be set to zero. These values are then used to update the marginal *a posteriori* distribution of the coded bits, as given in (6).

The principal block diagram of the proposed iterative receiver, for a two-user example, is presented in Fig. 2. Note that, unlike the proposed scheme with the MAP detector, shown in Fig. 1, the greedy detector is activated in every iteration to calculate the probabilities. Since the metric is a function of the updated *a priori* distribution obtained from the single-user turbo decoders, in every iteration the greedy detector is forced to explore a different subset of the bit combinations $\mathbf{b}(i)$ and the associated metric values.

The complexity of the greedy multiuser detector is $O(K^2(\log K + L))$ [4]. Therefore, the complexity of the proposed scheme is significantly reduced to $O(K^2(\log K + L) + 2^{\nu+1})$ per bit per user per iteration.

V. Numerical Results and Discussion

In all the simulation results in this section, all users employ the same PCCC encoders of rate 1/3, obtained by using systematic, recursive, four-state convolutional encoders of rate 1/2 with generator matrix $G = (1, 5_8/7_8)$ as constituent codes. Different users employ different pseudorandom interleavers that are changed for every new packet transmission. Although the gain in complexity reduction by utilizing the greedy multiuser detector is more evident for large values of number of users, K, as demonstrated in [3], [4], the simulation results are obtained for small values of K for the sake of the computational speed. Nevertheless, the scheme is examined for moderately to heavily loaded systems.

To illustrate the potential of the proposed scheme, we first consider a synchronous eight-user symmetric AWGN channel that is characterized by the following cross-correlation matrix, $R_{ii} = 1$ and $R_{ij} = \rho = 0.575$ for $i \neq j$; $i, j = 1, \ldots, K$. All users have the same power. In Fig. 3, we show the results for the greedy algorithm with $L = 1$ and different numbers of iterations and compare it with the original scheme with the MAP detector [1], [2] as well as scheme suggested in [14]. Also, the multiuser capacity limit for Gaussian signaling and equal amplitudes is computed from [24] as $C = 1/2 \log(\det[\mathbf{I} + \text{snr}\mathbf{R}])$, where $\mathbf{I}$ is the identity matrix and snr is the SNR per coded bit. In this case, the limit is at 0.86 dB, which means that for the interleaver size of 2,000 bits, it is possible to operate only 1.1 dB away from the limit at the packet error rate of 1%. The gap should be somewhat smaller if these results are compared to the multiuser capacity for binary signaling. For eight users, the greedy multiuser detector examines in every iteration between 6.25% ($100M_{\min}/2^K$) and 33.6% ($100M_{\max}/2^K$) of the possible bit vectors. The results show that for a profound reduction in complexity the loss in performance for the 32 iterations is less than 0.2 dB. Moreover, the results of the proposed iterative scheme with the greedy detector for different numbers of iterations indicate that the number of iterations is crucial in determining the performance of the scheme; the difference between 32 iterations and 16, 8, and 4 iterations are 0.25, 0.8, and 2 dB, respectively. It appears that the performance can further be improved by increasing the number of iterations beyond 32, although the incremental gain diminishes. In Fig. 3, the performance of the scheme suggested in [14] (Reed 8/32) is obtained for 32 turbo-decoding iterations and eight MAP multiuser iterations. The issue of optimum number of multiuser iterations is discussed in [14]. For the considered example, simulation results that we obtained indicate that eight-MAP multiuser iterations is the optimum number of iterations. Compared to the scheme proposed in this paper with the same number of multiuser/decoding iterations (Greedy 8/32), Reed's scheme outperforms the proposed greedy scheme by about 0.2 dB. Nevertheless, the complexity of Reed's scheme is exponential in the number of users while it is polynomial for the greedy approach.
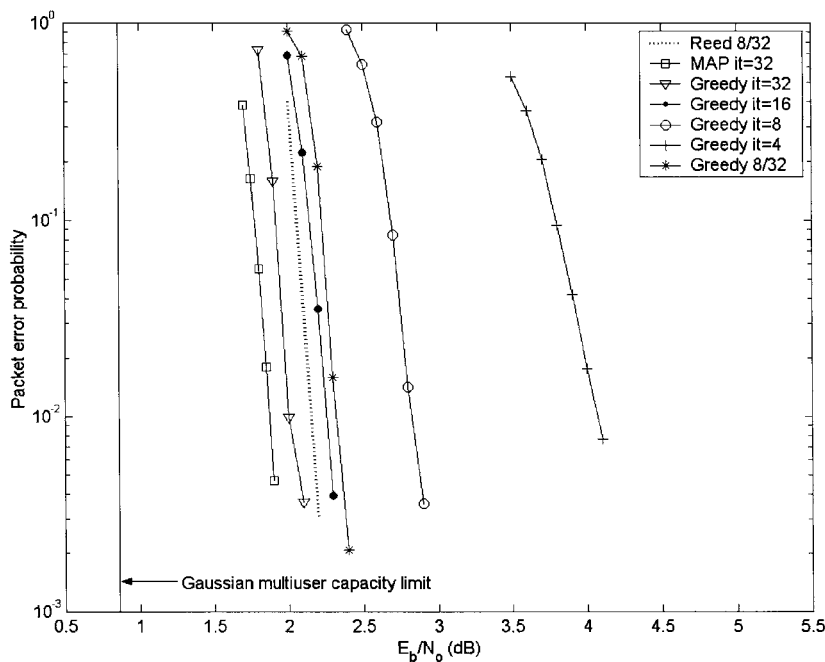
Fig. 3. Packet error probability with the greedy multiuser detection, for eight-user case, $\rho = 0.575$, interleaver size is 2000 bits, over AWGN channel.
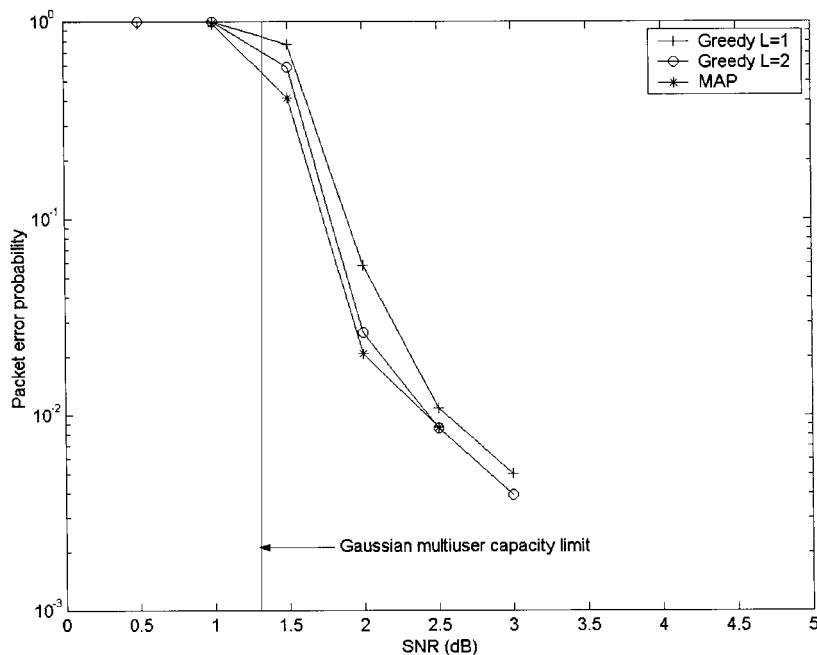

Fig. 4. Packet error probability with the greedy multiuser detection, for eight-user case, random spreading sequences with spreading factor of 15, interleaver size is 2000 bits, over frequency-nonselective fading channel.

Once the complexity of the greedy approach is increased to be comparable to that of Reed's scheme, its performance becomes better.

The simulation results indicate that at low SNR the proposed iterative detection/decoding algorithm can operate close to the multiuser capacity limit, even with the proposed complexity reduction technique. This is in contrast to the iterative receivers suggested in [13] and [17] that need high SNR to overcome the detrimental effects of multiuser interference, since at low SNR, it is limited by a relatively poor performance of the employed convolutional code.

In Fig. 4, the performance of the proposed scheme with the greedy algorithm and the MAP detector in the frequency-non-selective fading channel is presented, for eight-user case with 32 iterations, and interleaver size of 2000 bits. As explained in Section II, the fading amplitudes of each user are modeled as Rayleigh fading, assuming independent fading from user to user. The fading amplitudes are assumed to be constant for the whole symbol duration and independent from symbol to symbol. Also, perfect knowledge of the value of these amplitudes is assumed at the receiver. Random signature waveforms with a spreading gain $N_c = 5$ and code rate 1/3 are used (total bandwidth ex-
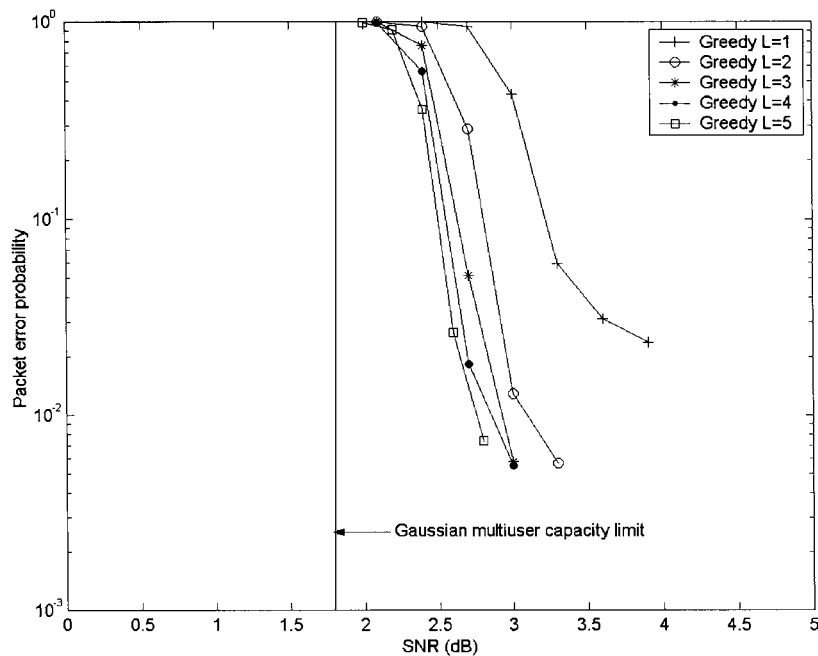
Fig. 5. Packet error probability with the greedy multiuser detection, for ten-user case, spreading factor of 5, interleaver size is 1000 bits, over frequency-nonselective fading channel.

pansion is 15). The multiuser capacity limit for Gaussian signaling is simulated using the capacity formula from [24], $C = 1/2 \log(\det[\mathbf{I}+\text{snr}\,\mathbf{ARA}])$, where $\mathbf{A}$ is the diagonal amplitude matrix with unit power Rayleigh faded amplitudes. In this case, for rate 1/3 code, the limit is at 1.3 dB. For the optimum rate, $r = 1/15$ (no spreading), the multiuser capacity is at 0.3 dB. At packet error probabilities of 10^{-1} and 10^{-2}, the performance of the original scheme [1], [2] is only 0.5 and 1.1 dB away from the multiuser capacity, respectively. Compared to the scheme with the MAP detector, the loss in performance when the greedy detector with $L = 1$ is employed is less than 0.3 dB and is negligible for $L = 2$. For this example, the greedy multiuser detector examines in every iteration between 6.25% and 33.6% for $L = 1$ and between 6.25% and 67.2% for $L = 2$ of the possible bit vectors.

The performance of the scheme for a system with 10 users, with 32 iterations and interleaver size of 1000 bits is shown in Fig. 5. In this case, the multiuser capacity limit for rate 1/3 code is at 1.8 dB. For the optimum code rate (no spreading), the multiuser capacity is at 0.7 dB. The signature waveforms of the users, with binary components and spreading gain of $N_c = 5$, are chosen such that the sum of the square of the cross-correlation values in $\mathbf{R}$ is minimized, i.e., $\min\left(\sum_{i=1}^{K}\sum_{j=1}^{K}\rho_{ij}^2\right)$. This criterion is chosen to minimize interference and to construct spreading sequences with the total squared cross-correlation close to the Welch's bound [25]. In this scenario, the greedy method examines at least 2% of all the possible vectors and at most 13.4%, 26.8%, 40.1%, 53.5%, and 66.9% of all the possible bit vectors for $L = 1$, 2, 3, 4, and 5, respectively. Compared to the results obtained for $L = 5$, the loss in performance at 10^{-1} is only 0.7, 0.3, 0.2, and 0.1 dB for $L = 1$, 2, 3, and 4, respectively. The same loss in performance reported at 10^{-1} is observed

for lower packet error probabilities for $L > 1$. However, we observe that at high SNR, approximately at 3.5 dB, the greedy scheme with $L = 1$ reaches an error floor, which is also observed in [3] and [4]. This is because error-free demodulation in the absence of noise is not achievable by the algorithm for low values of L since the algorithm considers a subset of the possible bit combinations and might not converge to the global maximum. However, the error floor is reduced or vanishes for $L > 1$. For this example the performance of the algorithm is 1.0 dB away from the multiuser capacity.

VI. CONCLUSION

A novel iterative approach for multiuser detection and single-user turbo decoding was proposed and described in [1] and [2]. Soft output MAP multiuser detection and turbo decoding are executed disjointly, but are coupled in each iteration by exchanging the extrinsic and *a posteriori* information. It has been shown that for binary signaling and low SNR the proposed detector/decoder can approach the multiuser capacity limit within approximately 1 dB. However, the proposed scheme suffers from an exponential complexity in the number of users. In this paper, we proposed a new complexity reduction technique that replaces the MAP detector with the greedy detector proposed in [3] and [4], whose complexity is proportional to $K^2 \log K$. The results for both AWGN and frequency-nonselective Rayleigh fading channels show that by replacing the MAP detector by the greedy detector, a substantial reduction in complexity is obtained at the expense of a slight degradation in performance, ≤ 0.5 dB compared to MAP detector, for moderately to heavily loaded systems. The complexity gain of greedy approach, relative to the MAP scheme, rapidly increases with the number of users.

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