On the Capacity of CDMA with Linear Successive Interference Cancellation

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Abstract— **Combined power control and interference cancellation in CDMA systems can be a very efficient resource management tool. While conventional power control tries to maintain equal received power or balanced SIR, Successive Interference Cancellation (SIC) in Multi-User Detection (MUD) relies more on the disparities between the powers of the different users. The combination can save more power into the system and thus a room for better capacity.**

This paper investigates the interaction between power control and linear SIC in a single rate CDMA system and its impact on the system capacity. The obtained results show that interference cancellation can improve the capacity of CDMA and relax power control requirements. The full integration of power control and SIC is shown to provide excellent resource management in CDMA systems. Limited cancellation can be a good solution for CDMA systems as it provides considerable capacity gain with reduced complexity. An upper bound on the system capacity as a function of the number of canceled signals is derived. Investigation of the optimum decoding order is also provided in this paper.

I. INTRODUCTION

WCDMA has been adopted in the third generation wireless communication systems, which improves the capacity over the second generation system as well as other performances [1]. For the current version of WCDMA specification, conventional single user receiver, which is matched to users' spreading codes, is used. However, Multi-User Detection (MUD) is an effective method to improve the capacity of WCDMA system. Successive Interference Cancellation (SIC) is a form of MUD in which signals are detected in order of perceived reliability [2]-[4]. Typically the strongest signals are detected first and canceled from the received signal which mitigates the interference seen by the remaining signals. Linear SIC is a type of SIC that produces decision statistics which are linear combinations of the received signal and matched filter outputs [3]. This type of SIC is practical to implement and does not introduce error propagation.

In the work of [5], MUD was shown to obtain the full potential of the CDMA technique in addition to interference reduction by power control. A possible upper bound for capacity gain of SIC system was presented in [6], but their model did not consider noise accumulation. In fact, every stage of the SIC detector always experiences noise accumulation even under perfect signal cancellation, as shown in [2], [3]. Taking into account the respective Quality of Service (QoS) requirements and impact of imperfect interference cancellation, a new power allocation has been derived in [4] for a multi-rate DS-CDMA system, but the capacity gain or capacity upper bound under this SIC model is still unknown for these possible power control schemes.

In this paper, we investigate system capacity by considering combination of power control algorithms and SIC detection. We also evaluate the effect of limited cancellation (applying the linear SIC on the strongest users only) on the system capacity.

The organization of this paper is as follows: Section II introduces CDMA system with single user detection and successive interference cancellation. Power control scheme and their interaction with linear SIC detection are given in Section III. Section IV discusses limited linear cancellation and gives an upper bound on the system capacity. Simulation results as well as comparisons and discussions are presented in Section V. Section VI gives some conclusions.

II. CDMA CELLULAR SYSTEM

Consider a CDMA system with a total bandwidth of W Hz and user data rate of R bps. We consider the uplink and assume that the QoS requirement can be specified by an equivalent received bit-energy-to-interference-spectral-density ratio (E_b/I_0, or SIR), which can be defined as

$$\Gamma_k = \left(\frac{E_b}{I_0}\right)_k = \frac{g_k p_k}{R\sigma_k^2} \tag{1}$$

where k is the user number, p_k is the transmit power of user k, $g_k = cS_k/r^\alpha$ is the total path gain with c representing the path gain constant, α is the propagation coefficient, r is the distance between the mobile unit and the base station, and S_k is lognormally distributed random variable corresponding to shadow fading. σ_k^2 is the total interference affecting signal k.

A. Conventional Single User Detection

In CDMA systems, detectors at the base station receive a signal composed of the sum of all users' signals, which overlap in time and frequency. The conventional CDMA system uses single user detection where each user is detected separately without considering other users [1]. The SIR of a given user k ($1 \leq k \leq N$) can be expressed as follows:

$$\Gamma_k = \frac{W}{R}\frac{g_k p_k}{\sum_{i=1,i\neq k}^{N} g_i p_i + N_0 W + I_e}. \tag{2}$$

where N is the total number of active users within the same cell, N_0 is the AWGN power spectral density and I_e is the inter-cell interference affecting this cell.

A user k is said to be in outage (does not get the QoS required) if $\Gamma_k < \gamma$, where γ is the required SIR threshold. As Γ_k is random variable, the QoS can be measured by the outage probability defined as

$$P_{\text{out}} = P\left(\Gamma_k \leq \gamma\right). \tag{3}$$

B. Linear Successive Interference Cancellation

A better CDMA system detection strategy is to employ Successive Interference Cancellation (SIC), where multiple users' information is jointly used to better detect each individual user. The SIC detector is a sub-optimum but simple scheme and may be implemented in real systems.

A linear SIC detector consists of detecting the strongest signal first, removing it from the received signal, detecting the next strongest, and so on. Due to signal cancellation at the different stages of the receiver, the total interference experienced by the different users will be different and depends on the decoding order of that particular user. The SIR of the kth user can be written as [4]

$$\Gamma_k = \frac{W}{R}\frac{g_k p_k}{\sum_{i=k+1}^{N} g_i p_i + N_0 W + \sum_{i=1}^{k-1}\sigma_i^2 R + I_e}. \tag{4}$$

We notice that the SIC detection reduces the intra-cell interference compared to conventional single user detection. But there is noise accumulation from the previous user detection stages. The total interference experienced by Signal k is a function of the interference experienced by the signals of the previous stages. With some manipulations we get the following relation:

$$\sigma_k^2 = \beta\sigma_{k-1}^2 = \beta^{k-1}\sigma_1^2, \quad \text{with } \beta = \frac{1+\frac{W}{R}}{\gamma+\frac{W}{R}}. \tag{5}$$

Thus, the total interference is reduced from one stage to the next. However, when a robust modulation (small γ) is used the parameter β approaches 1 and the SIC detector may not add considerable performance gain.

III. POWER CONTROL

Power control is one of the most important system requirements for CDMA systems for both uplink and downlink, which tries to distribute the interference between the users of each cell such that the QoS requirements of most (if not all) of the active users are met.

A. Regular Power Control

Here regular power control refers to the power control algorithm that assumes single user detection is employed by the CDMA system. SIR balancing algorithm is a kind of regular power control algorithm, which tries to make every transmitter's received SIR balanced (equalized). Distributed Constrained Power Control (DCPC) is a decentralized iterative SIR balancing scheme described by [8]

$$p_k(n+1) = \min\left[p_{\max}, \frac{\gamma}{\Gamma_k(n)}p_k(n)\right] \tag{6}$$

where $p_k(n)$ is the transmit power of user k at iteration n, γ is the SIR threshold, $\Gamma_k(n)$ is the SIR of user k at iteration n as defined in (2).

B. Combined Power Control and Successive Interference Cancellation

We have see earlier that by removing users one by one in the linear SIC detector, the expression of the SIR for each user will be different from one stage to the next as indicated in (4). Balancing the received signal energy using regular power control will not be efficient and may waste power. Efficient power control methods need to balance the SIR taking into account the operations done by the linear SIC detector at the different stages. With such interaction, better power allocation can be achieved and more power can be saved.

In the work of [4], optimum power allocation for CDMA systems with linear SIC was investigated and derived. For a CDMA system with single-rate and perfect linear SIC detection, the required allocated power for user k is given by

$$\tilde{p}_k = \frac{N_0 W + I_e}{g_k}\frac{\gamma\beta^k}{1+\frac{W}{R}-\gamma\beta\frac{1-\beta^N}{1-\beta}} \tag{7}$$

where γ is the SIR threshold for good quality of reception.

The expression of the required power in a CDMA system with linear SIC detection can be used to derive an upper bound on the capacity of the system. It is observed from (7) that a power solution is possible only when the denominator of that expression is strictly positive. Using this fact and assuming that there is no power ceiling for the transmitted power, an upper bound on the capacity of CDMA systems with SIC detection can be derived and is given by

$$N_{\text{sic}} < \frac{\ln(\gamma)}{\ln(1/\beta)}. \tag{8}$$

For CDMA systems with single user detection (sud) , an upper bound on the system capacity has been derived in [7] and is given by

$$N_{\text{sud}} < 1 + \frac{W}{R}\frac{1}{\gamma}. \tag{9}$$

The capacity gain for SIC detection over single user detection ($G = N_{\text{sic}}/N_{\text{sud}}$) is illustrated in Figure 1 as a function of the threshold γ and for two different processing gains. We notice that the relative capacity gain increases with increasing the threshold γ which confirms what has been indicated in the previous section. We also notice that the relative capacity gain is not that sensitive to the processing gain.

In a multiple cellular system, it is desirable to employ an iterative power control scheme that can asymptotically converge to the required power level for each user within each cell. From (7), we notice that the required power for user k is a function of the inter-cell interference. When employing iterative power control, inter-cell interference will change from one iteration to the next. With a proper Disparity Control (DC) between the received powers of the different users, this interference should converge to a steady state level. Thus, a modified iterative power control algorithm for CDMA systems with linear SIC detection can be implemented. This power control algorithm, denoted here by DC-SIC, can be implemented based on the following iteration procedure [4]

$$p_k(n+1) = \min\left[p_{\max}, \tilde{p}_k(n)\right] \tag{10}$$

where

$$\tilde{p}_k(n) = \frac{N_0W + I_e(n)}{g_k} \frac{\gamma\beta^k}{1 + \frac{W}{R} - \gamma\sum_{j=1}^{N}\beta^j} \tag{11}$$

is the target power level for user k and $I_e(n)$ is the inter-cell interference experienced by user k during the nth power iteration. This power control inherently contains the SIC within the power control and thus better performance is expected.

As shown in [4], SIR balancing power control can also be combined with linear SIC detection. By noting that each stage of the linear SIC detector experiences a different interference level, a good power control scheme should take into account only the interference that affects the signal under consideration. Thus, a modified SIR balancing power control scheme, we denote by SIR-SIC, has the same expression as (6), but $\Gamma_k(n)$ is different and as defined in (4).

This algorithm tries to balance the SIR of each user after interference cancellation. The difference between regular SIR balancing and SIR-SIC is the way to calculate the local SIR, which results from the combination of SIC into power control. The power value for each user can be computed based on measurements of the total interference after each cancellation stage.

The above two power control schemes converge to the same point after a certain number of iterations. However, the DC-SIC scheme converges faster than the SIR-SIC. The price for this fast convergence is a higher computational complexity.

Decoding order is very important in SIC detection and has a large impact on the performance of the CDMA system. The decoding order in every cell within the system should minimize the total transmit power of all users of the cell and at the same time minimizes the interference created to the other users in the other cells.

Using (7), the optimum decoding order is the order of the signals that minimizes the following cost function:

$$(N_0W + I_e)\sum_{k=1}^{N}\frac{\beta^k}{g_k}. \tag{12}$$

For an isolated single cell, there is no inter-cell interference and the above cost function reduces to $\sum_{k=1}^{N}\frac{\beta^k}{g_k}$. Since $\beta < 1$, the above cost function is minimized when the users with largest link gains are decoded first. Thus, the optimum decoding order for the single cell case is obtained when the users are arranged for decoding such that $g_1 \geq g_2 \geq \cdots \geq g_N$.

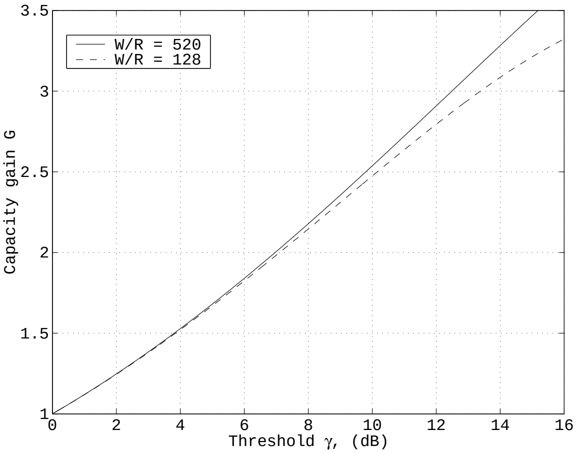

Fig. 1. Capacity gain for SIC detection over single user detection for different processing gains W/R.

In a multi-cell system, changing the decoding order in one cell will not only affect the total power within the cell but also the external interference experienced in the other cells. Thus, the minimization of the cost function given in (12) is not an easy task. One way to solve this problem is to try all possible combinations. However, such an approach is not practical as it increases exponentially with the number of users. In this paper, we will apply the single cell decoding order for a multi-cell system. This is a sub-optimum solution but it should not be far from the optimum because this decoding order minimizes intra-cell interference and we know that intra-cell interference dominates, specially for the uplink case.

IV. LIMITED SUCCESSIVE CANCELLATION

In some cases, it may not be necessary to cancel all interfering signals. Canceling some of the interfering signals may provide considerable capacity gain and at the same time keeps a low complexity for the CDMA receiver. In this section, we investigate the capacity of CDMA for different number of stages in the linear SIC detector.

A simplified block diagram of the limited linear SIC detector considered in this paper is shown in Figure 2 with $c_i(t)$ and b_i representing the spreading code and the information signal of user i, respectively. The integer L denotes the number of stages in the SIC detector. Thus, the first $L+1$ users will experience different interference levels and the remaining users will experience similar interference levels. In this case, the SIR is given by (4) for $1 \leq k \leq L+1$ and is given by

$$\Gamma_k = \frac{g_kp_k(W/R)}{\sum_{i=L+1}^{N} g_ip_i - g_kp_k + N_0W + \sum_{i=1}^{L}\sigma_i^2R + I_e} \tag{13}$$

for $L+2 \leq k \leq N$.

Again, if each user requires a maximum tolerable bit error rate that can be mapped into an equivalent minimum SIR, γ, the individual powers p_k of the different users can be determined from the set of linear equations $\Gamma_k = \gamma, \ 1 \leq k \leq N$.

From (13), the optimum power allocation for the last $N-L-1$ signals is constant received power, i.e.,

$$p_k = \frac{P}{g_k}, \quad L+2 \leq k \leq N \tag{14}$$

where P is the required received power at the base station receiver.

By using (5) and (14) into (13), the SIR of the first $L+1$ signals becomes

$$\Gamma_k = \frac{P(W/R)}{(N-L-1)P + N_0 W + \sum_{i=1}^{L} \beta^{i-1}\sigma_1^2 R + I_e}. \tag{15}$$

Setting $\Gamma_k = \gamma$ in the above equation and solving for P, the required power p_k in (14) becomes

$$p_k = \frac{1}{g_k} \frac{N_0 W + I_e + \frac{p_1 g_1}{\gamma}\frac{1-\beta^L}{1-\beta}}{\frac{W}{R\gamma} - N + L + 1} \tag{16}$$

where p_1 is the power allocated to user 1 and is derived below.

Using the relation given in (5), the SIR of Signal 1 can be rewritten as

$$\Gamma_1 = \frac{g_1 p_1 (W/R)}{\frac{p_1 g_1}{\Gamma_1} \sum_{i=2}^{L+1} \beta^{i-1}\Gamma_i + (N-L-1)P + N_0 W + I_e}.$$

Using (15) and (5) in the above expression and setting $\Gamma_i = \gamma$, the optimum power allocation for the different users is derived and is given by

$$p_k = \frac{\beta_k^{k-L-1}}{g_k} \frac{N_0 W + I_e}{\frac{W}{R\gamma} - N + L + 1 - \frac{1}{(1-\beta)\gamma}\left(\frac{1}{\beta^L} - 1\right)} \tag{17}$$

where

$$\beta_k = \begin{cases} \beta, & 1 \leq k \leq L+1 \\ 1, & L+2 \leq k \leq N \end{cases}. \tag{18}$$

It is observed from (17) that a power solution is possible only when the denominator of those expressions is strictly positive. Using this fact and assuming that there is no power ceiling for the transmitted power, an upper bound on the capacity of CDMA with limited linear SIC detection is derived and is given by

$$N_{\text{sic}}(L) \leq 1 + L + \frac{W}{R\gamma} - \frac{1}{(1-\beta)\gamma}\left(\frac{1}{\beta^L} - 1\right). \tag{19}$$

It is easy to verify that the two bounds indicated earlier in the paper are special cases of the above upper bound. For instance, the upper bound of (9) is obtained by just letting $L=0$ in the above expression. Similarly, the upper bound of (8) is obtained by letting $L = N-1$ in the above expression. Figure 3 shows the relative capacity gain of the SIC detector over single user detection for different number of stages L. It is observed that the capacity gain improves as the number of stages in the SIC detector increases. However, most of the improvement comes from the first stages (small L). This can be seen by investigating the slope of the capacity,

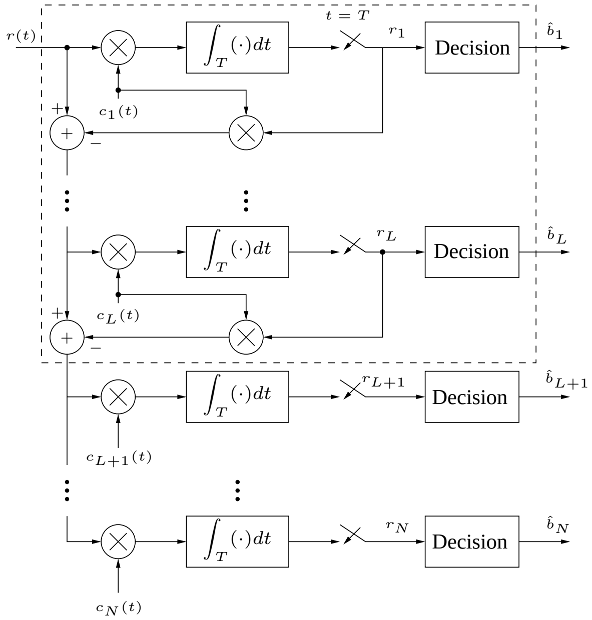

Fig. 2. Simplified block diagram of a limited linear SIC detector for CDMA signals.

$$\frac{dN_{\text{sic}}(L)}{dL} = 1 + \frac{\ln(\beta)}{(1-\beta)\gamma\beta^L}, \tag{20}$$

which is a decreasing function of L and is positive for all values of L satisfying the following:

$$L \leq \frac{\ln(\gamma)}{\ln(1/\beta)} - 1.$$

Thus, full cancellation provides the maximum capacity gain and most of the gain comes from the first stages. The number of stages in the SIC detector can then be reduced without affecting its potential performance gain.

V. SIMULATION RESULTS

Numerical evaluation is performed by system simulation of a two-tier (19 cells) hexagonal cellular system with omnidirectional antennas and the different parameters in Table I. The investigated algorithms are evaluated by snapshot simulation for the uplink of a CDMA cellular system. Active users are assumed uniformly distributed over the whole cells.

To assess the system performance we have looked at the outage probability for the different power control algorithms and detection schemes described in the previous sections.

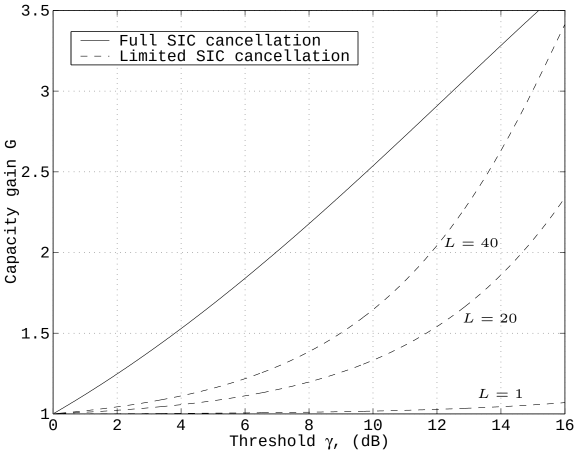

Fig. 3. Capacity gain of SIC detection with limited number of stages over single user detection for $W/R = 520$.

Table I: Simulation parameters.

Item	Value
Processing gain	$W/R = 520$
Path loss model	$L_p = -23.3 + 40\log_{10}(r)$ dB
Radius of the cell	$R_{\text{cell}} = 1000$ m
Transmit power	$p_{\min} = -50$ dBm, $p_{\max} = 33$ dBm
Noise floor	$N_0 = -105$ dBm
SIR threshold	$\gamma = 7$ dB
Lognormal standard deviation	$\sigma_s = 6$ dB
Constant received power	$p_{\text{crp}} = -121$ dBm
Number of PC iterations	5

Figure 4 illustrates the outage probability as a function of the number of users for the CDMA system with and without SIC detection. It is observed that, DC-SIC and SIR-SIC schemes have similar performance. It is also observed that SIC detection can provide considerable capacity gain when properly combined with power control. For instance, at an outage probability of 2% a total of 57 users/cell can be supported when single user detection is used. This total becomes 117 users/cell when linear SIC detection is used, a capacity gain of about 100% which is in line with the theoretical results of Figure 1. We also notice that the outage probability for CDMA with limited SIC gets better as L increases. This improvement is more pronounced for small values of L which also confirms the analytic results obtained in Section IV.

VI. CONCLUSIONS

In this work we have studied the capacity of a CDMA system with linear successive interference cancellation. We have looked at the interaction between power control and SIC detection. The effect of limited cancellation, where only part of the signals are involved in the SIC, has also been addressed. We have seen that it is not necessary to involve all signals to achieve the potential gain that can be provided by the linear SIC detector. A simple upper bound on the capacity of CDMA with linear SIC has been derived. This bound is very flexible as it is expressed in terms of the number of users, L, involved in the SIC detection.

Simulation results showed that the integration of linear SIC detection and power control improves the capacity of CDMA considerably. Limited linear cancellation appears as a good compromise between complexity and capacity gain. The obtained simulation results showed that by canceling the strongest signal in each cell, the capacity can be increased by about 14%!

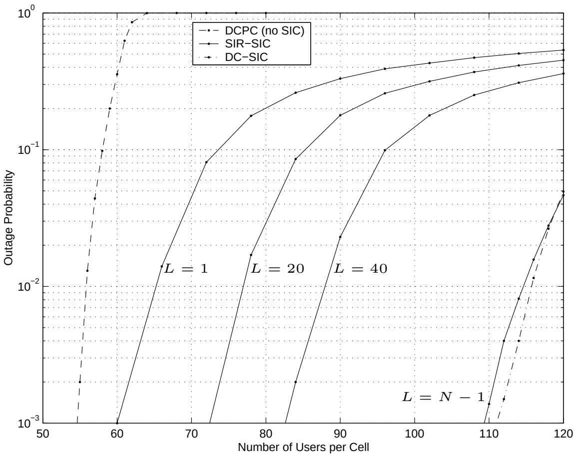

Fig. 4. Outage probability as function of number of users for CDMA with limited linear SIC detection.

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