# A Sliding Window PDA for Asynchronous CDMA, and a Proposal for Deliberate Asynchronicity

Jie Luo, *Member, IEEE*, Krishna R. Pattipati, *Fellow, IEEE*, and Peter K. Willett, *Fellow, IEEE*

***Abstract*—This letter contains two parts. In the first part, the probabilistic data association (PDA) method is extended to multiuser detection over symbol-asynchronous code-division multiple access (CDMA) communication channels. A direct extension as well as a sliding window processing method are introduced. While achieving near-optimal performance with $O(K^3)$ computational complexity in synchronous CDMA, $K$ being the number of users, it is shown that, in asynchronous CDMA, the probability of group detection error of the proposed PDA method is very close to the performance lower bound provided by an ideal clairvoyant optimal detector, and the computational complexity is only marginally increased to $O([h/s]K^3)$ per symbol where $h$ and $s$ are the width and the sliding rate of the processing window, respectively. In the second part, due to the outstanding performance of the PDA detector in heavily overloaded asynchronous systems, it is observed that an optimally designed synchronous system can be easily outperformed by an arbitrarily designed asynchronous system. Hence, it is proposed to use asynchronous transmission deliberately, even when synchronous transmission is possible—asynchronous is better than synchronous!**

***Index Terms*—Asynchronous code-division multiple access (CDMA), code-division multiple access (CDMA), multiuser detection, overloaded system, probabilistic data association (PDA).**

## I. Introduction

**D**UE TO THE NP-hard nature of the general multiuser detection problems in code-division multiple access (CDMA) communications [1], suboptimal algorithms that provide reliable decisions and that ensure polynomial computational costs have been extensively studied for over 15 years. Linear detectors [1] and decision-driven detectors [2], [3] improve the performance of the conventional matched filter significantly, while limiting their computational complexities to $O(K^2)$. Other advanced detectors of polynomial complexity have also been proposed recently to provide near-optimal performance [4], [5]. Among the advanced detectors, the probabilistic data association (PDA) detector [4] shows outstanding computational efficiency and near-optimal performance in almost all cases in synchronous CDMA [6]. Furthermore, since PDA works with probabilities and provides "soft" outputs, it is naturally flexible and relatively easy to extend to realistic CDMA situations including fading, coding, etc.

Paper approved by G. Cherubini, the Editor for CDMA Systems of the IEEE Communications Society. Manuscript received July 17, 2002; revised November 12, 2002 and May 3, 2003. This work was supported in part by the Office of Naval Research under Contract N00014-98-1-0465 and Contract 00014-00-1-0101, and in part by NUWC under Contract N66604-01-1-1125. This paper was presented in part at the Conference on Information Sciences and Systems, Princeton, NJ, March 2002.

J. Luo is with the Institute for Systems Research, University of Maryland, College Park, MD 20742 USA (e-mail: rockey@isr.umd.edu).

K. R. Pattipati and P. K. Willett are with the Electrical and Computer Engineering Department, University of Connecticut, Storrs, CT 06269 USA (e-mail: krishna@engr.uconn.edu; willett@engr.uconn.edu).

Digital Object Identifier 10.1109/TCOMM.2003.820726

Most of the multiuser detection algorithms for synchronous CDMA can be directly extended to asynchronous CDMA, based on the fact that asynchronous CDMA can be viewed as a large synchronous system [1]. A major concern of this direct extension is the computational complexity, since the size of the big synchronous system, $MK$, where $M$ is the number of transmitted symbols, is much larger than $K$. For linear detectors [1] and decision-driven detectors [2], [3], due to the special structure of the correlation matrix, direct extension increases the computational complexity only marginally. However, this is not necessarily the case for other detection algorithms. The complexity for the optimal detector, for example, increases dramatically. It is hard to perform computer simulations with optimal solution for even small-sized problems [1]; and, indeed, in this letter, we must resort to a lower bound on optimal performance for our comparisons.

In this letter, we consider a $K$-user $M$-symbol asynchronous CDMA system over nonfading channels. We first treat the asynchronous CDMA system as a $KM$-user synchronous system. By exploiting the special structure of the correlation matrix, the PDA multiuser detector for synchronous CDMA is simplified and applied to the equivalent synchronous system. Although such a direct extension requires the receipt of all the $M$ symbols of all users in order to do iterations, we show that the computational complexity per symbol is the same as that of a $K$-user synchronous system. To avoid considering the entire transmission data, which might cause substantial delay, a truncated processing window is introduced. This is further extended to a sliding window version. The complexity per symbol is shown to be $O([h/s]K^3)$, where $h$ is the width of the processing window and $s$ is the sliding rate. Simulation results for both regular and overloaded systems are presented to show the remarkable performance of the PDA detector.

In synchronous CDMA, having more users than the signature length results in a singular correlation matrix, which makes the direct implementation of many multiuser detection algorithms untenable. Synchronous overloaded systems have been studied mostly in terms of the user capacity [11] and signature design [9]. The optimal signature design that maximizes the channel capacity for synchronous CDMA as well as the chip-synchronized asynchronous CDMA system have been proposed in [9] and [12], respectively. It is shown that, if the optimal signature sequences are used, the channel capacities of the synchronous system and the chip-synchronized asynchronous system are identical. However, optimal signature sequences and the corresponding channel capacity for a general chip-asynchronous system remain unknown. In this letter, we present examples to show that a symbol-synchronous system with both optimal signature and maximum-likelihood (ML) multiuser detector can be easily outperformed by a chip-asynchronous system with an arbitrary signature design and a suboptimal detector. Therefore, instead of transmitting signals synchronously, we propose using asynchronous transmission with prespecified delay profiles on user signals. Moreover, with the help of the proposed PDA detector, we show that it is not difficult for an asynchronous system to achieve an outstanding performance under reasonable signal-to-noise ratio (SNR) even when the system is heavily overloaded (the number of users is three times the spreading factor, for example).

## II. Review of PDA in Synchronous CDMA

A discrete-time equivalent model for the matched-filter outputs at the receiver of a $K$-user synchronous CDMA channel is given by the $K$-length vector [1]

$$\boldsymbol{y} = \boldsymbol{RWb} + \boldsymbol{v} \tag{1}$$

where $\boldsymbol{R}$ is the normalized signature correlation matrix; $\boldsymbol{W}$ is a diagonal matrix whose $i$th diagonal element, $w_i$, is the square root of the received signal energy per bit of the $i$th user; $\boldsymbol{b} \in \{-1,+1\}^K$ denotes the vector of bits transmitted by the $K$ active users; and $\boldsymbol{v}$ is a zero-mean colored Gaussian noise vector with a covariance matrix $\sigma^2\boldsymbol{R}$.

When all the user signals are equally probable, the optimal solution of (1) is the output of a ML detector

$$\Phi_{\mathrm{ML}} : \hat{\boldsymbol{b}} = \arg\min_{\boldsymbol{b}\in\{-1,+1\}^K} (\boldsymbol{b}^T\boldsymbol{WRWb} - 2\boldsymbol{y}^T\boldsymbol{Wb}). \tag{2}$$

Obtaining the ML solution is generally NP-hard [1].

The PDA method from target tracking [7] was introduced to multiuser detection for synchronous CDMA in [4]. PDA suggests that we treat the decision variables $\boldsymbol{b}$ as binary random variables. For any user $i$, associate a probability $P_{bi}$ with user signal $b_i$ to express the current estimate of the probability that $b_i = 1$. Now, for an arbitrary user signal $b_i$, treat the other user signals $b_j (j \neq i)$ as binary random variables, and treat the Gaussian noise together with the multiple-access interference (MAI) as an "effective" Gaussian ambient noise (which, of course, it is not). Consequently, $P(b_i = 1|\boldsymbol{y}, \{P_{bj}\}_{j\neq i})$ can be obtained from (1); this serves as updated information for user signal $b_i$. The multistage PDA detector for synchronous CDMA proceeds as follows.

1) Sort users according to the user-ordering criterion proposed for decision-feedback (DF) detector [4].
2) $\forall i$, initialize the probabilities as $P_{bi} = 0.5$. Initialize the stage counter $k = 1$.
3) Initialize the user counter $i = 1$.
4) Based on the current value of $P_{bj} (j \neq i)$ for user $i$, update $P_{bi}$ by
$$P_{bi} = P\left\{b_i = 1|\boldsymbol{y}, \{P_{bj}\}_{j\neq i}\right\}. \tag{3}$$
5) If $i < K$, let $i = i + 1$ and goto step 4).
6) If $\forall i$, $P_{bi}$ has converged, goto step 7). Otherwise, let $k = k + 1$ and return to step 3).
7) $\forall i$, make a decision on user signal $i$ via $b_i = \mathrm{sign}(P_{bi} - 0.5)$.

Computationally efficient numerical schemes as well as several other refinements are presented in [4].

Although a similar probability update was proposed in [13] for coded CDMA, the PDA method updates the probabilities sequentially in a specific order. These serve as the key steps for PDA to achieve superior performance in synchronous CDMA.

## III. PDA for Asynchronous CDMA System

### A. System Model

Similar to the system model of (1), an asynchronous CDMA system can be described in the $z$ domain by [1]

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{Wb}(z) + \boldsymbol{v}(z) \tag{4}$$

where $\boldsymbol{v}$ is a colored Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{R}(z)$. The signature correlation matrix $\boldsymbol{R}(z)$ can be expressed and factorized as [3]

$$\begin{aligned}\boldsymbol{R}(z) &= \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1]z^{-1}\\ &= \left(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z\right)\left(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}\right).\end{aligned} \tag{5}$$

Here, $\boldsymbol{R}[0]$ is a symmetric matrix whose components represent the correlations between user signatures at the same symbol index, and $\boldsymbol{R}[1]$ is a singular matrix whose components represent the signature correlations relating to successive symbols. In the factorization, $\boldsymbol{F}[0]$ is a lower triangular matrix, and $\boldsymbol{F}[1]$ is singular. Applying the anticausal feed-forward filter $\left(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z\right)^{-1}$ to both sides of (4), we obtain the white noise model [3]

$$\tilde{\boldsymbol{y}}(z) = \left(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}\right)\boldsymbol{Wb}(z) + \tilde{\boldsymbol{v}}(z) \tag{6}$$

where $\tilde{\boldsymbol{y}}(z) = \left(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z\right)^{-1}\boldsymbol{y}(z)$ and $\tilde{\boldsymbol{v}}$ is a white Gaussian noise vector with zero mean and covariance matrix $\sigma^2\boldsymbol{I}$. The corresponding time-domain representation of the white noise model is

$$\tilde{\boldsymbol{y}}(n) = \boldsymbol{F}[0]\boldsymbol{Wb}(n) + \boldsymbol{F}[1]\boldsymbol{Wb}(n-1) + \tilde{\boldsymbol{v}}(n). \tag{7}$$

### B. Direct Extension

Suppose there are, overall, $M$ symbols in the transmission (i.e., $M$ symbols for each user). We can view the asynchronous system as an $MK$-user synchronous system and rewrite (7) as

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{W}}\tilde{\boldsymbol{b}} + \tilde{\boldsymbol{V}}. \tag{8}$$

Here

$$\begin{aligned}\tilde{\boldsymbol{Y}} &= \left[\tilde{\boldsymbol{y}}(0)^T, \tilde{\boldsymbol{y}}(1)^T, \ldots, \tilde{\boldsymbol{y}}(M-1)^T\right]^T\\ \tilde{\boldsymbol{b}} &= \left[\boldsymbol{b}(0)^T, \boldsymbol{b}(1)^T, \ldots, \boldsymbol{b}(M-1)^T\right]^T\\ \tilde{\boldsymbol{V}} &= \left[\tilde{\boldsymbol{v}}(0)^T, \tilde{\boldsymbol{v}}(1)^T, \ldots, \tilde{\boldsymbol{v}}(M-1)^T\right]^T\\ \tilde{\boldsymbol{W}} &= \begin{bmatrix}\boldsymbol{W} & \boldsymbol{0} & \ldots\\ \boldsymbol{0} & \ldots & \boldsymbol{0}\\ \ldots & \boldsymbol{0} & \boldsymbol{W}\end{bmatrix}\\ \tilde{\boldsymbol{L}} &= \begin{bmatrix}\boldsymbol{F}[0] & \boldsymbol{0} & \ldots & \ldots\\ \boldsymbol{F}[1] & \boldsymbol{F}[0] & \boldsymbol{0} & \ldots\\ \boldsymbol{0} & \ldots & \ldots & \ldots\\ \ldots & \boldsymbol{0} & \boldsymbol{F}[1] & \boldsymbol{F}[0]\end{bmatrix}.\end{aligned} \tag{9}$$

Although it appears that the computational cost of directly applying the PDA method to the equivalent $MK$-user system is $O(M^2K^3)$ per symbol (which can be very high if $M$ is not small), it can be substantially simplified due to the special structure of matrix $\tilde{\boldsymbol{L}}$.

Consider updating the probability associated with user $i$ in symbol $n$. From (7), we have

$$\begin{aligned} P_{bi}(n) &= P\left\{ b_i(n)=1 \middle| \tilde{\boldsymbol{Y}}, \{P_{bj}(n)\}_{j\neq i}, \{P_{bl}(k)\}_{k\neq n} \right\} \\ &= P\left\{ b_i(n)=1 \middle| \begin{matrix} \tilde{\boldsymbol{y}}(n), & \tilde{\boldsymbol{y}}(n+1), \\ \{P_{bj}(n)\}_{j\neq i}, & \{P_{bl}(k)\}_{k=n-1,n+1} \end{matrix} \right\}. \end{aligned} \quad (10)$$

Therefore, to update the probability $P_{bi}(n)$, only two observation vectors, $\tilde{\boldsymbol{y}}(n)$ and $\tilde{\boldsymbol{y}}(n+1)$, are required. The corresponding observation model from (7) is

$$\begin{bmatrix} \tilde{\boldsymbol{y}}(n) \\ \tilde{\boldsymbol{y}}(n+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}[0] \\ \boldsymbol{F}[1] \end{bmatrix} \boldsymbol{W}\boldsymbol{b}(n) + \begin{bmatrix} \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(n-1) \\ \boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(n+1) \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{v}}(n) \\ \tilde{\boldsymbol{v}}(n+1). \end{bmatrix} \quad (11)$$

Motivated by the PDA idea, when we consider the user signals in symbol $n$, we approximate user signals in all other symbols by Gaussian random variables with matched means and variances. That is, $\forall i$, approximate $b_i(n-1)$ by a Gaussian random variable with mean $[2P_{bi}(n-1)-1]$, and variance $4P_{bi}(n-1)(1-P_{bi}(n-1))$. Perform similar approximation to $b_i(n+1)$, $\forall i$. Consequently, if we define $\boldsymbol{P}_b(n)$ to be a $K$-length column vector whose $i$th component is $P_{bi}(n)$, (11) can be equivalently written as

$$\boldsymbol{A}\left\{ \begin{bmatrix} \tilde{\boldsymbol{y}}(n) \\ \tilde{\boldsymbol{y}}(n+1) \end{bmatrix} - \begin{bmatrix} \boldsymbol{F}[1]\boldsymbol{W}\left(2\boldsymbol{P}_b(n-1)-1\right) \\ \boldsymbol{F}[0]\boldsymbol{W}\left(2\boldsymbol{P}_b(n+1)-1\right) \end{bmatrix} \right\} = \boldsymbol{W}\boldsymbol{b}(n) + \xi(n) \quad (12)$$

where $\boldsymbol{A} = \left\{ \begin{bmatrix} \boldsymbol{F}[0] \\ \boldsymbol{F}[1] \end{bmatrix}^T \begin{bmatrix} \boldsymbol{F}[0] \\ \boldsymbol{F}[1] \end{bmatrix} \right\}^{-1} \begin{bmatrix} \boldsymbol{F}[0] \\ \boldsymbol{F}[1] \end{bmatrix}^T$ and $\xi(n)$ is the effective zero-mean Gaussian noise.

Apparently, (12) is equivalent to a $K$-user synchronous CDMA system. Therefore, according to the methods introduced in [4], the complexity of updating $\boldsymbol{P}_b(n)$ can be reduced to $O(K^3)$. However, since we treat the $K$-user $M$-symbol asynchronous system as an $MK$-user synchronous system, the probability update and iterations must be performed over all $MK$ users. Therefore, the entire set of observations must be received before we can perform PDA iterations and make final decisions on any of the user signals; this would cause a significant delay at the receiver output.

### C. PDA With Sliding Window Processing

Suppose we are only interested in decisions on user signal vector $\boldsymbol{b}(n)$. Consider a truncated processing window of width $h$ that contains user signal vectors $\boldsymbol{b}(m)$, $(n - \lfloor (h+1)/2 \rfloor < m \le n + \lceil h/2 \rceil)$, where the floor function $\lfloor (h+1)/2 \rfloor$ denotes the largest integer that is smaller than or equal to $(h+1)/2$, and the ceiling function $\lceil h/2 \rceil$ denotes the smallest integer that is larger than or equal to $h/2$. Due to the limited error propagation in practical systems, it is reasonable to assume that if $h$ is large enough, the effects of values of user signals outside the processing window on the decisions of $\boldsymbol{b}(n)$ are negligible. Therefore, when making decisions on $\boldsymbol{b}(n)$, one can apply the PDA method and perform iterations only within the truncated processing window.

Noting that the processing windows for user signals in successive symbol indexes differ only slightly, we can use the probabilities from a processing window as the initial conditions of the PDA method for the next processing window to further simplify and speed up the iterative updates. This modifies the truncated-window PDA to a sliding-window PDA. The detailed procedure is described below.

1) Sort users according to the user ordering and time labeling criterion proposed for DF detector in [10].
2) $\forall i$ and $\forall n$, initialize the probabilities as $P_{bi}(n) = 0.5$. Initialize the window counter $k = 0$.
3) Initialize the symbol counter $n = \max\{0, k - \lfloor (h+1)/2 \rfloor + 1\}$.
4) Initialize the user counter $i = 1$.
5) Based on the current values of the associated probabilities, update $P_{bi}(n)$ according to (10).
6) If $i < K$, let $i = i + 1$ and goto step 5).
7) If $n < \min\{M-1, k + \lceil h/2 \rceil\}$, let $n = n + 1$ and goto step 4).
8) If $k + 1 > \lfloor (h+1)/2 \rfloor - 1, \forall i$, define

$$m = k - \left\lfloor \frac{(h+1)}{2} \right\rfloor + 1 \quad (13)$$

make a decision on user signal $b_i(m)$ via $b_i(m) = \mathrm{sgn}(P_{bi}(m) - 0.5)$
9) Let $k = k + 1$. If $k < M + \lfloor (h+1)/2 \rfloor - 1$, goto step 3). Otherwise, stop.

Similar to the synchronous case, the overall performance of PDA detector in asynchronous CDMA is affected by the user ordering as well as the time labeling. As shown in step 1), we propose to use the optimal user ordering and time labeling for the ideal DF detector derived in [10].

The computational complexity of the above PDA detector is $O(hK^3)$ per symbol, and the associated probability of each user signal bit is updated $h$ times. Intuitively, the window width, $h$, should be large so that the performance of the sliding-window PDA can approach the performance of the original PDA presented in Section III-B. Nevertheless, due to the fast convergence of the PDA method, a large number of iterations on the probability updates is unnecessary (typically only 2 $\sim$ 5 iterations are required). Therefore, we introduce the sliding rate $s$, $0 < s \le h$, and further modify the steps 8) and 9) in the above procedure to the following.

8) If $k + s > \lfloor (h+1)/2 \rfloor - 1, \forall i$, and for all $m$ that satisfy

$$\begin{aligned} \max\left\{0, k - \left\lfloor \frac{(h+1)}{2} \right\rfloor + 1\right\} &\le m \\ &< \min\left\{M, k + s - \left\lfloor \frac{(h+1)}{2} \right\rfloor + 1\right\} \end{aligned} \quad (14)$$

make decision on user signal $b_i(m)$ via $b_i(m) = \mathrm{sgn}(P_{bi}(m) - 0.5)$
9) Let $k = k + s$. If $k < M + \lfloor (h+1)/2 \rfloor - 1$, goto step 3). Otherwise, stop.

The relations between the indexes $i$, $n$, $k$, the window width $h$, and the sliding rate $s$ in the above procedure are illustrated

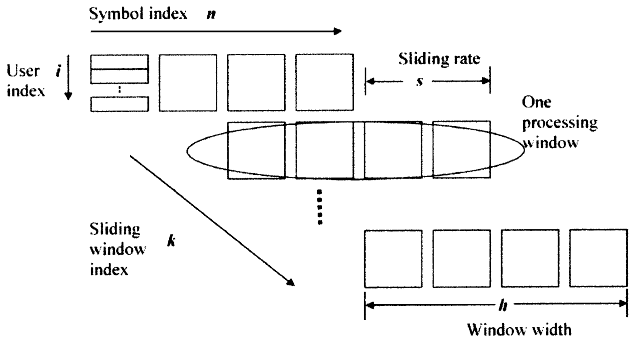


Fig. 1. Illustration of the sliding-window PDA. ($h = 4, s = 2$).

in Fig. 1. The computational complexity of the modified PDA detector is $O([h/s]K^3)$ per symbol.

## IV. Simulation Results

In this section, we compare the performances of the decorrelator, the DF detector and the PDA detector in terms of the probability of group detection error in various situations. The probability of group detection error is defined as the probability that all user signals with the specific symbol index are detected correctly, and is averaged among all symbol indexes. The optimal user ordering and time labeling rules proposed in [10] are applied to both the DF and the PDA detectors. Since the asynchronous ML detector is extremely complex, a performance lower bound is provided by (clairvoyantly) plugging the true values of $\boldsymbol{b}(n-1)$ and $\boldsymbol{b}(n+1)$ into (11) and applying the fast ML detection for synchronous CDMA [14] to detect $\boldsymbol{b}(n)$. The width of the processing window and the sliding rate for the PDA detector are set at three and one, respectively.

*Example 1:* In the first three-user example, the correlation matrices $\boldsymbol{R}[0]$, $\boldsymbol{R}[1]$, and the square roots of user signal powers $\boldsymbol{W}$ are randomly chosen as

$$\boldsymbol{R}[0] = \begin{bmatrix} 1.0 & -0.27 & -0.49 \\ -0.27 & 1.0 & 0.55 \\ -0.49 & 0.55 & 1.0 \end{bmatrix}$$
$$\boldsymbol{R}[1] = \begin{bmatrix} 0 & 0 & 0 \\ -0.06 & 0 & 0 \\ 0.16 & -0.01 & 0 \end{bmatrix}$$
$$\boldsymbol{W} = \mathrm{diag}(4.48, 4.36, 4.1). \quad (15)$$

Fig. 2 shows the performance comparison of different algorithms. The probability of error of the PDA detector is very close to the performance lower bound.

Additional computer similations of both synchronous and asynchronous CDMA systems with up to 60 users can be found in [15].

## V. A Deliberate Asynchronous Overloaded System With Designed Delay Profiles

In both synchronous and asynchronous CDMA, overloaded systems that accommodate a number of users greater than the signature length is attracting significant interest. However, the performance of a synchronous system degrades significantly when the number of users exceeds the signature length. When the system is heavily overloaded, both the synchronous system and the chip-synchronized asynchronous system suffer from the singular-correlation matrix problem, which prohibits the direct extension of many multiuser detection algorithms. For a synchronous system and chip-synchronized asynchronous system, optimal signature sequences (the Welch bound equality (WBE) signature) that maximize the channel capacity are derived in [9] and [12], respectively. Nevertheless, the optimal signature sequences for a general chip-asynchronous system and the resulting channel capacity remain unknown.

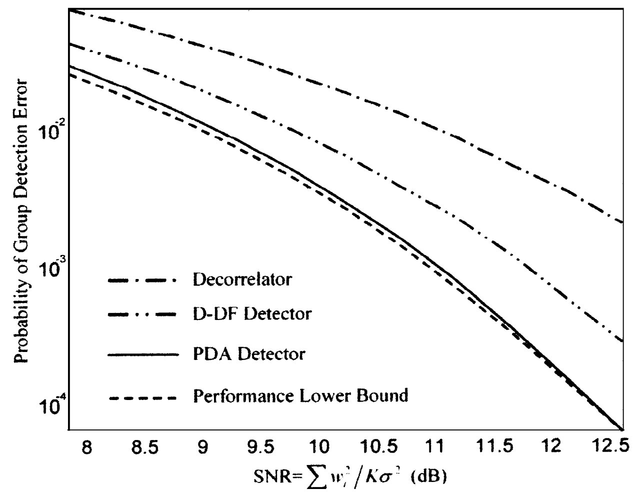


Fig. 2. Performance comparison, three users, $10^6$ Monte–Carlo runs.

In the following example, we show that, with the help of asynchronicity, a chip-asynchronous system can achieve a superior performance easily without using either the optimal signature design or the optimal multiuser detector.

*Example 2:* In this heavily overloaded system, we compare the performances of an optimally designed synchronous system and the corresponding asynchronous system. We have 21 users while the signature length is only seven. The powers of the signals are set to be equal. For the synchronous system, the seven-length WBE signature sequences are obtained from the iterative procedure introduced in [9]. For the asynchronous system, we use only three different signature sequences. Users $1 \sim 7$ use signature $[-1,-1,1,-1,1,1,1]/\sqrt{7}$, users $8 \sim 14$ use signature $[-1,1,-1,-1,1,1,1]/\sqrt{7}$, and users $15 \sim 21$ use signature $[1,1,-1,1,-1,1,1]/\sqrt{7}$. The delay of user signals are randomly generated as

$$T_c \times [0, 4.33, 1.49, 6.24, 6.58, 3.37, 5.17, 0.57, 5.8, 4.53, 3.39, 3.7, 2.5, 1.89, 6.56, 0.86, 0.35, 0.62, 4.6, 2.89, 2.73]$$

where $T_c$ is the chip duration. The correlation matrices are computed according to the system model of [8].

Fig. 3 shows the performances of different detectors for the asynchronous system together with that of the optimal detector for the optimal synchronous system. The performance of the optimal detector for the corresponding synchronous system is just slightly better than the decorrelator and is substantially worse than the PDA detector for the asynchronous system.

In this example, the synchronous system uses the WBE signatures, which are optimally designed and real valued. The

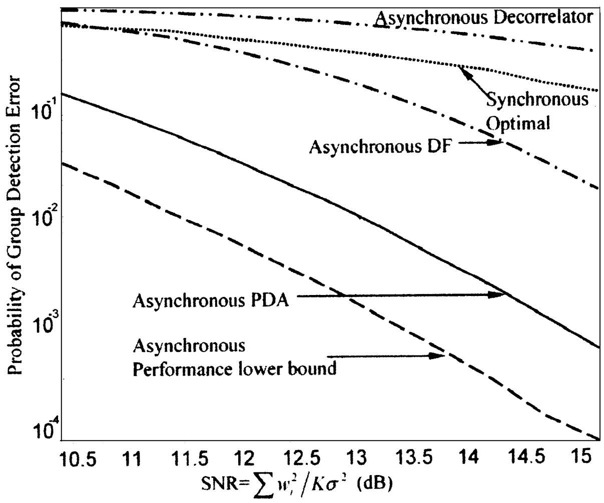


Fig. 3. Performance comparison, 21 users, seven-length signatures, random delay, $10^6$ Monte–Carlo runs. (Synchronous optimal was run for $10^4$ Monte–Carlo runs only due to excessive computational time).

asynchronous system, however, uses the randomly chosen binary-valued signatures. The synchronous system uses the NP-hard ML detector, while the asynchronous system uses the $O(K^3)$ complexity PDA detector. Apparently, the substantial differences between the performances of the two systems are due to the delay profiles introduced in the asynchronous system. In addition, the PDA detector for the asynchronous system achieves a bit-error rate of $10^{-3}$ at a SNR of 14.8 dB, which is encouraging considering that the number of users is three times the signature length. (Note that the performance lower bound shown in the figure may not be reachable.)

Although it appears that the asynchronicity may increase the complexity of the multiuser detectors, many asynchronous suboptimal detectors do maintain similar complexity as their synchronous versions. Note that if signal synchronization is possible in a synchronous system, it is always possible to design delay profiles for the user signals and transmit this information to the receiver so that the delays are known precisely to the multiuser detector. We suggest that one should use the symbol-asynchronous transmission with designed delay profiles even when synchronization is possible. The theoretical analysis and the optimality of the delay profile design, however, remain open and are outside the scope of this letter.

## VI. Conclusion

This letter has two main contributions. The first is an evolution of the PDA to asynchronous CDMA via a sliding-block approach. The performance of the proposed PDA detector is near the clairvoyant lower bound (the optimal solution is essentially uncomputable), and the computational complexity is $O([h/s]K^3)$, where $h$ is the window length and $s$ is the sliding rate.

The second is the observation that an arbitrarily designed asynchronous system can easily outperform the synchronous system with optimal signatures when the number of users is larger than the spreading factor. In addition, the PDA algorithm works remarkably well in asynchronous CDMA, even when the system is heavily overloaded. Thus, we propose that, at least in nonfading channels, CDMA should be asynchronous.

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