GROUP-ORTHOGONAL MULTI-CARRIER CDMA†

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ABSTRACT

Code division multiple access (CDMA) over frequency-selective fading channels faces challenges in suppressing multiuser interference (MUI), but can also benefit from the channel-induced multipath diversity. On the other hand, uncoded orthogonal frequency division multiple access (OFDMA) is MUI-free by design, but faces challenges arising from the fading-induced loss of multipath diversity. In this paper, we wed the advantages of CDMA with those of OFDMA. We design a group-orthogonal multi-carrier CDMA (GO-MC-CDMA) scheme that minimizes MUI subject to the constraint that the full multipath diversity is enabled for all users. Each group of GO-MC-CDMA users shares a properly selected set of system subcarriers. MUI only exists among users in the same small-size group, which renders application of low-complexity multi-user detection per group, practically feasible. Simulation results illustrate the merits of GO-MC-CDMA over competing multi-access alternatives.

I. INTRODUCTION

In direct sequence (DS) and multicarrier (MC) CDMA systems [7], [10], each user's signal is spread by a user-specific code, which expands the bandwidth compared to the data rate. Because of the large signal bandwidth, the receiver gains frequency diversity to combat fading effects of wireless channels. Although orthogonal CDMA spreading codes can be designed, their orthogonality is destroyed in the presence of multipath, which causes multiuser interference (MUI). MUI seriously affects the performance of CDMA with matched filter (MF) reception. Multiuser detection (MUD), which usually comes at the price of increased computation complexity, can be employed to deal with MUI.

Another very promising multiple access technique is orthogonal frequency division multiple access (OFDMA) [11]. Every OFDMA user is allocated one or more subcarriers. Since subcarriers retain their orthogonality even after multipath propagation, MUI is eliminated deterministically. OFDMA converts the multipath fading channel to an equivalent set of frequency flat fading channels. Unfortunately, uncoded OFDMA loses the multipath-induced diversity. A Mutually-Orthogonal User-code Receiver (AMOUR) [1] structure is developed in [3], which not only eliminates MUI deterministically but also retains multipath diversity. While AMOUR was originally designed for full load operation, dynamic load changes in the system can be exploited as discussed in [2] and [15]. The approaches in [2] and [15] however, need to change user code assignment and block length dynamically according to the load, which may not be always feasible.

In order to exploit the maximum possible channel diversity while being able to accommodate dynamic load changes in the system, we herein develop a group orthogonal (GO) MC-CDMA scheme that does not require complex dynamic code assignment operations. We partition the set of subcarriers into groups; the users who are assigned subcarriers of the same group are separated via spreading codes. By judiciously grouping subcarriers, we guarantee that all the users achieve full multipath diversity. The users in a particular group do not suffer from the interference from the other groups (this is the reason why we name our scheme group orthogonal MC-CDMA). Selecting groups of small size, we then apply MUD per group, which is practically feasible. Diversity allocation in multiuser MC systems was also studied in [4] and [5]. Our work herein systematically develops a novel system that achieves full diversity, and can afford low receiver complexity.

II. DESIGN OF GROUP-ORTHOGONAL MC-CDMA

Our design targets the uplink of a quasi-synchronous system, where the mobile users have means of aligning their timing to a common reference time, as is for example the case in IS-95. Suppose that the system bandwidth is W, and denote with $T_c = 1/W$ the chip duration. After the propagation delay is taken into account, the chip-sampled discrete-time baseband equiv-

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[1]In this paper, we refer to a special case of AMOUR which relies on IFFT user codes.

alent multipath channel (which also includes transmit-receive filters) of user μ can be modeled as an FIR filter with delay T_c between two consecutive taps. The total number of taps is $\lceil(\tau_{a,\max}+\tau_{s,\max})/T_c\rceil+1$, where $\tau_{a,\max}$ is the maximum relative user asynchronism, and $\tau_{s,\max}$ is the maximum delay spread of all multipath channels [9, p. 797]. The impulse response vector of user μ is written as $\mathbf{h}_\mu := [h_\mu(0), h_\mu(1), \ldots, h_\mu(L)]^T$, where $L = \lceil(\tau_{a,\max}+\tau_{s,\max})/T_c\rceil$ is the channel order. Denote by $\tau_{a,\mu}$ the propagation delay of user μ, and by $\tau_{s,\mu}$ the delay spread of user μ's channel. The first $L_{a,\mu} = \lfloor\tau_{a,\mu}\rfloor/T_c$ taps are zero; the next $L_{s,\mu} = \lceil(\tau_{a,\mu}+\tau_{s,\mu})/T_c\rceil - L_{a,\mu}$ taps are nonzero, while the last $L+1-L_{a,\mu}-L_{s,\mu}$ taps are zero. We denote by $L_{s,\max}$ the maximum of $L_{s,\mu}, \forall\mu$, and assume that the $L_{s,\mu}$ nonzero taps in $\mathbf{h}_\mu$ are independent zero mean complex Gaussian random variables; their variances are not necessarily identical, and the channels of different users are assumed independent.

A. User Grouping

We consider the symbol-spread case, where active users transmit only one symbol over a block of M chips. Let the symbol period be $T = MT_c$. The entire available bandwidth is utilized with M subcarriers that are spaced $1/T$ far apart from each other. A cyclic prefix of length TL/M is inserted between consecutive blocks. Accounting for the cyclic prefix, the augmented block contains $P = M + L$ chips. Define the $M \times M$ fast Fourier transform (FFT) matrix $\mathbf{F}$ with $(m+1, n+1)$st element $[\mathbf{F}]_{m,n} = (1/\sqrt{M})\exp(-j2\pi mn/M), m, n \in [0, M-1]$. If $\mathbf{f}_i$ denotes the ith column of matrix $\mathbf{F}$, then $\mathbf{f}_i^*$ is the ith digital subcarrier, where $()^*$ stands for conjugation. We partition the M subcarriers into N_g groups with each group having $Q = M/N_g$ subcarriers. By properly choosing M and N_g, we can ensure that Q is an integer and $Q \geq L_{s,\max}$. A user belonging to a specific group transmits information bearing symbols on the subcarriers corresponding to this group; Q users share Q subcarriers of each group which ensures no spectral efficiency loss. The issue of optimally grouping the subcarriers will be addressed in the next section.

Define an $M \times M$ permutation matrix $\mathbf{\Phi} := [\mathbf{\Phi}_0, \mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_{N_g-1}]$, where each $M \times Q$ sub-matrix $\mathbf{\Phi}_n$ consists of a specific pattern of $\{0, 1\}$ entries determining the Q subcarriers allocated to the nth group. We will specify $\mathbf{\Phi}$ in the next section. The Q digital subcarriers in the nth group are columns of the matrix $\mathbf{F}^H\mathbf{\Phi}_n$. Define now a $Q \times Q$ matrix $\mathbf{C} := [\mathbf{c}_0, \mathbf{c}_1, \ldots, \mathbf{c}_{Q-1}]$ whose qth column $\mathbf{c}_{q-1}$ serves as the code mapping each symbol of the qth user in the nth group onto Q subcarriers. The code matrix $\mathbf{C}$ does not have to be identical for different groups. But since there is no MUI among users of different groups, we choose the same code matrix for all the groups. We further design $\mathbf{C}$ so that:

AS1) *All the user codes are linearly independent, with* $|c_q(i)|^2 = 1/Q, \forall q, \forall i \in [1\ Q]$, *where* $c_q(i)$ *is the ith element of* $\mathbf{c}_q$.

This design condition is satisfied when $\mathbf{c}_q$ is any binary code such as Wash-Hadamard, Gold [9], or, unitary [14]. Define a $P \times M$ cyclic prefix inserting matrix $\mathbf{T}_{cp} := [\mathbf{I}_{cp}^T, \mathbf{I}_M]^T$, where $\mathbf{I}_{cp}$ denotes the matrix formed by the last L rows of the $M \times M$ identity matrix $\mathbf{I}_M$. Suppose that there are $N_{a,n}$ active users in the nth group. Thanks to the cyclic prefix, there is no inter-block interference (IBI) between different blocks; and thus, it is sufficient to decode on a block by block basis. Each $P \times 1$ transmitted block of the mth user in the nth group can be expressed as:

$$\mathbf{x}_{n,m} = \mathbf{T}_{cp}\mathbf{F}^H\mathbf{\Phi}_n\mathbf{c}_m s_{n,m}. \quad (1)$$

At the receiver end, after removing the IBI and FFT processing the IBI-free signal, we obtain an $M \times 1$ block [13]

$$\mathbf{y} = \sum_{n=0}^{N_g-1}\sum_{m=0}^{N_{a,n}-1}\boldsymbol{\mathcal{D}}(\tilde{\mathbf{h}}_{n,m})\mathbf{\Phi}_n\mathbf{c}_m s_{n,m} + \mathbf{w}, \quad (2)$$

where the $M \times 1$ vector $\tilde{\mathbf{h}}_{n,m}$ contains the frequency response samples on the FFT grid of the FIR channel of the mth user in the nth group; $\boldsymbol{\mathcal{D}}(\cdot)$ stands for a diagonal matrix with the vector in parentheses on its diagonal, and $\mathbf{w}$ is zero mean complex additive white Gaussian noise (AWGN) with variance $N_0/2$ per dimension. We then pick up the received samples on the subcarriers of the nth group using the group-specific selector matrix $\mathbf{\Phi}_n^H$ as follows:

$$\mathbf{y}_n = \mathbf{\Phi}_n^H\mathbf{y} = \sum_{m=0}^{N_{a,n}-1}\boldsymbol{\mathcal{D}}(\bar{\mathbf{h}}_{n,m})\mathbf{c}_m s_{n,m} + \mathbf{w}_n \quad (3)$$

where the $Q \times 1$ vector $\bar{\mathbf{h}}_{n,m}$ is given by $\bar{\mathbf{h}}_{n,m} = \mathbf{\Phi}_n^H\tilde{\mathbf{h}}_{n,m}$. As we shall see later, MUD can be applied to detect the information bearing symbols $\{s_{n,m}\}_{m=0}^{N_{a,n}-1}$ in the nth group based on $\mathbf{y}_n$.

Remark 1: It appears that GO-MC-CDMA with symbol spreading is similar to MC-CDMA [8] with one symbol transmitted per user per block, and with the number of subcarriers equal to the processing gain P_G. However, in MC-CDMA [8], all the users share P_G subcarriers, and thus, MUD is computationally prohibitive. In our GO-MC-CDMA design, a small group

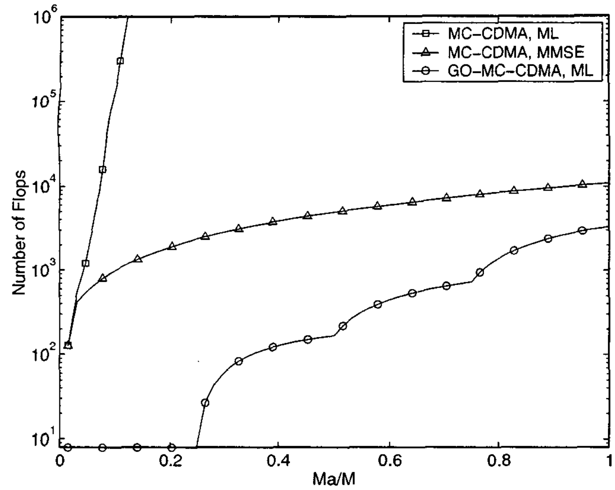

Fig. 1. Receiver complexity comparison, $|\mathcal{A}| = 4$, $M = 64$ and $Q = 4$.

of users shares a set of subcarriers. Actually, it will be shown in the next section that the group size only needs to be $\geq L_{s,\max}$ to achieve maximum diversity. For such sizes, the optimal MUD becomes practically feasible.

Fig. 1 compares the receiver complexity of MC-CDMA and GO-MC-CDMA, where the complexity is defined as the average number of complex flops per user [6, p.18], and M_a is the number of active users. With a processing gain $M = 64$, group size $Q = 4$, and constellation size $|\mathcal{A}| = 4$, the maximum likelihood (ML) detector of GO-MC-CDMA has lower complexity than the MMSE detector of MC-CDMA, even when the load is full.

B. User Allocation

Dynamic subcarrier assignment is employed in [2], [15] to improve the performance when the system load is low. In GO-MC-CDMA, the base station uniformly allocates active users to different groups. Denote by M_a the number of active users in the system. Let $\tilde{N}_a = \lfloor M_a/N_g \rfloor$, and $N_{g1} = M_a - N_g\tilde{N}_a$. If $N_{g1} = 0$, every group has $\tilde{N}_a$ active users. If $N_{g1} \neq 0$, the number of active users allocated to the nth group, $N_{a,n}$, is

$$N_{a,n} = \begin{cases} \tilde{N}_a + 1 & n \in [1,\ N_{g1}],\ N_{g1} \neq 0 \\ \tilde{N}_a & n \in [N_{g1}+1,\ N_g],\ N_{g1} \neq 0. \end{cases} \quad (4)$$

When a new user arrives, it is assigned to group $N_{g1}+1$. If $M_a \leq N_g$, every active user enjoys single user performance, because it is the only active user in its group. Since the performance of a user and the computational complexity of the ML detector depends on the number of active users in a group, this simple user allocation policy enhances the performance, and also reduces the average receiver computation complexity.

III. GROUP SUBCARRIER ASSIGNMENT

Intuitively thinking, the subcarriers in the same group should be separated as much as possible to enhance diversity gains, since the channel on neighboring subcarriers exhibits large correlation. For this reason, we assign a set of subcarriers $\mathbf{F}_n^H = [\mathbf{f}_n^*, \mathbf{f}_{Ng+n}^*, \mathbf{f}_{2Ng+n}^*, \dots, \mathbf{f}_{(Q-1)N_g+n}^*]$ to the nth group, where vector $\mathbf{f}_i$ is the ith column of $M \times M$ FFT matrix, and $Q \geq L_{s,\max}$. Equivalently, we choose the subcarrier selection matrix as:

$$\mathbf{\Phi}_n = [\mathbf{v}_n, \mathbf{v}_{Ng+n}, \mathbf{v}_{2Ng+n}, \dots, \mathbf{v}_{(Q-1)N_g+n}], \quad (5)$$

where $\mathbf{v}_i$ is the $M \times 1$ unit vector with 1 in its $(i+1)$st entry and 0's in all other entries. The nth group subcarriers are the columns of the matrix $\mathbf{F}_n^H = \mathbf{F}^H \mathbf{\Phi}_n$. It will turn out that this simple subcarrier assignment achieves maximum diversity for all the users as long as $Q \geq L_{s,\max}$.

Using Matlab's notation, let us define a $Q \times L_{s,n,m}$ matrix $\mathbf{F}_{n,m} := \sqrt{M}\mathbf{F}_n(L_{a,n,m}+1 : L_{a,n,m}+L_{s,n,m}, :)$, where $L_{a,n,m}$, $L_{s,n,m}$ account respectively for the propagation delay and the delay spread of the mth user in the nth group, as discussed in Section II. Let vector $\mathbf{h}_{n,m}$ contain the $L_{s,n,m}$ nonzero taps of the channel impulse response of user m in the nth group. Then, the channel frequency response vector $\bar{\mathbf{h}}_{n,m}$ in (3) can be expressed as

$$\bar{\mathbf{h}}_{n,m} = \mathbf{F}_{n,m}\mathbf{h}_{n,m}. \quad (6)$$

Because the subcarriers in a group are chosen to be equi-spaced, it can be shown that [1]

$$\mathbf{F}_{n,m}^H \mathbf{F}_{n,m} = Q\,\mathbf{I}_{L_{s,n,m}}. \quad (7)$$

IV. PERFORMANCE ANALYSIS

We first consider single-user performance. When only user 0 is active in the nth group, the received block after OFDM demodulation is simplified from (3) to

$$\mathbf{y}_n = \mathcal{D}(\mathbf{c}_0)\bar{\mathbf{h}}_{n,0}s_0 + \mathbf{w}_n. \quad (8)$$

The optimal single-user receiver is the matched filter. Under AS1), the matched filter output is given by

$$z = \bar{\mathbf{h}}_{n,0}^H \mathcal{D}(\mathbf{c}_0^*)\mathbf{y} = ||\bar{\mathbf{h}}_{n,0}||^2 s_0/Q + \eta, \quad (9)$$

where $||\cdot||$ stands for Euclidean norm, and η is a Gaussian random variable with variance $||\bar{\mathbf{h}}_{n,0}||^2 N_0/Q$. Using (6) and (7), the decision variable z in (9) becomes

$$z = ||\mathbf{h}_{n,0}||^2 s_0 + \eta. \quad (10)$$

The SNR in (10) is $|s_0|^2||\mathbf{h}_{n,0}||^2/N_0$. It is seen from (10) that the final decision variable z coherently combines the transmitted symbol s_0 from all the paths. Thus, the maximum diversity is achieved. From this analysis for the single user performance, we deduce that a user does not need to transmit over the entire bandwidth to achieve maximum diversity, which makes it possible to limit MUI without sacrificing diversity. This observation is not surprising, because it is well known that frequency diversity can be obtained by transmitting on carrier frequencies which are separated from each other by the coherence bandwidth of the channel. However, together with the subcarrier grouping scheme we proposed, it provides within the MC-CDMA framework a means of achieving maximum diversity with as few interfering users as possible.

We next study multi-user performance. Since the size of each group is small, the ML detector can be employed to jointly detect the symbols of all the active users in a group based on (3). To study the performance of the ML detector, we resort to the Chernoff bound of the pairwise error probability (PEP). Suppose that we are interested in the symbol error rate (SER) of user 0 in the nth group. Write symbols in (3) in a vector form $\mathbf{s} := [s_{n,0}, \ldots, s_{n,N_a-1}]^T$, and define $s_0 := s_{n,0}$, and $\mathbf{s}_I := [s_{n,1}, \ldots, s_{n,N_a-1}]^T$. Let $\tilde{\mathbf{s}}$ be a symbol vector such that $\tilde{s}_0 \neq s_0$, and let $\mathbf{h}$ comprise the channel impulse responses of all active users in the nth group, i.e., $\mathbf{h} := [\mathbf{h}_{n,0}^T, \mathbf{h}_{n,1}^T, \ldots, \mathbf{h}_{n,N_a-1}^T]^T$. The PEP, conditioned on the channel $\mathbf{h}$ is defined as the probability that the detector decides $\tilde{\mathbf{s}}$ when the block $\mathbf{s}$ is actually transmitted. The PEP has the following upper bound [12]

$$P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}|\mathbf{h}) \leq \exp\left(-\frac{d^2(\tilde{\mathbf{u}}, \mathbf{u})}{4N_0}\right), \quad (11)$$

where $\mathbf{u} := \sum_{m=0}^{N_a-1} \boldsymbol{\mathcal{D}}(\mathbf{c}_m)\bar{\mathbf{h}}_{n,m}s_m$, $\tilde{\mathbf{u}} := \sum_{m=0}^{N_a-1} \boldsymbol{\mathcal{D}}(\mathbf{c}_m)\bar{\mathbf{h}}_{n,m}\tilde{s}_m$, and $d(\tilde{\mathbf{u}}, \mathbf{u}) = ||\tilde{\mathbf{u}} - \mathbf{u}||$ is the Euclidean distance between $\tilde{\mathbf{u}}$ and $\mathbf{u}$. Defining the matrix $\mathbf{S} := [s_0\boldsymbol{\mathcal{D}}(\mathbf{c}_0)\mathbf{F}_{n,0}, s_1\boldsymbol{\mathcal{D}}(\mathbf{c}_1)\mathbf{F}_{n,1}, \ldots, s_{N_a-1}\boldsymbol{\mathcal{D}}(\mathbf{c}_{N_a-1})\mathbf{F}_{n,N_a-1}]$, we have $\mathbf{u} = \mathbf{S}\mathbf{h}$. Consider the error vector $\mathbf{e} = \tilde{\mathbf{s}} - \mathbf{s} = [e_0, e_1, \ldots, e_{N_a-1}]^T$, and define the error matrix $\mathbf{S}_e = \tilde{\mathbf{S}} - \mathbf{S}$. The square of the Euclidean distance between $\tilde{\mathbf{u}}$ and $\mathbf{u}$ can be expressed as $d^2(\tilde{\mathbf{u}}, \mathbf{u}) = \mathbf{h}^H\mathbf{S}_e^H\mathbf{S}_e\mathbf{h}$. Letting matrix $\mathbf{S}_{e,m} := e_m\boldsymbol{\mathcal{D}}(\mathbf{c}_m)\mathbf{F}_{n,m}$, the error matrix can be written as $\mathbf{S}_e = [\mathbf{S}_{e,0}, \mathbf{S}_{e,1}, \ldots, \mathbf{S}_{e,N_a-1}]$. To proceed with our analysis, we prove the following lemma regarding the rank of $\mathbf{S}_e$ in [1].

Lemma 1: *Denote by N_e the number of symbols in error, and by r_e the rank of matrix $\mathbf{S}_e$. If all the users' channels have the same delay spread L_s, then AS1) implies that, if $Q > L_s$, we have $r_e = L_s$ for $N_e = 1$, and $r_e > L_s$ for $N_e > 1$.*

Define two subsets of the vector $\tilde{\mathbf{s}}$ as follows $\tilde{\mathcal{S}}_0 := \{\tilde{\mathbf{s}} : \tilde{s}_0 \neq s_0, \tilde{\mathbf{s}}_I = \mathbf{s}_I\}$ and $\tilde{\mathcal{S}}_1 := \{\tilde{\mathbf{s}} : \tilde{s}_0 \neq s_0, \tilde{\mathbf{s}}_I \neq \mathbf{s}_I\}$. Let $\tilde{\mathbf{s}}_0 \in \tilde{\mathcal{S}}_0$ be the single error vector, and $\tilde{\mathbf{s}}_1 \in \tilde{\mathcal{S}}_1$ be the multiple error vector. From (11), the single error conditional PEP, $P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_0|\mathbf{h})$, is bounded by

$$P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_0|\mathbf{h}) < \exp\left(-\frac{|e_0|^2||\mathbf{h}_{n,0}||^2}{4N_0}\right), \quad (12)$$

and the unconditional PEP is obtained by averaging $P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_0|\mathbf{h})$ over all the realizations of $\mathbf{h}$, which yields

$$P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_0) < \prod_{i=1}^{L_s}(1 + |e_0|^2\sigma_i^2/4N_0)^{-1}. \quad (13)$$

where σ_i^2 is the variance of $\mathbf{h}_{n,0}(i)$. The multiple error PEP, $P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_1|\mathbf{h})$, can be found as follows. Let the square root of the covariance matrix of channel impulse response be $\mathbf{R}^{1/2} := \text{diag}(\sigma_0, \sigma_1, \ldots, \sigma_{L_s-1})$, and $\tilde{\mathbf{S}}_e = [\mathbf{S}_{e,0}\mathbf{R}^{1/2}, \ldots, \mathbf{S}_{e,N_a-1}\mathbf{R}^{1/2}]$. The matrix $\mathbf{A}_e := \tilde{\mathbf{S}}_e^H\tilde{\mathbf{S}}_e$ is Hermitian, and can be expressed as $\mathbf{A}_e = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H$, where $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{r_e})$ is an $r_e \times r_e$ diagonal matrix containing the eigenvalues of $\mathbf{A}_e$ in non-increasing order, and the columns of $N_eL_s \times r_e$ matrix $\mathbf{U}$ are the corresponding eigenvectors of $\mathbf{A}_e$. Then, $d^2(\tilde{\mathbf{u}}, \mathbf{u})$ can be written in terms of the eigenvalues of $\mathbf{A}_e$ as

$$d^2(\tilde{\mathbf{u}}, \mathbf{u}) = \sum_{i=0}^{r_e} \lambda_i|\breve{h}_i|^2, \quad (14)$$

where $\breve{h}_i$ is a complex Gaussian random variable with zero mean and unit variance. The upper bound of the multiple error PEP is then given by

$$P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_1|\mathbf{h}) \leq \exp\left(-\sum_{i=0}^{r_e} \lambda_i|\breve{h}_i|^2/4N_0\right), \quad (15)$$

and the unconditional PEP $P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_1)$ is bounded by [12]

$$P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_1) < \left(\prod_{i=1}^{r_e}(1 + \lambda_i/4N_0)\right)^{-1} \quad (16)$$

A more insightful bound on multiple error PEP is found as [1]

$$P(\mathbf{s} \rightarrow \tilde{\mathbf{s}}_1) < \prod_{i=1}^{L_s}(1 + |e_{\max}|^2\sigma_i/4N_0)^{-1} \times \prod_{i=L_s+1}^{r_e}(1 + \lambda_i/4N_0)^{-1}, \quad (17)$$

where $|e_{\max}|^2 := \max\{|e_0|^2, \ldots, |e_{N_a-1}|^2\}$. If $|e_{\max}|^2 > |e_0|^2$, i.e., a strong interfering user has a symbol in error, then comparing (13) with (17) reveals that the upper bound on the multiple error PEP is smaller than the upper bound on the single error PEP. This means that strong interference does not degrade ML performance, and the ML detector is near-far resistant. If $|e_{\max}|^2 = |e_0|^2$, we know from (16) and (17) that $P(\mathbf{s} \to \tilde{\mathbf{s}}_1)$ is in the same order as $P(\mathbf{s} \to \tilde{\mathbf{s}}_0)$ when $\{\lambda_i\}_{i=L_s}^{r_e}$ are smaller than or comparable to N_0, or, it is much smaller than $P(\mathbf{s} \to \tilde{\mathbf{s}}_0)$, when $\{\lambda_i\}_{i=L_s}^{r_e}$ are much larger than N_0. In any case, the multiple error PEP, $P(\mathbf{s} \to \tilde{\mathbf{s}}_1)$, is smaller than the single error PEP $P(\mathbf{s} \to \tilde{\mathbf{s}}_0)$. Since the group size Q is small, the symbol error rate in a multiuser environment is close to the single user SER bound. This is confirmed by simulation results in Section V.

When the delay spreads of different users' channels are different, we choose $Q \geq L_{s,\max}$. Lemma 1 is straightforward to extend in this case. When only one symbol error of user m occurs, i.e., $e_m \neq 0$, the rank of $\mathbf{S}_e$ is $L_{s,n,m}$. When there are more than one error symbols, say $e_{m_1} \neq 0$ and $e_{m_2} \neq 0$, we have $r_e \geq \max(L_{s,n,m_1}, L_{s,n,m_2})$. Hence, whenever user m has a symbol in error, we have $r_e \geq L_{s,n,m}$. Since the diversity order a user achieves is equal to the rank of $\mathbf{S}_e$ when this user's symbol is in error, we have established the following proposition:

Proposition 1: *Using the subcarrier group assignment matrix in (5), every user can achieve the maximum diversity order provided by its multipath channel. The maximum diversity order for a user equals the number of taps of its channel impulse response.*

V. SIMULATIONS

In this section, we test GO-MC-CDMA via computer simulations. In our simulation, the delay spreads of different users' channels are all chosen equal to $L_s = 3$; the different taps of each multipath channel are independently generated, and their variances are $1/L_s$. QPSK modulation is adopted; and the bit energy is defined as $\mathcal{E}_b = E(|s|^2)/2$. GO-MC-CDMA employs ML detection, and the number of subcarriers in a block is $M = 64$. The group size is chosen to be $Q = 4$. So, the maximum number of active users in each group is $N_a = 4$. When there are $N_a > 1$ active users in a group, we assume that all users $m \in [1, \;\; N_a]$ have the same power, while the user 0 of interest may have different power. We define the near-far ratio as $\nu_f := P_m/P_0$, $m > 0$, where P_m denotes user power.

Performance comparison (MC-CDMA, AMOUR and GO-MC-CDMA): In MC-CDMA, the processing gain is equal to the number of subcarriers $M = 64$, and the MMSE detector is employed. AMOUR is simulated with fixed block length and load adaptation [2]. In this setup, the maximum number of users is 8, and each user transmits $K = 8$ symbols. Since the delay spread is $L_s = 3$, each user should use at least $J = 11$ subcarriers to guarantee symbol detectability according to [2]. Hence, the number of subcarriers is chosen to be $M = 88$. When the number of active users M_a changes, each active user is allocated $J = \lfloor M/M_a \rfloor$ subcarriers. The MMSE detector is employed because the ML detector has high complexity to jointly detect eight symbols per user. Fig. 2 shows BER versus load for these three systems, where the load is defined as the number of active users divided by the maximum number of users. For comparison, the single-user theoretical BER curve with diversity order three is also displayed. We see that the BER of GO-MC-CDMA is very close to the single user bound, while the performance of both MC-CDMA and AMOUR degrades when the load increases.

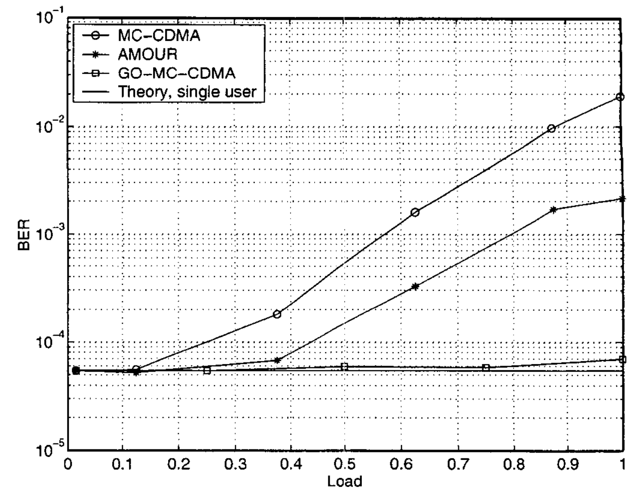

Fig. 2. BER comparison between MC-CDMA, AMOUR and GO-MC-CDMA, $\mathcal{E}_b/N_o = 16\ dB$.

GO-MC-CDMA for different near-far ratios: Fig. 3 depicts the BER of GO-MC-CDMA with $\nu_f = 0\ dB$. We see that when $N_a = 2$, the BER almost coincides with the single user bound across the $\mathcal{E}_b/N_o$ region. When $N_a = 4$, BER reaches the single user bound when $\mathcal{E}_b/N_o$ is high, which justifies our analysis in Section III. Note that $N_a = 2$ corresponds to 27% - 50% load, while $N_a = 4$ corresponds to 77% - 100% load. Fig. 4 depicts the BER of GO-MC-CDMA with $\nu_f = 3\ dB$. The horizontal axis is the $\mathcal{E}_b/N_o$ of the weakest user 0. From Fig. 4, we observe that the weak user's BER

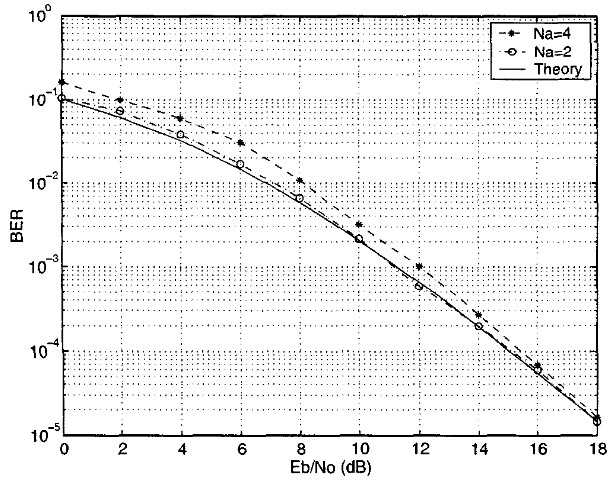

Fig. 3. BER versus $\mathcal{E}_b/N_o$, $\nu_f = 0\ dB$

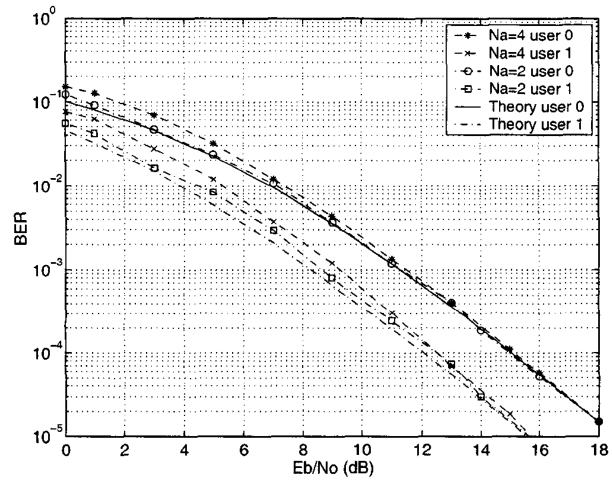

Fig. 4. BER versus $\mathcal{E}_b/N_o$, $\nu_f = 3\ dB$

performance is not degraded by the strong interference of other users; actually, it is slightly better than when all the users have the same power.

VI. CONCLUSIONS

We developed a group orthogonal MC-CDMA system with affordable receiver complexity. We showed that the performance of GO-MC-CDMA comes very close to the single user performance, even when the system is fully loaded. The superior performance of GO-MC-CDMA is achieved by transmitting signals of a small group of users on a set of judiciously selected subcarriers, which allows usage of ML multiuser detection. In short, GO-MC-CDMA is a practically feasible system with a number of attractive features.

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