Exact BER Analysis of Bandlimited BPSK With EGC and SC Diversity in Cochannel Interference and Nakagami Fading

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***Abstract*—Equal gain and selection combining for band-limited binary phase-shift keying systems in Nakagami fading with cochannel interference are considered. Spectrum raised-cosine and Beaulieu–Tan–Damen pulse shapes are employed. Average bit error rates are derived for arbitrary orders of diversity and fading parameters. The computational complexity of the solution does not grow with the diversity order. Slow flat fading, asynchronous timing and independent fading gains are assumed.**

***Index Terms*—Bit error rate (BER), cochannel interference (CCI), equal gain combining (EGC), Nakagami fading channels, selection combining (SC).**

I. INTRODUCTION

PERFORMANCE analysis of wireless communication systems is normally based on average bit error probability (BER) and/or outage probability [1]. Normally the BER analysis is much more involved than outage probability analysis. The BER performance of equal gain combining (EGC) and selection combining (SC) in fading channels without cochannel interference (CCI) was well studied in [2]–[5], and the references therein. In [2], a series method was proposed for the computation of the probability density function of the sum of independent random variables to analyze EGC performance in Nakagami fading. A numerical method based on Parseval's theorem was proposed in [3] for EGC in wireless channels. Two types of SC schemes were studied in [4] for CPSK and NCFSK in Rayleigh fading channels. A unified integral solution treatment of fading channels and diversity schemes was presented in [5]. In [6], the BER performance of dual branch EGC and SC in band-limited binary phase-shift keying (BPSK), quaternary PSK (QPSK), and 8PSK systems with CCI in Nakagami–Rayleigh fading was analyzed using an approximate Fourier series method. The BER performance of bandlimited BPSK systems with CCI in Nakagami fading was investigated using a characteristic function (CF) method in [7]. However, diversity schemes were not considered.

In this letter, we derive the exact BER of coherent bandlimited BPSK in asynchronous CCI and Nakagami flat fading with EGC using a CF method and with SC using a Fourier series method for arbitrary diversity orders and Nakagami fading parameters. Spectrum raised-cosine and Beaulieu-Tan-Damen [8] pulse shapes are employed. Slow fading and independent fading gains are assumed. The BER analysis of [7] is extended to EGC with CCI using the method based on Parseval's theorem presented in [3]. The Fourier series method of [6] applied for a Nakagami–Rayleigh fading model with dual branch SC is extended to independent identically distributed (i.i.d.) Nakagami fading with arbitrary diversity order.

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II. SYSTEM MODEL

Consider a coherent band-limited BPSK system with cochannel interference in a slowly fading environment. We adopt the system model of [7] for the diversity receiver structure. There are L space diversity branches and $K+1$ active users in the system. The j-th branch received signal of the desired (zeroth) user is

$$R_j(t) = \sqrt{2P_0T}\, R_{0,j}\, s_d(t)\, \cos(\omega_c t) + n(t) + \sum_{i=1}^{K} \sqrt{2P_iT} \times R_{i,j}\, s_i(t-\tau_i)\, \cos(\omega_c(t-\tau_i) + \theta_{i,j}) \quad (1)$$

where P_i is the transmitted power of the ith user, ω_c is the carrier frequency, $s_d(t) = \sum_{k=-\infty}^{+\infty} a[k] g_T(t-kT)$, $s_i(t) = \sum_{k=-\infty}^{+\infty} b_i[k] g_T(t-kT)$, $1/T$ is the symbol transmission rate and $g_T(\cdot)$ is the transmitter signal baseband pulse with its energy normalized according to $\int_{-\infty}^{+\infty} g_T^2(t)\mathrm{d}t = 1$, $a[k] \in \{+1, -1\}$ with equal probabilities and τ_i represents the symbol timing offset of the ith user signal with respect to the desired user signal, assumed to be uniform over $[0, T)$. The background noise $n(t)$ is a zero-mean white Gaussian process with two-sided power spectral density $N_0/2$; the phases $\theta_{i,j}$ are assumed to be mutually independent and uniformly distributed over $[0, 2\pi)$. The random variables $R_{i,j}$ represent the fading channel gains and each follows the Nakagami-m distribution with parameters (m, Ω)[1]. The fading gains are assumed to be independent and identically distributed. The desired user average signal-to-noise ratio (SNR) and signal-to-interference ratio are defined as $\mathrm{SNR(dB)} = 10\log_{10}(P_0T\Omega/2)$, and $\mathrm{SIR(dB)} = 10\log_{10}\left(P_0/(\sum_{i=1}^{K} P_i)\right)$, where without loss of generality, we assume the noise variance is one. As in [7], two types of bandlimited Nyquist pulse shapes are considered, the spectrum raised-cosine (SRC) and the Beaulieu-Tan-Damen (BTD) [8] pulse shapes. We assume 100% excess bandwidth pulse shapes in this letter.

III. Equal Gain Combining (EGC)

After demodulation, matched filtering and cophasing, the decision statistic for the desired user data symbol $a[0]$, on the jth branch, is given by

$$Z_j[0] = \sqrt{\frac{P_0T}{2}} R_{0,j} a[0] + \sum_{i=1}^{K} \sqrt{\frac{P_iT}{2}} R_{i,j} \cos(\phi_{i,j}) \rho_i + n_j \tag{2}$$

where $\phi_{i,j} = (\theta_{ij} - \omega_c \tau_i) \sim \mathbf{U}[0, 2\pi]$, n_j is zero-mean Gaussian noise with variance one

$$\rho_i = \sum_{k=-\infty}^{+\infty} b_i[k] g(-kT - \tau_i) \tag{3}$$

and $g(\cdot)$ is the pulse shape at the receiver. After the EGC, the decision statistic is given by

$$Z[0] = \sqrt{\frac{P_0T}{2}} a[0] \sum_{j=1}^{L} R_{0,j} + \sum_{j=1}^{L} n_j + \sum_{j=1}^{L} \sum_{i=1}^{K} \sqrt{\frac{P_iT}{2}} R_{i,j} \cos(\phi_{i,j}) \rho_i. \tag{4}$$

The third term of (4) is the cochannel interference component usefully rewritten as $I_C = \sum_{j=1}^{L} \sum_{i=1}^{K} I_{i,j}$, where

$$I_{i,j} = \sqrt{\frac{P_iT}{2}} R_{i,j} \cos(\phi_{i,j}) \rho_i. \tag{5}$$

Now, as shown in [6] and [7], the conditional characteristic function (CF) of the $I_{i,j}$ conditioned on τ_i and $X_{i,j}$, ($X_{i,j} = R_{i,j} \cos \phi_{i,j}$) is written as

$$\Phi_{I_{i,j}|X_{i,j},\tau_i}(\omega) = \prod_{k=-M}^{M} \cos\left(\sqrt{\frac{P_iT}{2}} X_{i,j} \omega \, g(-kT - \tau_i) \right) \tag{6}$$

where the cross intersymbol interference contribution is assumed to be from $2M + 1$ symbols only. The probability density function (pdf) of the inphase component of the Nakagami random variable is given by [7], [10]

$$f_{X_{i,j}}(x) = \frac{\Gamma\left(m - \frac{1}{2}\right)}{\pi \Gamma(m)} \sqrt{\frac{m}{\Omega}} {}_1\mathbf{F}_1\left(\frac{1}{2}; \frac{3}{2} - m; \frac{-mx^2}{\Omega}\right) + \frac{\Gamma\left(\frac{1}{2} - m\right)}{\pi\sqrt{\pi}} \left(\frac{m}{\Omega}\right)^m \left(x^2\right)^{m-1/2} \sin(m\pi) \times {}_1\mathbf{F}_1\left(m; m + \frac{1}{2}; \frac{-mx^2}{\Omega}\right) \tag{7}$$

where the second term is zero when the Nakagami fading parameter m, takes on integer values and ${}_1\mathbf{F}_1(\cdot)$ and $\Gamma(\cdot)$ are the confluent hypergeometric function and gamma function, respectively [10]. Now, forming the double product over i and j from (6) and then averaging out τ_i and $X_{i,j}$

$$\Phi_{I_C}(\omega) = \prod_{i=1}^{K} \frac{1}{T} \int_0^T \left[\prod_{j=1}^{L} 2 \int_0^{\infty} f_{X_{i,j}}(x) \prod_{k=-M}^{M} \cos\left(\sqrt{\frac{P_iT}{2}} x \, \omega \, g(-kT - u) \right) \mathrm{d}x \right] \mathrm{d}u. \tag{8}$$

As the n_j's are independent zero-mean Gaussian random variables with unit variance, the background noise term $n = \sum_{j=1}^{L} n_j$, in (4) is zero-mean Gaussian distributed with variance L. The CF of n is $\Phi_n(\omega) = e^{-\omega^2 L/2}$. The CF of the total CCI plus background noise is given by

$$\Phi_T(\omega) = \Phi_{I_C}(\omega) \Phi_n(\omega). \tag{9}$$

The average BER, conditioned on R_E, is written as [7]

$$P_{e1|R_E} = \Pr\left[\sqrt{\frac{P_0T}{2}} R_E + I_C + n < 0 \middle| a[0] = +1 \right] = \frac{1}{2} - \frac{1}{\pi} \int_0^{+\infty} \frac{\sin\left(\sqrt{\frac{P_0T}{2}} R_E u \right)}{u} \Phi_T(u) \mathrm{d}u \tag{10}$$

where $R_E = \sum_{j=1}^{L} R_{0,j}$. The unconditional BER can be obtained using several methods. One approach is to average out the $R_{0,j}$s one by one or similarly, using $L - 1$-fold convolution find the pdf of R_E and then average it out. This method is computationally very complex when L is large. Another approach is to find the pdf of R_E using the series method proposed in [2] and average it out. A third approach uses the Parseval's theorem based method proposed in [3]. Equation (10) has the desired format to apply the latter method. Now, the unconditional BER is given by

$$P_{e1} = \frac{1}{2} - \frac{1}{\pi} \int_0^{\infty} \int_0^{\infty} \frac{\sin\left(\sqrt{\frac{P_0T}{2}} r \, u \right) \Phi_T(u) f_{R_E}(r)}{u} \mathrm{d}u \mathrm{d}r. \tag{11}$$

Applying the method proposed in [3], (11) becomes (12), shown at the bottom of the next page, where $\mathbb{R}\mathrm{e}[x]$ and $\mathbb{I}\mathrm{m}[x]$ is the real and imaginary part of x, respectively, $\Phi^*_{R_E}(\omega)$ is the complex conjugate of the CF of R_E ($\Phi_{R_E}(\omega)$), $\mathrm{j} = \sqrt{-1}$ and $\delta(\cdot)$ is the Dirac delta function. Equation (12) with (9) is an exact solution for the general case including pulse shaping. A triple numerical integration is required, but the computational complexity does not grow with L. The CF of $R_{0,j}$ is written as [2], [9]

$$\Phi_{R_{0,j}}(\omega) = {}_1\mathbf{F}_1\left(m; \frac{1}{2}; \frac{-\Omega\omega^2}{4m}\right) + \mathrm{j}\omega \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)} \times \sqrt{\frac{\Omega}{m}} {}_1\mathbf{F}_1\left(m + \frac{1}{2}; \frac{3}{2}; \frac{-\Omega\omega^2}{4m}\right). \tag{13}$$

Then, the CF of R_E is $\Phi_{R_E}(\omega) = (\Phi_{R_{0,j}}(\omega))^L$.

IV. Selection Combining (SC)

In this section, the performance of L branch predetection SC in Nakagami fading is considered. The decision statistic for the desired user data symbol $a[0]$ after coherent demodulation and matched filtering in all branches, is written as

$$Z[0] = \sqrt{\frac{P_0T}{2}} a[0] R_S + \sum_{i=1}^{K} \sqrt{\frac{P_iT}{2}} R_{i,j} \cos(\phi_{i,j}) \rho_i + n_j \tag{14}$$

where $R_S = \max_{j \in [1 \cdots L]} R_{0,j}$. The CF of the total CCI $\Phi^s_{I_C}(\omega)$, is then obtained by substituting $L = 1$ in (8). As in

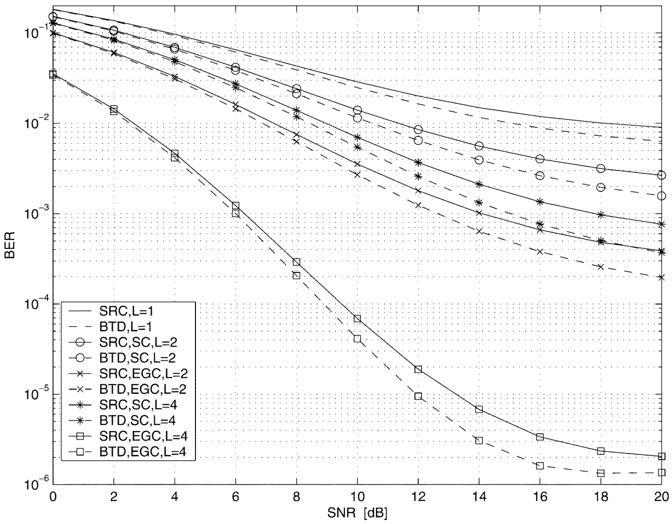

Fig. 1. Performance of the band-limited BPSK system with EGC and SC in Nakagami fading ($m = 5$), with CCI, $K = 6$, and $\mathrm{SIR} = 5$ dB.

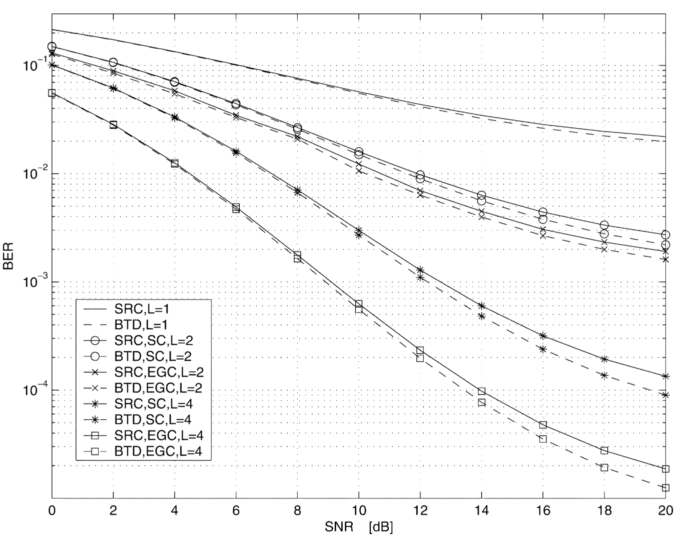

Fig. 2. Performance of the band-limited BPSK system with EGC and SC in Rayleigh fading ($m = 1$), with CCI, $K = 6$, and $\mathrm{SIR} = 10$ dB.

[6], using a Fourier series method [11], the unconditional BER is given by

$$P_{e2} = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{l=1 \\ l \text{ odd}}}^{\infty} \frac{e^{-l^2\omega_0^2/2}\Phi_{I_C}^{s}(l\omega_0)}{l} \times \int_0^{\infty} \sin\left(\sqrt{\frac{P_0 T}{2}} r\, l\, \omega_0\right) f_{R_S}(r)\mathrm{d}r \qquad (15)$$

where $f_{R_S}(r)$ is the pdf of R_S, $\omega_0 = 2\pi/T_0$ and T_0 is selected to obtain the required accuracy. Now, the pdf of R_S can be obtained using order statistics [1]. For L independent branch SC in Nakagami fading, the pdf of R_S is [10]

$$f_{R_S}(r) = \frac{2L\left[\gamma\left(m, \frac{mr^2}{\Omega}\right)\right]^{L-1}}{[\Gamma(m)]^L} \left(\frac{m}{\Omega}\right)^m e^{-mr^2/\Omega} r^{2m-1} \qquad (16)$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function [10].

V. Numerical Results

Numerical results are computed using MATHEMATICA and MATLAB softwares. We assume there are 6 interfering users ($K = 6$) with equal powers and $M = 10$. Figs. 1 and 2 depict the performances of bandlimited BPSK with CCI in Nakagami fading ($m = 5$) and Rayleigh fading ($m = 1$), respectively, for EGC and SC with diversity orders $L = 1$,2 and 4 with SRC and BTD pulse shapes.

VI. Conclusions

The accurate performances of coherent bandlimited BPSK systems with EGC and SC in CCI and fading have been investigated. Exact integral results were obtained for arbitrary diversity orders and Nakagami fading parameters. The superiority of the BTD pulse over the SRC pulse is once again observed and the superiority increases with the diversity order.

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$$\begin{aligned} P_{e1} &= \frac{1}{2} - \int_0^{\infty} \frac{\Phi_T(u)}{\pi^2 u} \int_0^{\infty} \mathbb{R}\mathrm{e}\left[\left[\mathrm{j}\pi\delta\left(\omega + \sqrt{\frac{P_0 T}{2}} u\right) - \mathrm{j}\pi\delta\left(\omega - \sqrt{\frac{P_0 T}{2}} u\right)\right] \Phi_{R_E}^{*}(\omega)\right] \mathrm{d}\omega \mathrm{d}u \\ &= \frac{1}{2} - \frac{1}{\pi} \int_0^{\infty} \frac{\Phi_T(u)}{u} \mathbb{I}\mathrm{m}\left[\Phi_{R_E}\left(\sqrt{\frac{P_0 T}{2}} u\right)\right] \mathrm{d}u \end{aligned} \qquad (12)$$