A PDA-Kalman Approach to Multiuser Detection in Asynchronous CDMA

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***Abstract*—The Probabilistic Data Association (PDA) method for multiuser detection in synchronous code-division multiple-access (CDMA) communication channels is extended to asynchronous CDMA, where a Kalman filter or smoother is employed to track the correlated noise arising from the outputs of a decorrelator. The estimates from the tracker, coupled with an iterative PDA, result in impressively low bit error rates. Computer simulations show that the new scheme significantly outperforms the best decision feedback detector. The algorithm has $O(K^3)$ complexity per time frame, where K is the number of users.**

***Index Terms*—Code-division multiple access, filtering, multiuser detection, probabilistic data association.**

I. Introduction

THE exponential computational complexity of the optimal maximum-likelihood (ML) [1] multiuser detector for asynchronous code-division multiple-access (CDMA) communication channels has motivated research into suboptimal algorithms that yield low bit error rates and that ensure polynomial computational costs. Examples of these include the decorrelator [1] and the decision feedback (DF) detector [2], [4]. While these detectors provide bit error rates that are lower than the conventional matched-filter detector, there is still a large performance gap between these detectors and the optimal ML detector.

In this letter, a new multiuser detection scheme for asynchronous CDMA channels is introduced. The detection scheme uses the Probabilistic Data Association (PDA) method for multiuser detection in synchronous CDMA [3] in conjunction with a Kalman technique for tracking the correlated noise arising from the outputs of a decorrelator. Simulation results show that the new detection scheme significantly outperforms the DF detector. We also remark that since the PDA and Kalman algorithms are both $O(K^3)$ in complexity [3], [5], the new algorithm, which is a serial combination of the two (see Fig. 1), also possesses $O(K^3)$ complexity per time frame, where K is the number of users. (In the new asynchronous algorithm, PDA typically converges in three iterations or less; this is similar to synchronous PDA in [3].)

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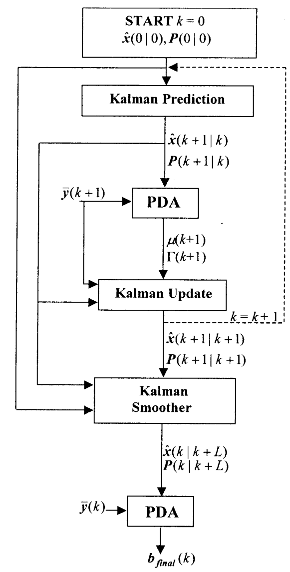

Fig. 1. Block diagram of PDA-Kalman detector. ($L=0$: filter only, $L=1$: one-step smoothing).

II. PDA-Kalman Multiuser Detector for Asynchronous CDMA

A. The Model

The discrete-time equivalent model for the matched-filter outputs at the receiver of a K-user asynchronous CDMA channel is described in the z domain [1] by the K-length vector

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{W}\boldsymbol{b}(z) + \boldsymbol{n}(z) \tag{1}$$

where $\boldsymbol{W}$ is a diagonal matrix whose ith diagonal element, w_i, is the square root of the received signal energy per bit of the ith user; $\boldsymbol{b}(z)$ is the z transform of the bit sequence transmitted by the K active users; $\boldsymbol{n}(z)$ is colored Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{R}(z)$; and $\boldsymbol{R}(z)$ is the signature correlation matrix, which is given by [1]

$$\boldsymbol{R}(z) = \boldsymbol{R}^T(z^{-1}) = \boldsymbol{R}_1^T z + \boldsymbol{R}_0 + \boldsymbol{R}_1 z^{-1}. \tag{2}$$

In (2), $\boldsymbol{R}_0$ is a symmetric matrix with unity diagonal elements and whose off-diagonal elements represent the correlations between user signatures at the same time index; and $\boldsymbol{R}_1$ is a singular matrix whose elements represent the signature correlations relating to successive time frames. The (i,j)th element of $\boldsymbol{R}_1$ is denoted by $R_{1,ij}$. Since user signal i in time frame k cannot simultaneously be correlated with that of j in time frame $k-1$ and in time frame $k+1$, we have $R_{1,ij}R_{1,ji}=0$.

The decorrelated model for asynchronous CDMA in the z domain is obtained by multiplying both sides of (1) from the left by $\boldsymbol{R}^{-1}(z)$

$$\bar{\boldsymbol{y}}(z) = \boldsymbol{W}\boldsymbol{b}(z) + \boldsymbol{x}(z) \tag{3}$$

where $\bar{\boldsymbol{y}}(z) = \boldsymbol{R}^{-1}(z)\boldsymbol{y}(z)$ and $\boldsymbol{x}(z) = \boldsymbol{R}^{-1}(z)\boldsymbol{n}(z)$ is colored Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{R}^{-1}(z) = \sigma^2(\boldsymbol{R}(z^{-1}))^{-T}$. The corresponding time-domain representation of (3) is

$$\bar{\boldsymbol{y}}(k) = \boldsymbol{W}\boldsymbol{b}(k) + \boldsymbol{x}(k) \tag{4}$$

where $\boldsymbol{b}(k) \in \{-1, +1\}^K$. By rearranging (4) as

$$\boldsymbol{W}^{-1}\bar{\boldsymbol{y}}(k) = b_i(k)\boldsymbol{e}_i + \sum_{j\neq i} b_j(k)\boldsymbol{e}_j + \boldsymbol{W}^{-1}\boldsymbol{x}(k) \tag{5}$$

where $b_j(k)$ is the jth element of $\boldsymbol{b}(k)$ and $\boldsymbol{e}_j$ is a column vector whose jth element is 1 and whose other elements are zero, we observe that the "effective noise" interfering with user i is

$$\boldsymbol{N}_i(k) = \sum_{j\neq i} b_j(k)\boldsymbol{e}_j + \boldsymbol{W}^{-1}\boldsymbol{x}(k). \tag{6}$$

It is shown in [2] that the correlation matrix $\boldsymbol{R}(z)$ can be factored as

$$\boldsymbol{R}(z) = (\boldsymbol{F}_0 + \boldsymbol{F}_1 z)^T(\boldsymbol{F}_0 + \boldsymbol{F}_1 z^{-1}) \tag{7}$$

where $\boldsymbol{F}_0$ is a lower triangular matrix and $\boldsymbol{F}_1$ is a singular matrix [2]. Since the covariance of $\boldsymbol{x}(z)$ is $\sigma^2\boldsymbol{R}^{-1}(z)$, then from (7), $\boldsymbol{x}(z)$ can be transformed to white noise $\boldsymbol{v}(z)$ via

$$\begin{aligned}(\boldsymbol{F}_0 + \boldsymbol{F}_1 z^{-1})\boldsymbol{x}(z) =& \boldsymbol{v}(z)\\ \boldsymbol{x}(z) =& -\boldsymbol{F}_0^{-1}\boldsymbol{F}_1 z^{-1}\boldsymbol{x}(z) + \boldsymbol{F}_0^{-1}\boldsymbol{v}(z).\end{aligned} \tag{8}$$

The corresponding time-domain representation of (8) is

$$\boldsymbol{x}(k+1) = -\boldsymbol{F}_0^{-1}\boldsymbol{F}_1\boldsymbol{x}(k) + \boldsymbol{F}_0^{-1}\boldsymbol{v}(k+1) \tag{9}$$

where $\boldsymbol{v}(k) \sim N(\boldsymbol{0}, \sigma^2\boldsymbol{I})$.

B. The PDA Detector

The key behind the new detection scheme lies in the dynamics of the correlated noise $\boldsymbol{x}(k)$. Since it is evident from (9) that $\boldsymbol{x}(k)$ evolves according to a linear Gauss–Markov process, a Kalman approach can be used to track $\boldsymbol{x}(k)$. Using the conditional mean and covariance of $\boldsymbol{x}(k)$ from the tracker, the PDA method of [3] can be applied to obtain $\boldsymbol{b}(k)$ with only a few modifications (also see Fig. 1): approximating (6) as the effective Gaussian noise, the statistics of $\boldsymbol{N}_i(k)$ are given by

$$\begin{aligned}E[\boldsymbol{N}_i(k)] =& \sum_{j\neq i}\boldsymbol{e}_j(2P_{bj}(k) - 1) + \boldsymbol{W}^{-1}\hat{\boldsymbol{x}}(k|k+L)\\ \mathrm{cov}[\boldsymbol{N}_i(k)] =& \sum_{j\neq i}[4P_{bj}(k)(1 - P_{bj}(k))\boldsymbol{e}_j\boldsymbol{e}_j^T]\\ &+ \boldsymbol{W}^{-1}\boldsymbol{P}(k|k+L)\boldsymbol{W}^{-1}\end{aligned} \tag{10}$$

where $\hat{\boldsymbol{x}}(k|k+L)$ and $\boldsymbol{P}(k|k+L)$ denote the conditional mean and covariance of $\boldsymbol{x}(k)$ given the observations up to and including time index $k+L$ (if L is a positive integer, a Kalman-smoother is employed; if $L = 0$, only a Kalman filter is used) and $P_{bj}(k)$ denotes the probability that the jth element of $\boldsymbol{b}(k)$ equals $+1$. The likelihood ratio of the belief that $b_i(k) = +1$ is

$$\frac{P_{bi}(k)}{1 - P_{bi}(k)} = \exp(-2\theta_i^T(k)\Omega_i^{-1}(k)\boldsymbol{e}_i) \tag{11}$$

where the Gaussian (by assumption) random variable $\theta_i(k)$ and its covariance $\Omega_i(k)$ are given by

$$\begin{aligned}\theta_i(k) =& E[\boldsymbol{N}_i(k)] - \boldsymbol{W}^{-1}\bar{\boldsymbol{y}}(k)\\ \Omega_i(k) =& \mathrm{cov}[\boldsymbol{N}_i(k)].\end{aligned} \tag{12}$$

Using the modifications of (10)–(12), the procedure for updating $P_{bi}(k)$ remains unchanged from [3].

C. Kalman Filter Implementation

By treating (9) as the state equation and (4) as the measurement equation, a Kalman filter is used in conjunction with the PDA method of [3] to compute $\hat{\boldsymbol{x}}(k|k)$ and $\boldsymbol{P}(k|k)$. The Kalman update is given by

$$\begin{aligned}\hat{\boldsymbol{x}}(k|k) =& \hat{\boldsymbol{x}}(k|k-1) + \boldsymbol{M}(k)[\bar{\boldsymbol{y}}(k) - \boldsymbol{W}\mu(k) - \hat{\boldsymbol{x}}(k|k-1)]\\ \boldsymbol{P}(k|k) =& (\boldsymbol{I} - \boldsymbol{M}(k))\boldsymbol{P}(k|k-1)\end{aligned} \tag{13}$$

where $\boldsymbol{M}(k) = \boldsymbol{P}(k|k-1)[\boldsymbol{P}(k|k-1) + \boldsymbol{W}\Gamma(k)\boldsymbol{W}]^{-1}$; and $\mu(k)$ and $\Gamma(k)$ denote the expectation and conditional covariance of $\boldsymbol{b}(k)$ obtained by the PDA method of [3] using $\hat{\boldsymbol{x}}(k|k-1)$ and $\boldsymbol{P}(k|k-1)$ in place of $\hat{\boldsymbol{x}}(k|k+L)$ and $\boldsymbol{P}(k|k+L)$ in (10). We have $\mu(k)$ and $\Gamma(k)$ computed via

$$\begin{aligned}\mu(k) =& \sum_i (2P_{bi}(k) - 1)\boldsymbol{e}_i\\ \Gamma(k) =& \sum_i 4P_{bi}(k)(1 - P_{bi}(k))\boldsymbol{e}_i\boldsymbol{e}_i^T.\end{aligned} \tag{14}$$

In (13), the predicted state estimate $\hat{\boldsymbol{x}}(k|k-1)$ and the predicted state covariance $\boldsymbol{P}(k|k-1)$ are given by

$$\begin{aligned}\hat{\boldsymbol{x}}(k|k-1) =& -\boldsymbol{F}_0^{-1}\boldsymbol{F}_1\hat{\boldsymbol{x}}(k-1|k-1)\\ \boldsymbol{P}(k|k-1) =& \boldsymbol{F}_0^{-1}[\boldsymbol{F}_1\boldsymbol{P}(k-1|k-1)\boldsymbol{F}_1^T + \sigma^2\boldsymbol{I}]\boldsymbol{F}_0^{-T}.\end{aligned} \tag{15}$$

After (13) has been computed, the PDA method of [3] (with the modifications) is applied to obtain the final decision on $\boldsymbol{b}(k)$. (Note that in obtaining $\boldsymbol{b}(k)$, the user probabilities at time index k are reinitialized to 0.5 before PDA is applied.)

D. Kalman Smoother Implementation

To improve the performance of the detector, smoothing can be performed to obtain more information on $\boldsymbol{x}(k)$ (i.e., $\hat{\boldsymbol{x}}(k|k+L)$ and $\boldsymbol{P}(k|k+L)$ where L is a positive integer). It was observed from simulations that smoothing beyond one-step resulted in no significant improvement when compared to the increased computational cost needed for the task. For this reason, we set $L = 1$ in (10). Then $\hat{\boldsymbol{x}}(k|k+1)$ and $\boldsymbol{P}(k|k+1)$ are given by [5]

$$\begin{aligned}\hat{\boldsymbol{x}}(k|k+1) =& \hat{\boldsymbol{x}}(k|k) + \boldsymbol{C}(k)[\hat{\boldsymbol{x}}(k+1|k+1) - \hat{\boldsymbol{x}}(k+1|k)]\\ \boldsymbol{P}(k|k+1) =& \boldsymbol{P}(k|k) + \boldsymbol{C}(k)[\boldsymbol{P}(k+1|k+1) - \boldsymbol{P}(k+1|k)]\boldsymbol{C}^T(k)\end{aligned} \tag{16}$$

where $\boldsymbol{C}(k) = -\boldsymbol{P}(k|k)\boldsymbol{F}_1^T\boldsymbol{F}_0^{-T}\boldsymbol{P}^{-1}(k+1|k)$. The PDA-Kalman detector is illustrated in Fig. 1.

E. Initialization

The PDA-Kalman detector requires the initial state estimate and covariance to start; this information is obtained by recognizing that $\boldsymbol{x}(0)$ is

$$\boldsymbol{x}(0) = \boldsymbol{F}_0^{-1}\boldsymbol{v}(0). \tag{17}$$

Equation (17) is based on the assumption that because the first information packet arrives at $k = 0$, the output of the decorrelator is zero prior to that time. Hence, $\boldsymbol{x}(-1)$ is zero. From (17), the predicted state estimate and state covariance at $k = 0$ is

$$\begin{aligned}\hat{\boldsymbol{x}}(0|-1) &= 0\\ \boldsymbol{P}(0|-1) &= \sigma^2\boldsymbol{F}_0^{-1}\boldsymbol{F}_0^{-T}.\end{aligned} \tag{18}$$

By substituting (18) into (13), $\hat{\boldsymbol{x}}(0|0)$ and $\boldsymbol{P}(0|0)$ is obtained.

III. Simulation Results

In this section, we compare the performance of the Decorrelator, the DF detector, and the PDA-Kalman detector. The optimal user ordering and time labeling rule proposed in [6] is applied to both the DF and the PDA-Kalman detectors. A performance lower bound is also provided by the ideal optimal detector that assumes no error propagation. The simulation was performed on an overloaded system of 25 users and 15-length randomly generated codes. The time delays of the user signals are random and uniformly distributed within a symbol duration and we use the system model introduced in [7] to generate the signature correlation matrix. The square roots of user signal powers are generated randomly $w_i \sim N(4.5, 4)$ and are limited to the range of [3], [6] to avoid domination of results by deep fade errors.

Fig. 2 shows the performance comparison of different detectors. The performance of the new detector is significantly better than that of the DF detector and reasonably close to the performance lower bound. (Note that the lower bound provided by the ideal optimal detector is not necessarily reachable.)

The effect of smoothing was examined by eliminating the smoothing stage from the detector and using $\hat{\boldsymbol{x}}(k|k)$ and $\boldsymbol{P}(k|k)$ to obtain the final decision on $\boldsymbol{b}(k)$. At every SNR value, the PDA-Kalman filter-based detector had a higher error probability than in the case when smoothing was employed.

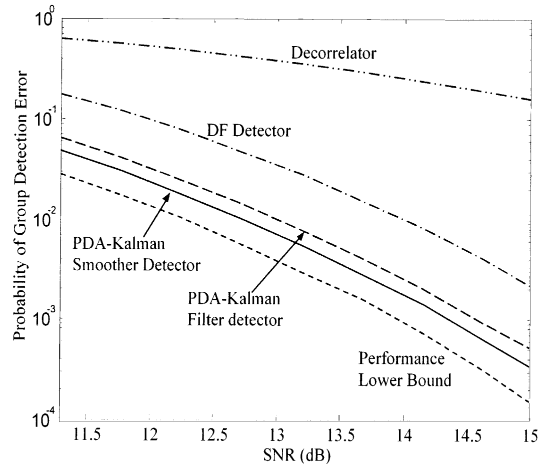

Fig. 2. Performance comparison, 25-users, 15-length randomly generated codes, 10^6 Monte Carlo runs.

IV. Conclusions

Transformation to a decorrelated signal model for asynchronous CDMA results in a vector noise process that evolves according to a linear Gaussian system. This would suggest the use of a Kalman technique to estimate it; however, unfortunately, this noise is added to a very non-Gaussian process arising from the users' bits. It turns out that the iterative "Gaussianization" of these other users by the PDA detector of [3] makes it a natural fit with a Kalman filter or smoother; the Kalman estimator, in turn, benefits PDA by reducing the effective noise. The smoothing, particularly, amounts to a one-step soft look-ahead to time $k+1$ by the PDA, with an accompanying Kalman smoother to mitigate Gaussian noise at time k. (Deeper look-aheads are possible, but have shown little improvement compared to their added complexity.) Simulation results show that the performance of the PDA-Kalman Smoother detector, in terms of the probability of group detection error, is significantly better than those of the decorrelator and the DF detectors. We also remark that after (13) has been computed, any of the algorithms discussed in [8] for synchronous CDMA can be used to obtain $\boldsymbol{b}(k)$.

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