Trellis - Coded Unitary Space - Time Modulation

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Abstract

Space-time coding is well established for high data rate communications over wireless channels with perfect channel state information. On the other hand, the case where the channel state information is unknown has received limited attention. Recently, a new signaling scheme called unitary space-time modulation that is suitable for the latter case has been proposed. In this paper, we describe the use and design of trellis-coded space-time modulation schemes that use unitary space-time constellations. We construct these codes using a novel suboptimal code design criteria and study the performance of trellis-coded unitary space-time modulation for block fading channels under the assumption of no channel state information. Simulation results show that the proposed schemes improve the performance compared to uncoded transmission with the same spectral efficiency. The results are also compared with the turbo-coded modulation scheme devised in [1] and the differential detection scheme described in [2] under the same assumptions.

I. Introduction

The future wireless communication systems are to be driven by high date rate applications such as video and multimedia applications. Services such as video on demand and video-conferencing over wireless links require a few orders of magnitude higher bandwidth compared to that provided by current wireless standards including CDMA 2000, GSM, IS-136 etc. The demand for systems with such capabilities initiated an intensive research for spectrally efficient high data rate transmission. Among various strategies developed, one such method to meet this goal is to deploy multiple antennas at the transmitter and the receiver. The information theoretic considerations demonstrate that the capacity of a channel with multiple antennas increases linearly with the smaller of the number of the transmit and receive antennas, provided that exact channel state information is available to the receiver [3], [4]. In order to exploit this capacity, space-time trellis codes, space-time block codes and turbo-coded modulation systems with antenna diversity have been proposed and shown to be suitable for bandwidth efficient transmission providing many bits per channel use, see for example, [5–13]. We emphasize that the operation of these schemes depends on the availability of channel state information at the receiver which can be estimated from the received signal provided fading is sufficiently slow.

Estimating the fading coefficients between each pair of transmit and receive antenna elements becomes difficult, if not impossible, if the fading is fast or a large number of antenna elements are used.

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Consider, for instance, a mobile with a speed of 90 mi/h operating at a frequency 1.9 GHz and a symbol rate of 30 kHz. The coherence time for the channel is about 2 ms and corresponds to 60 symbols. If multiple antennas are used, the channel coefficients between each pair of transmit and receive antennas must be estimated. If 5 training symbols were used for each sub-channel, a system with 10 transmit antennas would require 50 training symbols, constituting a significant overhead. The problem is over-burdened if number of antennas is increased in an effort to lower probability of error or increase the transmission rate. Furthermore, for higher speeds, the fading would be so fast that it would prohibit accurate estimation of the channel. For such cases, it is necessary to develop modulation techniques that do not need channel estimation at the receiver.

Assuming that no channel state information is available at the transmitter or at the receiver, Marzetta and Hochwald present the information theoretic limits of multi antenna systems for Rayleigh block fading channels in [14]. They prove that there is no need to make the number of transmit antennas, M, larger than the length of the coherence time of the channel, T. In [15], it is shown that, when when $T \gg M$ or signal-to noise ratio is large with $T > M$, the capacity achieving space-time signals (in conjunction with coding) are $T \times M$ matrices with orthogonal columns. The resulting constellations are called unitary space-time constellations. Such constellations have been designed and shown to perform well for uncoded (unitary space–time) transmission [16].

Other recently proposed methods that do not require channel state information are the differential space-time modulation schemes [2], [17], [18], [19]. The schemes in [17], [18] can be considered as extensions of the standard differential phase-shift keying where the transmitted signals are space-time symbols in the form of complex matrices. In [17], the design of such signals are based on unitary space-time modulation. In [18], a similar differential encoding/decoding scheme with a more general approach is described in which the signals are generated by differentially encoding the group codes. In [18], the unitary space-time modulation is also considered with reference to [15].

We know that the use of channel codes is necessary to achieve channel capacity. With this motivation, we proposed turbo-coded modulation with unitary space-time constellations and showed that the use of turbo codes improves the system performance by about $10 - 15$ dB at a bit error rate (BER) of 10^{-5} compared to uncoded transmission with same spectral efficiency [1]. Other related work on the design of iteratively decodable (turbo based) codes for multiple antennas with no channel state information is reported in [20]. Although the turbo based scheme is well tailored to block fading channels with no channel state information for delay tolerant applications, it is not suitable for delay sensitive applications, such as speech communications. Therefore, we need to consider coding schemes with lower delay requirements as well. As a step in this direction, in this paper, we design trellis-coded modulation schemes for wireless systems with multiple transmit and receive antennas using unitary space-time constellations. Even though one could employ a general signal constellation, we observe that the design is analytically tractable and decoding is much simpler if we employ unitary space-time constellations. Specifically, with signals drawn from a unitary space-time constellation, the decoding (of the trellis code) can be performed by an application of the Viterbi algorithm. Simulations show that $5-11$ dB coding gains at bit error rate of 10^{-4} can be obtained compared to uncoded transmission with the same spectral efficiency. Although inferior to the turbo-coded modulation systems in terms of the bit error rate at a given signal to noise ratio, we emphasize that these schemes are applicable for delay sensitive applications where turbo-coded unitary space–time modulation is not appropriate. We also note that decoding is much simpler.

The paper is organized as follows: In Section II, we describe the signal model for the multiple antenna link. In Section III, we first summarize necessary background for the unitary space-time modulation and then we describe the trellis-coded unitary space-time modulation scheme. Section IV describes several code design techniques. We present several numerical examples in Section V and finally summarize our conclusions in Section VI.

II. Multiple Antenna Channel Model

Consider a link with M transmit and N receive antennas. We assume that the channel is a Rayleigh block fading channel [21], that is, the propagation coefficient between each antenna pair is a zero mean complex Gaussian random variable, and they remain constant for T symbols, i.e., for the coherence time of the channel. In general this block fading channel assumption can be considered as a first order approximation to the time varying fading channels. In addition, it provides us with a useful and accurate model for the channel encountered in GSM (with frequency hopping) where the differently faded blocks are obtained by the use of different frequency bands, and the channel encountered in IS-54 with time hopping.

Using complex baseband representation, the received signal at the n^{th} receive antenna at time t denoted by $x_n(t)$ can be written as

$$x_n(t) = \sqrt{\frac{\rho}{M}} \sum_{m=1}^{M} h_{mn} s_m(t) + w_n(t), \quad t = 1, \ldots, T, \quad n = 1, \ldots, N \tag{1}$$

where h_{mn} and $w_n(t)$ are the circularly symmetric complex Gaussian random variables with zero mean and unit variance representing fading coefficients from m^{th} transmit antenna to n^{th} receive antenna, and additive white Gaussian noise (AWGN), respectively. We assume that the fading coefficients and noise are independent among sub-channels and from one block to the next. $s_m(t)$ denotes the transmitted signal from m^{th} antenna element at time t. We normalize the average expected power of the transmitted signal to be equal to 1, i.e.,

$$\frac{1}{M} \sum_{m=1}^{M} E|s_m(t)|^2 = 1, \quad t = 1, ..., T$$

where E is the expectation operator. Therefore, ρ is the expected signal to noise ratio at each receiver antenna, independently of the number of the transmit antennas. We can write the above equation in matrix form as [15]:

$$X = \sqrt{\frac{\rho}{M}} SH + W \tag{2}$$

where X is the $T \times N$ received signal matrix, S is the $T \times M$ transmitted signal matrix, H is the $M \times N$ channel transfer matrix and W is the $T \times N$ additive white Gaussian noise matrix. We assume that the channel state information (i.e., the H matrix) is not available at the receiver or at the transmitter.

III. Trellis-Coded Unitary Space-Time Modulation

A. Review of Unitary Space-Time Signals

A constellation of L unitary space-time signals, $\{S_1, ..., S_L\}$, are defined to comprise of $T \times M$ complex-valued matrices such that $S_i = \sqrt{T}\phi_i$,$i = 1, ..., L$, with the constraints $\phi_1^H \phi_1 = ... = \phi_L^H \phi_L = I_M$ [16]. When the unitary space-time modulation is used over a Rayleigh flat fading channel where the channel gain is constant for a period of T symbols, the basic criteria to measure the distance between the signal pairs, and hence the performance of the scheme, is determined in terms of the singular values, $d_m,\ m = 1, \cdots, M$, of the matrices $\phi_l^H \phi_{l'}$. More specifically, in [15], the design parameter to be optimized among all signal pairs, denoted by δ, is given as

$$\delta = \max_{1 \leq l, l' \leq L} \left\| \phi_l^H \phi_{l'} \right\|$$

where $\| \cdot \|$ denotes the (scaled) Frobenius norm, i.e., $\|A\| = \sqrt{\frac{1}{M} tr\{A^H A\}}$, where $tr\{\cdot\}$ denotes the trace of the matrix. It is shown that a unitary space-time constellation, when employed in an uncoded transmission, performs well if δ is minimized. For large values of SNR, another (related) parameter is the *diversity product*, ζ, which is defined using the singular values d_m as [17]

$$\zeta = \min_{0 \leq l \leq l' \leq L-1} \zeta_{ll'}$$

where

$$\begin{aligned} \zeta_{ll'} &= \prod_{m=1}^{M} \sin(\theta_m)^{(1/M)} \\ &= \left[\prod_{m=1}^{M} (1 - d_m^2) \right]^{1/2M} \end{aligned}$$

For improved performance ζ should be maximized. Considering the d_m as the cosine of the principle angle θ_m between the subspaces spanned by the columns of ϕ_l and $\phi_{l'}$ [22], the diversity product can be thought of as the geometric means of the squares of the sines of the m principle angles. We note that this result is analogous to the Euclidean product distance criteria for the case of known CSI.

The criterion of minimizing δ, or maximizing ζ, is a useful parameter in designing constellation but it is still cumbersome to generate $L = 2^{TR}$ signals using these criteria alone. For example, if we seek for a data rate of 2 b/s/Hz over a channel with $T = 8$ symbols of coherence time, then we need $L = 65536$ $T \times M$ complex matrices with the given structure. An iterative algorithm to find a good constellation is described in [15] but it involves a complicated search for optimization. Constraining the $T \times M$ signals such that they have a block circulant structure, a systematic way of designing large constellations has been proposed in [16]. Although this algorithm is not optimal, due to its simplicity, we will employ it to generate the signal constellations necessary for our trellis code design.

B. Trellis-Coded Modulation with Unitary Space-Time Constellations

In this section, we describe the trellis-coded modulation scheme (TCM) employing unitary space-time constellations.

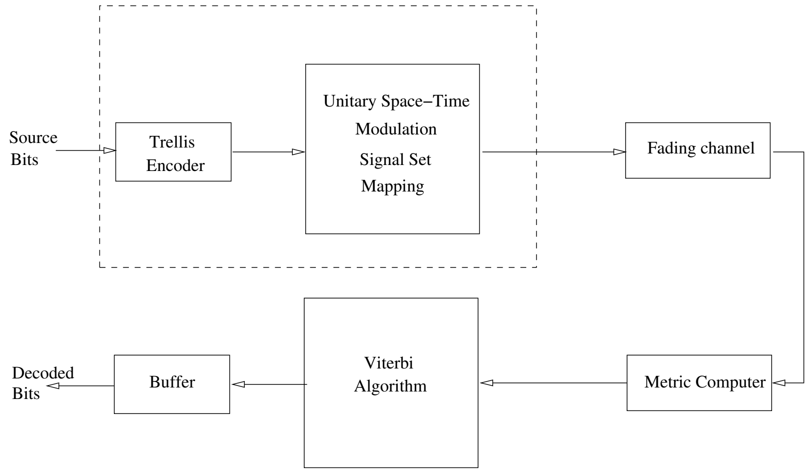

Fig. 1. Block diagram of trellis-coded unitary space-time modulation system.

B.1 Encoding

The block diagram for the encoder and the decoder of the system is presented in Figure 1. In the modulation process, the source bits are mapped to signals selected from a space-time constellation using a finite state encoder which decides the space-time symbol to be transmitted. The finite state encoder can be represented by a trellis with its nodes denoting the states. Depending on the current encoder state and the input bits, a space-time symbol is selected from the constellation and transmitted for duration of T symbols. The size of the constellation is $L = 2^{TR}$ to achieve a spectral efficiency of R b/s/Hz for uncoded transmission. In the trellis-coded modulation scheme, we expand the signal set in order to provide redundancy for coding and we perform coding and signal mapping jointly so as to improve the performance without any bandwidth expansion.

B.2 Decoding

We employ a soft decision maximum likelihood decoder. Consider the transmission of a sequence $\boldsymbol{\phi} = \left\{\phi_{l_j}\right\}_{j=0}^{J-1}$ of J unitary space-time signals where ϕ_{l_j} is the transmitted signal during the j^{th} block. Assuming that the fading is independent from block to block, the conditional probability density function of the received sequence $\boldsymbol{X} = \{X_j\}_{j=0}^{J-1}$ given the transmitted signal sequence is

$$p(X_0, ..., X_{J-1}|\phi_{l_0}, ..., \phi_{l_{J-1}}) = \prod_{j=0}^{J-1} p(X_j|\phi_{l_j}) = \prod_{j=0}^{J-1} \frac{e^{-tr\left\{\left[I_T - \frac{1}{1+M/\rho T}\phi_{l_j}\phi_{l_j}^H\right]X_j X_j^H\right\}}}{\pi^{TN}(1+\rho T/M)^{MN}} \tag{3}$$

which is easily obtained by extending the expression for the conditional probability in [15]. Second equality follows since for each transmitted block j, conditioned on the transmitted signal ϕ_{l_j}, the received signals are statistically independent of each other. Taking the logarithms of both sides, we obtain:

$$\begin{aligned}\log p(X_0, ..., X_{J-1}|\phi_{l_0}, ..., \phi_{l_{J-1}}) &= -JNT \log \pi - JMN \log(1 + \rho T/M) \\ &\quad - \sum_{j=0}^{J-1} tr\left\{X_j X_j^H\right\} + \sum_{j=0}^{J-1} \frac{1}{1 + M/\rho T} tr\left\{X_j^H \phi_{l_j} \phi_{l_j}^H X_{l_j}\right\}. \qquad (4)\end{aligned}$$

Assume that all the sequences are equally likely to be transmitted — which is the case when the transmitted bits are equally likely — the optimal receiver (i.e., the receiver that minimizes average probability of error) computes the likelihood in (4) for all possible transmitted sequences using the received signals and decides in favor of the sequence that maximizes this metric.

Observing that the first three terms are identical for all possible transmitted sequences, we can further simplify the decoding process as:

$$\boldsymbol{\phi}_{opt} = \arg\max_{\boldsymbol{\phi}} m(\boldsymbol{X}, \boldsymbol{\phi}) \qquad (5)$$

where

$$m(\boldsymbol{X}, \boldsymbol{\phi}) = \sum_{j=0}^{J-1} tr\left\{X_j^H \phi_{l_j} \phi_{l_j}^H X_j\right\}. \qquad (6)$$

Comparing this decoding rule with the decoding of uncoded unitary space–time modulation, we observe that we need to maximize the sum of a sequence of "trace metrics" as opposed to a single "trace metric". This should be compared to the case of decoding for AWGN channels where for the uncoded case we minimize the Euclidean distance between the received signal and possible constellation points, while for the coded case, we minimize the squared Euclidean distance between the received vector and the possible transmitted sequences. Clearly, since the metric to be maximized is additive, we can use soft decision Viterbi algorithm to perform the decoding in an efficient manner — just as in the case of trellis-coded modulation for AWGN channels.

IV. Code Design

To perform the code design, it would be useful to evaluate pairwise error probabilities (or, tight upper bounds on the pairwise error probabilities), and then find the optimal design criteria. However,

TABLE I

PARAMETERS FOR $L = 4, 8, 16, 32, 64, 256, 512$-ARY UNITARY SPACE-TIME CONSTELLATIONS. $T = 8,\ M = 2,\ N = 1$.

L	δ	ζ	K	q	U
4	0.0000	1.0000	2	2	[1 0 0 1 1 0 0 1] [0 1 0 1 0 1 0 1]
8	0.3835	0.9198	1	8	[2 1 7 5 0 1 6 3]
16	0.4751	0.8631	1	17	[1 12 11 9 14 6 10 0]
32	0.5319	0.8133	1	32	[1 18 11 2 22 8 0 5]
64	0.5889	0.7446	1	67	[1 7 31 15 3 29 20 0]
256	0.6693	0.6533	1	257	[1 7 60 79 187 125 198 154]
512	0.7339	0.5671	2	23	[1 0 15 3 10 9 15 17] [0 1 22 16 14 4 21 21]

for our case, the evaluation of such expressions turn out to be quite difficult. Therefore, we resort to a suboptimal code design technique elaborated in the following.

Assume that the necessary (expanded) unitary space–time constellation is already generated, for instance, using the technique proposed in [16]. With the systematic method described in [16], one can produce the signals comprising the constellation starting from a $T \times M$ matrix, ϕ_1, with orthonormal columns, and rotating it in the M dimensional complex matrix space by multiplying it with suitably chosen unitary matrices (that is, L^{th} roots of identity.). We select the multiplication matrix such that the singular values of the resulting correlation matrices, $\phi_{l'}^H \phi_l$, are small. Using the algebraic method described in [16], we can generate the unitary space-time signals ϕ_l, $l = 1, \cdots, L$ with

$$\phi_l = \Theta_1^{l_1} ... \Theta_K^{l_K} \phi_1, \quad 0 \le l_1, ..., l_K < q \tag{7}$$

where $\Theta_1, .., \Theta_K$ are unitary matrices such that the diagonal elements $[\Theta_k]_{tt} = exp(j2\pi U_{kt}/q)$ where U_{kt} is the element in the generator matrix $U_{K \times T}$ whose entries are the elements of the ring of integers in $0, 1, ..., q-1$ with modulo q operations. Table I provides the parameters necessary for the unitary space-time constellations with $M = 2$ used in this paper [16].

Given the constellations, let us describe the mapping strategies of the constellation points to the trellis branches to provide improved code design. We employ a technique similar to the Ungerboeck's mapping by set partitioning to design the trellis-coded unitary space-time modulation scheme. Although it may be suboptimal, due to its simplicity, we use δ as the metric to be minimized. We first partition the constellation into smaller subsets that have smaller δ values for the signal pairs in the subsets. Then, we

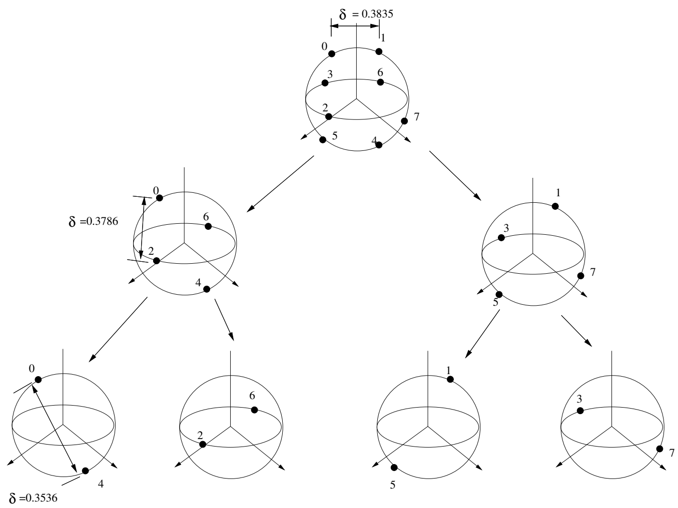

Fig. 2. Set partitioning for $L = 8$ unitary space-time constellation.

- assign the signals in the lowest level partition to the parallel transitions,
- assign the signals in the next available partition to adjacent transitions, i.e., the transitions with the same originating or terminating state.

A simple example would be helpful to explain the design procedure.

Example 1: In Table II, we give the δ values for a unitary space-time constellation with $T = 8$, $L = 8$, and $M = 2$. The signals $\phi_0, .., \phi_7$ generated with a similar technique described in [16]. Due to their block circulant structure, the distance between signal points depends only on the differences in the indices of the signals, i.e., $\phi_l = \Theta^l \phi_1$ and $\phi_{l'} = \Theta^{l'} \phi_1$, $\delta_{ll'} = \sqrt{\frac{1}{M} tr\{\phi_l^H \phi_{l'} \phi_{l'}^H \phi_l\}} = f(\phi_0, \Theta, l - l')$. In the first partition, we select $A_o = \{\phi_0, \phi_2, \phi_4, \phi_6\}$ and $A_1 = \{\phi_1, \phi_3, \phi_5, \phi_7\}$ since $\delta_{20} < \delta_{10}$ and with this partitioning, the "distance" between the signals that are "closest" to each other is increased. In this step, the signals with $l - l' \bmod L = 2$ constitute the "closest" signals. Thus, the metric δ is reduced from 0.3835 to 0.3786. In the second step, observing that $\delta_{40} < \delta_{20}$, we partition these two groups into $B_o = \{\phi_0, \phi_4\}, B_1 = \{\phi_2, \phi_6\}, B_2 = \{\phi_1, \phi_5\}, B_3 = \{\phi_3, \phi_7\}$, hence reducing δ to 0.3536, which is the minimum possible value of δ for this constellation. This set partitioning procedure is illustrated in Figure 2.

Now consider an 8-state trellis-coded modulation scheme using the unitary space-time constellation with $L = 8$. Using the design rules outlined above, the code described in Figure 3 is obtained. $\square$

For comparison purposes, in Figure 4, we present the bit error rate obtained for the trellis-coded modulation scheme with some random signal mapping together with that using our mapping as ex-

TABLE II

δ AND ζ VALUES AND FOR $L = 8$ UNITARY SPACE-TIME CONSTELLATION.

$l - l' \bmod L$	$\delta_{ll'}$	$\zeta_{ll'}$	$l - l' \bmod L$	$\delta_{ll'}$	$\zeta_{ll'}$
0	1.000	0.0000	4	0.3536	0.9306
1	0.3835	0.9232	5	0.3835	0.9198
2	0.3786	0.9253	6	0.3786	0.9253
3	0.3835	0.9198	7	0.3835	0.9232

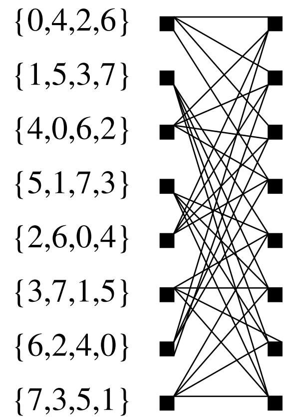

Fig. 3. Trellis structure for L=8 unitary space-time modulation.

plained in the example above (both employing the same set of signals). We observe that the code using the set partitioning performs several dBs better than the scheme with the unoptimized set partitioning, e.g., 7 dB better at a bit error rate of 10^{-4} . in Figure 4.

The design technique explained so far uses δ as the metric to be optimized. Another related parameter that can be used as the distance metric for the unitary space-time signal pairs ϕ_l and $\phi_{l'}$ is $\zeta_{ll'}$. In our experiments, we observed that using this method gives the similar results. In our simulations, we also found that the larger the minimum free distance of the code computed using the squares of the diversity product as the pairwise distance metric, the better the performance of the system. A criterion that can be used for the trellis code design is to construct the trellis such that

$$\zeta_{free}^2 = \zeta_1^2 + \zeta_2^2 + ... + \zeta_F^2$$

is maximized where F is the number of the transitions until a pair of paths that originate from the same state merge into the same state and ζ_i is the pairwise diversity product between the signals associated with the branches at the i^{th} transition. This is analogous to the Euclidean distance metric criteria for the case of known CSI.

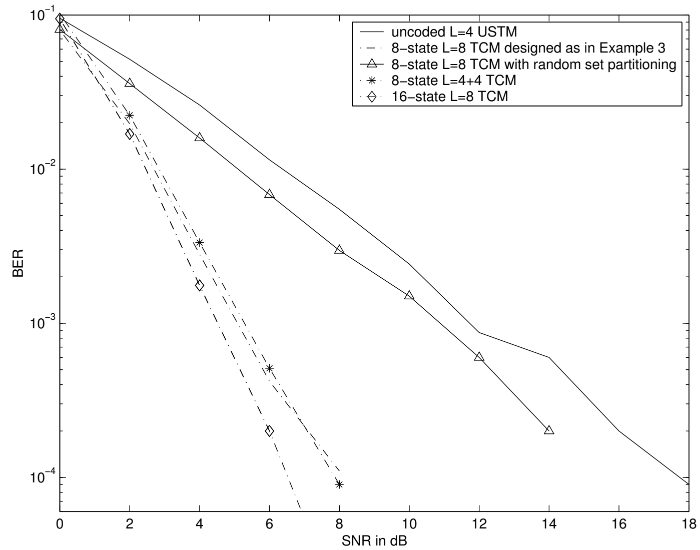

Fig. 4. Bit error rate for $L = 4$ uncoded modulation and 8-state $L = 8$ trellis-coded unitary space-time modulation system using two different trellis structures. $M = 2$ and $N = 1$.

Design using "Good" Sub-constellations

As an alternative to simply selecting the constellation using the technique in [16] in a brute force manner, we will consider the expanded signal constellation selection using two sub–constellations which have very good distance properties among its constellation points. We know that as the size of the constellation is increased, the δ values increase, i.e., the overlap between the subspaces spanned by the columns of the unitary space-time signals increases. With this technique, in order to expand the signal set, instead of designing a larger size constellation optimized for all pairs of signals, we design two or more unitary space-time signal sets, whose signal points have a smaller overlap among themselves, but the signals from different sets might have a larger overlap. Since the subsets have better distance properties, the signals in the subsets are assigned to the branches emanating from/to the same state using the set partitioning rules described earlier. In this way, although we expand the signal set and obtain a unitary space-time constellation with increased overlap, the careful design of the trellis-coded modulation scheme provides significant coding gains as we will see from our simulations. In Figure 4, we include the bit error rate result for the case when the signal constellation is designed using this method. For the TCM scheme under consideration, we design two $L = 4$ unitary space-time constellations such that the pairwise distances in each set are the optimum (i.e., $\zeta_{ll'} = 1.0$) while the distance between the signals in different sets are maximized as much as possible. We select two $L = 4$ unitary space-time constellations and rotate one of the subsets in the complex space using unitary matrices such that the δ metric is as small as possible, or ζ is as large as possible for the signals that do not belong to the same subset. With this technique, we have not seen a significant difference in the error rate performance compared to the one with the unitary space-time constellation that was designed such that the distances are optimized over the 8 signals in the constellation. However, for large constellations, the complexity of generating the expanded signal constellation is reduced.

V. Simulation Results

In Figures 4 – 7, we present the bit error rates for trellis-coded modulation schemes for various spectral efficiencies and compare them with the uncoded unitary space-time modulation schemes. We assume a system using two transmit antennas and one receive antenna. The fading coefficients remain constant for $T = 8$ symbols. The unitary space-time constellations are obtained using the technique described in the previous section. In Figure 4, the bit error rate plots for the uncoded transmission for a constellation size of $L = 4$ together with the performance of 8- and 16-state trellis-coded modulation with $L = 8$ unitary space-time constellation are given. We observe that a considerable performance improvement is achieved with trellis-coded unitary space-time modulation scheme compared to the uncoded transmission using unitary space–time constellations with the same spectral efficiency. In particular, we obtain 10 dB coding gain for 8-state and over 11 dB coding gain for 16-state TCM at a BER of 10^{-4}. We notice that the "free distances" for 8-state and 16-state TCM, respectively, are

$$\zeta_{free}^2 = \zeta_{02}^2 + \zeta_{04}^2 = 1.7221$$

and

$$\zeta_{free}^2 = \zeta_{06}^2 + \zeta_{07}^2 + \zeta_{07}^2 = 2.5583.$$

Additional coding gain obtained by 16-state TCM compared to 8-state TCM may be attributed to the increase in the "free distance".

In Figure 5, we plot the bit error rates for 32-state trellis-coded modulation scheme using a unitary space-time constellation of size $L = 16$. The spectral efficiency is 3/8 b/s/Hz, therefore this scheme is comparable to the uncoded unitary space-time modulation with $L = 8$. A coding gain of 8 dB at a BER of 10^{-4} is achieved as shown in Figure 5. In Figure 6, we also plot the results for a spectral efficiency of 1/2 b/s/Hz. We observe considerable coding gains over the uncoded system as

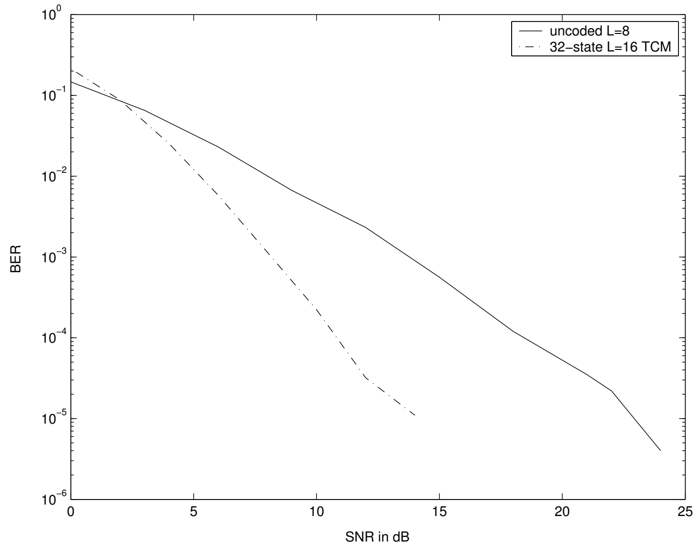

Fig. 5. Bit error rate for $L = 8$ uncoded modulation and 32-state $L = 16$ trellis-coded unitary space-time modulation system. $M = 2$ and $N = 1$.

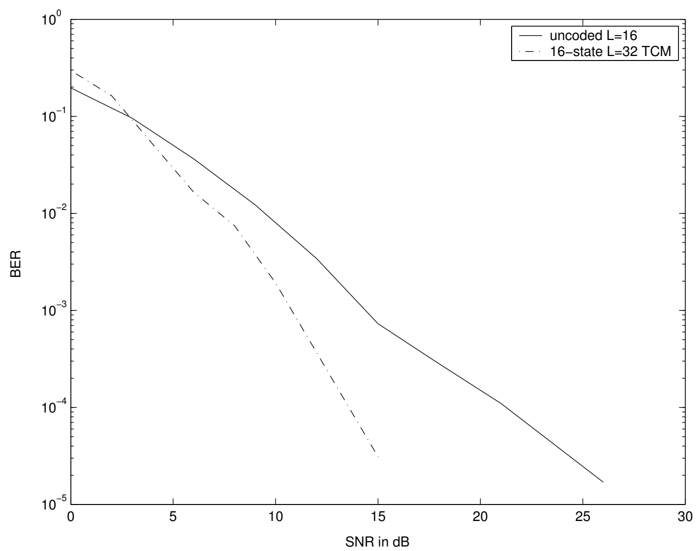

Fig. 6. Bit error rate for $L = 16$ uncoded modulation and 16-state $L = 32$ trellis-coded unitary space-time modulation system. $M = 2$ and $N = 1$.

well. Finally, the performance results for the system with a spectral efficiency of 1 b/s/Hz is shown in Figure 7. For this scheme, we use 256-state trellis which makes it difficult to obtain the optimal trellis structure. Although we employed a suboptimal set-partitioning and signal mapping for this trellis-coded modulation scheme, the performance improvement compared to the uncoded case is still significant. Of particular interest, the results of the trellis-coded modulation scheme with $r = 1$ b/s/Hz is comparable to the results of turbo-coded unitary space-time modulation proposed in [1] for the same spectral efficiency. We observe that the turbo-coded scheme significantly outperforms the trellis-coded scheme in terms of the bit error rates, however the decoding complexity of the trellis-coded modulation

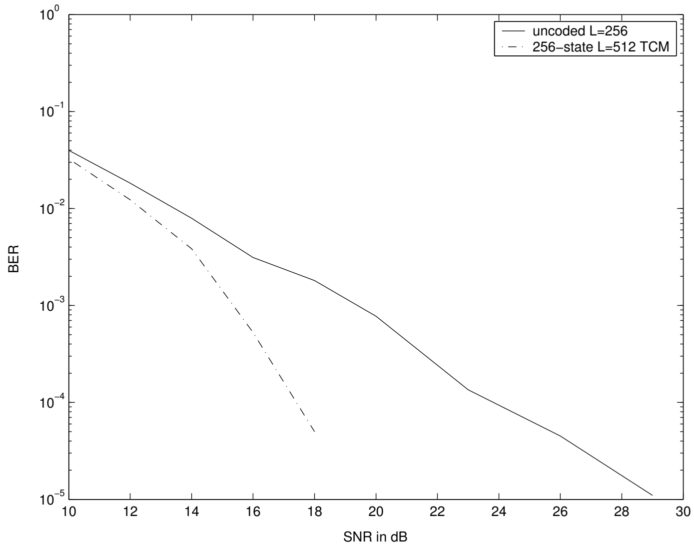

Fig. 7. Bit error rate for $L = 256$ uncoded modulation and 256-state $L = 512$ trellis-coded unitary space-time modulation system. $M = 2$ and $N = 1$.

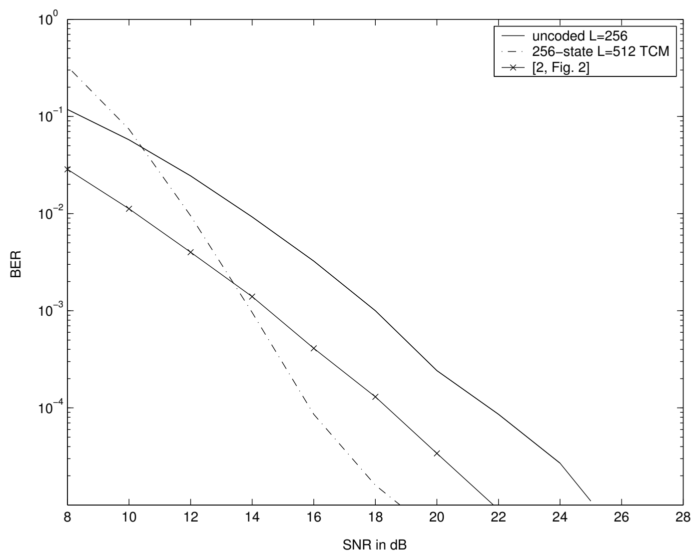

Fig. 8. Bit error rate for $L = 256$ uncoded modulation, 256-state $L = 512$ trellis-coded unitary space-time modulation system and Jafarkhani's scheme with noncoherent detection. $M = 3$ and $N = 1$.

is much smaller and it is applicable for delay sensitive applications.

In Figure 8, we present the performance results of a system with three transmit and one receive antennas ($M = 3$, $N = 1$). For comparison purposes, we have also plotted the bit error rate results of Jafarkhani and Tarokh's differential detection scheme from orthogonal designs [2]. The trellis-coded modulation scheme uses a 256-state trellis and a unitary space-time constellation of size $L = 512$ designed using $K = 2$, $q = 23$, $U_1 = [6, 13, 10, 17, 8, 0, 18, 7]$ and $U_2 = [1, 18, 13, 12, 19, 1, 4, 1]$. All three schemes compared in this plot achieve the same bit rate of 1 b/s/Hz. From the figure, we observe that the trellis-coded scheme outperforms the uncoded scheme by about 6 dB and the Jafarkhani and

Tarokh's scheme with noncoherent detection by about 3 dB at a bit error rate of 10^{-5}. However, we note that the decoding complexity of the trellis-coded modulation scheme is higher than the scheme presented in [2].

Finally, we note that it is more difficult to design trellis-coded schemes for higher spectral efficiencies since we are required to use very large constellations of unitary space-time signals. For very large constellations, set partitioning and signal mapping becomes more difficult to perform. However, in practice the design of the trellis has to be done only once and no real time computations are required.

VI. Conclusions

We proposed trellis-coded unitary space-time modulation schemes for block fading channels when there is no channel state information at the receiver. The code design principles are developed and the improved performance results are demonstrated via simulations. If the trellis is constructed properly, trellis-coded unitary space-time modulation performs significantly better than the uncoded unitary space–time modulation scheme with the same spectral efficiency by about $5-11$ dB at a bit error rate of 10^{-4}.

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