Diagonal Block Space-Time Code Design for Diversity and Coding Advantage Over Flat Fading Channels

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***Abstract*—The potential promised by multiple transmit antennas has raised considerable interest in space-time coding for wireless communications. In this paper, we propose a systematic approach for designing space-time trellis codes over flat fading channels with full antenna diversity and good coding advantage. It is suitable for an arbitrary number of transmit antennas with arbitrary signal constellations. The key to this approach is to separate the traditional space-time trellis code design into two parts. It first encodes the information symbols using a one-dimensional $(M, 1)$ nonbinary block code, with M being the number of transmit antennas, and then transmits the coded symbols diagonally across the space-time grid. We show that regardless of channel time-selectivity, this new class of space-time codes always achieves a transmit diversity of order M with a minimum number of trellis states and a coding advantage equal to the *minimum product distance* of the employed block code. Traditional delay diversity codes can be viewed as a special case of this coding scheme in which the repetition block code is employed. To maximize the coding advantage, we introduce an optimal construction of the nonbinary block code for a given modulation scheme. In particular, an efficient suboptimal solution for multilevel phase-shift-keying (PSK) modulation is proposed. Some code examples with 2–6 bits/s/Hz and two to six transmit antennas are provided, and they demonstrate excellent performance via computer simulations. Although it is proposed for flat fading channels, this coding scheme can be easily extended to frequency-selective fading channels.**

***Index Terms*—Block code, delay diversity, product distance, space-time code, transmit diversity.**

I. Introduction

THE design of future wireless communication systems is to offer a variety of multimedia services that require reliable transmissions at high data rates. This is a challenging task due to multipath fading, multiple access interference, as well as limited spectrum resource in wireless channels. Recently, a considerable amount of research work on multiple-input multiple-output (MIMO) antenna systems has been done using variable signal processing techniques to achieve this design goal. This was motivated by the information-theoretic results, studied by Foschini and Gans [1] and Telatar [2], that the channel capacity of a multiple-antenna system substantially exceeds that of a conventional single-antenna system. It is now acknowledged that multiple antennas can provide spatial diversity to combat fading and increase the degree of freedom for high-speed transmissions. One of the important signal processing solutions to achieve the great potential is space-time coding. By performing coding across both temporal and spatial dimensions, space-time codes can effectively utilize the maximum possible diversity advantage as well as coding advantage without sacrificing channel bandwidth. The fundamental design criteria for good space-time codes at high signal-to-noise ratio (SNR) were derived by Tarokh *et al.* [3] and Guey *et al.* [4]. These criteria have been commonly used to construct many classes of space-time codes.[1] Several handcrafted space-time trellis codes for systems with two transmit antennas were provided in [3]. Subsequent computer searches were carried out in [5]–[8] to find codes with enhanced error performance. Instead of following the traditional design criteria in [3] and [4], several efforts in [9]–[13] investigated different design criteria, where, in particular, the role of the Euclidean distance was studied. The corresponding codes in [10]–[13] with a large minimum Euclidean distance and nonfull transmit diversity perform well with enough receive antennas but poorly when the number of receive antennas is small. It is noticed that all of the above codes are limited to a small number of transmit antennas and low-level modulation due to the extraordinary complexity of exhaustive search. An algebraic approach to construct space-time trellis codes for systems with an arbitrary number of transmit antennas was presented in [14] and [15], but it is only for binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) modulation. Delay diversity transmission, which was proposed in [16] and [17], is a simple transmit diversity scheme. It transmits delayed copies of the same information signal sequence on multiple antennas and is seen at the receiver as a single-antenna transmission with increased channel delay spread. The spatial diversity is, thus, artificially transformed to multipath diversity, where the gain can be realized at the receiver using the Viterbi-algorithm based maximum likelihood sequence estimator (MLSE) [18]. This transmission scheme can be designed for an arbitrary number of transmit antennas with arbitrary signal constellations. From a coding perspective, it can be viewed as a systematic approach

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[1]We are primarily interested in space-time trellis codes in this paper.

of designing space-time code and is, hence, referred to as the delay diversity (DD) code in this paper. The drawback is that the coding advantage is inefficient.

In this paper, we are interested in the case when the number of receive antennas is limited so that transmit diversity is crucial. We propose a systematic coding approach for achieving maximum transmit diversity and good coding advantage. The key of this approach is to separate the traditional space-time code design into two parts. It first encodes the information symbols using a one-dimensional $(M, 1)$ nonbinary block code, with M being the number of transmit antennas, and then transmits the coded symbols through the M antennas in a diagonal pattern. Hence, we refer to this scheme as diagonal block space-time (DBST) coding. By construction, a DBST code forms a regular trellis diagram with $2^{b(M-1)}$ states, where b is the number of information bits per symbol. Delay diversity codes can be viewed as a special example in this category where a repetition block code is employed. Generally, based on the traditional criteria in [3], the space-time code design for quasistatic fading and rapid fading should be treated differently. Most of the previous work has focused on the quasistatic fading case. The proposed DBST coding is not only suitable for quasistatic fading but is suitable for rapid fading as well. Specifically, we show that in both fading models, DBST codes achieve a transmit diversity of order M and that its coding advantage is equal to the *minimum product distance* of the employed block code over a chosen modulation scheme. Therefore, the optimization of DBST codes reduces to the design of sophisticated block codes. We introduce an optimal construction of the $(M, 1)$ nonbinary block code: a permutation optimization problem that requires computer search with very high time complexity. Thus, we further propose an efficient suboptimal solution, particularly for multilevel PSK modulation.

Similar work on generalized delay diversity codes was done in [19], but only the quasistatic fading was considered, and it is not clear how to apply its design method to high-level modulation. A recent work [20] suggested another systematic design of space-time trellis codes for quasistatic fading involving manually assigning the channel output symbols for each trellis state transition with certain rules. This scheme can achieve the maximum possible antenna diversity order, but its coding advantage is less efficient, as will be shown in detail.

The rest of this paper is organized as follows. In Section II, we review the channel model of a multiple-antenna system and the fundamental performance criteria of space-time codes. In Section III, we describe the proposed diagonal block space-time code structure and discuss its pair-wise error probability. The nonbinary block code construction is presented in Section IV, along with some code examples. In Section V, the performance of the proposed codes is evaluated and compared with that of existing codes. Finally, Section VI concludes this paper.

II. Background

A. Channel Model

We consider a point-to-point wireless communication link equipped with M transmit antennas and N receive antennas. Let $c_m[t]$ denote the signal to be transmitted on antenna m, for $1 \leq m \leq M$, at discrete time index t. It is chosen from a signal constellation (e.g., PSK and QAM) with unit average energy. The received signal on antenna n, for $1 \leq n \leq N$, at time t is denoted by $r_n[t]$ and modeled as

$$r_n[t] = \sqrt{E_s} \sum_{m=1}^{M} h_{n,m}[t] c_m[t] + w_n[t] \tag{1}$$

where E_s denotes the average energy per symbol, $h_{n,m}[t]$ denotes the channel coefficient from transmit antenna m to receive antenna n at time t, and $w_n[t]$ denotes the additive complex white Gaussian noise with mean zero and variance N_0. It is assumed that the channel is flat Rayleigh fading and that the channel coefficients for different transmit-receive antenna pairs are statistically independent. Thus, $h_{n,m}[t], 1 \leq m \leq M$ and $1 \leq n \leq N$ are modeled as samples of independent complex Gaussian random processes with zero mean and unit variance. In quasistatic fading, the channel coefficients remain unchanged within each transmission frame and vary from frame to frame. Hence, within a frame, time index t in each $h_{n,m}[t]$ can be dropped. In rapid fading, the channel coefficients at different time t are independent.

B. Performance Criteria of Space-Time Codes

Consider a transmission frame of length T symbol periods. A space-time codeword is defined as an $M \times T$ matrix $\mathbf{C}$, which is formed as

$$\mathbf{C} = \begin{bmatrix} c_1[1] & c_1[2] & \cdots & c_1[T] \\ c_2[1] & c_2[2] & \cdots & c_2[T] \\ \vdots & \vdots & \ddots & \vdots \\ c_M[1] & c_M[2] & \cdots & c_M[T] \end{bmatrix} \tag{2}$$

in which the tth column $\mathbf{c}[t] = [c_1[t]c_2[t]\ldots c_M[t]]^T$ (superscript "T" denotes transpose operation) is the space-time signal transmitted at time t, and the mth row $\mathbf{c}_m = [c_m[1]c_m[2]\ldots c_m[T]]$ is the signal sequence transmitted from antenna m.

It is assumed that perfect channel state information (CSI) is available at the receiver and that the maximum likelihood (ML) decoder is applied. Then, the pair-wise error probability (PWEP) of mistaking codeword $\mathbf{E}$ for $\mathbf{C}$ is upper-bounded, at high SNR, by [3] [4]

$$p(\mathbf{C} \to \mathbf{E}) \leq \left(\frac{E_s}{4N_0}\right)^{-E_H \cdot N} (E_P)^{-N} \tag{3}$$

in which we have the following.

i) *Quasistatic fading:* E_H is the rank of the codeword difference matrix $\mathbf{C}-\mathbf{E}$, and E_P is the product of nonzero eigenvalues of matrix $(\mathbf{C}-\mathbf{E})(\mathbf{C}-\mathbf{E})^H$ (superscript "H" denotes transpose conjugate operation)

ii) *Rapid fading:* E_H is the size of the time index set of $1 \leq t \leq T$ with $\mathbf{c}[t] \neq \mathbf{e}[t]$, which is denoted by ζ, and E_P is the product of $\|\mathbf{c}[t] - \mathbf{e}[t]\|^2$ over $t \in \zeta$ (notation $\|\cdot\|^2$ denotes the squared Euclidean norm of a vector or matrix).

In both cases, E_H is called the *effective Hamming distance*, and E_P is called the *effective product distance*. These two parameters quantify the transmit (either in the space domain or time

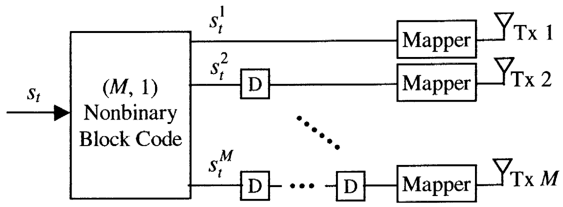

Fig. 1. Transmitter diagram of the diagonal block space-time codes, where "D" denotes one symbol delay.

domain) diversity order and the coding advantage, respectively, of a space-time code.

Thus, the classic design criteria of space-time codes are twofold: First, make the minimum of E_H over all pairs of distinct codewords $\mathbf{C}$ and $\mathbf{E}$ as large as possible; then, at the minimum of E_H, let the minimum of E_P be maximized.

III. Diagonal Block Space-Time Codes

A. Code Structure

Fig. 1 depicts the simplified transmission diagram of the proposed diagonal block space-time codes in a system with M transmit antennas. Assume that the information bit stream is divided into b-bit long blocks, forming P-ary ($P = 2^b$) source symbols, denoted by $s_t, s_t \in \{0, 1, \ldots, P-1\}$ at time t ($t = 1, 2, \ldots$). As can be seen from Fig. 1, the encoding framework can be separated into two parts. In the first part, each information symbol s_t is encoded using an $(M, 1)$ nonbinary block code with output codeword $[s_t^1 \; s_t^2 \ldots s_t^M]$. In the second part, the M elements of each output codeword are transmitted using the M antennas in a diagonal pattern across the space-time grid. That is, while the first element s_t^1 is transmitted by antenna one at time t, the second element s_t^2 is transmitted by antenna two at time $t+1$, the third one s_t^3 by antenna three at $t+2$, and so forth. As a consequence, the baseband version of the transmitted signal at time t on antenna m is given by $c_m[t] = f(s_{t-m+1}^m)$, for $m = 1, 2, \ldots, M$, where f is the modulator mapping function, and $s_t^m = s_t = 0$ when $t \leq 0$. Hence, the space-time codeword pattern is formulated

$$\mathbf{C} = \begin{bmatrix} \cdots & f\left(s_t^1\right) & f\left(s_{t+1}^1\right) & \cdots & f\left(s_{t+M-1}^1\right) & \cdots \\ \cdots & f\left(s_{t-1}^2\right) & f\left(s_t^2\right) & \cdots & f\left(s_{t+M-2}^2\right) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \cdots & f\left(s_{t-M+1}^M\right) & f\left(s_{t-M+2}^M\right) & \cdots & f\left(s_t^M\right) & \cdots \end{bmatrix}. \tag{4}$$

The design criterion of the $(M, 1)$ nonbinary block code will be discussed in detail in the next subsection. It is noted here that the Hamming distance between any two distinct block code outputs is equal to M, i.e., there is a one-to-one mapping from each input symbol to every element in the output codeword [19]. The original delay diversity code falls within the special case when this block code is a repetition code, i.e., $s_t^m = s_t$ for all $m = 1, 2, \ldots, M$.

Note that the space-time signal $\mathbf{c}[t]$ transmitted at a given time t is governed by the current input s_t and the $M-1$ most recent inputs $s_{t-1}, s_{t-2}, \ldots$, and s_{t-M+1}. The encoder thus forms a finite-state-machine, and we define the trellis state at time t as

$$S_t = (s_{t-1}, s_{t-2}, \ldots, s_{t-M+1}).$$

Given that the information symbols are P-ary, the total number of trellis states is equal to P^{M-1}, which is the minimum number of states for a space-time trellis code to achieve full antenna diversity over a quasistatic fading channel [3]. During the transition from state S_t to state S_{t+1} produced by input s_t, the encoder outputs M channel symbol indices $[s_t^1 s_{t-1}^2 \ldots s_{t-M+1}^M]$: one for each transmit antenna. This procedure is illustrated as

$$S_t \xrightarrow{s_t/\left(s_t^1 s_{t-1}^2 \ldots s_{t-M+1}^M\right)} S_{t+1}.$$

Therefore, the Viterbi algorithm can be applied at the receiver to do ML decoding.

B. Performance Measure

Proposition: In both quasistatic fading and rapid fading channels, the diagonal block space-time code with M transmit antennas satisfies

$$E_{H\,\min} = M \tag{5}$$

and

$$E_{P\,\min} = \min_{0 \leq s < \tilde{s} \leq P-1} \prod_{m=1}^{M} |f(s^m) - f(\tilde{s}^m)|^2 \tag{6}$$

where $E_{H\,\min}$ and $E_{P\,\min}$ are the minimum effective Hamming distance and the associated minimum effective product distance in the PWEP formula (3), $[s^1 \ldots s^M]$, and $[\tilde{s}^1 \ldots \tilde{s}^M]$ are the two block code outputs generated by inputs s and $\tilde{s}$, respectively.

Note from this proposition that the proposed DBST codes always achieve a transmit diversity of order M, and its coding advantage is governed by the *minimum product distance* $(PD_{\min})$ of the employed $(M, 1)$ nonbinary block code over a chosen modulation scheme.

Proof: Consider an error event in the Viterbi-algorithm-based ML decoder. The correct and estimated trellis states are denoted by S_t and $\tilde{S}_t$, respectively. Similarly, the transmitted information symbol sequence and the estimated sequence are denoted by $\mathbf{s} = \{s_t\}$ and $\tilde{\mathbf{s}} = \{\tilde{s}_t\}$, respectively. Suppose without loss of generality that, in this error event, the estimated path through the trellis diverges from the correct path at time k and remerges with the correct path at time $k+T$. Then, we have

$$\begin{cases} S_k = \tilde{S}_k \quad \text{and} \quad s_k \neq \tilde{s}_k \\ S_{k+t} \neq \tilde{S}_{k+t}, \quad \text{for } t = 1, \ldots, T-1 \, . \\ S_{k+T} = \tilde{S}_{k+T} \end{cases} \tag{7}$$

Because every single error in the information symbol sequence propagates M time intervals by the nature of diagonal transmission, it follows that $T \geq M$. The corresponding space-time codeword difference matrix within this time period is of the form

$$\mathbf{B} = \mathbf{C} - \mathbf{E} = \begin{bmatrix} \Delta_0^1 & \Delta_1^1 & \cdots & \Delta_{T-M}^1 & 0 & \cdots & 0 \\ 0 & \Delta_0^2 & \Delta_2^2 & \cdots & \Delta_{T-M}^2 & & 0 \\ \vdots & & \ddots & & & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta_0^M & \Delta_1^M & \cdots & \Delta_{T-M}^M \end{bmatrix}_{M \times T} \tag{8}$$

where $\Delta_i^m = f(s_{k+i}^m) - f(\tilde{s}_{k+i}^m)$, for $m = 1, 2, \ldots, M$ and $i = 0, 1, \ldots, T - M$.

a) *In quasistatic fading:* Partition the codeword difference matrix (8) into $\mathbf{B} = [\mathbf{U}\,\mathbf{R}]$, where $\mathbf{U}$ is an $M \times M$ upper triangular matrix, and $\mathbf{R}$ is an $M \times (T - M)$ matrix. Since the inequality $s_k \neq \tilde{s}_k$ results in $\Delta_0^m \neq 0$ for all $m = 1, 2, \ldots, M$, $\mathbf{U}$ is a full rank matrix. Then, $\mathbf{B}$ is also a full rank matrix. Hence, the effective Hamming distance E_H is equal to M. This also yields

$$E_P = \det(\mathbf{B}\mathbf{B}^H) = \det(\mathbf{U}\mathbf{U}^H + \mathbf{R}\mathbf{R}^H) \geq \det(\mathbf{U}\mathbf{U}^H) = |\det(\mathbf{U})|^2 = \prod_{m=1}^{M} |\Delta_0^m|^2 \qquad (9)$$

where "$\geq$" holds directly from the Minkowski's determinant inequality [27] as a Hermitian matrix of the form $\mathbf{R}\mathbf{R}^H$ is non-negative definite. The equality in (9), indeed, holds if, and only if, $\mathbf{R} = 0$, i.e., the sequence pair $\mathbf{s}$ and $\tilde{\mathbf{s}}$ are different at time k only. In the sequel, the final expression in (9) corresponds to the product distance between the two block code outputs $[s_k^1 \ldots s_k^M]$ and $[\tilde{s}_k^1 \ldots \tilde{s}_k^M]$ generated by input s_k and $\tilde{s}_k$, respectively. Thus, the results for the quasistatic fading case are proved.

b) *In rapid fading:* By definition, E_H is now equal to the number of nonzero columns in $\mathbf{B}$ shown in (8) associated with the sequence pair $\mathbf{s}$ and $\tilde{\mathbf{s}}$. It is observed, based on (7), that $E_H = T$. Therefore, we have $E_{H\,\min} = M$ as $T \geq M$. Indeed, the minimum occurs if, and only if, $\mathbf{s}$ and $\tilde{\mathbf{s}}$ are distinct at time k only. In this circumstance, the effective product distance E_P is equal to

$$E_P = \prod_{m=1}^{M} |\Delta_0^m|^2. \qquad (10)$$

The results in rapid fading are also proved. ■

As in the proof, the E_H of a DBST code in quasistatic fading channels is always equal to M for any distinct information sequences $\mathbf{s}$ and $\tilde{\mathbf{s}}$, but $E_{P\,\min}$ is obtained if, and only if, there is only one symbol error between $\mathbf{s}$ and $\tilde{\mathbf{s}}$. While, in rapid fading channels, E_H depends on the error sequence, and it can be greater than M. Nevertheless, $E_{H\,\min}$ is still equal to M, and it occurs if, and only if, there is only one symbol error between $\mathbf{s}$ and $\tilde{\mathbf{s}}$ as well. This is because, eventually, there is no outer coding across the information symbols. The quasistatic fading and the rapid fading are just the two extreme cases of a general time-varying fading model. It is, therefore, reasonable to expect that, regardless of channel time-selectivity, a DBST code always achieves the diversity advantage and coding advantage, as shown in (5) and (6), respectively. The simulation results presented in Section V demonstrate this statement.

With this *proposition*, the optimization of a DBST code simply amounts to finding the optimal block code that maximizes the minimum product distance given in (6). In particular, the minimum product distance of a repetition code is given by $PD_{\min}(\text{rep}) = d^{2M}$, where d is the minimum Euclidean distance of the signal constellation. Later on, to characterize the theoretical performance of our proposed coding scheme, we treat delay diversity codes as references and define the asymptotic improvement of a DBST code as

$$\Delta_\infty \equiv \frac{10}{M} \log_{10} \frac{PD_{\min}(\text{new})}{PD_{\min}(\text{rep})} \text{ [dB]}. \qquad (11)$$

C. Discussions on Diagonal Structure

The diagonal transmission pattern in the proposed DBST coding has been frequently utilized for MIMO systems, as seen in the literature. It first appeared in [21] as diagonally layered space-time architecture (D-BLAST). Recent work includes the trellis coded D-BLAST [22] and wrapped space-time coding [23], [24]. Most of existing work is designed for the case in which the number of receive antennas $N \geq M$ relies on the diagonal structure to perform a simple ZF or MMSE decision-feedback detection coupled with constituent decoder at the receiver. This work, instead, applies the diagonal structure to achieve the full transmit diversity that is essential for reliable transmissions when $N < M$. In the sequel, our scheme achieves a far lower error probability than the variants of D-BLAST when N takes a small value (in particular, $N = 1$ in the downlink of most personal wireless communication systems).

As another merit of the diagonal transmission pattern in this work, the proposed DBST coding scheme can be easily extended to frequency-selective fading channels. As done in [25] for delay diversity codes, we can change the delay step, shown in Fig. 1, from one symbol period to L symbol periods, with L being the number of the channel taps. Therefore, the maximum possible combined transmit diversity of order *LM* is achieved, as shown in [25].

IV. $(M, 1)$ Nonbinary Block Code Construction

In the last section, we introduced the DBST code structure and derived the minimum product distance criterion (6) for designing the employed $(M, 1)$ nonbinary block code with M transmit antennas. In this section, we discuss the construction of this 1-D code in detail.

A. Optimal Construction for Given Constellations

We first consider the optimal code construction. Let $c_m(s) = s^m$, which is the mth element in the block code output corresponding to input $s \in \{0, 1, \ldots, P-1\}$, with $m = 1, 2, \ldots, M$. Due to the one-to-one mapping between s and every $c_m(s)$, the P-long sequence $c_m(0), c_m(1), \ldots, c_m(P-1)$ forms a permutation of the numbers $0, 1, \ldots, P-1$. The ultimate code design is, therefore, to find these permutations for $m = 1, 2, \ldots, M$ that give the largest minimum product distance over a given modulator mapping function. As the numbers $0, 1, \ldots, P-1$ can be arranged in $P!$ different ways, the size of the exhaustive search space is $(P!)^M$. As each transmit antenna is statistically equivalent to every other in the space domain, the permutation on the first antenna can be fixed. Without loss of generality, we simply let it be the natural order $\{0, 1, \ldots, P-1\}$ and form a systematic block code with $s^1 = s$. For constellations that are symmetrical in shape, the size can be further reduced, as has been done for QPSK modulation in [19]. Yet, with an increase

TABLE I
OPTIMUM BLOCK CODES USED IN DBST CODING FOR $P = 4$ AND 8 WITH PSK MODULATION

P	M	Codeset	$PD_{\min}$(new)	$PD_{\min}$(rep)	Δ_∞ [dB]
4	2	$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix}$	4 (multi=2)	4(multi=4)	0
	3	$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 2 \\ 0 & 2 & 1 & 3 \end{bmatrix}$ ref. [19]	16	8	1.00
	4	$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 2 \\ 0 & 2 & 1 & 3 \end{bmatrix}$ ref. [20]	32	16	0.75
	5	$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 2 & 1 & 3 \end{bmatrix}$	64	32	0.60
	6	$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 2 & 1 & 3 \\ 0 & 2 & 1 & 3 \end{bmatrix}$ ref. [20]	256	64	1.00
8	2	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 3 & 6 & 1 & 4 & 7 & 2 & 5 \end{bmatrix}$ ref. [3]	2	0.3431	3.83
	3	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 2 & 5 & 7 & 3 & 1 & 6 & 4 \\ 0 & 3 & 7 & 4 & 1 & 6 & 2 & 5 \end{bmatrix}$	4	0.2010	4.33
	4	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 3 & 4 & 7 & 6 & 2 & 5 \\ 0 & 4 & 1 & 5 & 7 & 3 & 2 & 6 \\ 0 & 4 & 2 & 7 & 5 & 1 & 6 & 3 \end{bmatrix}$	4.6863	0.1177	4.00
	5	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 4 & 3 & 6 & 7 & 2 & 5 \\ 0 & 3 & 1 & 6 & 2 & 5 & 7 & 4 \\ 0 & 3 & 5 & 2 & 7 & 4 & 6 & 1 \\ 0 & 3 & 6 & 1 & 2 & 7 & 4 & 5 \end{bmatrix}$	13.6569	0.0690	4.59
	6	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 2 & 6 & 5 & 4 & 7 & 3 \\ 0 & 2 & 5 & 6 & 1 & 3 & 4 & 7 \\ 0 & 3 & 6 & 2 & 7 & 4 & 1 & 5 \\ 0 & 3 & 7 & 2 & 6 & 1 & 5 & 4 \\ 0 & 4 & 2 & 7 & 5 & 1 & 3 & 6 \end{bmatrix}$	32	0.0404	4.83

TABLE II
OPTIMUM BLOCK CODES USED IN DBST CODING FOR $P = 16$ WITH PSK/QAM MODULATION

M		Codeset	$PD_{\min}$(new)	Δ_∞ [dB]
2	PSK	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 5 & 11 & 2 & 7 & 14 & 4 & 9 & 1 & 12 & 6 & 15 & 10 & 3 & 13 & 8 \end{bmatrix}$	0.4210	6.30
	QAM	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 6 & 13 & 11 & 9 & 15 & 4 & 2 & 7 & 1 & 10 & 12 & 14 & 8 & 3 & 5 \end{bmatrix}$	0.64	7.21
3	PSK	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 4 & 9 & 14 & 2 & 11 & 5 & 15 & 8 & 12 & 1 & 6 & 10 & 3 & 13 & 7 \\ 0 & 7 & 13 & 3 & 10 & 6 & 1 & 12 & 8 & 15 & 5 & 11 & 2 & 14 & 9 & 4 \end{bmatrix}$	0.8929	8.01
	QAM	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 3 & 8 & 15 & 9 & 14 & 5 & 2 & 7 & 0 & 11 & 12 & 10 & 13 & 6 & 4 \\ 1 & 10 & 13 & 6 & 15 & 4 & 3 & 8 & 2 & 9 & 14 & 5 & 12 & 7 & 0 & 11 \end{bmatrix}$	1.28	8.53
4	PSK	$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 3 & 7 & 13 & 9 & 2 & 14 & 4 & 8 & 11 & 15 & 5 & 1 & 10 & 6 & 12 \\ 0 & 5 & 14 & 11 & 4 & 9 & 2 & 15 & 8 & 13 & 6 & 3 & 12 & 1 & 10 & 7 \\ 0 & 7 & 11 & 5 & 1 & 14 & 10 & 4 & 8 & 15 & 3 & 13 & 9 & 6 & 2 & 12 \end{bmatrix}$	2	8.93

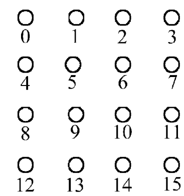

Fig. 2. 16QAM constellation.

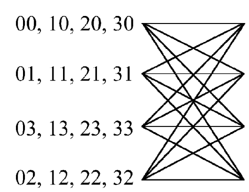

Fig. 3. Trellis diagram for the DBST code with QPSK and $M = 2$.

in P and M, the complexity of the exhaustive search still increases prohibitively.

To solve this permutation optimization problem efficiently, the general branch-and-bound algorithm [28] can be applied. The thrust of this algorithm is to form a tree structure (branching operation) and establish a lower bound (bounding operation). We take $M = 2$, for example, to illustrate its application in our problem. As discussed above, our problem is to find the permutation on the second antenna that can give the largest minimum product distance. In the first level of the tree, the root has P children, each denoting an integer number between $[0, P-1]$. Each node in the first level further has $P-1$ children, each denoting an integer selected from $[0, P-1]$. During the construction of the tree, a child node must be distinct from its ancestor nodes. The tree has P levels. Each path from the root to a leaf corresponds to a possible permutation, whereas the whole tree enumerates all $P!$ permutations. The algorithm then traverses the tree in the depth-first manner. When reaching a node of the tree, a local minimum product distance is calculated. If it is greater than a given lower bound of the largest minimum product distance, the search continues. If not, the remaining tree associated with this node is pruned. Once a permutation is found, it is used to form, or update, the lower bound of the largest minimum product distance. The extension to $M > 2$ is straightforward. The height of the tree is still P, but the number of nodes in each level grows exponentially with M.

Tables I and II list the search results. Due to space limitations, only codes with $P \leq 16$ and $M \leq 6$ are provided. Notice that the solution for $P = 2$ with BPSK modulation at any M is just the repetition code and that no more gain can be obtained using other permutations. The modulator mapping function for P-ary PSK modulation is given by $f(s) = e^{i(2\pi s/P)}$, whereas the mapping for 16QAM modulation is shown in Fig. 2. For each code, all the codewords are arranged in an $M \times P$ matrix, in which each $M \times 1$ column vector represents one codeword, and P is the total number of the codewords. To illustrate the mapping of the nonbinary block codes onto the proposed DBST codes, Fig. 3 gives an example of the trellis diagram for $P = 4$ (QPSK) and $M = 2$, in which the branch label *xy* denotes the symbols on antenna 1 and 2, respectively.

Tables I and II also shows the asymptotic improvement Δ_∞, defined in (11), for comparison. In particular, the Δ_∞s of the 16QAM codes in Table II are over the 16PSK repetition codes. As it can be seen, although no improvement is obtained in terms of $PD_{\min}$, the QPSK code with $M = 2$ has less multiplicity $(\text{multi} = 2)$ than the repetition code $(\text{multi} = 4)$.

Notice that the optimal code for each pair of P and M listed in these two tables is not unique due to the symmetric constellations. As a result, the 8PSK code with $M = 2$ is the same as the 8PSK 8-state code designed by Tarokh *et al.* for two transmit antennas in [3].

B. Linear Construction for PSK Modulation

The branch-and-bound algorithm can usually reduce the time of searching for optimal codes. However, as the search space grows exponentially, the search time could still be extremely long for large P and M, and this renders this algorithm impractical. Hence, we propose an efficient approach that may produce suboptimal solutions. This approach is particularly applied for PSK modulation. By observing that the signal points in a P-ary PSK constellation are evenly distributed on a unit circle, we can construct a linear block code over a ring of integers, which are denoted by $Z_P = \{0, 1, \ldots, P-1\}$. The mapping from ring Z_P onto the constellation is given by $f(s) = e^{i(2\pi s/P)}$, for $s \in Z_P$. Let the generator be formed as a $1 \times M$ row vector $\mathbf{G} = [g_1\ g_2\ \ldots\ g_M]$, in which $g_m \in Z_P$ for $m = 1, 2, \ldots, M$. With input $s \in Z_P$, the block code output $\mathbf{c}(s) = [s^1 s^2 \ldots s^M]$ is generated by

$$\mathbf{c}(s) = s \cdot \mathbf{G}(\text{mod} P). \tag{12}$$

The minimum product distance of this block code can thus be written as

$$PD_{\min} = \min_{0 \leq s < \tilde{s} \leq P-1} \prod_{m=1}^{M} |e^{j(2\pi s g_m/P)} - e^{j(2\pi \tilde{s} g_m/P)}|^2. \tag{13}$$

After simple manipulation, we further write $PD_{\min}$ as

$$PD_{\min} = \min_{0 < s \leq P-1} 4^M \prod_{m=1}^{M} \sin^2\left(\frac{s g_m \pi}{P}\right). \tag{14}$$

Thus, designing the optimal linear code becomes finding the solutions of $g_m, m = 1, 2, \ldots, M$ that maximize (14), and this can be done by performing a simple search in set Z_P for every g_m. The following four properties can be applied to further reduce the size of the search space. First, it is seen that (14) does not change if g_m is replaced by $P - g_m$. Hence, the search can be restricted in the new set $\{0, 1, \ldots, P/2\}$. Second, to guarantee nonzero $PD_{\min}$, each g_m must be relatively prime to P. Third, since each transmit antenna is statistically equivalent to every other in the space domain, we can impose the ordering $g_1 \leq g_2 \leq \cdots \leq g_M$. Last, the codewords generated by $[g_1\ g_2\ \ldots\ g_M]$ and $[\alpha g_1 \alpha g_2 \ldots \alpha g_M]$ are identical for any α that is relatively prime to P. An α exists in set Z_P such that $\alpha g_1 = 1(\text{mod} P)$. By multiplying $g_2, \ldots g_M$ with this same α, we can let $g_1 = 1$.

Table III shows some of the search results for $P = 16$ (16PSK), 32 (32PSK), and 64 (64PSK) with $M = 2, 3, \ldots, 6$. Again, due to the symmetrical shapes of the constellations, the optimum solution of the generator $\mathbf{G}$ is not unique. From this table, it is observed that this linear construction, though suboptimal, provides reasonably good results, besides having significantly low complexity of searching. It is further noticed that the code with $P = 16$ (16PSK) and $M = 4$ is indeed optimal in terms of $PD_{\min}$, as compared with the one in Table II.

Another suboptimal approach to combat the complexity of searching global optimal block codes was recently reported in [20]. This approach for QPSK modulation, as a matter of fact, provides the optimal solution that maximizes $PD_{\min}$, as indicated in Table I. However, at higher level modulation ($P \geq 8$), it becomes much less efficient. For example, the 8PSK code with $M = 3$ in [20] only achieves $PD_{\min} = 0.6863$, whereas the optimal code has $PD_{\min}(\text{new}) = 4$, as shown in Table I. Similarly, the 16PSK code with $M = 3$ in [20] only achieves $PD_{\min} = 0.1101$, whereas the linear code we present in Table III using our proposed suboptimal approach has $PD_{\min}(\text{new}) = 0.5198$.

TABLE III
LINEAR BLOCK RING CODES USED IN DBST CODING FOR $P = 16, 32$, AND 64 WITH PSK MODULATION

P	M	G	$PD_{\min}$(new)	$PD_{\min}$(rep)	Δ_∞ [dB]
16	2	[1 7]	0.3431	0.02318	5.85
	3	[1 3 5]	0.5198	3.529×10^{-3}	7.23
	4	[1 3 5 7]	2	5.372×10^{-4}	8.93
	5	[1 1 3 7 7]	0.4020	8.178×10^{-5}	7.38
	6	[1 1 3 5 7 7]	1.1716	1.245×10^{-5}	8.29
32	2	[1 7]	0.06186	1.477×10^{-3}	8.11
	3	[1 7 9]	0.08918	5.675×10^{-5}	10.65
	4	[1 7 9 15]	0.1177	2.181×10^{-6}	11.83
	5	[1 3 5 11 15]	0.1359	8.381×10^{-8}	12.42
	6	[1 3 5 11 13 15]	0.2702	3.221×10^{-9}	13.21
64	2	[1 19]	0.02485	9.275×10^{-5}	12.14
	3	[1 11 27]	0.02862	8.932×10^{-7}	15.02
	4	[1 11 17 19]	0.04561	8.602×10^{-9}	16.81
	5	[1 3 23 25 27]	0.02364	8.284×10^{-11}	16.91
	6	[1 7 9 15 17 23]	0.03624	7.978×10^{-13}	17.76

C. Discussion

As can be seen from Tables I–III, the asymptotic improvement of the new space-time codes over delay diversity codes increases significantly as constellation size P increases. This is because the minimum product distance of a repetition code is only a function of the minimum Euclidean distance of the given constellation. Nevertheless, the minimum product distance of a new code depends on the whole Euclidean distance profile, and the wider the profile distributes, the more the degree of freedom the new code can exploit. This also explains why no gain can be obtained at BPSK ($P = 2$) modulation. From Table II, it is also observed that the asymptotic advantage of using a 16QAM constellation rather than a 16PSK constellation is not as much as traditionally expected.

Generally, for a given constellation, the product distance profile of the block code should be made as dense as possible in order to maximize the minimum product distance. In the ideal case, the product distance between any two distinct codewords should be a constant. As a consequence, an upper bound of the optimum minimum product distance $PD_{\min}$ (opt) can be obtained. Suppose the signal constellation has the Euclidean distances $\{d_1, d_2, \ldots, d_k\}$ with multiplicity $\{n_1, n_2, \ldots, n_k\}$, respectively. Then, the $PD_{\min}$ (opt) of the $(M, 1)$ block code is upper bounded by

$$PD_{\min}(\text{opt}) \leq (d_1^{n_1} \times d_2^{n_2} \times \ldots \times d_k^{n_k})^{2M/\sum_{i=1}^{k} n_i}. \tag{15}$$

Up to now, we have only considered the block code design over conventional constellations, e.g., PSK and QAM. Notice, however, that our ultimate goal is to maximize the value in (6) over an arbitrary constellation with unit average energy. Therefore, a more general problem is to design the constellation shape at a given size P with unit average energy that achieves the maximum value of $PD_{\min}$. This, however, is beyond the scope of present research.

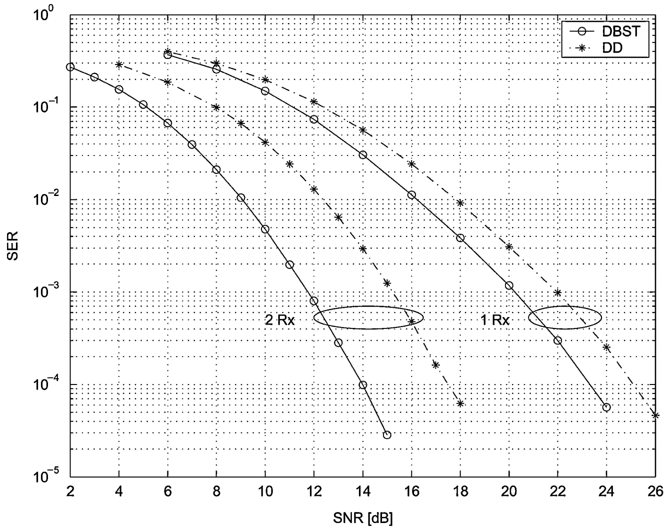

Fig. 4. SER performance of 8PSK codes with $M = 3$ transmit antennas over a quasistatic fading channel.

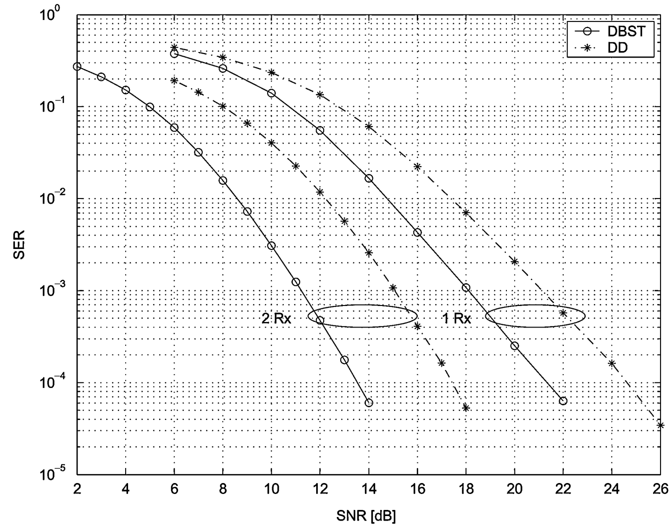

Fig. 5. SER performance of 8PSK codes with $M = 3$ transmit antennas over a rapid fading channel.

V. Simulation Results

The analysis in the previous section demonstrates the asymptotic performance improvement of the proposed DBST codes over DD codes. In this section, simulations are carried out to evaluate the actual performance gain in practical SNR regions. The channel is set to be flat Rayleigh fading, and the channel state information is available at the receiver but not at the transmitter. Unless specified otherwise, ML decoding is obtained by the Viterbi algorithm. The performances are plotted versus the total average transmitted SNR, which, by definition, is given as $E_s M/N_0$.

A. Comparison With Delay Diversity Codes

We first take the 8PSK code with $M = 3$ transmit antennas, as shown in Table I for example. Simulation is performed with three different channel autocorrelations in the time domain. Since the frame error rate (FER) depends on the transmission frame length and the bit error rate (BER) is a function of the bit-to-symbol mapping,[2] the information symbol error rate (SER) is selected as the performance measure.

Fig. 4 plots the SER performance comparison over a quasistatic fading channel (frame length $T = 130$). It is observed that the actual gain of the DBST code over the DD code at a SER of 10^{-4} is about 1.8 dB with one receive antenna. With two receive antennas, the gain increases to 3.5 dB, which is less than 1 dB away from the theoretically asymptotic improvement of 4.33 dB shown in Table I.

Fig. 5 shows the SER performance comparison over a rapid fading channel. Now, the actual gains at the SER of 10^{-4} are about 3.3 and 3.9 dB with one and two receive antennas, respectively, closer to the asymptotic improvement.

The SER performance comparison over a time-varying fading channel is illustrated in Fig. 6. The channel autocorrelation function is modeled as $J_0(2\pi f_d T_s k)$, where f_d is the maximum Doppler frequency, T_s is the symbol period, k is the discrete time index, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. The parameter $f_d T_s$ is set to 0.05 in this simulation. First, it is observed that the gain of the DBST code over the DD code at 10^{-4} SER is around 2.5 dB with one receive antenna. This is greater than the gain over a quasistatic fading channel but less than that over rapid fading. It is also observed that with the same number of receive antennas, the performance curve of the DBST code over this time-varying fading channel always lies somewhere between the curve in quasistatic fading and that in rapid fading. This observation demonstrates clearly the robustness of DBST codes over the time-selectivity of a fading channel.

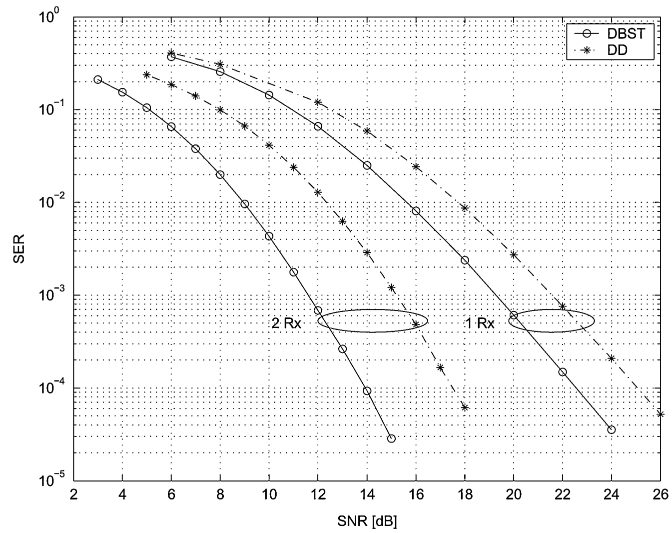

Fig. 6. SER performance of 8PSK codes with $M = 3$ transmit antennas over a time-varying fading channel with $f_d T_s = 0.05$.

To illustrate the performance enhancement of other DBST codes in Tables I–III over DD codes, we select the ones with $M = 2$ transmit antennas and evaluate their required operating SNRs, respectively, at a specified SER of 2×10^{-4} over a rapid fading channel. The results are reported in Table IV, from which

[2]Gray mapping is not necessarily the optimal mapping in space-time codes.

TABLE IV
OPERATING SNR [dB] AT SER $=$ FOR CODES WITH $M = 2$ TRANSMIT ANTENNAS OVER A RAPID FADING CHANNEL

	1 Rx			2 Rx		
	SNR_{DBST}	SNR_{DD}	gain	SNR_{DBST}	SNR_{DD}	gain
QPSK	22	22.24	0.24	12.86	13.13	0.27
8PSK	25.33	27.74	2.41	15.48	18.5	3.02
16PSK*	29.34	33.6	4.26	18.9	24.47	5.57
32PSK	33.15	39.56	6.41	21.84	30.42	8.58
64PSK	35.9	45.5	9.6	14.42	24.76	10.34

* suboptimal code from Table III

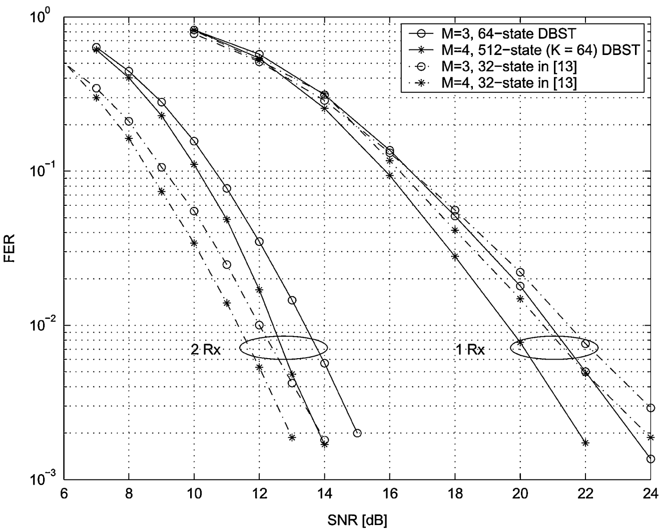

Fig. 7. FER performance of 8PSK codes with $M = 3, 4$ transmit antennas over a quasistatic fading channel.

it is seen that the asymptotic improvement Δ_∞ predicts the actual SNR reduction very well, especially when there is more than one receive antenna.

B. Comparison with Other Existing Codes

Through exhaustive search, the authors of [13] provided several QPSK and 8PSK codes with three and four transmit antennas for quasistatic flat fading channels based on the Euclidean distance criterion. In particular, they designed 8PSK codes with up to 32 trellis states with the order of transmit antenna diversity equal to 2. In this subsection, we discuss a comparison made between the 32-state 8PSK codes in [13] and our 8PSK codes at the same number of transmit antennas. The transmission rate is the same for all the codes, that is, 3 bits/s/Hz, but the number of trellis states is different. Our 8PSK codes with $M = 3$ and 4 have 64 and 512 trellis states, respectively. To make a fair comparison, a suboptimal tree decoding algorithm is applied in the simulation of our 512-state code: the M-algorithm [26]. In this algorithm, only a certain number of most likely states, denoted as K, are kept, and the remaining states are deleted at each decoding stage. Thus, the decoding complexity is $O(K)$. In our case, $K = 64$. Fig. 7 illustrates the FER performance comparison over a quasistatic fading channel with frame length $T = 130$. As can be seen in this figure, a higher diversity order is achieved using our DBST codes. As a result, even though our codes perform less well with two receive antennas, a superior performance is achieved with one receive antenna. This is because full transmit diversity is necessary at high SNR with a limited number of receive antennas, whereas the minimum Euclidean distance is the dominating factor at low SNR with a high enough total diversity order, as claimed in [10] and [13]. The codes in [13] have a much larger minimum Euclidean distance but a smaller transmit diversity order than our codes. Hence, the observation in Fig. 7 is not surprising.

VI. CONCLUSION

In this paper, we proposed an efficient and systematic space-time coding scheme: diagonal block space-time coding. It is basically a two-step approach: First, construct a 1-D nonbinary block code; then, apply the diagonal transmission pattern to send the block code outputs through multiple transmit antennas. It was shown that the diagonal transmission pattern promises a transmit (spatial and temporal) diversity of order M in a system with M transmit antennas under both quasistatic and rapid flat fading channels, whereas a carefully designed nonbinary block code assures good coding advantage. The conventional delay diversity code is a special case of this coding scheme when the block code is a repetition code. To design the optimal block code that maximizes the coding advantage, two general problems were formulated, namely, the permutation optimization for a given constellation and the constellation optimization. In particular, we proposed an efficient linear block code construction over rings for multilevel PSK modulation. Through simple computer search, we obtained some optimal and suboptimal code examples using PSK and QAM modulations with 2–6 bits/s/Hz transmission rate and two to six transmit antennas. Simulation results showed that they possess a significant advantage over the original delay diversity codes in not only quasistatic fading and rapid fading channels but also general time-varying fading channels. They also demonstrate superior performance over existing codes that are optimally designed based on the Euclidean distance criterion with one receive antenna.

The proposed coding scheme is suitable for an arbitrary number of transmit antennas with arbitrary signal constellations. It can also be easily extended to frequency-selective fading channels. The transmission efficiency, which is defined as the number of information symbols transmitted per signaling interval, is equal to 1 symbol/s/Hz. The edge effect due to the diagonal transmission pattern is only $(M-1)/T$, with T being the length of the transmission frame, and can be ignored when $T >> M$. Finally, by changing the rate of the employed block code, this two-step design approach is of high flexibility for future developments with various combinations of transmission efficiency and diversity order.

REFERENCES

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, no. 3, pp. 311–355, Mar. 1998.
[2] I. E. Telatar, "Capacity of multi-antenna gaussian channels," *Eur. Trans. Telecom.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[3] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.
[4] J. C. Guey, M. P. Fitz, M. R. Bell, and W. Y. Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 527–537, Apr. 1999.
[5] S. Baro, G. Bauch, and A. Hansmann, "Improved codes for space-time trellis coded modulation," *IEEE Commun. Lett.*, vol. 4, pp. 20–22, Jan. 2000.
[6] Q. Yan and R. S. Blum, "Optimum space-time convolutional codes," in *Proc. WCNC*, Sept. 2000.
[7] ——, "Improved space-time convolutional codes for quasistatic slow fading channels," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 563–571, Oct. 2002.
[8] X. Lin and R. S. Blum, "Systematic design of space-time codes employing multiple trellis coded modulation," *IEEE Trans. Commun.*, vol. 50, pp. 608–615, Apr. 2002.
[9] D. M. Ionescu, "New results on space-time code design criteria," in *Proc. WCNC*, Sept. 1999.
[10] M. Tao and R. S. Cheng, "Improved design criteria and new trellis codes for space-time coded modulation in slow flat fading channels," *IEEE Commun. Letter*, vol. 5, pp. 313–315, July 2001.
[11] Z. Chen, B. S. Vucetic, J. Yuan, and K. L. Lo, "Space-time trellis codes for 4-PSK with three and four transmit antennas in quasistatic flat fading channels," *IEEE Commun. Lett.*, vol. 6, pp. 67–69, Feb. 2002.
[12] Z. Chen, B. S. Vucetic, K. L. Lo, and J. Yuan, "Space-time trellis codes for 8-PSK with two, three and four transmit antennas in quasistatic flat fading channels," *Electron. Lett.*, vol. 38, no. 10, pp. 462–464, May 2002.
[13] Z. Chen, B. S. Vucetic, J. Yuan, and K. L. Lo, "Space-time trellis codes with two, three and four transmit antennas in quasistatic flat fading channels," in *Proc. ICC*, May 2002.
[14] H. El Gamal and A. R. Hammons Jr., "On the design and performance of algebraic space-time codes for BPSK and QPSK modulation," *IEEE Trans. Commun.*, vol. 50, pp. 907–913, June 2002.
[15] A. R. Hammons Jr. and H. El Gamal, "On the theory of space-time codes for PSK modulation," *IEEE Trans. Inform. Theory*, vol. 46, pp. 524–542, Mar. 2000.
[16] N. Seshadri and J. Winters, "Two signaling schemes for improving the error performance of frequency-division duplex (FDD) transmission systems using transmitter antenna diversity," in *Proc. VTC*, 1993.
[17] A. Wittneben, "A new bandwidth efficient transmit antenna modulation diversity scheme for linear digital modulation," in *Proc. ICC*, 1993.
[18] J. H. Winters, "The diversity gain of transmit diversity in wireless systems with Raleigh fading," *IEEE Trans. Veh. Technol.*, vol. 47, pp. 119–123, Feb. 1998.
[19] S. Li, X. Tao, W. Wang, P. Zhang, and C. Han, "Generalized delay diversity code: A simple and powerful space-time coding scheme," in *Proc. Int. Conf. Commun. Technol.*, 2000.
[20] Z. Safar and K. J. R. Liu, "Systematic design of space-time trellis codes for diversity and coding advantage," *EURASIP J. Applied Signal Process.*, pp. 221–235, Mar. 2002.
[21] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–59, Aug. 1996.
[22] A. Matache, R. D. Wesel, and J. Shi, "Trellis coding for diagonally layered space-time systems," in *Proc. ICC*, May 2002.
[23] G. Caire and G. Colvalope, "On space-time coding for quasistatic multiple-antenna channels," in *Proc. Globecom*, San Antonio, TX, Nov. 2001.
[24] ——, On low-complexity space-time coding for quasistatic channels. *IEEE Trans. Inform. Theory* [Online]. Available: http://www.eurecom.fr/~caire/
[25] D. Gore, S. Sandhu, and A. Paulraj, "Delay diversity code for frequency selective channels," *Electron. Lett.*, vol. 37, no. 20, Sept. 2001.
[26] J. B. Anderson and S. Mohan, "Sequential decoding algorithms: A survey and cost analysis," *IEEE Trans. Commun.*, vol. COM–32, pp. 169–176, Feb. 1984.
[27] R. A. Horn and C. R. Johnson, *Matrix Analysis*: Cambridge Univ. Press, 1985.
[28] G. L. Nemhauser, A. H. G. Rinnooy Kan, and M. J. Todd, *Optimization*. Amsterdam, The Netherlands: North-Holland, 1989.

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