Computationally Efficient Bandwidth Allocation and Power Control for OFDMA

Didem Kivanc, Guoqing Li, and Hui Liu

***Abstract*—This paper studies the problem of finding an optimal subcarrier and power allocation strategy for downlink communication to multiple users in an orthogonal-frequency-division multiplexing-based wireless system. The problem of minimizing total power consumption with constraints on bit-error rate and transmission rate for users requiring different classes of service is formulated and simple algorithms with good performance are derived. The problem of joint allocation is divided into two steps. In the first step, the number of subcarriers that each user will get is determined based on the users' average signal-to-noise ratio. The algorithm is shown to find the distribution of subcarriers that minimizes the total power required when every user experiences a flat-fading channel. In the second stage of the algorithm, it finds the best assignment of subcarriers to users. Two different approaches are presented, the rate-craving greedy algorithm and the amplitude-craving greedy algorithm. Numerical results demonstrate that the proposed low complexity algorithms offer comparable performance with an existing iterative algorithm.**

***Index Terms*—Multiuser, orthogonal-frequency-division multiplexing (OFDM), orthogonal-frequency-division multiplexing (OFDM)-based frequency-division multiple-access (OFDMA), power control, water filling.**

I. Introduction

With HIGH-SPEED wireless services increasingly in demand, there is a need for more throughput per bandwidth to accommodate more users with higher data rates while retaining a guaranteed quality of service. Multiuser power loading and resource allocation strategies allow available resources to be used more efficiently. In this paper, this problem is explored in the context of an orthogonal-frequency-division multiplexing (OFDM)-based frequency-division multiple-access (OFDMA) system. An OFDMA system is defined as one in which each user is assigned a subset of the subcarriers for use, and each carrier is assigned exclusively to one user.

One of the biggest advantages of OFDM systems is the ability to allocate power and rate optimally among subcarriers, using "water filling" over the inverse of the channel spectrum. Computationally efficient algorithms exist to perform discrete water filling for single-user communication [2]. Although it is easy to devise a multiuser water filling algorithm—just give each subcarrier to the user with highest gain on it [6], [7]—the algorithm does not support minimum rate requirements for individual users.

While OFDMA systems have been proposed, power control and bandwidth allocation for these systems is still largely unexplored. Code-division multiple-access (CDMA) coding over multiple carriers (MC-CDMA) can be used to exploit diversity and eliminate the problem of subcarrier allocation. OFDMA systems can potentially outperform MC-CDMA since instead of transmitting over all subcarriers, users can choose to transmit over their best channels. Rohling *et al.* [3] present a simple heuristic greedy algorithm, and show that it performs better than simple banded OFDMA. Wahlqvist *et al.* [8] show that dynamic resource allocation can improve quality of service.

Efforts to exploit the full extent of centralized resource allocation prove to be computationally hazardous. An innovative technique, introduced by Wong *et al.* [1] applies Lagrangian relaxation (LR) to this problem. In LR, the Lagrange method of optimization is used on an integer parameter, which is "relaxed" to take on noninteger values. In this case, the subcarrier assignment function $\rho_k(n)$, which yields one when a user k is assigned subcarrier n and zero otherwise, is allowed to take on any value between zero and one. Despite the significant gain over fixed assignment strategies, the algorithm is computationally intensive and is difficult to implement.

In this paper, a class of computationally inexpensive methods for power allocation and subcarrier assignment are proposed, which achieve comparable performance, but do not require intensive computation. A single cell with one base station and many mobile stations is considered. The algorithms assume perfect information about the channel state due to multipath fading as well as path loss and shadowing effects, and the presence of a multiple-access protocol which will convey information about channel state, subcarrier allocation, etc., to and from the base station and the mobile stations [10]. Section II outlines the system model and some known approaches to this problem [3], [1]. In Section III, the algorithms are described: Bandwidth assignment based on signal-to-noise ratio (SNR), rate-craving greedy (RCG) subcarrier assignment, and amplitude-craving greedy (ACG) subcarrier assignment. Section IV presents the simulation studies that support these results, and the paper is concluded with Section V.

II. System Model and Problem Formulation

The system under consideration is an OFDM system with frequency-division multiple access (FDMA). Perfect channel state information is assumed at both the receiver and the transmitter, i.e., the channel gain on each subcarrier due to path loss, shadowing, and multipath fading is assumed to be known. Channel

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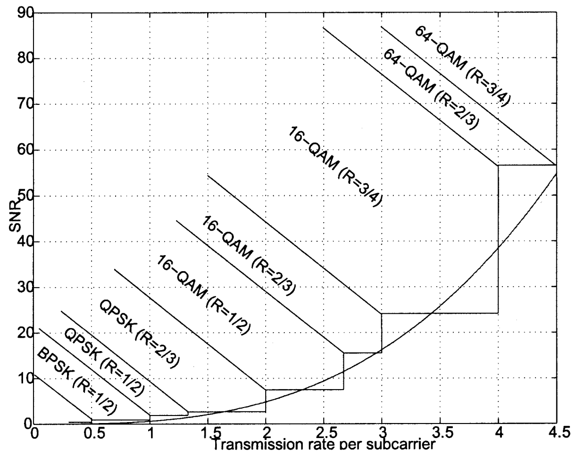

Fig. 1. $f(r)$ for $P_e = 1 \times 10^{-6}$ and continuous approximation $f(r) = 0.60r^3$.

parameters are assumed to be estimated by some other method, which is not specified in this paper. The system does not employ spreading in either time or frequency; each subcarrier can only be used by one user at any given time. Subcarrier allocation is performed at the base station and the users are notified of the carriers chosen for them. After the allocation, each user performs power allocation and bit loading across the subcarriers allocated to it to find the transmission power.

Consider a system with K users and N subcarriers. Each user k must transmit at least $R^k_{\min}$ bits per unit time. Let $H_k(n)$ be the channel gain, $p_k(n)$ the transmission power, and $r_k(n)$ the transmission rate for user k on subcarrier n. The quantities are related by a function $f(r_k(n)) = p_k(n)\,|H_k(n)|^2$. The rate-power function $f(\cdot)$ depends on the minimum bit-error rate that can be tolerated P_e and the available coding and modulation schemes. Fig. 1 shows the function used in simulations and the continuous function approximation used for some of the algorithms. A subcarrier can transmit at most $R_{\max}$ bits per unit time.

The objective is to find a subcarrier allocation which allows each user to satisfy its rate requirements while using minimum power

$$\min P_T = \sum_{n=0}^{N-1}\sum_{k=0}^{K-1} p_k(n) \text{ s.t. } \sum_{n=0}^{N-1} r_k(n) \geq R^k_{\min},\ \forall k$$
$$\sum_{k=1}^{K-1} \delta[r_k(n)] \leq 1,\ \forall n \tag{1}$$

where $\delta[\cdot]$ is the Kronecker delta function.

A. Some Previous Approaches

1) Fixed Assignment Methods: The simplest approach to subcarrier assignment is to ignore channel information and allocate carriers to users proportional to their rate requirements. Subcarriers can be allocated in consecutive chunks (bands) or interleaved to improve frequency diversity.

2) Single-Step Frequency Allocation (SSFA) Algorithm: The simple centralized frequency allocation algorithm proposed in [3] will be referred to as the SSFA algorithm in this paper. In this algorithm, the user requests N_k carriers, proportional to $R^k_{\min}$. The base station first makes a list V_k of favorite subcarriers for each user k. In each stage, a subcarrier is allocated to the user with the lowest ratio of allocated to requested carriers, going down the favorites list for the user. For each user k there is also a list A_k of the n_k carriers already allocated and the $N_k - n_k$ carriers that could still potentially be allocated. When a user requests a carrier that is already allocated, the carrier is given to the user with the highest accumulated relative power loss. The power lost by user k from not getting a subcarrier $V_k(i)$ is defined as $L(V_k(i), V_k(j)) = |H_k(V_k(i))|^2$, where $V_k(j)$ is the carrier user k will get instead of $V_k(i)$. The accumulated power loss h^k_{acc} is the sum of $L(V_k(i), V_k(j))$ for all carriers lost by user k until that stage, and the accumulated relative power loss is defined as

$$P^k = \frac{|H_k(V_k(i))|^2 - |H_k(V_k(j))|^2 + h^k_{\text{acc}}}{\sum_{n=1}^{n_k} |H_k(A_k(n))|^2}. \tag{2}$$

3) LR Algorithm: The mixed integer optimization problem in (1) can be solved using the LR algorithm [1]. The algorithm approaches the solution by slowly increasing the power level for each user. Each user is given a power coefficient λ_k, which determines their transmit power. This is not a cap on the total power allocated to the user, but related more closely to the "water level" in single-user water filling. λ_k has the dual role of regulating both the subcarrier allocation and the total transmission power for each user. The algorithm iterates, by incrementing λ_k by $\Delta\lambda$ for the user who needs the rate increase the most, reassigning channels, and finding the new rates [1].

Though iterative, this algorithm does converge to a good solution. Unfortunately, there are drawbacks to this algorithm due to the nonlinear nature of the integer problem. The algorithm requires a large number of iterations to converge. When the algorithm is forced to stop after a fixed number of iterations, even when the number of iterations is large, the resulting solution is not close to the final result since the convergence to the optimal result is not smooth. The accuracy of the result and the speed of convergence can be controlled by varying the increment $\Delta\lambda$. For small $\Delta\lambda$, the convergence is slow but the result is accurate; for larger $\Delta\lambda$, accuracy improves and smaller total power can be achieved but at greater cost.

III. Sensible Greedy Approach

The problem posed by (1) is computationally intractable, and as described above, a direct approach to solving it does not yield a good algorithm. This paper examines two algorithms which use information about users' channel and rate requirements to find a close approximation to the solution.

Intuitively, the problem is separated into two stages.

1) *Resource Allocation*: Decide the number of subcarriers each user gets—its bandwidth—based on rate requirements and the users' average channel gain.
2) *Subcarrier Allocation*: Use the result of the resource allocation stage and channel information to allocate the subcarriers to the users.

By solving these subproblems separately, a good, but not necessarily optimal, solution is found which guarantees a certain level of service for each user.

A. Resource Allocation Algorithm

In a wireless environment, some users will see a lower overall SNR than other users. These users tend to require the most power. Studying the subcarrier allocations from the LR algorithm shows that once users have enough subcarriers to satisfy their minimum rate requirements, giving more subcarriers to users with lower average SNR helps to reduce the total transmission power. This section describes the bandwidth assignment based on SNR (BABS) algorithm which uses the average SNR for each user to decide the number of subcarriers that user will be assigned.

Consider the problem described in Section II, but assume that each user k experiences a channel gain of $H_k = \left(\sum_{n=0}^{N-1} |H_k(n)|^2 / N\right)$ on every subcarrier. Let user k be allocated m_k subcarriers. When the gain on every subcarrier is the same, the optimal rate-power allocation is to transmit $R_{\min}^k / m_k$ bits on each subcarrier, resulting in total transmission power $m_k f(R_{\min}^k / m_k)/H_k$. The objective is to find a set of m_k subcarriers $k = 1, \ldots, K$ which satisfy

$$\begin{aligned} &\min \sum_{k=0}^{K-1} \frac{m_k}{H_k} f\left(\frac{R_{\min}^k}{m_k}\right) \\ &\text{s.t.} \sum_{k=0}^{K-1} m_k = N \\ &m_k \in \left\{ \left\lceil \frac{R_{\min}^k}{R_{\max}} \right\rceil, \ldots, N \right\}, \forall k. \end{aligned} \tag{3}$$

To find the optimal distribution of subcarriers among users given the flat channel assumption, a greedy descent algorithm is proposed, similar to discrete water filling, shown as Algorithm 1.

Algorithm 1 BABS Algorithm

$$m_k \leftarrow \left\lceil \frac{R_{\min}^k}{R_{\max}} \right\rceil, \qquad k = 0, \ldots, K-1$$

while $\sum_{k=0}^{K-1} m_k > N$ **do**

$$k^* \leftarrow \arg\min_{0 \le k \le K-1} m_k$$

$$m_{k^*} \leftarrow 0$$

end while

while $\sum_{k=0}^{K-1} m_k < N$, **do**

$$G_k \leftarrow \frac{m_k + 1}{H_k} f\left(\frac{R_{\min}^k}{m_k + 1}\right) - \frac{m_k}{H_k} f\left(\frac{R_{\min}^k}{m_k}\right), \qquad k = 0, \ldots, K-1$$

$$l \leftarrow \arg\min_{0 \le k \le K-1} G_k$$

$$m_l \leftarrow m_l + 1.$$

end while

In Appendix A, this algorithm is shown to converge to the distribution of subcarriers among users that solves (3), and thus, (1) for the special case when the subcarriers for a given user all experience identical fading gains $|H_k(n)|^2 = H_k \; \forall k, n$ given the following.

- There is enough bandwidth to satisfy the users' rate requirements
$$\sum_{k=0}^{K-1} \left\lceil \frac{R_{\min}^k}{R_{\max}} \right\rceil \le N.$$
- The total power that a user needs decreases as the number of subcarriers allocated increases, i.e., $\Delta_k(m_k) = (m_k + 1) f(R_{\min}^k / (m_k + 1)) - m_k f(R_{\min}^k / m_k)$ is negative definite.
- This decrease in power is largest when being allocated the first subcarrier, and decreases there on, i.e., $\Delta_k(m_k)$ is uniformly increasing in m_k.

The last two conditions are true for fading channels when the rate-power function $f(\cdot)$ is strictly convex and uniformly increasing, as shown in Fig. 1.

B. Subcarrier Assignment Algorithms

Once the number of subcarriers is determined, the next step is to assign specific subcarriers to users. As shown in Appendix B, the problem is still difficult to solve since different users see different channels. In this section, two suboptimal algorithms are proposed to allocate subcarriers to users. The RCG algorithm begins with an estimate of the users' transmission rate on each carrier and aims to maximize the total transmission rate. The ACG algorithm is a modification of RCG which achieves comparable performance at reduced computational complexity.

1) RCG Algorithm: Since the number of subcarriers assigned to user k by the BABS algorithm, m_k must be greater than $\lceil R_{\min}^k / R_{\max} \rceil$, the condition that $\sum_{n=0}^{N-1} r_k(n) \ge R_k$ can be satisfied whenever $\sum \delta[r_k(n)] = m_k$, and $p_k(n)$ is large enough on each carrier. After the subcarriers are allocated to the users, the optimal transmission rate for user k on carrier n will be $r_k^*(n) = f'^{-1}(\lambda_k |H_k(n)|^2)$, where λ_k is the familiar "water level" parameter from the single-user water filling algorithm. Let $u[\cdot]$ be the unit step function. Then the condition $\sum_{n=0}^{N-1} r_k(n) \ge R_k$ can be replaced by the conditions $\sum \delta[r_k(n)] = m_k$ and $r_k(n) = r_k^*(n) u[r_k^*(n)]$ whenever subcarrier n is allocated to user k. Since the rate constraint has been replaced by a power constraint, the objective function is also transformed from power to rate. This approximation simplifies the problem slightly, and it becomes a version of the well-known combinatorial set partitioning problem. Let $\mathcal{A}$ be any partition of the set of subcarriers, $\{1, \ldots, N\}$ into K sets, $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_N$. The problem can be rephrased as

$$\begin{aligned} &\max_{\mathcal{A}} \sum_{k=0}^{K-1} \sum_{\{n \in \mathcal{A}_k\}} r_k(n) \\ &\text{s.t.} \; \#\mathcal{A}_k = m_k, \qquad k = 0, \ldots, K-1 \end{aligned} \tag{4}$$

where $\#\mathcal{A}_k$ denotes the cardinatlity of set $\mathcal{A}_k$.

λ_k is not known in advance; however, it can be estimated based on the average SNR. For the model used in simulations

$f(r) = 0.6r^3$ $\lambda_k^* = \left(R_{\min}^k / \sum_{m=0}^{N-1} |H_k(m)|\right)^2$ is used, and $r_k(n) = |H_k(n)|\, R_{\min}^k / \sum_{m=0}^{N-1} |H_k(m)|$.

Problem (4) can be solved by the following algorithm:

- Allocate each carrier n to the user with maximum transmission rate $r_k(n)$.
- While there exists some user k such that $\#\mathcal{A}_k > m_k$, remove a subcarrier from user k and add a subcarrier to a user l such that $\#\mathcal{A}_l < m_l$ using a sequence of reallocations: give carrier n from user k to user k_1, give carrier n_1 from user k_1 to user k_2,..., give carrier n_p from user k_p to user l.

The above algorithm is computationally intensive, since it involves finding the shortest path through a K–node graph and recalculating graph weights in each iteration. However, a family of suboptimal rate-craving (RC) algorithms can be found where algorithm RC-p searches for only p-stage reallocations, with $p < K$. In particular for $p = 1$ (nearest neighbors search), the algorithm can be greatly simplified. The simplified version of the algorithm is called the RCG algorithm and is outlined in Algorithm 2.

Algorithm 2 RCG Algorithm

Ensure: m_k is the number of subcarriers allocated to each user, $r_k(n) = f'^{-1}(\lambda_k^* |H_k(n)|^2)$ is the estimated transmission rate of user k on subcarrier n, $C_k \leftarrow \{\}$, for $k = 0, \ldots, K-1$.

for each subcarrier $n = 0 : N-1$, **do**

$$k^* \leftarrow \arg\max_{0 \le k \le K-1} r_k(n)$$
$$C_{k^*} \leftarrow C_{k^*} \cup \{n\}$$

end for

for all users k such that $\#C_k > m_k$ **do**

while $\#C_k > m_k$ **do**

$$l^* \leftarrow \arg\min_{\{l:\#C_k < m_k\}} \min_{0 \le n \le N-1} -r_k(n) + r_l(n)$$
$$n^* \leftarrow \arg\min_{0 \le n \le N-1} -r_k(n) + r_{l^*}(n)$$
$$C_k \leftarrow C_k \backslash \{n^*\}, C_{l^*} \leftarrow C_{l^*} \cup \{n^*\}$$

end while

end for

Fig. 2 demonstrates an example run of the algorithm for $N = 8$ subcarriers and $K = 4$ users which require two subcarriers each. Columns represent different users, rows are different subcarriers. The number in row n column k represents an estimate of the rate at which user k would transmit on n if it were allocated that carrier. The circled numbers show to whom each subcarrier is allocated.

2) ACG Algorithm: The aim of the ACG algorithm is to see if an even simpler algorithm can provide comparable results to the RCG algorithm. If the individual users do not have to transmit at some minimum rate, and the goal was simply to maximize the overall volume of data transmitted, there is a simple algorithm which will accomplish this: For each carrier n, find the user $\widehat{k} = \arg\max_k |H_k(n)|$ with maximum gain on that carrier. User $\widehat{k}$ is allocated that carrier, and transmits at rate $R_{\max}$.

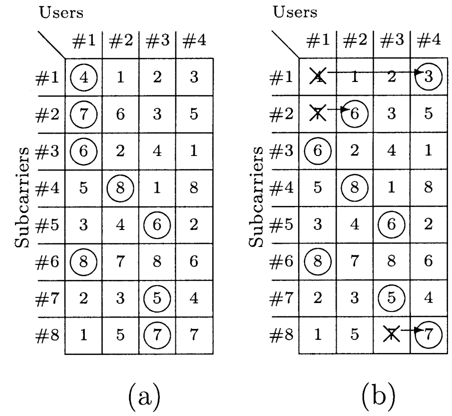

Fig. 2. Carrier allocation by RCG algrorithm at the end of (a) stage 1 and (b) the algorithm. The number in column k row n is the estimated rate of transmission of user k on subcarrier n, if it is allocated that subcarrier. The circled rate has been chosen.

The motivation behind the ACG algorithm (Algorithm 3) is to use this basic idea but modify it slightly to allow individual users to satisfy their minimum service requirements.

- Each user can only get m_k subcarriers. Once it is allocated m_k subcarriers, it cannot bid for any more.
- The users' average channel gains are normalized to one, so that users with lower power can have a fair chance when bidding against more powerful users. Otherwise, the results of the scheme will resemble a simple banded FDMA scheme with the most powerful user getting the first block of subcarriers.
- The carriers are not processed in order (from subcarrier 0 to $N-1$), but in some random order (e.g., 4, 111, 70, 35,...), to counteract correlation between adjacent subcarrier gains.

Algorithm 3 ACG Algorithm

Ensure: m_k is the number of subcarriers allocated to each user $C_k \leftarrow \{\}$ for $k = 0, \ldots, K-1$.

for each subcarrier $n = 0 : N-1$, **do**

$$k^* \leftarrow \arg\max_{0 \le k \le K-1} |H_k(n)|^2$$

Let $\#C_k$ denote the cardinality of set C_k

while $(\#C_{k^*} = m_{k^*})$ **do**

$$|H_{k^*}(n)|^2 \leftarrow 0$$
$$k^* \leftarrow \arg\max_{0 \le k \le K-1} |H_k(n)|^2$$

end while

$$C_{k^*} \leftarrow C_{k^*} \cup \{n^*\}$$

end for

TABLE I
ORDER OF THE COMPLEXITY OF THE ALGORITHMS

Method	Order of operations
BABS	$\mathcal{O}(KN)$
ACG	$\mathcal{O}(KN)$
RCG	$\mathcal{O}(KN + NlogN)$
SSFA	$\mathcal{O}(KNlogN + NK)$

C. Algorithmic Complexity

In this section, the worst case performance of each algorithm is studied as a function of the number of users K, and the number of subcarriers N.

LR: Each iteration of the LR algorithm requires $N - m_l$ inversions of a nonlinear function, and $m_l + 1$ evaluations of $f(\cdot)$ and $f'^{-1}(\cdot)$, where m_l is the number of subcarriers that is assigned to the user being evaluated in that iteration. While these function evaluations may be performed using table lookup, the number of iterations of the algorithm can grow large particularly as the number of users increases and as the users' powers become relatively unbalanced. The worst case cost of each iteration is $\mathcal{O}(N)$, but the number of iterations is much greater than both K and N.

SSFA Algorithm: The algorithm first sorts the users' channel gains, $\mathcal{O}(KN\log(N))$. The algorithm cycles through the favorites list of each user, which requires N iterations, with, at most, K comparisons in each iteration. The algorithm is $\mathcal{O}(KN\log N + KN)$.

BABS Algorithm: The BABS algorithm iterates N times, and in each stage requires K function evaluations and K comparisons. The algorithm is $\mathcal{O}(KN)$.

RCG Algorithm: The initialization step requires KN function evaluations to find $r_k(n)$, and KN comparisons. The second half of the algorithm, which involves the reassignment of subcarriers, can be shown to be $\mathcal{O}(KN + N\log N)$ overall. In practice, the number of subcarriers to be reassigned is usually much less than N.

ACG Algorithm: The ACG algorithm also iterates N times, but with only $2K$ comparisons in each step. It is $\mathcal{O}(KN)$.

Table I compares the order of the complexity of different algorithms.

IV. SIMULATIONS

The system under consideration has parameters given in Table II. The channel model used is based on the Saleh–Valenzuela multipath fading model [12]. A double exponential channel model is used, with independent fading based on distance from the base station

$$h(t) = \sum_{t=1}^{\infty}\sum_{k=0}^{\infty} \beta_{k,l} e^{j\phi_{k,l}} \delta(t - T_l - \tau_{k,l}) \qquad (5)$$

where $\phi_{k,l}$ is uniformly distributed on [0, 2π), $\beta_{k,l}$ is Rayleigh distributed with mean square value $\overline{\beta^2_{k,l}} = \overline{\beta^2(0,0)} e^{-T_l/\Gamma} e^{-\tau_{k,l}/\gamma}$, and the cluster and ray arrival times T_l and $\tau_{k,l}$ are distributed with exponential interarrival times

$$p\big(T_l | T_{l-1} = \Lambda e^{-\Lambda(T_l - T_{l-1})}\big)$$
$$p\big(\tau_{k,l} | \tau_{k-1,l} = \lambda e^{-\lambda(\tau_{k,l} - \tau_{k-1,l})}\big)$$

TABLE II
SYSTEM PARAMETERS

Bandwidth	4 MHz
No. of subcarriers	512
Rate per subcarrier	7.81 KSymbols/s
Modulation	BPSK, QPSK, 16-, 64-QAM
Convolutional codes	Rate 1/2, 2/3, 3/4

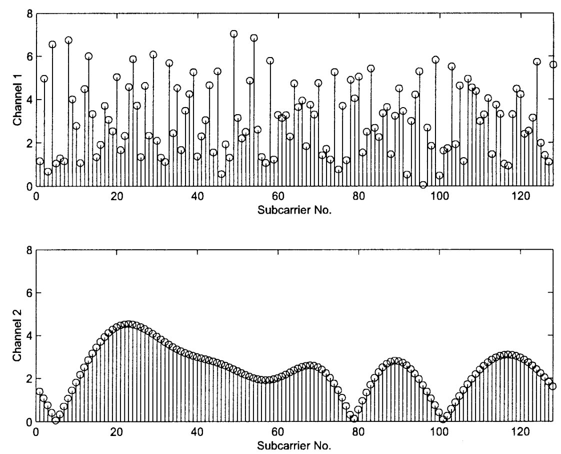

Fig. 3. Two example channel profiles for parameters used.

Two sets of channel parameters are studied.

- Channel 1 is highly uncorrelated with only a single cluster at $T_l = 0$ and ray arrival parameters $\gamma = 3.75 \times 10^{-6}$, $\lambda = 3.0 \times 10^{-6}$.
- Channel 2 is highly correlated with parameters $\Gamma = 336 \times 10^{-9}$, $1/\Lambda = 168 \times 10^{-9}$, $\gamma = 286 \times 10^{-9}$, $1/\lambda = 51 \times 10^{-9}$.

The channel profile is then normalized to model slow fading for users who are distributed with a two-dimensional Gaussian distribution around the base station. The total power received by the base station from user k is given by $P_k = \sum_{n=0}^{N-1} |H_k(n)|^2 = P_0 d_k^{-2.5}$, where d_k is the ratio distance between the user and the base station to the cell radius. Choosing $P_0 = 3.578$ allows an SNR ≥ 10 dB for 75% of all users. Two example channel profiles are shown in Fig. 3.

The coding and modulation schemes used are shown in Table II. Adaptive modulation is used both with and without power loading of subcarriers in simulations. While power-based loading increases the throughput, it is computationally costly and may not be used in a practical system [11].

Three classes of users are considered: data, voice, and video. It is assumed that 10% of the users will transmitting video, 40% will be transmitting voice, and the remaining 50% of the users will be transmitting data. Video and voice traffic are given a constant transmission rate of 64 and 16 kb/s, respectively, whereas

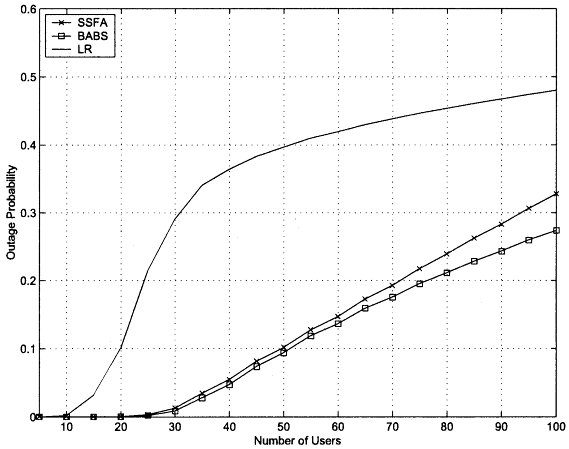

Fig. 4. Adaptive modulation with water filling outage probability versus number of users.

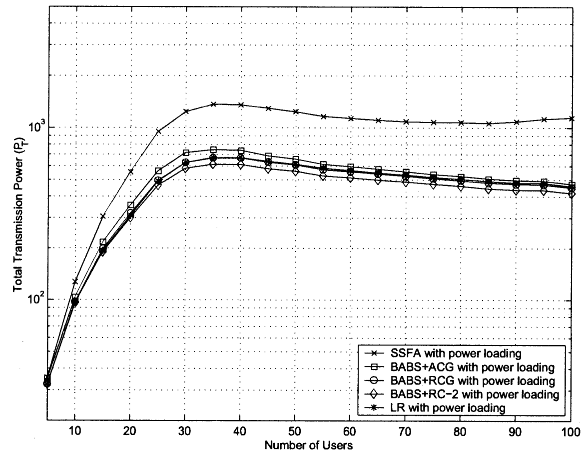

Fig. 5. Adaptive modulation for Channel 1 with power-based water filling, total power transmitted versus number of users, same outage probability for all methods.

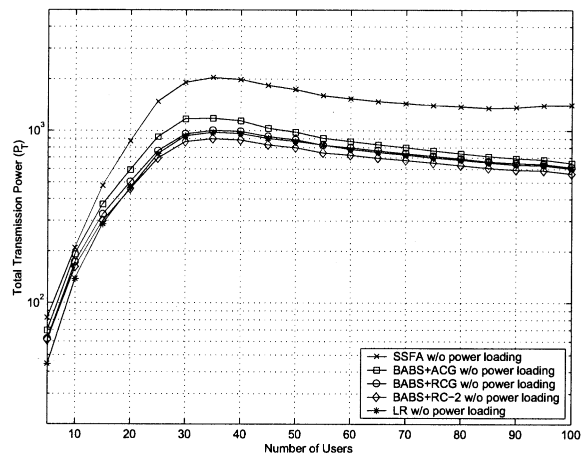

Fig. 6. Adaptive modulation for Channel 1 without power-based water filling, total power transmitted versus number of users, same outage probability for all methods.

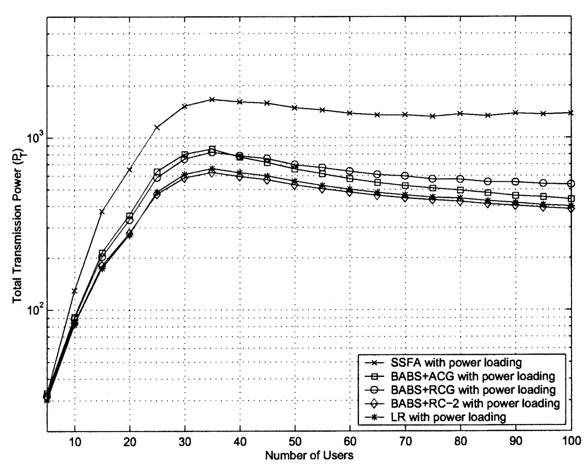

Fig. 7. Adaptive modulation for Channel 2 with water filling, total power transmitted versus number of users, same outage probability for all methods.

data traffic is assumed to be exponentially distributed, with a mean of 30 kb/s.

In simulations, the sensible greedy algorithms, BABS with ACG subcarrier assignment (BABS-ACG), BABS with RCG subcarrier assignment (BABS-RCG), and BABS-RC-2, described in Section III, are compared to the SSFA and the LR algorithm described in Section II-A. To speed up the LR algorithm, the nonlinear function inversion have been implemented through table lookup, and constraints relaxed to $\rho_k(n) \in \{0, 1/2, 1\}$ during iterations, but only integer values are retained in the final solution [1]. As in [1], all algorithms are followed by running the single-user power allocation algorithm for each user. A slightly modified version of the RCG algorithm, which searches all two-step reallocation procedures as well as the one-step reallocations searched by RCG was also implemented. This algorithm is labeled RC-2, and since it is closer to the full search required by the RCG algorithm, it results in a better subcarrier distribution.

The algorithms are compared based on three criteria: the probability that a user k is transmitting less than $R^k_{\min}$ bits per unit time, the total transmission power P_T, and the computational complexity used in central processing unit (CPU) cycles. A total of 1×10^4 frames are simulated, each with a different channel response and user profile.

The outage probability is about the same for all channel and subcarrier loading strategies used. The probability for Channel 1, with power loading is shown in Fig. 4. As shown, the outage probability of LR is 2–3 times that of BABS and SSFA. The relaxation of the $\rho_k(n)$ parameter causes a problem here, and a user which attains its rate by sharing subcarriers may end up with too few when the integer constraint is tightened. The outage probability of the greedy algorithms depends only on the number of subcarriers allocated to each user in the BABS algorithm, since there is no constraint on power. While the outage probability of SSFA is higher, the total number of bits transmitted per unit time is the same for SSFA and the BABS algorithms. By isolating resource assignment from subcarrier assignment, the BABS algorithm allows the system designer to decide whether to drop users or reduce rates, and who to drop based on fairness and other requirements.

The discrepancy between the outage probabilities makes it difficult to compare different methods based on total power requirements, since the LR algorithm simply "drops" users with high power requirements, resulting in much lower total

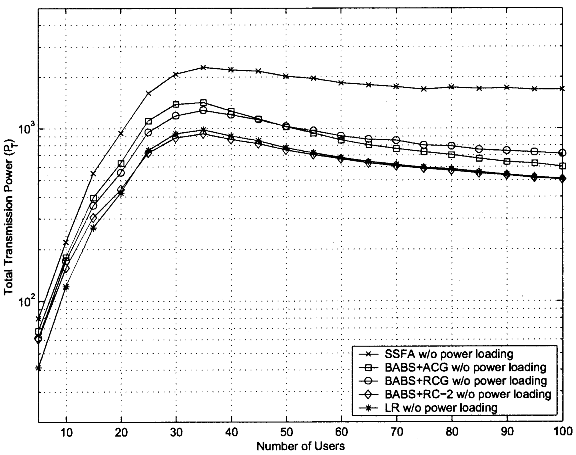

Fig. 8. Adaptive modulation for Channel 2 without water filling, total power transmitted versus number of users, same outage probability for all methods.

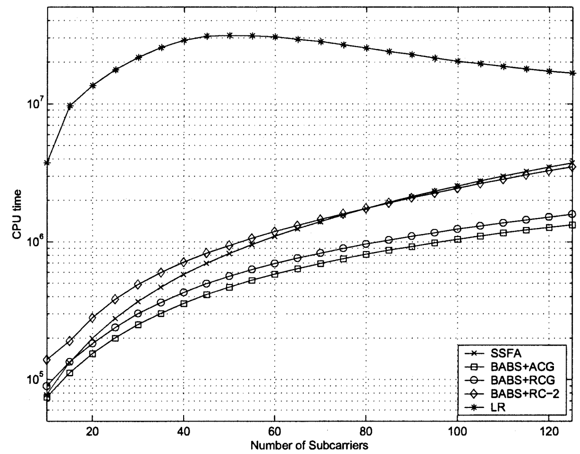

Fig. 10. Adaptive modulation with water filling, average CPU time required versus number of subcarriers (ten users).

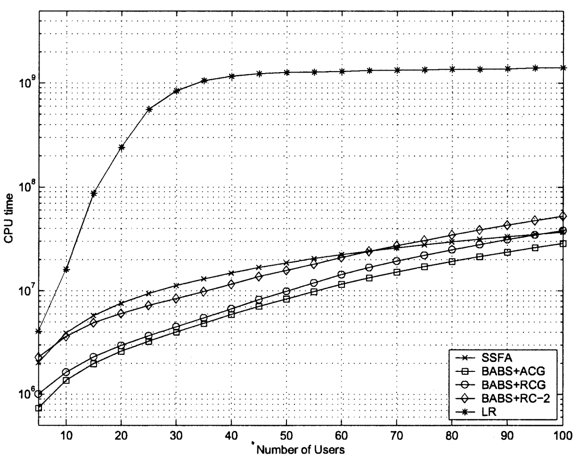

Fig. 9. Adaptive modulation with water filling, average CPU time required versus number of users (128 subcarriers).

transmit power. To allow a fair comparison, the LR algorithm was run first to determine the rates that the algorithm could satisfy. Then SSFA, BABS-ACG, BABS-RCG, and BABS-RC-2 were run for the same channel conditions, to transmit the same rates (Figs. 5–8). The LR and RC-2 algorithms require lowest power. In certain cases, RC-2 finds a better allocation than LR, due to factors such as a high $\Delta\lambda$ parameter, and the effect of first allowing users to share carriers then allocating the shared carrier to a single user. The BABS-RCG algorithm finds good allocations for channels with low subcarrier correlation, but not for Channel 2, where the nearest neighbors search used by the RCG algorithm is not thorough enough. By also searching for two-stage reallocations, the BABS-RC-2 algorithm still finds a subcarrier allocation with very low power. Both BABS-ACG and BABS-RCG do considerably better than SSFA.

Figs. 9 and 10 compare the computational complexity of the algorithms. These plots aid the comparison in two respects. First, the number of iterations required by the LR algorithm is difficult to predict, and thus, a theoretical comparison is not possible. Second, while the upper bound for the number of iterations the RCG algorithm requires is $\mathcal{O}(KN + N\log N)$, in practice, the number of subcarrier reassignments required in Step 2 is not very large, and the algorithm executes much more quickly than could be expected. The two methods were coded in C, the platform for implementation was an AMD Duron-based personal computer running Linux with 42.9 SPECint_95 and 29.4 SPECfp_95 ratings. The algorithms were tested on the same 300 channel conditions and rate requirements. Plots of the required CPU time show that both greedy algorithms perform an order of magnitude faster than the iterative LR algorithm, but BABS-ACG performs about twice as fast as BABS-RCG, demonstrating that the worst case performance is a pessimistic lower bound for the RCG algorithm. The SSFA algorithm is about twice as slow as the others, and has about the same computational complexity as BABS-RC-2.

V. Conclusion

Fast and efficient multiple-access algorithms allow mobile networks to adapt quickly to changes in the environment. In this paper, a computationally efficient class of algorithms for allocating subcarriers and power among users in a multicarrier system has been described. Dividing the problem into two stages enabled the design of algorithms with low computational complexity, which operate well under realistic channel and data traffic assumptions. This approach allows efficient use of system resources in terms of transmission power, bandwidth efficiency, and computational time. Simulations show that the algorithms yield low outage probability and low power requirements at reasonable complexity, showing that a good resource allocation strategy can be achieved by efficient algorithms in a practical system. The feasibility of the algorithms will depend on factors such as how quickly the channel varies, the accuracy and overhead of the channel estimation algorithm, and the latency in the multiple-access protocol. The interaction of power and rate control protocols with such factors is a course for further study.

Appendix A
Proof That the BABS Algorithm is Optimal

Assumption: $(m_k+1)f(R_{\min}^k/(m_k+1)) - m_k f(R_{\min}^k/m_k)$ is a negative definite, monotonically increasing function of m_k for all users k.

Let $\{m_k\}$ be the distribution of carriers found by the algorithm, define $G_k(m) = (m/H_k)f(R^k_{\min}/m)$. Assume that there exists another distribution $\{n_k\}$ such that $\sum_{k=0}^{K-1} G_k(n_k) < \sum_{k=0}^{K-1} G_k(m_k)$, which differs from $\{m_k\}$ in at least one term. Then $\exists p, q$ such that $n_p > m_p$ and $n_q < m_q$. By assumption, $G_q(m_q+1)-G_q(m_q) > G_q(n_q+1)-G_q(n_q)$ and $G_p(m_p) - G_p(m_p-1) \leq G_p(n_p) - G_p(n_p-1)$.

Since distribution, $\{n_k\}$ is optimal if a carrier is reassigned from user p to user q, the total power will increase, i.e., $G_q(n_q+1) - G_q(n_q) > G_p(n_p) - G_p(n_p-1)$. Thus, $G_q(m_q+1) - G_q(m_q) > G_p(m_p) - G_p(m_p-1)$. But at some stage in the algorithm, user p was allocated one final carrier, and at that stage, user p had $\widetilde{m}_p = m_p$ carriers and user q had $\widetilde{m}_q \leq m_q$ carriers. By virtue of the algorithm and the assumption

$$\begin{aligned} G_p(m_p) - G_p(m_p-1) &\geq G_p(\widetilde{m}_p+1) - G_p(\widetilde{m}_p) \\ &\geq G_q(m_q+1) - G_q(m_q). \end{aligned}$$

Thus, there is a contradiction. The BABS algorithm finds a minimum power subcarrier allocation for flat-fading channels.

APPENDIX B
OPTIMAL RATE-CRAVING ALGORITHM

A particular interpretation of the problem and the pseudocode in Section III-B1 is used to derive the RCG algorithm. In this section, this algorithm is described and proven to find the allocation with maximum transmission rate.

Let $\mathcal{A}$ be a partition of subcarriers $\{1,\ldots,N\}$ into K sets $\mathcal{A}_1,\ldots,\mathcal{A}_K$ (an allocation of N subcarriers to K users). Define $\mathcal{G}(\mathcal{A}) = (V, E(\mathcal{A}))$ to be a directed graph with K nodes and $K(K-1)$ edges. Each node k represents a set $\mathcal{A}_k$ of the partition (a user). Each edge $e_{k,l}$ represents taking some subcarrier n from node k and reassigning it to node l and has weight $w(e_{k,l}) = \min_{n\in\mathcal{A}_k} r_k(n) - r_l(n)$, since this is the rate lost if the reallocation is carried out. Any path through this graph is denoted by a string of nodes $k_1, k_2, \ldots, k_P$. Define $Rate(\mathcal{A}) = \sum_{k=0}^{K-1}\sum_{n\in\mathcal{A}_k} r_k(n)$.

Theorem: The following algorithm results in the partition of N subcarriers into K sets $\mathcal{A}_k$ such that each set $\mathcal{A}_k$ contains exactly m_k subcarriers and $Rate(\mathcal{A})$ is maximized.

1) Initialize the partition $\mathcal{A}$ by assigning subcarrier n to user $k^* = \arg\max_k \ r_k(n)$.
2) While there exists a node (user) k such that $\#\mathcal{A}_k > m_k$, follow the minimum weight reassignment in $\mathcal{G}(\mathcal{A})$ from node k to any node l such that $\#\mathcal{A}_l < m_l$, update $\mathcal{G}(\mathcal{A})$.

Lemma 1: For any two partitions $\mathcal{A}$ and $\mathcal{B}$ of N subcarriers into K sets such that $\#\mathcal{A}_k = \#\mathcal{B}_k \ \forall k$, if $\exists n_1 \in \mathcal{A}_1$ such that $n_1 \notin \mathcal{B}_1$, then there exist $2 \leq P \leq K$ subcarriers $n_k \in \mathcal{A}_k$ and $n_k \in \mathcal{B}_{\Pi(k)}$ for $k = 1, 2, \ldots, P$, where Π is some permutation of $(1, 2, \ldots, P)$ such that $\Pi(k) \neq k$.

Proof (by Construction): Construct a table with N rows, and two columns. The first column of row r contains $k_{\mathcal{A}}$ such that subcarrier $r \in \mathcal{A}_{k_{\mathcal{A}}}$; the second column contains k_B such that $r \in \mathcal{B}_{k_{\mathcal{B}}}$. Let $T(n,q)$ denote the element in row n and column q. Cross out all rows of the table which have the same number k in both columns.

Form a list of subcarriers (S_{car}) and users (S_Π): Start with $S_{car} = \{\}$ and $S_\Pi = \{n_1\}$. Let $k_{\text{next}} = T(n_1, 2)$. Find an occurrence of k_{next} in column 1 (row n_2). Add n_2 to S_{car} and k_{next} to S_Π. Let $k_{\text{next}} = T(n_2, 2)$, and continue until $k_{\text{next}} = T(n_1, 1)$.

The lists, S_{car} and S_Π, are ordered so that $S_{car}(k) \in \mathcal{A}_{S_\Pi(k)}$, $S_{car}(k) \in \mathcal{B}_{S_\Pi(k+1)}$, and $S_\Pi(k) \neq S_\Pi(k+1)$, for $k = 1, \ldots N'$. Since every n must appear in both columns at least once and $\mathcal{A} \neq \mathcal{B}$, $N' \geq 2$.

Lemma 2: Let $\mathcal{A}^0$ denote the partition at the end of Stage 1. The updated graph after the Mth iteration of the second step, $\mathcal{G}(\mathcal{A}^M)$ does not contain any circuits with negative weight. As a result, a standard shortest path algorithm, such as the Ford–Bellman algorithm, will find the minimum weight path from node k to a node l such that $\#\mathcal{A}_l < m_l$.

Proof (by Induction): Consider $\mathcal{G}(\mathcal{A}^0)$. A link connecting l_1 to l_2 will have weight $r_{l_1}(n) - r_{l_2}(n)$, which is strictly positive because the algorithm is greedy. Since no link in $\mathcal{G}(\mathcal{A}^0)$ can have negative weight, no circuit can have negative total weight.

Assume that after the $M-1$th iteration of Step 2, the graph does not contain any circuits with negative weight. The Mth iteration of the algorithm chooses the path $\mathcal{R} = (r_1, r_2, \ldots, r_R)$. Assume there exists a circuit $\mathcal{C} = (c_1, c_2, \ldots, c_C, c_1)$ on $\mathcal{G}(\mathcal{A}^M)$ with negative weight. Since $\mathcal{C}$ is not on $\mathcal{G}(\mathcal{A}^{M-1})$, $\exists r_b \in (\mathcal{R} \cap \mathcal{C})$. Without loss of generality, assume $r_b = c_1$. Carrier n is assigned from r_{b-1} to c_1 in $\mathcal{R}$, and from c_1 to c_2 in $\mathcal{C}$.

$\mathcal{R}$ is the minimum weight path from r_1 to r_R by definition of the algorithm. But consider the path $\mathcal{R} \cup \mathcal{C} = \{r_1, \ldots, r_{q-1}, c_2, \ldots, c_C, c_1, r_{q+1}\}$. $Rate(\mathcal{R} \cup \mathcal{C}) = Rate(\mathcal{R}) + Rate(\mathcal{C}) < Rate(\mathcal{R})$. This raises a contradiction.

Proof of the Theorem: Suppose a partition $\mathcal{B}$ maximizes $Rate(\mathcal{B}) > Rate(\mathcal{A}^M)$ and $\exists\ n_1 \in \mathcal{A}_1^M$, where $n_1 \notin \mathcal{B}_1$. Then, by Lemma 1, $\exists P$ subcarriers $n_1, \ldots, n_P$ such that $n_k \in \mathcal{A}_k^M$ and $n_k \in \mathcal{B}_{\Pi(k)}$, where $\Pi(k) \neq k$. Since $\mathcal{B}$ has the maximum rate

$$\sum_{k=1}^{P} r_{\Pi(k)}(n_k) > \sum_{k=1}^{P} r_k(n_k). \tag{6}$$

Consider $\mathcal{G}(\mathcal{A}^M)$, where M is the last iteration of Step 2. From (6), $n_1, \Pi(n_1), \ \Pi(\Pi(n_1)), \ldots, \ \Pi(\Pi(\ldots\Pi(n_1)\ldots))$ form a circuit with negative weight on this graph. But by Lemma 2, a circuit with negative weight cannot exists. Thus, there is no subcarrier allocation which has a higher rate than $\mathcal{A}^M$.

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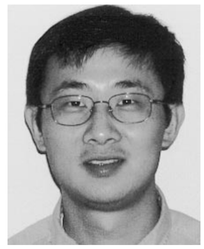

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