A Reduced Complexity Frequency Offset Estimation Technique for Flat Fading MIMO Channels

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***Abstract* - In this contribution we consider the carrier frequency offset estimation in the case of linear digital modulation, transmitted over a flat-fading Multiple-Input Multiple-Output (MIMO) Channel. For this problem, Maximum-Likelihood (ML) estimation gives rise to a carrier frequency offset algorithm that involves the maximization of the magnitude of a Fourier transform. Since no closed-form solution can be derived and numerical calculation of the estimate is computationally hard, we propose a sub-optimal solution. Our simulation results show that the proposed estimator yields a performance that is comparable with the Cramer-Rao bound.**

***Keywords* - Frequency offset estimation, synchronization, MIMO.**

I. INTRODUCTION

Using multiple transmit and receive antennas in wireless fading channels has been advocated as a means to increase capacity and to mitigate fading effects thanks to the provided diversity [1]. It is also well known that Space Time coding is an efficient strategy to explore both transmit and receive diversity, while maintaining a simple linear decoding structure [2-3]. However, the performance of space time coding may dramatically degrade in the presence of a carrier frequency offset. Hence accurate frequency offset estimation in Space-Time MIMO configurations is important in a view to compensate for it.

Similar to single antenna configurations, optimal frequency offset estimation using training data requires a numerical calculation, which is a time consuming task [4-5]. To overcome this inconvenience, some computationally efficient estimation techniques have been developed for Single Input Single Output (SISO) systems, which result in near optimum performance [6-7]. As will be demonstrated in section III, this good performance cannot be maintained by straightforwardly extending these algorithms to a MIMO fading environment. Therefore, we propose a new MIMO frequency offset estimation technique using training data, providing near optimum performance with little computational complexity (Section IV).

II. CHANNEL MODEL

Let us consider a flat-fading channel with N_t transmit antennas and N_r receive antennas affected by a carrier frequency mismatch and/or Doppler shift. The sampled output, assuming the timing to be known, at instant kT can be modelled as

$$\mathbf{r}(k) = \mathbf{H}\mathbf{a}(k)\exp\big(j(2\pi kFT + \theta)\big) + \mathbf{w}(k)\ ,\ k=1,\ldots,L \quad (1)$$

where $\mathbf{r}(k)$ is a $N_r\times 1$ vector of received signal samples, $\mathbf{H}$ denotes the $N_r\times N_t$ channel matrix, $\mathbf{a}(k)$ is a $N_t\times 1$ vector of training symbols with variance E_s transmitted at instant kT, F is the carrier frequency offset, T is the symbol interval, θ is the carrier phase, and $\mathbf{w}(k)$ is a $N_r\times 1$ noise vector. The components of the noise vector $\mathbf{w}(k)$ are statistically independent and complex-valued, with independent Gaussian zero-mean real and imaginary parts, each having a variance of $N_0/2$; also, noise vectors at different instants of time are statistically independent. The components $H_{m,n}$ of the channel matrix are statistically independent and complex-valued, with independent Gaussian zero-mean real and imaginary parts, each having a variance of 1/2; this yields $E[|H_{m,n}|^2] = 1$. In this contribution, we assume the channel matrix to be known, and by this the phase relation between the different antennas. However the global carrier phase θ remains an unknown random variable with uniform probability density.

III. MIMO-EXTENDED SISO ESTIMATORS

In this section, we show that extending the SISO closed-form frequency estimators [6-7] to a MIMO set-up gives rise to a serious noise enhancement and thus performance degradation. To circumvent this problem we propose a new closed-form frequency offset estimator in section IV.

The problem of frequency offset estimation for Additive White Gaussian Noise (AWGN) SISO-channels is well documented in literature [4,8,9]. In a MIMO context, Maximum Likelihood (ML) estimation is very similar to SISO when the channel matrix **H** is considered to be known. A similar derivation as for SISO yields the following ML estimate:

$$\hat{F} = \arg\max_{F} \left| \sum_{k=1}^{L} \mathbf{a}^{\mathrm{H}}(k)\mathbf{H}^{\mathrm{H}}\mathbf{r}(k)\exp(-j2\pi FkT) \right| \tag{2}$$

The Mean-Square Error (MSE) of this estimate is very close to the Cramer-Rao Bound (CRB), which is a theoretical lower bound on the MSE. For a not too small number of random pilot symbols, this bound is easily found to be

$$\sigma_{CR}^2 = \frac{3}{2\pi^2 T^2}\frac{N_0}{E_s\|\mathbf{H}\|^2 L(L^2-1)} \tag{3}$$

where $\|\cdot\|$ denotes the Frobenius norm of a matrix, defined as the square root of the sum of the absolute squares of the matrix's elements. Unfortunately, a simple closed-form solution to the problem of maximizing (2) does not exist. The exact determination of $\hat{F}$ would require a time consuming search over a large set of frequency values. Hence we look for a simpler frequency offset estimation algorithm.

In [6] and [7], the suboptimal Luise&Regiannini (L&R) and Fitz algorithms have been presented in the context of PSK transmission over a static AWGN SISO channel. In spite of their simplicity, the performance of those algorithms is close to ML performance (and to the CRB). Straightforward extension of the algorithms from [6] and [7] to a MIMO context can be obtained by multiplication of (1) with the pseudo-inverse of **Ha**. This yields the extended versions of the respectively L&R (4) and Fitz (5) algorithms:

$$\hat{F} = \frac{1}{\pi(P+1)T}\arg\left(\sum_{m=1}^{P} R(m)\right) \tag{4}$$

$$\hat{F} = \frac{1}{2\pi T}\frac{\sum_{m=1}^{Q} m\arg\big(R(m)\big)}{\sum_{m=1}^{Q} m^2} \tag{5}$$

where R(m) denotes the following time-correlation:

$$R(m) = \frac{1}{L-m}\sum_{k=m+1}^{L} z(k)z^*(k-m) \tag{6}$$

with $z(k) = y(k)/x(k)$, $y(k) = \mathbf{a}^{\mathrm{H}}(\mathrm{k})\mathbf{H}^{\mathrm{H}}\mathbf{r}(\mathrm{k})$, $x(k) = \mathbf{a}^{\mathrm{H}}(\mathrm{k})\mathbf{H}^{\mathrm{H}}\mathbf{Ha}(\mathrm{k})$, P and Q are design parameters. In a SISO environment, $P,Q \cong L/2$ yields a MSE close to the CRB. In a MIMO environment however, such estimators will not lead to optimal performance. The correction for channel matrix and symbols gives rise to a noise enhancement, which can substantially degrade the performance of the estimation. This effect will be important in particular when receive diversity is small, which for instance occurs when applying Alamouti 's space-time transmit diversity scheme [3].

Noise enhancement will however decrease as the number of receive antennas – and diversity – grows. Consequently, the MSE of both estimators will closely approach to the CRB only for high receive diversity, as illustrated in Fig 1.

IV. NEW CLOSED-FORM MIMO ESTIMATOR

We now introduce a closed-form frequency offset estimation technique for fading MIMO channels, exploring full multi-antenna diversity. This resulting new technique provides estimates with a variance that is close to the CRB for *every* antenna configuration.

Let us define the following time-correlation function:

$$R(m) = \frac{1}{L-m}\sum_{k=m+1}^{L} y(k)y^*(k-m) \tag{7}$$

with $y(k) = \mathbf{a}^{\mathrm{H}}(\mathrm{k})\mathbf{H}^{\mathrm{H}}\mathbf{r}(\mathrm{k})$. This time-correlation function has the form

$$R(m) = r(m)\exp(j2\pi FmT) + noise \tag{8}$$

where $r(m) = \frac{1}{L-m}\sum_{k=m+1}^{L} x(k)x(k-m)$, and where x(k) equals y(k) in the absence of noise: $x(k) = \mathbf{a}^{\mathrm{H}}(\mathrm{k})\mathbf{H}^{\mathrm{H}}\mathbf{Ha}(\mathrm{k})$. Our approach is to determine the value of F, such that $r(m)\exp(j2\pi FmT)$ is a least-squares fit of the time-correlation function *R(m)*. Therefore we minimize with respect to F the following function:

$$\Lambda(F) = \sum_{m=1}^{L-1}\left|R(m) - r(m)\exp(j2\pi FmT)\right|^2 \tag{9}$$

As in (2), a simple closed-form solution to the problem of minimizing (9) does not exist. But since *r(m)* is real-valued we can however derive a closed-form approximation of this solution. Setting the derivative of (9) with respect to *F* equal to zero, we find the following equation

$$\sum_{m=1}^{L-1}\mathrm{Im}\left[2\pi mTR(m)r(m)\exp(-j2\pi\hat{F}mT)\right] = 0 \tag{10}$$

Now replacing the exponential by its truncated Tailor series expansion ($\exp(x) \cong 1+x$), we obtain, after grouping terms appropriately,

$$\hat{F} = \frac{1}{2\pi T} \frac{\sum_{m=1}^{L-1} \mathrm{Im}\left[m r(m) R(m)\right]}{\sum_{m=1}^{L-1} \mathrm{Re}\left[m^2 r(m) R(m)\right]} \qquad (11)$$

Assuming large L or small noise in (8) and making use of the fact that r(m) is real-valued we carry out the following approximations

$$\mathrm{Im}\left[m r(m) R(m)\right] \cong m \left|R(m)\right|^2 \arg\left[R(m)\right] \qquad (12)$$

$$\mathrm{Re}\left[m^2 r(m) R(m)\right] \cong m^2 \left|R(m)\right|^2 \qquad (13)$$

To reduce complexity furthermore, we can limit the summation interval to $(1,M)$, with $M<L$. As will be shown in the sequel, this can be done without significant degradation in performance, as long as $M>L/2$. Finally we obtain the following estimator.

$$\hat{F} = \frac{1}{2\pi T} \frac{\sum_{m=1}^{M} m \left|R(m)\right|^2 \arg\left[R(m)\right]}{\sum_{m=1}^{M} m^2 \left|R(m)\right|^2} \qquad (14)$$

It should be noted that in case of an AWGN SISO channel with PSK symbols, this estimator is equivalent to the estimator presented by Fitz [7].

We further indicate that due to the presence of the arg (.) function in (12), $\hat{F}$ is unambiguous as long as the argument of $R(m)$ does not exceed $\pm\pi$. This limits the operating range of our frequency offset estimation technique to the interval

$$\left|F\right| < [2MT]^{-1} \qquad (15)$$

We illustrate the performance of our frequency offset estimation algorithm (12) and compare it with performance of the modified L&R (4) and Fitz (5) estimators. In all simulations, 10000 Monte Carlo trials were run to obtain the empirical Mean Square Error (MSE) or Mean Estimated Frequency. The training symbols were randomly chosen from a QPSK constellation.

Fig. 1 shows plots of the MSE, emphasizing the improved performance of the new estimator with respect to the extended SISO estimators. The influence of parameter M is illustrated in Fig 2, where it can be seen that the truncation in (14) induces little degradation in performance as long as $M>L/2$. Fig 3 shows that the estimator's performance is remarkably independent of the particular value of F and we also found the estimator to be unbiased in a broad range around $F=0$, according to (15).

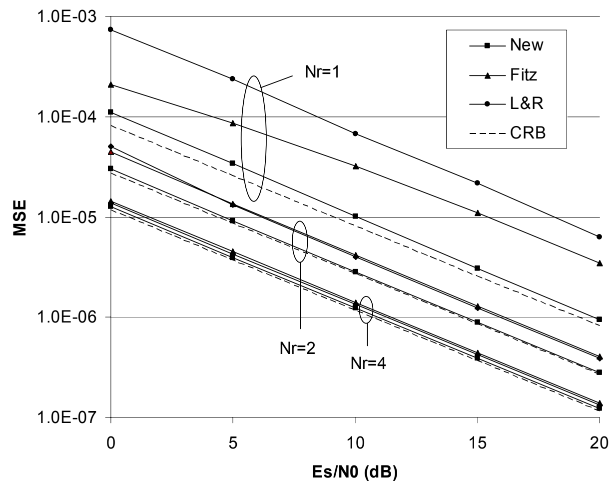

Fig 1: Performance of new frequency estimator (14), modified L&R estimator (4) and modified Fitz estimator (5) compared to CRB (L=10, N_t=2, P=Q=5, M=6).

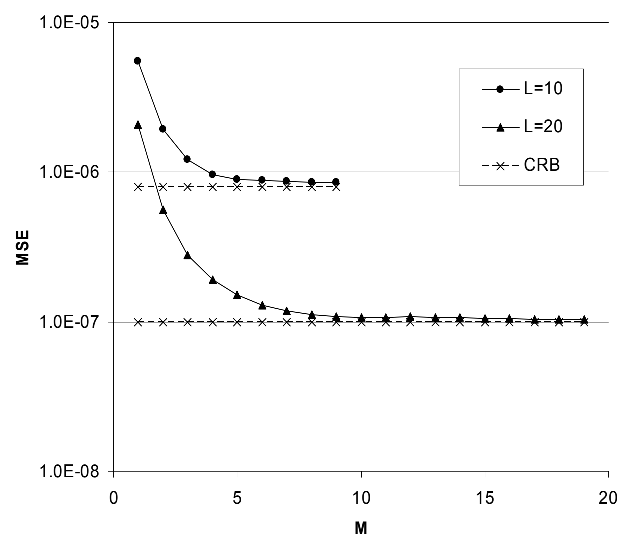

Fig 2: Performance of new frequency estimator (14) for different values of parameter M (Es/N0=15dB, Nr=Nt=2).

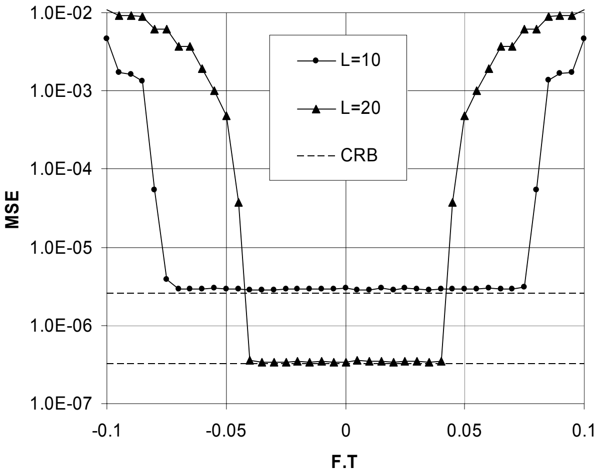

Fig 3: Performance of new frequency estimator (14) for different frequency offset values (Es/N0=10dB, Nr=Nt=2, M=(L/2)+1).

V. CONCLUDING REMARKS

In this contribution we derived a new frequency offset estimation algorithm for flat fading MIMO channels. We showed that the proposed algorithm is computationally efficient, and performs well in comparison with extended SISO-estimators.

The proposed algorithm assumes knowledge of the channel matrix, which is usually not available in practice. However, the frequency estimation algorithm can be used in the following way:

- First, the channel matrix is estimated, assuming F=0, for example according to [2,10].
- The channel matrix estimate is used in the proposed frequency estimation algorithm, as if it were the correct value.
- Further improvement can be obtained by feeding back the frequency estimate to the channel estimation algorithm, and then using the updated channel estimate in the frequency estimation algorithm. Several iterations can be performed, exchanging information between channel estimation and frequency estimation algorithms.

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