Performance of Transmit Precoding in Time-Varying Point-to-Point and Multi-User MIMO Channels

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Abstract— **Transmit precoding strategies in multiple-input multiple-output (MIMO) systems provide a mechanism for increasing the performance of point-to-point links and enable spatial division multiple access in multi-user networks. However, communication node mobility in such systems can lead to rapid channel variation which limits the quality of attainable channel state information (CSI). This paper explores the performance of point-to-point and multi-user precoding strategies based on CSI which goes out of date and channel distribution information which provides a more average channel representation. Results based on experimentally obtained MIMO channels in outdoor environments are presented.**

I. Introduction

The time-varying, multi-user, multiple-input multiple-output (MIMO) wireless channel promises significant gains over conventional single-antenna systems [1]. Multiplexing gains are achieved through the use of multiple antennas while temporal diversity gains can be acquired as a result of the time variation of the channel. Unfortunately, this temporal variation can significantly degrade the quality of channel state information (CSI) at the transmitter (CSIT) due to lag in channel updates. Transmit precoding (TP) techniques that use CSIT [2] must accommodate the spatial and temporal variation of the channel; consequently, algorithms that are robust to the channel parameter variations are desirable for more reliable systems.

For the MIMO broadcast channel (BC), a TP technique so-called "dirty-paper coding" (DPC) maximizes the sum-capacity of the BC [1] for stationary applications. DPC is a non-linear algorithm that is considered computationally complex for standard systems [2]. Linear techniques, such as transmit beamforming (BF), are simple and easily implemented but are sub-optimal in a sum-capacity sense compared to DPC. Finally, multiple access interference (MAI) can be completely removed from the BC by employing time-sharing (TS) at the transmitter.

In order to focus on the temporal and spatial variation in the channel, this paper uses measurements acquired from an experimental MIMO channel sounder [3]. These measurements allow insight into the sensitivity and throughput of the various TP techniques in selected physical channels. For this study, two environments are chosen including a large open field (Open Field) with a few large scatterers on the perimeter and a more tightly packed urban environment (Urban) containing multiple scattering structures. Each measurement was taken by test equipment developed at Brigham Young University (BYU) [4].

The paper will be organized as follows. Section II will detail the test measurement setup and the environments explored in this paper. Section III explains the metrics used in characterizing the measured MIMO channel. Sections IV and V review the TP techniques analyzed in this paper and present the results of using TP in the multi-user BC, respectively. The paper is concluded in Section VI.

II. Channel Measurements

The received vector given the standard time-varying system model for the K-user, MIMO broadcast channel can be written as

$$\mathbf{y}_j(n) = \mathbf{H}_j(n)\mathbf{x}_j(n) + \sum_{i \neq j}^{K} \mathbf{H}_j(n)\mathbf{x}_i(n) + \boldsymbol{\eta}(n) \qquad (1)$$

for the j^{th} user, where $\mathbf{H}_j(n)$ is the channel sample at integer time index n, $\mathbf{x}_i$ is the signal destined for the i^{th} user, and $\boldsymbol{\eta}(n)$ is additive white Gaussian noise (AWGN). Equation (1) will be modified later in the paper to include the effects of transmit precoding.

The test equipment at BYU allows sampling of an 8×8, point-to-point wireless link. The measurements are configured for up to 100 MHz of instantaneous bandwidth at a center frequency of 2-8 GHz. The mobility is constrained to pedestrian velocities; however, the channel is sampled at up to 3200 times per second allowing decimation or interpolation of the data in order to create an effective nodal velocity. Specific details of the "open hardware" measurement equipment are available in [5].

Prior to any measurement campaign, calibration measurements are taken with the transmitter "off" in order to measure background interference. This external interference was found to be negligible at a center frequency of 2.45 GHz. A second calibration is performed with both the transmitter and receiver "on", but stationary, in order to observe the time variation in the channel caused by ambient changes such as pedestrians, atmospheric conditions, and other natural disturbances. It was

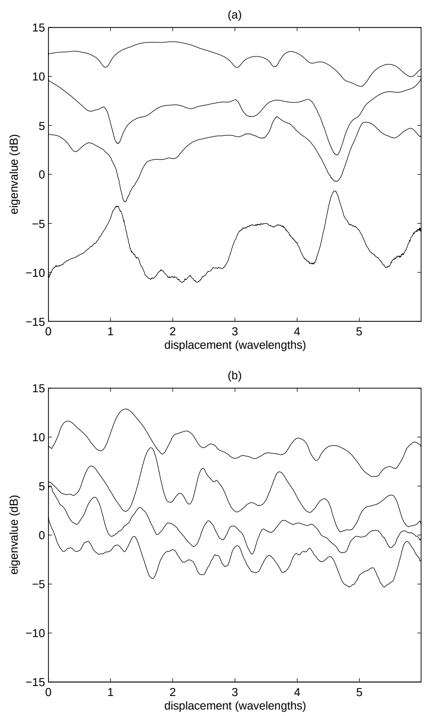

Fig. 1. Eigenvalue spread and deviation for the (a) Open Field and (b) Urban environments.

found that temporal variation of the channel when the nodes are stationary was insignificant for the datasets examined in this paper. Finally, the receiver is moved at a constant velocity with channel sampling occurring every 2.5-3.2 ms. Again, for the selected environments, it was observed that the time variation in the channel results from node mobility itself.

These results allow the obtained datasets from the single-user point-to-point links to be expanded to model the multi-user channel. To model the multi-user channel, the above experiment was repeated multiple times in the same environment but at different receiver locations. This allows for acquisition of independent channel realizations for different users in the simulated network. Since it was observed that the time variation occurs almost exclusively from node movement, the superposition of these asynchronous measurements into a single synchronized multi-user channel seems justified. Once acquired, the j^{th} measured dataset can be used in the system model (1) for $\mathbf{H}_j(n)$.

III. Channel Metrics

After providing a framework for the measured data in the single- and multi-user channels it is important to provide meaningful metrics that define performance and sensitivity of the MIMO channel to various precoding strategies.

A. Eigenvalue Spread

A simple method of determining the richness of multipath is to look at the eigenvalue spread and deviation over time. A low spread of the eigenvalues of the channel matrix suggests high scattering, while a large spread implies a strong line-of-sight (LOS) component. Consider the following spectral decomposition of the channel

$$\mathbf{H}(n) = \mathbf{U}(n)\mathbf{\Lambda}(n)\mathbf{U}^{-1}(n) \tag{2}$$

where $\mathbf{\Lambda}(n)$ represents the diagonal matrix of eigenvalues per sample n and $\mathbf{U}(n)$ are the corresponding matrices of eigenvectors. Figure 1 shows a plot of the four largest eigenvalues versus displacement from datasets taken from each of the environments studied in this work. Notice that the Open Field environment has a larger spread in the eigenvalues compared to the Urban environment. This is suggestive of the corresponding strong LOS component in Open Field and the higher scattering in the Urban environment. Both environments demonstrate few dominant eigenmodes which will result in relatively low rank channel matrices.

B. Channel Capacity

The channel capacity is another important metric in describing the characteristics of a channel. Capacity quantifies the maximum achievable throughput of a link given various assumptions about channel knowledge [1]. Since this study is focused on channel variation and not estimation, we will assume the receiver always has perfect CSI while the transmitter either uses error-free, though outdated, CSIT from node mobility, or is unaware of the channel state.

The capacity of a single-user MIMO link given some input covariance matrix $\mathbf{Q} = E[\mathbf{x}\mathbf{x}^H]$ is

$$C_T = \log\left|\mathbf{I} + \mathbf{H}\hat{\mathbf{Q}}\mathbf{H}^H\right| \tag{3}$$

where $\mathbf{I}$ is the identity matrix and $\hat{\mathbf{Q}}$ is used to emphasize that the input covariance matrix was calculated using CSIT acquired from a previous location. If no knowledge of the channel is used at the transmitter then the uninformed transmit capacity can be written as

$$C_{UT} = \log\left|\mathbf{I} + \frac{P}{N_t}\mathbf{H}\mathbf{H}^H\right| \tag{4}$$

where equal power is split between all sub-channels. As the channel estimate becomes more and more outdated the transmit capacity will tend to approach the uninformed transmit capacity [4].

C. Effectiveness of CSIT

In this section, the effectiveness of CSI at the transmitter will be determined as a function of: node mobility, transmit power, number of transmit antennas N_t, and number of receive antennas N_r. This effectiveness of CSIT can be written, using the defined capacity metrics, as

$$U_T = \frac{C_T}{C_{UT}} - 1. \tag{5}$$

Equation 5 will be zero when CSIT does not increase capacity over the uninformed transmitter and $U_T = 1$ when CSIT doubles the available rate. Equation 5 can also become negative

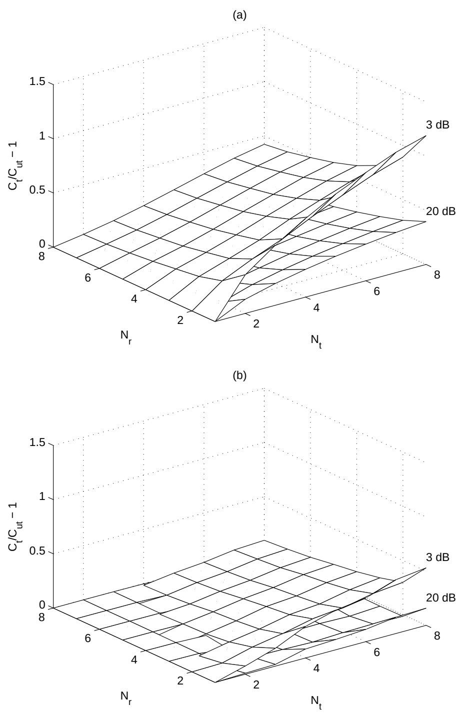

Fig. 2. Effectiveness of channel state information U_t for (a) perfect CSIT at zero displacement and (b) CSIT delayed by 10 wavelengths for N_t transmit antennas and N_r receive antennas.

when the outdated channel knowledge is such that using it is actually detrimental and lowers the capacity below C_{UT}.

Figure 2 plots single-user U_T in the Urban environment for perfect CSIT and delayed CSIT at SNRs of 3 and 20 dB, respectively. Fig. 2(a) uses perfect CSIT while Fig. 2(b) has the same CSIT but is displaced by approximately 10 wavelengths or 1.2 meters. This outdated CSIT is equivalent to erroneous CSIT and hence the effectiveness of channel knowledge drops. Another parameter that reduces the effectiveness of CSIT is the number of available receiver antennas. This satisfies intuition since receive nodes with more degrees of freedom are better able to recover the signal and obviate the need for CSI at the transmitter. Conversely, more antennas at the transmitter provides mode possible spatial processing thus increasing the utility of CSIT. Finally, CSIT is more important for power constrained systems which is confirmed by noting the optimal input covariance matrix approaches the scaled identity (i.e. equivalent optimal input covariance for the uninformed transmitter) as power is increased.

IV. Transmit Precoding

Transmit precoding schemes in broadcast channels attempt to utilize knowledge of the channel in order to increase data throughput and minimize interference between the multiple users and data streams. Such schemes are often categorized into non-linear and linear types [2] with non-linear methods considered more complex than their linear counterparts. This study focuses on dirty-paper coding as the sum-capacity optimal non-linear transmit precoding algorithm and beamforming as the standard linear scheme. Also, as a benchmark, a scheduled system using fair time-sharing is compared to the multiple-access schemes.

A. Dirty-Paper Coding

Dirty-paper coding (DPC) is a transmit precoding technique that successively removes encoded users as interference from other users prior to transmission [1]. For example, consider User 1 as the first encoded user who is interfered with by all other users. Using DPC, the signal for User 1 is then pre-subtracted at the transmitter and does not cause interference to any other user. The algorithm then continues this pre-subtraction for each successive user until User K receives no interference from any of the other users. DPC requires full channel state information at the transmitter.

When the DPC algorithm has outdated or erroneous CSIT, interference will propagate through the encoding of each user since the transmitter will not effectively pre-subtract the entire signal. This self-interference can be observed by first writing the received signal for User j as

$$\mathbf{y}_j = \mathbf{H}_j \sum_{i=j}^{K} \mathbf{x}_i + (\mathbf{E}_j - \mathbf{E}_{\mu,j}) \sum_{i=1}^{j-1} \mathbf{x}_i + \boldsymbol{\eta} \qquad (6)$$

where $\mathbf{E}_i$ is a random matrix representing the channel error for the i^{th} user and $\mathbf{E}_{\mu,i}$ is the channel error mean. With these received vectors, a lower bound on the sum-capacity, given known error statistics, becomes

$$C_{DPC} = \sum_{i=1}^{K} \log \frac{\left| I + \sum_{j=i}^{K} \boldsymbol{\Psi}_{j,j}^{H} + \sum_{j=i}^{i-1} \left(\boldsymbol{\Psi}_{i,j}^{E} - \boldsymbol{\Psi}_{i,j}^{E_\mu} \right) \right|}{\left| I + \sum_{j=i+1}^{K} \boldsymbol{\Psi}_{i,j}^{H} + \sum_{j=i}^{i-1} \left(\boldsymbol{\Psi}_{i,j}^{E} - \boldsymbol{\Psi}_{i,j}^{E_\mu} \right) \right|} \qquad (7)$$

where for some matrix $\mathbf{V}$, $\boldsymbol{\Psi}_{i,j}^{V} = E\left[\mathbf{V}_i^H \mathbf{Q}_j \mathbf{V}_i \right]$.

B. Linear Transmit Precoding

Linear transmit and receive coding or beamforming (BF), is a simple spatial method to encode and decode the data in order to suppress interference from other users. The transmitter has different options when deciding the type of precoding to use based on system criteria [6]. Channel inversion or zero-forcing (ZF) creates multiple and independent subspaces for transmission, but is sub-optimal and power inefficient for low rank channel matrices. Regularized channel inversion (RCI) is a more robust method of linear transmit-precoding and is able to optimize the sum-capacity. Since this study will also consider receiver beamforming, if the transmitter sends multiple data streams to the same user then each stream is also considered as interference to the other streams destined to that same user.

Consider reception of the j^{th} data stream

$$\mathbf{y}_j = \mathbf{H}_j \mathbf{b}_j \mathbf{x}_j + \mathbf{H}_j \sum_{i \neq j}^{N_s} \mathbf{b}_i \mathbf{x}_i + \boldsymbol{\eta} \qquad (8)$$

where $\mathbf{H}_j$ is the appropriate channel for the j^{th} data stream, $\mathbf{b}_j$ are the corresponding beamformer weights, and N_s is the total number of streams transmitted. For this scenario, the individual signal-to-interference plus noise ratios (SINR) can be calculated as

$$SINR_j = \frac{p_j \mathbf{b}_j^H \mathbf{H}_j^H \mathbf{H}_j \mathbf{b}_j}{1 + \sum_{i \neq j} p_i \mathbf{b}_j^H \mathbf{H}_j^H \mathbf{H}_j \mathbf{b}_j} \tag{9}$$

assuming unit noise variance and a signal power of p_j. The receiver will attempt to maximize (9) and produce receive beamforming weights $\mathbf{w}_j$. Since BF is simplified to multiple simultaneous SISO transmissions with some interference level, the sum capacity can be expressed as

$$C_{BF} = \sum_{i=1}^{N_s} \log |1 + SINR_i|. \tag{10}$$

In order to calculate the beamforming weights $\mathbf{w}_j$ and $\mathbf{b}_j$, the so-called "Coordinated Transmitter/Receiver Beamforming Algorithm" is used [2]. At one step in the iteration, received weights $\mathbf{b}_j$ are found using the minimum mean square error (MMSE) criteria. In order to find $\mathbf{w}_j$, the duality between uplink and downlink is used with $\mathbf{b}_j$ now the transmit weights and $\mathbf{w}_j$ found using the same MMSE criteria. Convergence to some threshold is guaranteed since the iterations will produce non-decreasing values of SINR. However, the problem itself is non-convex resulting in different solutions based on initial estimates. The iterations are similar to the joint transmit/receiver beamforming method in [7]; however, this paper will assume power is uniformally distributed between all data streams and the number of streams will be optimized iteratively to maximize the expected SINR. This method of transmit beamforming using the iterative algorithm with CSIT and MMSE transceivers will be called MMSE-CSIT.

Since the time-varying channel introduces errors in the channel estimates, it is worthwhile to examine the use of channel distribution information (CDI) rather than CSI on which to code [1]. Using the measured data, the transmit correlation matrix over N samples for User j can be estimated with

$$\mathbf{R}_j = \frac{1}{N} \sum_{n=1}^{N} \mathbf{H}_j^H(n) \mathbf{H}_j(n) \tag{11}$$

where n is the integer sample index. For simulation purposes, the window size N will span the region of interest.

Consider the beamforming problem where, instead of maximizing the SINR of each stream, one maximizes the average signal power over the average interference power

$$\begin{aligned} \overline{SINR}_j &= \frac{\frac{1}{N}\sum_{n=1}^{N} p_j \mathbf{b}_j^H \mathbf{H}_j^H(n) \mathbf{H}_j(n) \mathbf{b}_j}{1 + \frac{1}{N}\sum_{n=1}^{N} \sum_{i \neq j} p_i \mathbf{b}_i^H \mathbf{H}_j^H(n) \mathbf{H}_j(n) \mathbf{b}_i} \\ &= \frac{p_j \mathbf{b}_j^H \sqrt{\mathbf{R}_j}^H \sqrt{\mathbf{R}_j} \mathbf{b}_j}{1 + \sum_{i \neq j} p_i \mathbf{b}_i^H \sqrt{\mathbf{R}_j}^H \sqrt{\mathbf{R}_j} \mathbf{b}_i}. \end{aligned} \tag{12}$$

The average SINR (12) is analogous to the instantaneous SINR (9) where CDIT is used rather than CSIT. This suggests a similar iteration can be performed to find beamforming weights as used for the MMSE-CSIT beamforming. The method of maximizing the average SINR with the above iterative algorithm using the estimated channel statistics will be called MMSE-CDIT.

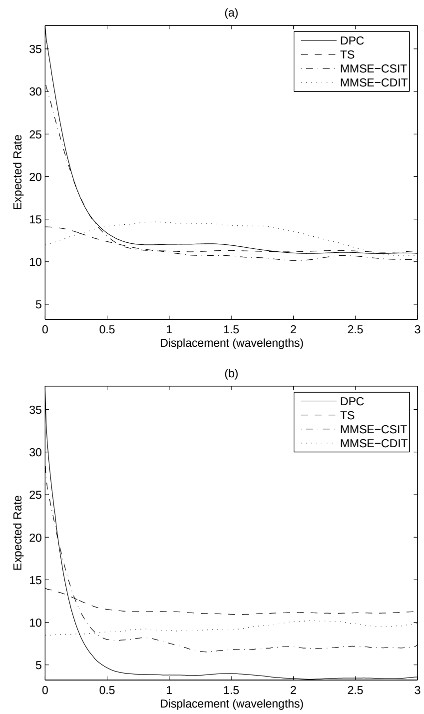

Fig. 3. Capacity degradation versus displacement for DPC, fair TS, MMSE-CSIT, and MMSE-CDIT for the cellular-type network in a) Open Field and b) Urban environments. Simulations were performed at 10 dB SNR per user with $N_t = 8$ transmit antenna, $N_r = 2$ receive antennas, and $K = 6$ users.

C. Fair Time-Sharing

With fair time-sharing (TS), a scheduler is used to give each user exclusive access to that channel at any given time. This reduces MAI to zero, but can result in sub-optimal performance compared with the perfectly informed transmitter. The sum-capacity with fair TS is simply the average capacity over the resulting single-user channels

$$C_{\text{TS}} = \frac{1}{K} \sum_{i=1}^{K} \log \left| \mathbf{I} + \hat{\mathbf{H}}_i \mathbf{Q}_i \hat{\mathbf{H}}_i^H \right|. \tag{13}$$

For fair TS, multiple data streams destined to a single user do not cause self-interference.

V. Results

Simulation results will use the system defined in (1) with measured datasets from the Open Field and Urban environments. Each of the precoding strategies discussed in this paper (DPC, fair TS, MMSE-CSIT, and MMSE-CDIT) will be examined in both environments. The performance metric for each scenario will be based on the sum-capacity lower bounds (7) (10) (13) in the the broadcast channel with a single transmitter and six receivers each with multiple antennas. Specific system parameters such as power and number of antennas will depend further on whether the simulated network is considered cellular or ad hoc.

A. Cellular Network

Results for this section reflect simulations that attempt to capture behavior in a single-cell network. The transmitter is equipped with eight antennas while receivers each contain only two. The power available to the base station is equivalent to a single-user SNR of 10 dB times the number of users. This amount of power satisfies the sum-capacity equivalence between the dual multiple-access or uplink channel [1] when error-free channel estimates are available.

Fig. 3 shows the results for the single-cell network in the two different environments for each of the transmit precoding techniques discussed. At zero displacement the channel is assumed to be perfectly known at the transmitter. Using this initial channel estimate, the transmitter performs the respective linear or non-linear transmit precoding and then uses the resulting input covariance matrix throughout the simulation. Note that the optimal precoding strategy DPC is extremely sensitive to node mobility and rapidly drops to sub-optimal levels. This mimics the single-user behavior of CSIT effectiveness from Fig. 2; a large number of transmit antennas and small number of receive antennas implies that accurate CSIT is important to maintain significant capacity gains. Also of interest is the small loss in capacity when the linear MMSE-CSIT beamformer is used instead of non-linear DPC.

B. Ad Hoc Network

For the ad hoc network, both transmitter and receivers are equipped with four antennas. In this simulation, the transmit power does not scale with the number of users and is considered fixed at an SNR of 10 dB. These parameters suggest a typical ad hoc setting with equally equipped nodes and no base station.

Fig. 4 shows the results for the ad hoc network simulations in the Open Field and Urban environments for each of the four transmit precoding techniques. Though not as drastic as the cellular network, significant loss in capacity is still observed for small displacements which again reflects behavior seen in Fig. 2. Another interesting result is the performance of MMSE-CDIT in the ad hoc network as nodes move. Beamforming on the transmit correlation matrix produces a stable subspace on which transmission is possible over large distances. In fact, after a small displacement MMSE-CDIT outperforms both DPC and MMSE-CSIT.

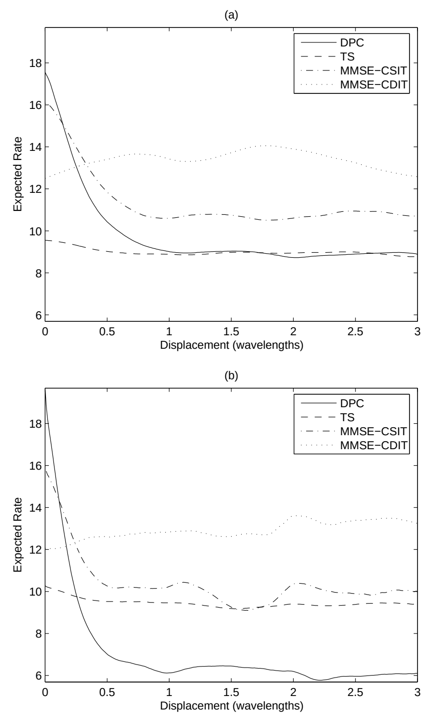

Fig. 4. Capacity degradation versus displacement for DPC, fair TS, MMSE-CSIT, and MMSE-CDIT for the ad hoc network in a) Open Field and b) Urban environments. Simulations were performed at 10 dB SNR with $N_t = 4$ transmit antenna, $N_r = 4$ receive antennas, and $K = 6$ users.

VI. Conclusion

For the metric used in this work, in single-user MIMO systems, the usefulness of CSIT is an increasing function of number of transmit antennas and a decreasing function of number of receive antennas, available power, and outdatedness due to mobility. It was also shown that these trends occur to some degree in the multi-user broadcast channel as well. The sum-capacity optimal transmit precoding scheme for these broadcast channels is DPC; however, DPC is extremely sensitive to time variations in the channel due to the self-interference caused by non-linear processing and outdated CSIT. Linear precoding, such as beamforming, captures the majority of available information rate and is also less sensitive to fluctuations in the channel. In fact, there exists stable subspaces in the wireless channel on which transmission is possible as reflected in the performance of the non-causal MMSE-CDIT transmit beamforming.

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