Compensation of Phase Noise in OFDM Wireless Systems

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***Abstract*—Phase noise causes significant degradation in the performance of orthogonal frequency division multiplexing (OFDM)-based wireless communication systems. The presence of phase noise can reduce the effective signal-to-noise ratio (SNR) at the receiver, and consequently, limit the bit error rate (BER) and data rate. In this paper, the effect of phase noise on OFDM wireless systems is studied, and a compensation scheme is proposed to mitigate the common phase error and intercarrier interference (ICI) caused by phase noise. In the proposed scheme, the communication between the transmitter and receiver blocks consists of two stages. In the first stage, block-type pilot symbols are transmitted and the channel coefficients are jointly estimated with the phase noise in the time domain. In the second stage, comb-type OFDM symbols are transmitted such that the receiver can jointly estimate the data symbols and the phase noise. It is shown both by theory and computer simulations that the proposed scheme can effectively mitigate the ICI caused by phase noise and improve the BER of OFDM systems. Another benefit of the proposed scheme is that the sensitivity of OFDM receivers to phase noise can be significantly lowered, which helps simplify the oscillator and circuitry design in terms of implementation cost and power consumption.**

***Index Terms*—Common phase error, compensation scheme, equalization, intercarrier interference (ICI), orthogonal frequency division multiplexing (OFDM), performance analysis, phase-locked loop (PLL), phase noise.**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is a widely recognized modulation technique for high data rate communications over wireless links. Because of its capability to capture multipath energy and eliminate intersymbol interference, OFDM has been chosen as the transmission method for several standards, including the IEEE 802.11a wireless local area network (WLAN) standard in the 5-GHz band, the IEEE 802.11g WLAN standard in the 2.4-GHz band, and the European digital video broadcasting system (DVB-T). Also, the OFDM-based physical layer is being considered by several standardization groups, such as the IEEE 802.15.3 wireless personal area network (WPAN) and the IEEE 802.20 mobile broadband wireless access (MBWA) groups. The heightened interest in OFDM has resulted in tremendous research activities in this field to make the real systems more reliable and less costly in practice.

One limitation of OFDM systems is that they are highly sensitive to the phase noise introduced by local oscillators. Phase noise is the phase difference between the phase of the carrier signal and the phase of the local oscillator, and its effect on OFDM receivers has been investigated in many previous works, such as [2]–[5]. The distortion caused by phase noise is characterized by a common phase error (CPE) term and an intercarrier interference (ICI) term. The CPE term represents the common rotation of all constellation points in the complex plane, while the ICI term behaves like additive Gaussian noise. Compared to the single-carrier modulation methods that can track the fast variation in phase noise adaptively in a decision-directed manner, e.g., [6], OFDM transmits data symbols over many low-rate subcarriers, which makes it more difficult to track and compensate for phase noise.

A. Prior Work on Phase Noise Compensation

The high sensitivity of OFDM receivers to phase noise imposes a stringent constraint on the design and fabrication of oscillators and the supplementary circuitry, such that the generated phase noise level will not cause the system to fail [7]–[9]. This requirement increases the implementation cost of OFDM receivers because impairments associated with fabrication variations are usually either unpredictable or uncontrollable. There have been works in the literature to mitigate the effects of phase noise in the digital domain. This approach provides an efficient, low-cost, and reliable solution to the phase noise problem. Some authors have proposed methods to compensate for the CPE term, in which the constellation rotation is estimated using pilot tones embedded in OFDM symbols and then corrected by the demodulator [10]. Since the ICI effect is either ignored or treated as additive noise in these schemes, they perform poorly if the phase noise varies fast in comparison to the OFDM symbol rate. To overcome this difficulty, some ICI compensation schemes have been proposed. The method in [11] utilizes the pilot tones, which are sufficiently separated away from the data tones, to transmit pilot symbols for phase noise estimation. In the self-cancellation method presented in [12] and [13], each data symbol is transmitted using two adjacent subcarriers and the received symbols are linearly combined to suppress ICI by exploiting the fact that the ICI coefficients change slowly over adjacent subcarriers. This technique has the advantage of low implementation complexity, but it reduces the spectral efficiency by one half. In [14], a finite-impulse response (FIR)-type equalizer is employed to compensate for

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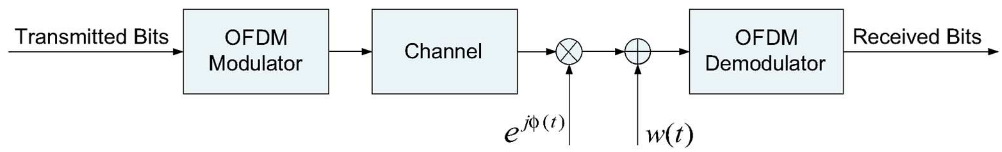


Fig. 1. Model of the OFDM system with phase noise.

phase noise and the filter coefficients are determined by the method of least squares. Since the filter length is limited by the number of pilot tones, it can only compensate for the ICI that is from adjacent subcarriers. A time-domain phase noise estimation and correction scheme is proposed in [15], where the phase noise process is parameterized by using sinusoidal waveforms as the bases and the parameters are estimated by the method of least squares. However, using sinusoidal waveforms to approximate phase noise may not be optimal, and how to jointly estimate the model parameters and the transmitted symbols is not addressed in [15]. Similarly, Liu and Zhu [16] approximate phase noise by using a small number of sinusoidal components, and it is suggested to insert some pilot tones outside the spectrum occupied by data transmission and estimate the model parameters of phase noise using the received pilot signals. This scheme requires extra bandwidth and can only correct the ICI from adjacent subcarriers because of the approximation made in modeling. Recently, a method that utilizes *a priori* information about phase noise spectrum to suppress phase noise is presented in [17]. All these ICI compensation schemes assume that the receiver has perfect channel state information; however, in wireless communications, the channel is time varying and the receiver has to estimate the channel in the presence of phase noise, which makes the scenario more complicated than what has been studied before. In [18], joint channel estimation and phase noise suppression are achieved by using the expectation–maximization (EM) algorithm; however, it simply models the ICI term as additive white noise. Also, Wu and Bar-Ness [19] propose a channel estimation method with the aid of the cyclic prefix symbols, and Kim and Kim [20] propose a joint channel estimation scheme using soft decision decoding. In [21], the maximum *a posteriori* (MAP) channel estimator is derived for the case when both frequency offset and phase noise are present.

B. This Work

In this paper, we propose a new phase noise compensation scheme for OFDM-based wireless communications with improved performance. The scheme consists of a channel estimation stage and a data transmission stage. In the channel estimation stage, block-type pilot symbols are transmitted so that the receiver can jointly estimate the channel coefficients and the phase noise.[1] Instead of estimating the channel coefficients and phase noise in the frequency domain, we estimate them in the time domain by using interpolation techniques to reduce the number of unknowns. The joint channel and phase noise estimation gives a more accurate estimate of the channel than other conventional methods that either ignore phase noise or simply treat it as additive Gaussian noise. The slowly varying nature of wireless channels allows us to use the channel estimate in the subsequent data transmission stage for detecting the data symbols. In the data transmission stage, comb-type pilot symbols, which contain both data symbols and pilot symbols, are transmitted and the data symbols and the phase noise components are jointly estimated at the receiver in order to mitigate both the CPE and ICI effects of phase noise. The proposed approach outperforms other methods for two main reasons. First, instead of approximating the ICI term as additive noise, we use the exact data model given in Section II and treat the phase noise components as unknown parameters to be determined over two stages. Second, the correlation property of phase noise is exploited to reduce the number of unknowns. This is similar to the idea of modeling the phase noise process by the sum of several sinusoidal waveforms, but the use of linear interpolation in the time domain makes our approach less sensitive to variations in phase noise models.

This paper is organized as follows. Section II briefly describes the system model with phase noise and formulates the effects of phase noise on OFDM receivers. The proposed algorithm is presented in Section III and analyzed in Section IV in terms of the effective signal-to-noise ratio (SNR). Simulation results and performance comparison of different algorithms are given in Section V.

Throughout this paper, we adopt the following notations: $(\cdot)^T$ denotes the matrix transpose, $(\cdot)^*$ denotes the matrix conjugate transpose, $\mathrm{diag}\{\cdot\}$ represents the diagonal matrix whose diagonal entries are determined by its argument, $\mathrm{Tr}\{\cdot\}$ returns the trace of a matrix, $\mathrm{Re}\{\cdot\}$ denotes the real part of its argument, and $\mathbf{E}\{\cdot\}$ is the expected value with respect to the underlying probability measure.

II. System Model

An OFDM system with phase noise is illustrated in Fig. 1. At the transmitter, the bits from information sources are first mapped into constellation symbols, and then converted into a block of N symbols $x[k]$, $k = 0, 1, \dots, N-1$, by a serial-to-parallel converter. The N symbols are the frequency components to be transmitted using the N subcarriers of the OFDM modulator and they are converted to OFDM symbols $X[n]$, $n = 0, 1, \dots, N-1$, by the unitary inverse fast Fourier transform (IFFT), i.e.,

$$X[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x[k] e^{j\frac{2\pi nk}{N}}, \qquad n = 0, 1, \dots, N-1.$$

A cyclic prefix of length $P (P \leq N)$ is added to the IFFT output in order to eliminate the intersymbol interference caused by multipath propagation. The resulting $N + P$ symbols are

[1]All standardized OFDM systems today provide such full pilot symbols at the beginning of every packet. Therefore, the proposed scheme does not require any modification to the packet structure and can be applied to existing standards.

converted into a continuous-time baseband signal $x(t)$ for transmission. If the continuous-time impulse response function of the baseband channel is $h(t)$ and the phase noise at the local oscillator is $\phi(t)$,[2] then the received baseband signal is given by

$$y(t) = e^{j\phi(t)} \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau + w(t)$$

where $w(t)$ is additive white Gaussian noise. Let T_s be the symbol time of the system. At the demodulator, $y(t)$ is sampled at period T_s. After removing the cyclic prefix, a block of N symbols $Y[n]$, $n = 0, 1, \ldots, N-1$, is obtained, whose elements are related to $X[n]$, $n = 0, 1, \ldots, N-1$, by

$$\begin{aligned} Y[n] &= e^{j\phi(nT_s)} \left(X[n] \circledast h[n]\right) + w[n] \\ &= e^{j\phi(nT_s)} \sum_{r=0}^{N-1} X[r] h\left[(n-r)_N\right] + w[n] \end{aligned} \quad (1)$$

where $\circledast$ denotes circular convolution, $h[n]$ is the discrete-time baseband channel impulse response, $(n-r)_N$ stands for $((n-r) \bmod N)$, and $w[n]$ is the additive noise component. Assume that $h[n]$ has length L, i.e., $h[n] = 0$ if $n \notin \{0, 1, \ldots, L-1\}$. OFDM modulation requires $P \geq L-1$ in order to eliminate the intersymbol interference. The unitary fast Fourier transform (FFT) is then performed on $Y[n]$, $n = 0, 1, \ldots, N-1$, to obtain $y[k]$, $k = 0, 1, \ldots, N-1$. Let

$$\alpha[k] = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi(nT_s)} e^{-j\frac{2\pi kn}{N}}, \qquad k = 0, 1, \ldots, N-1 \quad (2)$$

and

$$H[k] = \sum_{n=0}^{L-1} h[n] e^{-j\frac{2\pi kn}{N}}, \qquad k = 0, 1, \ldots, N-1.$$

It follows from (1) that

$$\begin{aligned} y[k] &= \alpha[k] \circledast \left(H[k]x[k]\right) + w[k] \\ &= \sum_{r=0}^{N-1} \alpha[r] H\left[(k-r)_N\right] x\left[(k-r)_N\right] + w[k], \\ &\qquad k = 0, 1, \ldots, N-1 \end{aligned} \quad (3)$$

because multiplication in the time domain implies convolution in the frequency domain and convolution in the time domain implies multiplication in the frequency domain. Here, $H[k]$ is the channel response in the kth subcarrier and $w[k]$ is the additive noise component in the kth subcarrier. Expression (3) can be rewritten as

$$y[k] = \alpha[0]H[k]x[k] + \sum_{r=1}^{N-1} \alpha[r] H\left[(k-r)_N\right] x\left[(k-r)_N\right] + w[k] \quad (4)$$

[2]In Appendix I, we give a brief review of the existing models for phase noise, and derive some useful statistical measures that will be used in assessing the performance improvement of the proposed compensation scheme in Section IV.

where the term $\alpha[0]H[k]x[k]$ is called the common phase error (CPE) and $\sum_{r=1}^{N-1} \alpha[r]H[(k-r)_N]x[(k-r)_N]$ is called the intercarrier interference (ICI). In the absence of phase noise, we have the traditional relation

$$y[k] = H[k]x[k] + w[k]$$

which follows by setting $\alpha[0] = 1$ and $\alpha[r] = 0$ for $r \neq 0$. Using matrix notation, expression (3) can be represented as

$$\mathbf{y} = \mathbf{AHx} + \mathbf{w} \quad (5)$$

where

$$\begin{aligned} \mathbf{y} &= \left[\, y[0] \quad y[1] \quad \cdots \quad y[N-1] \,\right]^T \\ \mathbf{x} &= \left[\, x[0] \quad x[1] \quad \cdots \quad x[N-1] \,\right]^T \\ \mathbf{w} &= \left[\, w[0] \quad w[1] \quad \cdots \quad w[N-1] \,\right]^T \\ \mathbf{A} &= \begin{bmatrix} \alpha[0] & \alpha[N-1] & \ldots & \alpha[1] \\ \alpha[1] & \alpha[0] & \ldots & \alpha[2] \\ \vdots & \vdots & \ddots & \vdots \\ \alpha[N-1] & \alpha[N-2] & \ldots & \alpha[0] \end{bmatrix} \\ \mathbf{H} &= \begin{bmatrix} H[0] & 0 & \ldots & 0 \\ 0 & H[1] & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & H[N-1] \end{bmatrix}. \end{aligned} \quad (6)$$

When phase noise is not present, the matrix $\mathbf{A}$ in (5) is replaced by the identity matrix.

III. Proposed Algorithm for Phase Noise Compensation

If both $\mathbf{A}$ and $\mathbf{H}$ in (5) were known, then the data vector $\mathbf{x}$ could be recovered, e.g., by solving [22]

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{AHx}\|^2.$$

However, in practice, neither the channel matrix $\mathbf{H}$ nor the phase noise matrix $\mathbf{A}$ are known to the receiver. In this section, we propose a solution to deal with the situation when both $\mathbf{A}$ and $\mathbf{H}$ are unknown at the receiver. The proposed algorithm consists of two stages: One is the channel estimation stage and the other is the data transmission stage. In the channel estimation stage, we use block-type pilot symbols to jointly estimate $\mathbf{H}$ and $\mathbf{A}$. In the data transmission stage, comb-type symbols are transmitted such that $\mathbf{x}$ can be jointly estimated with $\mathbf{A}$ by using the $\mathbf{H}$ estimated in the channel estimation stage. The motivation for this algorithm is based on the fact that wireless channels are usually slowly time varying compared to phase noise. Since the phase noise components may change significantly from one OFDM symbol to another, it is harmful to use the previous estimate of phase noise to help detect the data symbols in the subsequently received OFDM symbols. However, we can use the channel estimate for a few subsequent OFDM symbols due to the slowly time-varying nature of wireless channels. This motivates our approach to compensate for phase noise by using the joint channel estimation (with phase noise) first and then followed by the joint

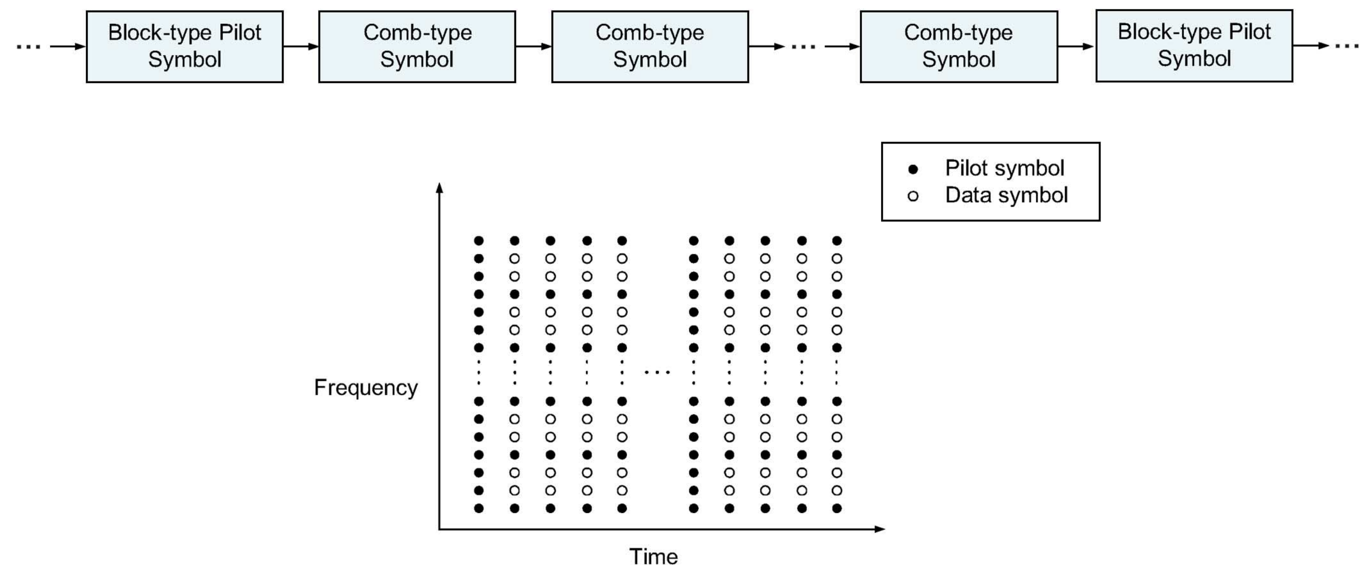


Fig. 2. Proposed algorithm.

data symbol estimation (with phase noise). The algorithm is illustrated in Fig. 2.

A. Joint Channel and Phase Noise Estimation

In the block-type pilot symbols, all subcarriers are used to transmit pilot symbols. For convenience of exposition, we assume that each time only one OFDM symbol is used as the block-type pilot symbol for channel estimation. Since there are only N pilot tones in each block-type pilot symbol, it is underdetermined to directly estimate the $2N$ unknowns, i.e., $\alpha[k]$ and $H[k]$, $k = 0, 1, \ldots, N-1$. To overcome this difficulty, we can reduce the number of unknowns by properly modeling the channel and the phase noise process with fewer parameters as follows. Since the length L of the discrete-time baseband channel impulse response is normally less than the OFDM symbol size N, we can relate $H[k], k = 0, 1, \ldots, N-1$, to $h[n]$, $n = 0, 1, \ldots, L-1$, through

$$\mathbf{h} = \mathbf{F}_h \mathbf{h}' \tag{7}$$

where

$$\mathbf{h} = \begin{bmatrix} H[0] \\ H[1] \\ \vdots \\ H[N-1] \end{bmatrix} \quad \mathbf{h}' = \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[L-1] \end{bmatrix}$$

$$\mathbf{F}_h = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j\frac{2\pi}{N}} & \cdots & e^{-j\frac{2\pi(L-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(N-1)}{N}} & \cdots & e^{-j\frac{2\pi(N-1)(L-1)}{N}} \end{bmatrix}.$$

Instead of estimating $\mathbf{h}$, we can estimate $\mathbf{h}'$. This reduces the number of unknown channel coefficients from N to L.

For the phase noise, instead of estimating $\alpha[k]$, $k = 0, 1, \ldots, N-1$, we can estimate the phase noise components in the time domain, i.e., $e^{j\phi(nT_s)}$, $n = 0, 1, \ldots, N-1$. In order to reduce the number of unknowns, we can estimate $e^{j\phi(m(N-1)T_s/(M-1))}$ for $m = 0, 1, \ldots, M-1 (M < N)$, and then obtain the approximation of $e^{j\phi(nT_s)}$, $n = 0, 1, \ldots, N-1$, by interpolation. Let

$$\mathbf{c} = \begin{bmatrix} e^{j\phi(0)} \\ e^{j\phi(T_s)} \\ \vdots \\ e^{j\phi((N-1)T_s)} \end{bmatrix} \quad \mathbf{c}' = \begin{bmatrix} e^{j\phi(0)} \\ e^{j\phi\left(\frac{(N-1)T_s}{M-1}\right)} \\ \vdots \\ e^{j\phi((N-1)T_s)} \end{bmatrix}. \tag{8}$$

Then

$$\mathbf{c} \approx \mathbf{P}\mathbf{c}'$$

where $\mathbf{P}$ is an interpolation matrix to be determined. Using (2), we have

$$\mathbf{a} = \frac{1}{N}\mathbf{F}_a \mathbf{c} \approx \frac{1}{N}\mathbf{F}_a \mathbf{P}\mathbf{c}' \tag{9}$$

where

$$\mathbf{a} = [\,\alpha[0] \quad \alpha[1] \quad \cdots \quad \alpha[N-1]\,]^T,$$

$$\mathbf{F}_a = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j\frac{2\pi}{N}} & \cdots & e^{-j\frac{2\pi(N-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(N-1)}{N}} & \cdots & e^{-j\frac{2\pi(N-1)^2}{N}} \end{bmatrix}.$$

Consequently, knowing $\mathbf{x}$ during training, we can estimate $\mathbf{A}$ and $\mathbf{H}$ by solving

$$\min_{\mathbf{c}', \mathbf{h}'} \|\mathbf{y} - \mathbf{A}\mathbf{H}\mathbf{x}\|^2 \tag{10}$$

where $\mathbf{A}$ is related to $\mathbf{c}'$ by (9) and $\mathbf{H}$ is related to $\mathbf{h}'$ by (7). The estimates of $\mathbf{A}$ and $\mathbf{H}$ from (10) will have an ambiguity of a scaling factor, which can be resolved by constraining $\alpha[0]$ to be 1. If $L + M \leq N$, then (10) may have a unique solution for $\mathbf{c}'$ and $\mathbf{h}'$ (with the constraint $\alpha[0] = 1$). However, finding the necessary and sufficient conditions for which (10) has a unique solution, in general, belongs to the class of problems concerned with the global or local identifiability of a system [23], which is beyond the scope of this paper.

The optimization problem given by (10) is nonlinear and nonconvex. Suppose that $\mathbf{A}$ is known. The optimal $\mathbf{h}'$ is then given by

$$\mathbf{h}'_o = (\mathbf{F}_h^*\mathbf{X}^*\mathbf{A}^*\mathbf{A}\mathbf{X}\mathbf{F}_h)^{-1}\mathbf{F}_h^*\mathbf{X}^*\mathbf{A}^*\mathbf{y}$$

where $\mathbf{X} = \mathrm{diag}\{\mathbf{x}\}$. By substituting $\mathbf{H} = \mathrm{diag}\{\mathbf{F}_h\mathbf{h}'_o\}$ into (10), the optimal $\mathbf{c}'$ is given by the solution to the following nonconvex optimization problem:

$$\begin{aligned}\mathbf{c}'_o &= \arg\min_{\mathbf{c}'} \|\mathbf{y} - \mathbf{A}\cdot\mathrm{diag}\{\mathbf{F}_h\mathbf{h}'_o\}\cdot\mathbf{x}\|^2 \\ &= \arg\min_{\mathbf{c}'}\Big\|\mathbf{y} - \mathbf{A}\cdot\mathrm{diag}\Big\{\mathbf{F}_h(\mathbf{F}_h^*\mathbf{X}^*\mathbf{A}^*\mathbf{A}\mathbf{X}\mathbf{F}_h)^{-1} \\ &\qquad\qquad \cdot\mathbf{F}_h^*\mathbf{X}^*\mathbf{A}^*\mathbf{y}\Big\}\cdot\mathbf{x}\Big\|^2.\end{aligned}$$

In implementation, we use the two-step iterative algorithm presented in Algorithm 1 to find a suboptimal solution to (10). In Step 3, given a previously estimated $\mathbf{c}'$, we find an optimal $\mathbf{h}'$. Expression (11) can be rewritten as

$$\begin{aligned}\widehat{\mathbf{h}}'_{i-1} &= \arg\min_{\mathbf{h}'}\|\mathbf{y} - \widehat{\mathbf{A}}_{i-1}\mathbf{H}\mathbf{x}\|^2 \\ &= \arg\min_{\mathbf{h}'}\|\mathbf{y} - \widehat{\mathbf{A}}_{i-1}\mathbf{X}\mathbf{h}\|^2 \\ &= \arg\min_{\mathbf{h}'}\|\mathbf{y} - \widehat{\mathbf{A}}_{i-1}\mathbf{X}\mathbf{F}_h\mathbf{h}'\|^2.\end{aligned}$$

This is because $\mathbf{H} = \mathrm{diag}\{\mathbf{h}\}$ and $\mathbf{h} = \mathbf{F}_h\mathbf{h}'$. The problem is now in the form of a standard least squares problem and its solution is

$$\widehat{\mathbf{h}}'_{i-1} = \left(\mathbf{F}_h^*\mathbf{X}^*\widehat{\mathbf{A}}_{i-1}^*\widehat{\mathbf{A}}_{i-1}\mathbf{X}\mathbf{F}_h\right)^{-1}\mathbf{F}_h^*\mathbf{X}^*\widehat{\mathbf{A}}_{i-1}^*\mathbf{y}.$$

In Step 4, given a previously estimated $\mathbf{h}'$, we find an optimal $\mathbf{c}'$. Expression (12) can be reformulated as

$$\begin{aligned}\widehat{\mathbf{c}}'_i &= \arg\min_{\mathbf{c}'}\|\mathbf{y} - \mathbf{A}\widehat{\mathbf{H}}_{i-1}\mathbf{x}\|^2 \\ &= \arg\min_{\mathbf{c}'}\|\mathbf{y} - \mathbf{T}\mathbf{a}\|^2 \\ &= \arg\min_{\mathbf{c}'}\left\|\mathbf{y} - \frac{1}{N}\mathbf{T}\mathbf{F}_a\mathbf{P}\mathbf{c}'\right\|^2\end{aligned}$$

where $\mathbf{T}$ is the matrix whose elements are given by the vector $\widehat{\mathbf{H}}_{i-1}\mathbf{x}$. It then follows that

$$\widehat{\mathbf{c}}'_i = N(\mathbf{P}^*\mathbf{F}_a^*\mathbf{T}^*\mathbf{T}\mathbf{F}_a\mathbf{P})^{-1}\mathbf{P}^*\mathbf{F}_a^*\mathbf{T}^*\mathbf{y}.$$

It can be seen that Step 3 finds the optimal $\widehat{\mathbf{h}}'_{i-1}$ by setting $\mathbf{c}'$ to be the predetermined $\widehat{\mathbf{c}}'_{i-1}$, and hence

$$\|\mathbf{y} - \widehat{\mathbf{A}}_{i-1}\widehat{\mathbf{H}}_{i-1}\mathbf{x}\|^2 \le \|\mathbf{y} - \widehat{\mathbf{A}}_{i-1}\widehat{\mathbf{H}}_{i-2}\mathbf{x}\|^2.$$

Step 4 finds the optimal $\widehat{\mathbf{c}}'_i$ by setting $\mathbf{h}'$ to be the predetermined $\widehat{\mathbf{h}}'_{i-1}$, and hence

$$\|\mathbf{y} - \widehat{\mathbf{A}}_i\widehat{\mathbf{H}}_{i-1}\mathbf{x}\|^2 \le \|\mathbf{y} - \widehat{\mathbf{A}}_{i-1}\widehat{\mathbf{H}}_{i-1}\mathbf{x}\|^2.$$

Therefore, the objective function is decreasing with $i = 1, 2, \ldots$ and eventually converges to a local minimum. The obtained $\widehat{\mathbf{H}}$ will be used in the data transmission stage when comb-type symbols are transmitted. In Section III-B, we assume that $\mathbf{H}$ is known to be $\widehat{\mathbf{H}}$.

Algorithm 1 Joint Channel and Phase Noise Estimation

0: Start with an initial guess $\widehat{\mathbf{c}}'_0$. For example

$$\widehat{\mathbf{c}}'_0 = [1 \quad 1 \quad \ldots \quad 1]^T.$$

1: $i = 1$

2: **repeat**

3: Let $\widehat{\mathbf{a}}_{i-1} = (1/N)\mathbf{F}_a\mathbf{P}\widehat{\mathbf{c}}'_{i-1}$ and find the associated optimal $\widehat{\mathbf{h}}'_{i-1}$ by solving the following least squares problem:

$$\widehat{\mathbf{h}}'_{i-1} = \arg\min_{\mathbf{h}'}\|\mathbf{y} - \widehat{\mathbf{A}}_{i-1}\mathbf{H}\mathbf{x}\|^2 \tag{11}$$

where $\mathbf{x}$ and $\mathbf{y}$ are known and $\widehat{\mathbf{A}}_{i-1}$ is determined by $\widehat{\mathbf{a}}_{i-1}$ according to (6).

4: Let $\widehat{\mathbf{h}}_{i-1} = \mathbf{F}_h\widehat{\mathbf{h}}'_{i-1}$, and find the associated optimal $\widehat{\mathbf{c}}'_i$ by solving the following least squares problem:

$$\widehat{\mathbf{c}}'_i = \arg\min_{\mathbf{c}'}\|\mathbf{y} - \mathbf{A}\widehat{\mathbf{H}}_{i-1}\mathbf{x}\|^2 \tag{12}$$

where $\widehat{\mathbf{H}}_{i-1} = \mathrm{diag}\{\widehat{\mathbf{h}}_{i-1}\}$.

5: $i = i + 1$

6: **until** there is no significant improvement in the objective function $\|\mathbf{y} - \widehat{\mathbf{A}}_i\widehat{\mathbf{H}}_i\mathbf{x}\|^2$.

B. Joint Data Symbol and Phase Noise Estimation

In each comb-type OFDM symbol, assume that Q carriers are used for pilot tones and $N - Q$ carriers for data symbols. Similarly, instead of estimating $\alpha[k]$, $k = 0, 1, \ldots, N-1$, we estimate the phase noise components in the time domain, i.e., $e^{j\phi(m(N-1)T_s/(M-1))}$ for $m = 0, 1, \ldots, M-1$, and then obtain $e^{j\phi(nT_s)}$, $n = 0, 1, \ldots, N-1$, by interpolation. The estimation problem can be formulated as

$$\min_{\mathbf{c}', \mathbf{x}_{\mathrm{data}}}\|\mathbf{y} - \mathbf{A}\mathbf{H}\mathbf{x}\|^2$$

which can be rewritten as

$$\min_{\mathbf{c}', \mathbf{x}_{\mathrm{data}}}\|\mathbf{y} - \mathbf{A}_{\mathrm{pilot}}\mathbf{H}_{\mathrm{pilot}}\mathbf{x}_{\mathrm{pilot}} - \mathbf{A}_{\mathrm{data}}\mathbf{H}_{\mathrm{data}}\mathbf{x}_{\mathrm{data}}\|^2 \tag{13}$$

where $\mathbf{x}_{\mathrm{pilot}}$ is the subvector of $\mathbf{x}$ that consists of all the pilot symbols, $\mathbf{A}_{\mathrm{pilot}}$ and $\mathbf{H}_{\mathrm{pilot}}$ are its associated submatrices from $\mathbf{A}$ and $\mathbf{H}$, $\mathbf{x}_{\mathrm{data}}$ is the subvector of $\mathbf{x}$ that consists of all the data symbols, and $\mathbf{A}_{\mathrm{data}}$ and $\mathbf{H}_{\mathrm{data}}$ are its associated submatrices from $\mathbf{A}$ and $\mathbf{H}$. Since there are $N - Q$ unknown data symbols in $\mathbf{x}_{\mathrm{data}}$ and M unknown phase noise components in $\mathbf{c}'$, then (13) may have a unique solution if $Q \ge M$.

Since the optimization problem given by (13) is similar to the joint channel estimation problem we just solved, we can follow the same procedure to solve it. If $\mathbf{A}$ is known, the optimal $\mathbf{x}_{\rm data}$ is given by

$$\mathbf{x}_{{\rm data},o} = \mathbf{H}_{\rm data}^{-1}\left(\mathbf{A}_{\rm data}^{*}\mathbf{A}_{\rm data}\right)^{-1}\mathbf{A}_{\rm data}^{*} \cdot (\mathbf{y} - \mathbf{A}_{\rm pilot}\mathbf{H}_{\rm pilot}\mathbf{x}_{\rm pilot}).$$

Then, the optimal $\mathbf{c}'$ is given by the following nonconvex optimization problem:

$$\begin{aligned}\mathbf{c}'_o &= \arg\min_{\mathbf{c}'} \|\mathbf{y} - \mathbf{A}_{\rm pilot}\mathbf{H}_{\rm pilot}\mathbf{x}_{\rm pilot} - \mathbf{A}_{\rm data}\mathbf{H}_{\rm data}\mathbf{x}_{{\rm data},o}\|^2\\ &= \arg\min_{\mathbf{c}'} \|\mathbf{y} - \mathbf{A}_{\rm pilot}\mathbf{H}_{\rm pilot}\mathbf{x}_{\rm pilot}\\ &\quad - \mathbf{A}_{\rm data}\left(\mathbf{A}_{\rm data}^{*}\mathbf{A}_{\rm data}\right)^{-1}\\ &\quad \cdot \mathbf{A}_{\rm data}^{*}(\mathbf{y} - \mathbf{A}_{\rm pilot}\mathbf{H}_{\rm pilot}\mathbf{x}_{\rm pilot})\|^2.\end{aligned}$$

An iterative method for finding a suboptimal solution is presented in Algorithm 2, in which we first estimate the CPE coefficient $\alpha[0]$ in order to correct the scaling ambiguity in $\widehat{\mathbf{H}}$. Finally, the obtained $\widehat{\mathbf{x}}_{\rm data}$ is mapped into bits.

Algorithm 2 Joint Data Symbol and Phase Noise Estimation

0: Use the method given in [10] to estimate the CPE coefficient $\alpha[0]$. Assume that $k_{{\rm pilot},j}$, $j = 1, 2, \ldots, Q$, are the subcarrier indices of the Q pilot tones. Then, $\widehat{\alpha}[0]$ is given by

$$\widehat{\alpha}[0] = \frac{\sum_{j=1}^{Q}\left(H[k_{{\rm pilot},j}]\right)^{*}\left(x[k_{{\rm pilot},j}]\right)^{*} y[k_{{\rm pilot},j}]}{\sum_{j=1}^{Q}|H[k_{{\rm pilot},j}]|^2\,|x[k_{{\rm pilot},j}]|^2}.$$

Let

$$\widehat{\mathbf{c}}'_0 = [\widehat{\alpha}[0] \quad \widehat{\alpha}[0] \quad \ldots \quad \widehat{\alpha}[0]]^T.$$

1: $i = 1$

2: **repeat**

3: Let $\widehat{\mathbf{a}}_{i-1} = (1/N)\mathbf{F}_a\mathbf{P}\widehat{\mathbf{c}}'_{i-1}$ and find the associated optimal $\widehat{\mathbf{x}}_{{\rm data},i-1}$ by solving the following least squares problem:

$$\widehat{\mathbf{x}}_{{\rm data},i-1} = \arg\min_{\mathbf{x}_{\rm data}} \|\mathbf{y} - \widehat{\mathbf{A}}_{{\rm pilot},i-1}\mathbf{H}_{\rm pilot}\mathbf{x}_{\rm pilot} - \widehat{\mathbf{A}}_{{\rm data},i-1}\mathbf{H}_{\rm data}\mathbf{x}_{\rm data}\|^2$$

where $\widehat{\mathbf{A}}_{{\rm pilot},i-1}$ and $\widehat{\mathbf{A}}_{{\rm data},i-1}$ are determined by $\widehat{\mathbf{a}}_{i-1}$ according to (6).

4: Find the optimal $\widehat{\mathbf{c}}'_i$ by solving the following least squares problem:

$$\widehat{\mathbf{c}}'_i = \arg\min_{\mathbf{c}'} \|\mathbf{y} - \mathbf{A}_{\rm pilot}\mathbf{H}_{\rm pilot}\mathbf{x}_{\rm pilot} - \mathbf{A}_{\rm data}\mathbf{H}_{\rm data}\widehat{\mathbf{x}}_{{\rm data},i-1}\|^2.$$

5: $i = i + 1$

6: **until** there is no significant improvement in the objective function $\|\mathbf{y} - \widehat{\mathbf{A}}_{{\rm pilot},i}\mathbf{H}_{\rm pilot}\mathbf{x}_{\rm pilot} - \widehat{\mathbf{A}}_{{\rm data},i}\mathbf{H}_{\rm data}\widehat{\mathbf{x}}_{{\rm data},i}\|^2$.

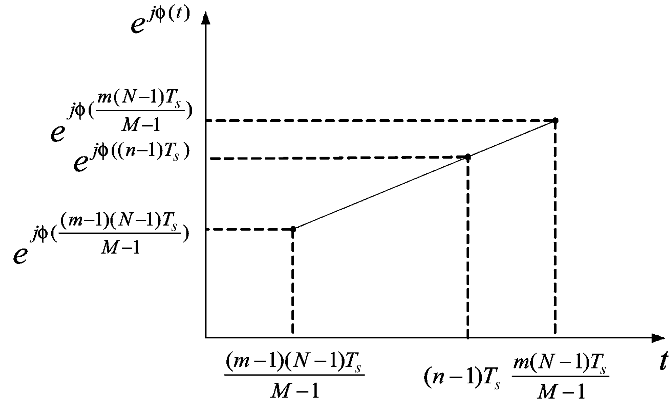


Fig. 3. Linear interpolation.

C. Selection of the Interpolation Matrix $\mathbf{P}$

1) PSD of the Phase Noise is Known: If the power spectral density (PSD) of the phase noise is known, the optimal interpolation matrix $\mathbf{P}_o$ can be obtained by minimizing the mean square error of interpolating $\mathbf{c}$ from $\mathbf{c}'$, i.e.,

$$\mathbf{P}_o = \arg\min_{\mathbf{P}} \mathbf{E}\|\mathbf{c} - \mathbf{P}\mathbf{c}'\|^2$$

from which the optimal $\mathbf{P}_o$ is given by

$$\mathbf{P}_o = \mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1} \tag{14}$$

where $\mathbf{R}_{\mathbf{c}\mathbf{c}'} = \mathbf{E}\{\mathbf{c}\mathbf{c}'^*\}$ and $\mathbf{R}_{\mathbf{c}'} = \mathbf{E}\{\mathbf{c}'\mathbf{c}'^*\}$.

2) PSD of the Phase Noise is Unknown: In this case, an interpolation matrix $\mathbf{P}_L \in \mathbb{R}^{N\times M}$ is constructed from linear interpolation as is illustrated in Fig. 3, and its element at the nth row and mth column is given by (15) shown at the bottom of the page, where $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$.

It can be shown that for free-running oscillators with linewidth ν, the optimal interpolator $\mathbf{P}_o$ is approximately equal to the linear interpolator $\mathbf{P}_L$ if $\nu T_s \ll 1$ (see Appendix II for a proof). In wireless systems, $T_s \sim 1$ μs and ν ranges from 10 Hz to at most 10 kHz, for which $\nu T_s \leq 0.01$ makes the assumption valid.

IV. Performance Analysis

In this section, we analyze the effect of phase noise on OFDM receivers and the performance of different compensation schemes in terms of effective SNR degradation. The effective SNR at the receiver is an important measure to characterize the system performance and can be used to predict the bit error rate (BER) [4], [5]. The expressions of the effective SNR are derived by assuming that the receiver has perfect

$$\mathbf{P}_L(n,m) = \begin{cases} m - \dfrac{(n-1)(M-1)}{N-1}, & \text{if } \dfrac{(m-1)(N-1)}{M-1} \leq n-1 < \dfrac{m(N-1)}{M-1}\\ \dfrac{(n-1)(M-1)}{N-1} - (m-2), & \text{if } \dfrac{(m-2)(N-1)}{M-1} \leq n-1 < \dfrac{(m-1)(N-1)}{M-1}\\ 0, & \text{otherwise}\end{cases} \tag{15}$$

information about the channel matrix $\mathbf{H}$. The following is also assumed: 1) the data symbols $x[k]$ are independent and identically distributed with zero mean and 2) the data symbols, the phase noise, the channel coefficients, and the additive noise are independent of each other. Moreover, let

$$\sigma_x^2 = \mathbf{E}\left\{|x[k]|^2\right\} \quad \sigma_H^2 = \mathbf{E}\left\{|H[k]|^2\right\} \quad \sigma_w^2 = \mathbf{E}\left\{|w[k]|^2\right\}.$$

A. No Compensation for Phase Noise

In this scenario, the receiver assumes no phase noise in the system. Expression (4) can be rewritten as

$$y[k] = H[k]x[k] + (\alpha[0]-1)\,H[k]x[k] + \sum_{r=1}^{N-1} \alpha[r]H\left[(k-r)_N\right]x\left[(k-r)_N\right] + w[k]$$

$H[k]x[k]$ is the desired signal component and the other terms are regarded as noise. Hence, the effective SNR at the receiver is given by

$$\begin{aligned}\mathrm{SNR}_{\mathrm{no}} &= \frac{\sigma_H^2\sigma_x^2}{\mathbf{E}\left\{|\alpha[0]-1|^2\right\}\sigma_H^2\sigma_x^2 + \sum_{k=1}^{N-1}\sigma_{\alpha,k}^2\sigma_H^2\sigma_x^2 + \sigma_w^2}\\ &= \frac{\mathrm{SNR}_0}{\left(\mathbf{E}\left\{|\alpha[0]-1|^2\right\} + \sum_{k=1}^{N-1}\sigma_{\alpha,k}^2\right)\mathrm{SNR}_0 + 1}\end{aligned} \tag{16}$$

where effective $\mathrm{SNR}_0 = \sigma_H^2\sigma_x^2/\sigma_w^2$ is the effective SNR when there is no phase noise in the system and $\sigma_{\alpha,k}^2 = \mathbf{E}\{|\alpha[k]|^2\}$. For the phase noise models presented in Appendix I, the expressions for $\mathbf{E}\{|\alpha[0]-1|^2\}$ and $\sum_{k=1}^{N-1}\sigma_{\alpha,k}^2$ are given by (29) and (30), respectively.

B. CPE Compensation Scheme

In this case, assume that the receiver has perfect information about $\alpha[0]$ but has no information about $\alpha[k], k=1,2,\ldots,N-1$. With the CPE correction scheme, $\alpha_0 H[k]x[k]$ in (4) represents the signal component and the other terms are regarded as noise. Hence, the effective SNR is given by

$$\begin{aligned}\mathrm{SNR}_{\mathrm{CPE}} &= \frac{\sigma_{\alpha,0}^2\sigma_H^2\sigma_x^2}{\sum_{k=1}^{N-1}\sigma_{\alpha,k}^2\sigma_H^2\sigma_x^2 + \sigma_w^2}\\ &= \frac{\sigma_{\alpha,k}^2\mathrm{SNR}_0}{\left(\sum_{k=1}^{N-1}\sigma_{\alpha,k}^2\right)\mathrm{SNR}_0 + 1}.\end{aligned} \tag{17}$$

C. Ideal Compensation for Both CPE and ICI

In expression (5), if both $\mathbf{A}$ and $\mathbf{H}$ are known at the receiver, $\mathbf{AHx}$ is the desired signal component and $\mathbf{w}$ is the noise component. Hence

$$\begin{aligned}\mathrm{SNR}_{\mathrm{ideal}} &= \frac{\mathbf{E}\{\mathbf{x}^*\mathbf{H}^*\mathbf{A}^*\mathbf{AHx}\}}{\mathbf{E}\{\mathbf{w}^*\mathbf{w}\}}\\ &= \frac{\sigma_H^2\sigma_x^2\mathrm{Tr}\left\{\mathbf{E}\{\mathbf{AA}^*\}\right\}}{N\sigma_w^2}\\ &= \left(\sum_{k=0}^{N-1}\sigma_{\alpha,k}^2\right)\mathrm{SNR}_0\\ &= \mathrm{SNR}_0.\end{aligned} \tag{18}$$

D. Proposed Algorithm—Nonideal Compensation for CPE and ICI

Using the proposed algorithm, we can get an estimate $\widehat{\mathbf{A}}$ of the phase noise matrix $\mathbf{A}$. Expression (5) can be rewritten as

$$\mathbf{y} = \widehat{\mathbf{A}}\mathbf{Hx} + (\mathbf{A}-\widehat{\mathbf{A}})\mathbf{Hx} + \mathbf{w}.$$

Then, the effective SNR can be expressed as

$$\begin{aligned}\mathrm{SNR}_{\mathrm{prop}} &= \frac{\mathbf{E}\{\mathbf{x}^*\mathbf{H}^*\widehat{\mathbf{A}}^*\widehat{\mathbf{A}}\mathbf{Hx}\}}{\mathbf{E}\left\{\mathbf{x}^*\mathbf{H}^*(\mathbf{A}-\widehat{\mathbf{A}})^*(\mathbf{A}-\widehat{\mathbf{A}})\mathbf{Hx}\right\} + \mathbf{E}\{\mathbf{w}^*\mathbf{w}\}}\\ &= \frac{\sigma_H^2\sigma_x^2\mathrm{Tr}\left\{\mathbf{E}\{\widehat{\mathbf{A}}\widehat{\mathbf{A}}^*\}\right\}}{\sigma_H^2\sigma_x^2\mathrm{Tr}\left\{\mathbf{E}\left\{(\mathbf{A}-\widehat{\mathbf{A}})(\mathbf{A}-\widehat{\mathbf{A}})^*\right\}\right\} + N\sigma_w^2}\\ &= \frac{\mathbf{E}\left\{\|\widehat{\mathbf{a}}\|^2\right\}\cdot\mathrm{SNR}_0}{\mathbf{E}\left\{\|\mathbf{a}-\widehat{\mathbf{a}}\|^2\right\}\cdot\mathrm{SNR}_0 + 1}.\end{aligned}$$

Since $\widehat{\mathbf{a}} = (1/N)\mathbf{F}_a\mathbf{P}\widehat{\mathbf{c}}'$, we have

$$\begin{aligned}\mathbf{E}\left\{\|\widehat{\mathbf{a}}\|^2\right\} &= \mathbf{E}\left\{\frac{1}{N^2}\widehat{\mathbf{c}}'^*\mathbf{P}^*\mathbf{F}_a^*\mathbf{F}_a\mathbf{P}\widehat{\mathbf{c}}'\right\}\\ &= \frac{1}{N}\mathbf{E}\{\widehat{\mathbf{c}}'^*\mathbf{P}^*\mathbf{P}\widehat{\mathbf{c}}'\}\\ &= \frac{1}{N}\mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\widehat{\mathbf{c}}'}\mathbf{P}^*\}\end{aligned}$$

where $\mathbf{R}_{\widehat{\mathbf{c}}'} = \mathbf{E}\{\widehat{\mathbf{c}}'\widehat{\mathbf{c}}'^*\}$. Moreover

$$\begin{aligned}\mathbf{E}\left\{\|\mathbf{a}-\widehat{\mathbf{a}}\|^2\right\} &= \mathbf{E}\left\{\left\|\frac{1}{N}\mathbf{F}_a\mathbf{c} - \frac{1}{N}\mathbf{F}_a\mathbf{P}\widehat{\mathbf{c}}'\right\|^2\right\}\\ &= \frac{1}{N}\mathbf{E}\left\{\|\mathbf{c}-\mathbf{P}\widehat{\mathbf{c}}'\|^2\right\}\\ &= \frac{1}{N}\mathbf{E}\left\{\|\mathbf{c}-\mathbf{P}\mathbf{c}'+\mathbf{P}\mathbf{c}'-\mathbf{P}\widehat{\mathbf{c}}'\|^2\right\}\\ &\approx \frac{1}{N}\mathbf{E}\left\{\|\mathbf{c}-\mathbf{P}\mathbf{c}'\|^2\right\} + \frac{1}{N}\mathbf{E}\left\{\|\mathbf{P}\mathbf{c}'-\mathbf{P}\widehat{\mathbf{c}}'\|^2\right\}\end{aligned}$$

where $\mathbf{E}\{\|\mathbf{c}-\mathbf{P}\mathbf{c}'\|^2\}$ accounts for the modeling error caused by the interpolation, $\mathbf{E}\{\|\mathbf{P}\mathbf{c}'-\mathbf{P}\widehat{\mathbf{c}}'\|^2\}$ accounts for the estimation error of $\mathbf{c}'$, and the approximation assumes that the modeling error is uncorrelated with the estimation error. Letting

$$\mathbf{R}_{\mathbf{c}} = \mathbf{E}\{\mathbf{cc}^*\} \quad \text{and} \quad \mathbf{R}_{\mathbf{c}'-\widehat{\mathbf{c}}'} = \mathbf{E}\left\{(\mathbf{c}'-\widehat{\mathbf{c}}')(\mathbf{c}'-\widehat{\mathbf{c}}')^*\right\}$$

we have

$$\begin{aligned}\mathbf{E}\left\{\|\mathbf{a}-\widehat{\mathbf{a}}\|^2\right\} &\approx \frac{1}{N}\Big[\mathrm{Tr}\{\mathbf{R}_{\mathbf{c}}\} - 2\mathrm{Re}\left\{\mathrm{Tr}\{\mathbf{R}_{\mathbf{cc}'}\mathbf{P}^*\}\right\}\\ &\qquad + \mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'}\mathbf{P}^*\} + \mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'-\widehat{\mathbf{c}}'}\mathbf{P}^*\}\Big]\\ &= \frac{1}{N}\Big(N - 2\mathrm{Re}\left\{\mathrm{Tr}\{\mathbf{R}_{\mathbf{cc}'}\mathbf{P}^*\}\right\}\\ &\qquad + \mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'}\mathbf{P}^*\} + \mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'-\widehat{\mathbf{c}}'}\mathbf{P}^*\}\Big)\end{aligned}$$

because $\mathrm{Tr}\{\mathbf{R}_{\mathbf{c}}\} = N$. Therefore, we have the equation shown at the bottom of the next page.

To further simplify that expression, we assume that $\mathbf{c}' \approx \widehat{\mathbf{c}}'$, which gives that $\mathbf{R}_{\widehat{\mathbf{c}}'} \approx \mathbf{R}_{\mathbf{c}'}$ and $\mathbf{R}_{\mathbf{c}'-\widehat{\mathbf{c}}'} \approx 0$. The effective SNR can then be approximated by

$$\mathrm{SNR}_{\mathrm{prop}} \approx \frac{\frac{1}{N}\mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'}\mathbf{P}^*\}\cdot\mathrm{SNR}_0}{\frac{1}{N}\left(N-2\mathrm{Re}\left\{\mathrm{Tr}\{\mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{P}^*\}\right\}+\mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'}\mathbf{P}^*\}\right)\cdot\mathrm{SNR}_0+1}.$$

If the interpolation matrix is given by $\mathbf{P} = \mathbf{P}_o = \mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}$, then

$$\mathrm{SNR}_{\mathrm{prop}} \approx \frac{\frac{1}{N}\mathrm{Tr}\left\{\mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}\mathbf{R}_{\mathbf{c}\mathbf{c}'}^*\right\}\cdot\mathrm{SNR}_0}{\left(1-\frac{1}{N}\mathrm{Tr}\left\{\mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}\mathbf{R}_{\mathbf{c}\mathbf{c}'}^*\right\}\right)\cdot\mathrm{SNR}_0+1}. \quad (19)$$

In order to see more clearly how $\mathrm{SNR}_{\mathrm{prop}}$ varies with M, namely, the length of $\mathbf{c}'$, we assume that $\mathbf{c}'$ is a subvector of $\mathbf{c}$, although it is true only when $N-1$ is divisible by $M-1$ according to (8). Since $\mathbf{c}'$ is a subvector of $\mathbf{c}$, then $\mathbf{R}_{\mathbf{c}'}$ is a submatrix of $\mathbf{R}_{\mathbf{c}\mathbf{c}'}$. Hence

$$\mathrm{Tr}\left\{\mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}\mathbf{R}_{\mathbf{c}\mathbf{c}'}^*\right\} \geq \mathrm{Tr}\left\{\mathbf{R}_{\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}\mathbf{R}_{\mathbf{c}'}^*\right\} = \mathrm{Tr}\{\mathbf{R}_{\mathbf{c}'}\} = M. \quad (20)$$

Combining (19) with (20) gives

$$\begin{aligned}\mathrm{SNR}_{\mathrm{prop}} &\approx \frac{\frac{1}{N}\mathrm{Tr}\left\{\mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}\mathbf{R}_{\mathbf{c}\mathbf{c}'}^*\right\}\cdot\mathrm{SNR}_0}{\left(1-\frac{1}{N}\mathrm{Tr}\left\{\mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}\mathbf{R}_{\mathbf{c}\mathbf{c}'}^*\right\}\right)\cdot\mathrm{SNR}_0+1}\\ &\geq \frac{\frac{M}{N}\cdot\mathrm{SNR}_0}{\left(1-\frac{M}{N}\right)\cdot\mathrm{SNR}_0+1}. \quad (21)\end{aligned}$$

The bound given by (21) indicates that increasing M will make the effective SNR closer to SNR_0.

In Fig. 4, the theoretical effective SNR is plotted for different compensation schemes and for different types of phase noise by using expressions (16)–(19). Fig. 4(a) plots the effective SNR versus effective SNR_0 for the free-running oscillators with linewidth $\nu = 5$ kHz, and Fig. 4(b) plots the effective SNR versus the oscillator linewidth ν when $\mathrm{SNR}_0 = 25$ dB. It is seen in Fig. 4(a) and (b) that $\mathrm{SNR}_{\mathrm{no}}$ is about -3.0 dB for free-running oscillators. This is because

$$\begin{aligned}\mathbf{E}\left\{|\alpha[0]-1|^2\right\}+\sum_{k=1}^{N-1}\sigma_{\alpha,k}^2 &= 2-2\mathrm{Re}\left\{\mathbf{E}\left\{\alpha[0]\right\}\right\}\\ &= 2-2\mathrm{Re}\left\{\mathbf{E}\left\{c_{\mathrm{Free}}(t)\right\}\right\}\\ &= 2.\end{aligned}$$

From (16), $\mathrm{SNR}_{\mathrm{no}} \approx 0.5 = -3.0$ dB if $\mathrm{SNR}_0 \gg 1$. It can be also interpreted as follows. The power of the carrier signal is $\mathbf{E}\{|c_{\mathrm{Free}}(t)|^2\} = 1$ and the variance of the carrier noise is

$$\mathbf{E}\left\{|c_{\mathrm{Free}}(t)-1|^2\right\} = 2-2\mathrm{Re}\left\{\mathbf{E}\left\{c_{\mathrm{Free}}(t)\right\}\right\} = 2$$

which implies the -3.0-dB uncompensated SNR. In Fig. 4(d) and (f), the phase noise variance is determined by the linewidth ν and the loop bandwidth f_L or f_n according to expressions (23) and (25), respectively, and is measured in decibels with respect to the carrier power, namely, dBc. In Section V, we will compare the theoretical system performance with the simulated system performance and discuss the observations from the plots.

V. Computer Simulations

The proposed scheme is simulated in comparison with the ideal OFDM receiver with perfect phase noise compensation, the CPE correction scheme [10], and the ICI compensation schemes proposed in [12], [14], and [16]. The system bandwidth is 20 MHz, i.e., $T_s = 0.05$ μs, and the constellation used for symbol mapping is 16-QAM. The OFDM symbol size is $N = 64$ and the prefix length is $P = 20$. The channel length is 6, and each tap is independently Rayleigh distributed with the power profile specified by 3-dB decay per tap. Two scenarios are studied in the simulations. One scenario assumes that the receiver has perfect channel information, while the other scenario assumes that the receiver has to estimate the channel using pilot symbols. Only one block-type pilot symbol is used in the proposed scheme for each time of channel estimation, while two are used in the CPE correction scheme. The assumed channel length in the time domain for channel estimation is $L = 12$, the length of the phase noise vector to be estimated is $M = 4$ or 8, and the number of pilot tones in the comb-type symbols is $Q = 8$ or 16. The phase noise generated by the free-running oscillators and the phase-loop locked oscillators is simulated according to the models given in Appendix I.

Fig. 5 shows the simulated system performance when the phase noise is generated by a free-running oscillator. The phase noise spectrum for $\nu = 5$ kHz is plotted in Fig. 5(a). Fig. 5(b)–(e) compares different schemes in terms of the effective SNR and the uncoded BER for the phase noise spectrum with $\nu = 5$ kHz. Compared to the CPE correction scheme, the proposed scheme can achieve 5–8 dB more improvement in the effective SNR for the high SNR regime (more than 20 dB). Fig. 5(f) and (g) shows the system performance in terms of the effective SNR for different oscillator linewidth. The larger the linewidth, the more random the phase noise is, and consequently, the more difficult it is to compensate for the phase noise. It can be seen that the proposed algorithm with $Q = 16$ and $M = 8$ can improve the effective SNR by about 5 dB if perfect channel information is available (i.e., no channel estimation) and 8 dB if the receiver has to estimate the channel. In other words, the proposed algorithm can reduce the sensitivity of OFDM receivers to phase noise by about 8 dB. Moreover, the simulated system performance displayed here is consistent with the theoretical performance shown in Fig. 4. It is also demonstrated in Figs. 6 and 7 that the proposed scheme

$$\mathrm{SNR}_{\mathrm{prop}} \approx \frac{\frac{1}{N}\mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\widehat{\mathbf{c}}'}\mathbf{P}^*\}\cdot\mathrm{SNR}_0}{\frac{1}{N}\left(N-2\mathrm{Re}\left\{\mathrm{Tr}\{\mathbf{R}_{\mathbf{c}\mathbf{c}'}\mathbf{P}^*\}\right\}+\mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'}\mathbf{P}^*\}+\mathrm{Tr}\{\mathbf{P}\mathbf{R}_{\mathbf{c}'-\widehat{\mathbf{c}}'}\mathbf{P}^*\}\right)\cdot\mathrm{SNR}_0+1}$$

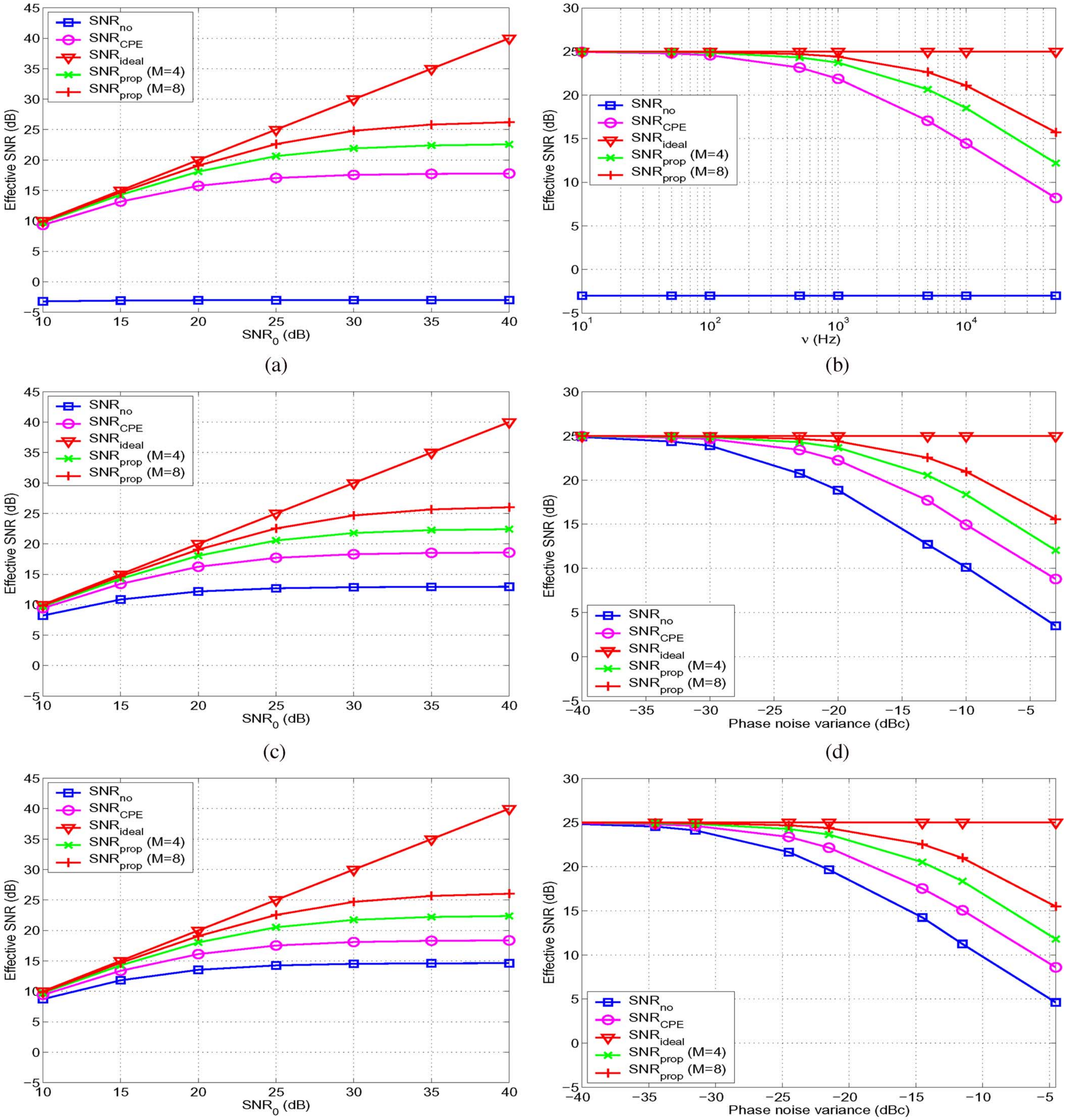


Fig. 4. Plots of theoretical effective SNR by using (16)–(19). (a) and (b) Plots for the free-running oscillators. (c) and (d) Plots for the first-order phase-locked loop (PLL) oscillators with $f_L = 50$ kHz. (e) and (f) Plots for the second-order PLL oscillators with $f_n = 50$ kHz. (a) $\nu = 5$ kHz. (b) $\mathrm{SNR}_0 = 25$ dB. (c) $\nu = 5$ kHz. (d) $\mathrm{SNR}_0 = 25$ dB. (e) $\nu = 5$ kHz. (f) $\mathrm{SNR}_0 = 25$ dB.

can significantly improve the effective SNR and uncoded BER for the phase-loop locked oscillators. The improvement in the effective SNR can be more than 8 dB. In order to further improve the effective SNR and uncoded BER, we can add more pilot tones to increase Q and M; however, this reduces the data rate and spectral efficiency.

In Fig. 8, the proposed phase noise compensation scheme is compared with the self-cancellation scheme proposed in [12], the frequency-domain FIR-type equalizer proposed in [14], and the ICI suppression method using sinusoidal approximation proposed in [16]. For fairness, we assume that the receivers have perfect channel information, without worrying about the

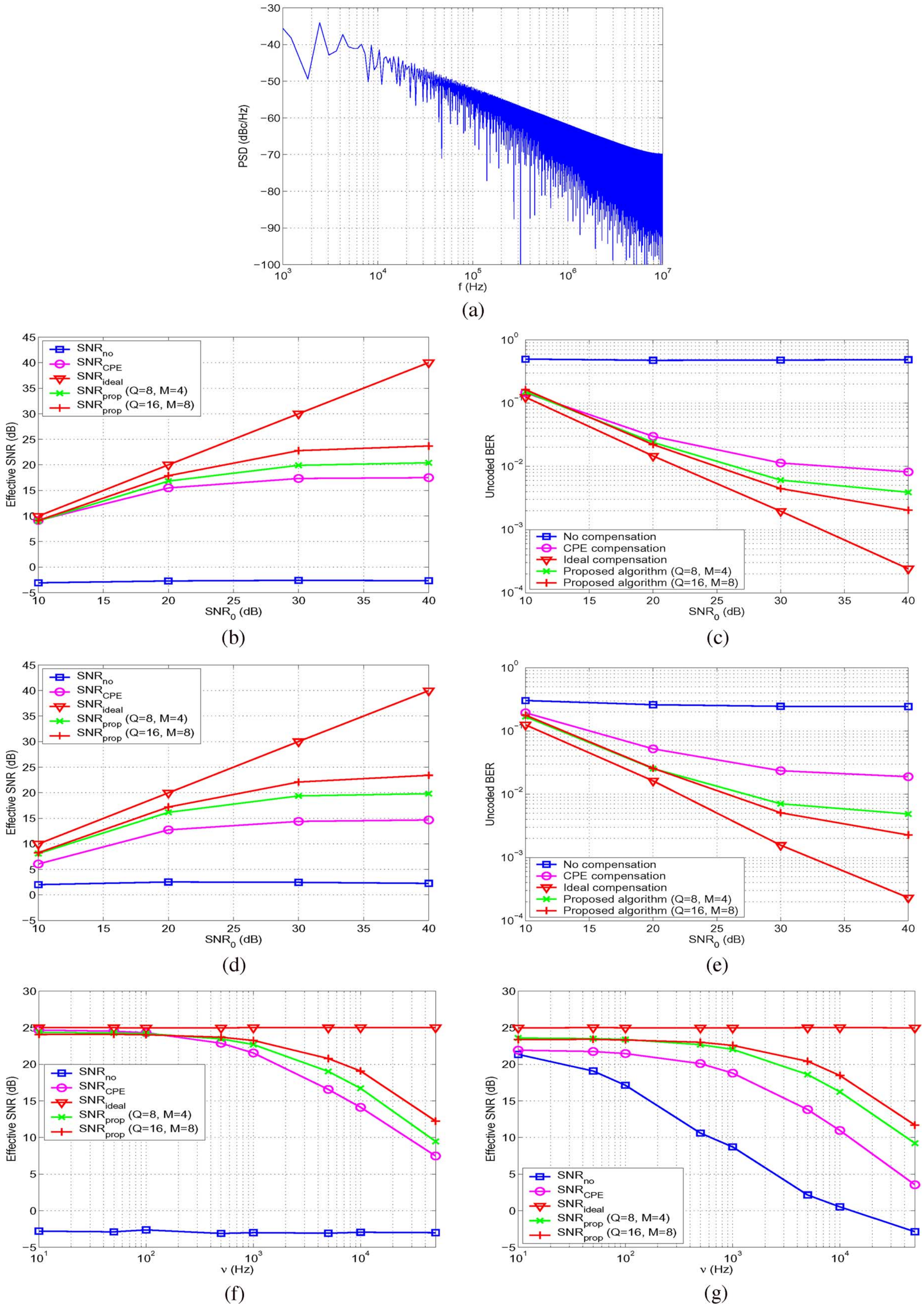
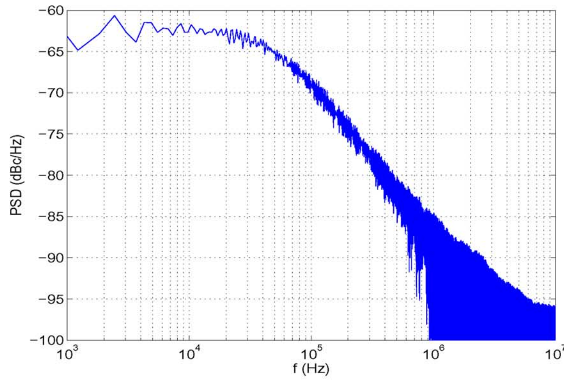
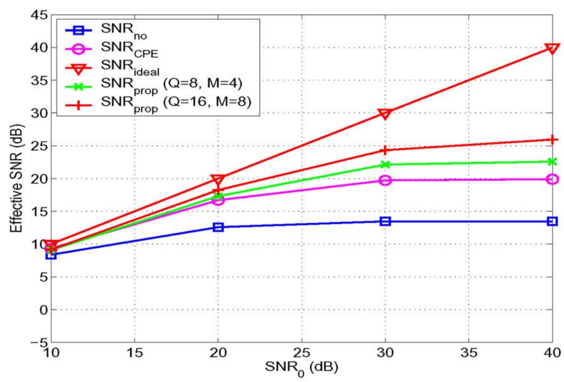


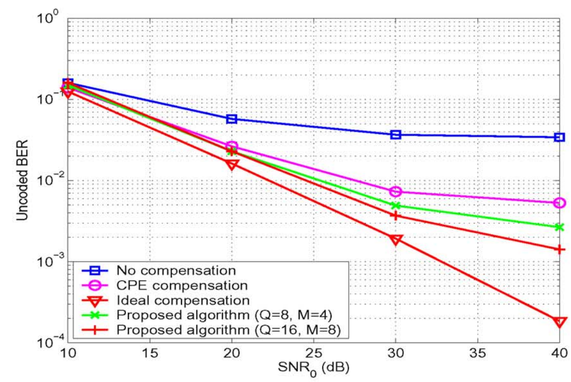
Fig. 5. Simulation results when the phase noise is generated by a free-running oscillator. (b)–(e) Effective SNR and uncoded BER versus SNR_0 when the oscillator linewidth is $\nu = 5$ kHz. (f) and (g) Effective SNR versus ν when $\mathrm{SNR}_0 = 25$ dB. (a) PSD of phase noise with $\nu = 5$ kHz. (b) Effective SNR with perfect channel information. (c) Uncoded BER with perfect channel information. (d) Effective SNR with estimated channel information. (e) Uncoded BER with estimated channel information. (f) Effective SNR versus ν with perfect channel information. (g) Effective SNR versus ν with estimated channel information.



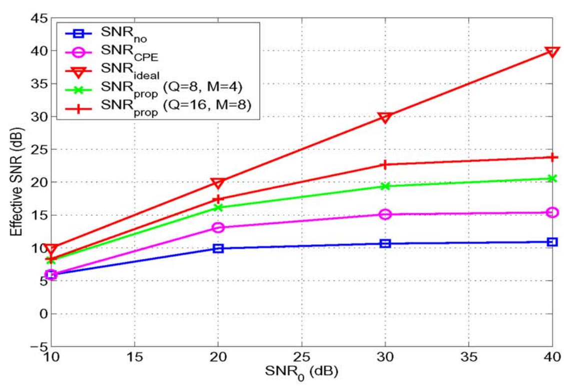
(a)



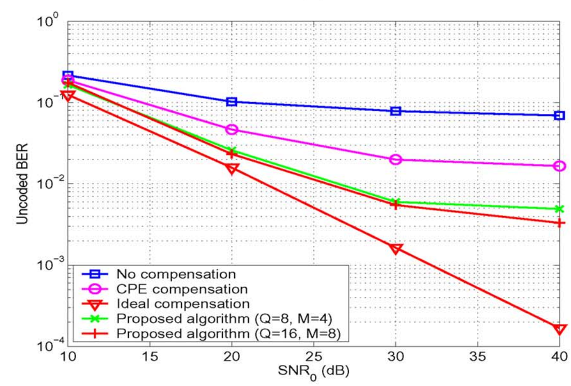
(b)



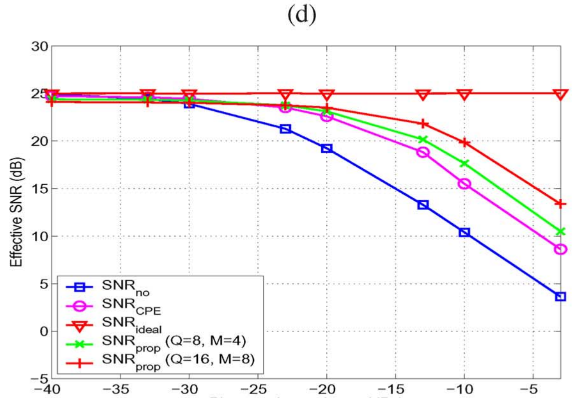
(c)



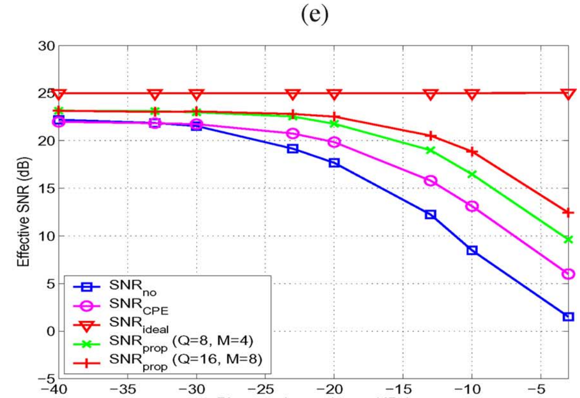
(d)



(e)



(f)



(g)

Fig. 6. Simulation results when the phase noise is generated by a first-order PLL oscillator. (b)–(e) Effective SNR and uncoded BER versus SNR_0 when $\nu = 5$ kHz and $f_L = 50$ kHz. (f) and (g) Effective SNR versus the phase noise variance when $\mathrm{SNR}_0 = 25$ dB and $f_L = 50$ kHz. (a) PSD of phase noise with $\nu = 5$ kHz and $f_L = 50$ kHz. (b) Effective SNR with perfect channel information. (c) Uncoded BER with perfect channel information. (d) Effective SNR with estimated channel information. (e) Uncoded BER with estimated channel information. (f) Effective SNR versus phase noise variance with perfect channel information. (g) Effective SNR versus phase noise variance with estimated channel information.

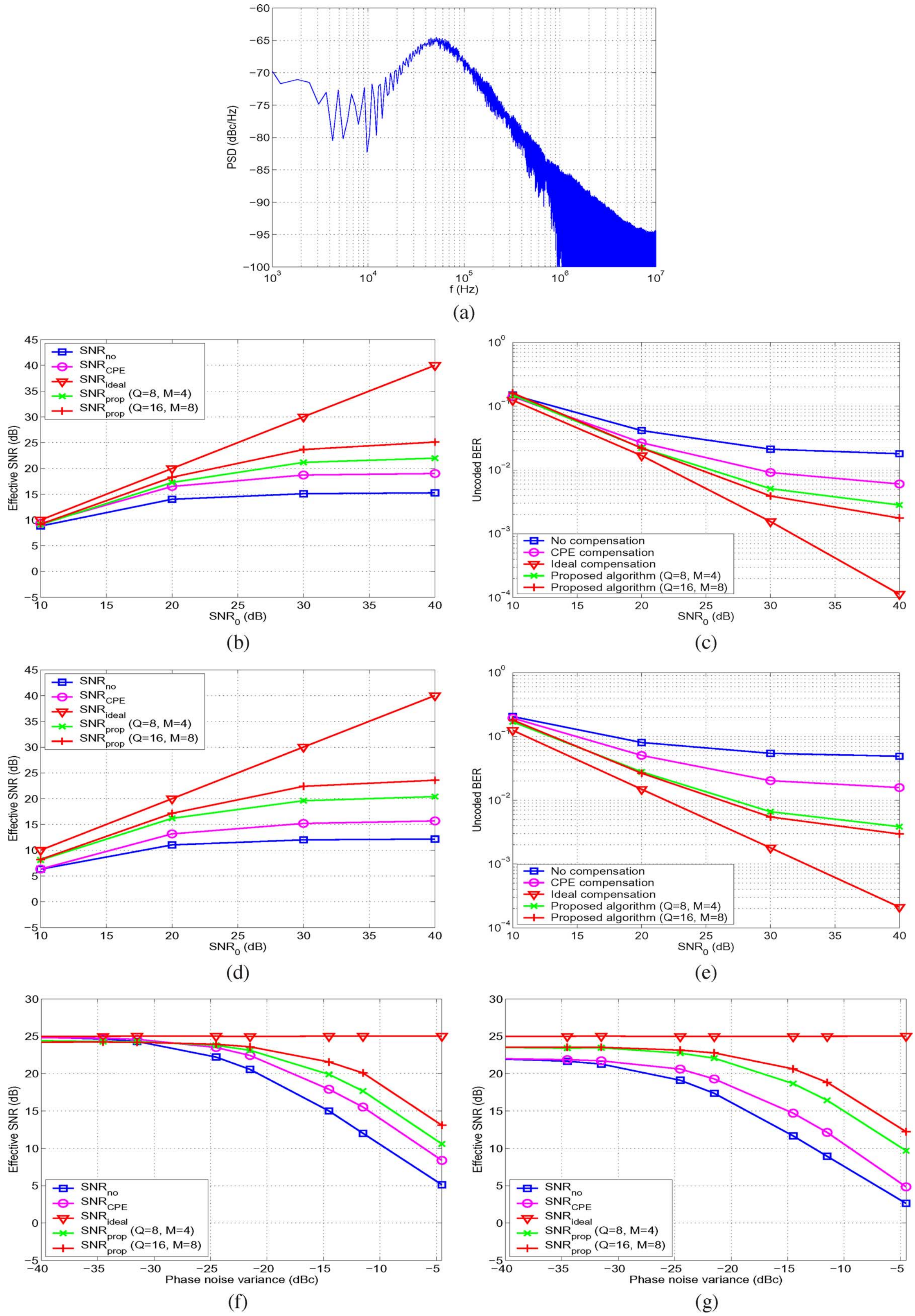


Fig. 7. Simulation results when the phase noise is generated by a second-order PLL oscillator. (b)–(e) Effective SNR and uncoded BER versus SNR_0 when $\nu = 5$ kHz and $f_n = 50$ kHz. (f) and (g) Effective SNR versus the phase noise variance when $\mathrm{SNR}_0 = 25$ dB and $f_n = 50$ kHz. (a) PSD of phase noise with $\nu = 5$ kHz and $f_n = 50$ kHz. (b) Effective SNR with perfect channel information. (c) Uncoded BER with perfect channel information. (d) Effective SNR with estimated channel information. (e) Uncoded BER with estimated channel information. (f) Effective SNR versus phase noise variance with perfect channel information. (g) Effective SNR versus phase noise variance with estimated channel information.

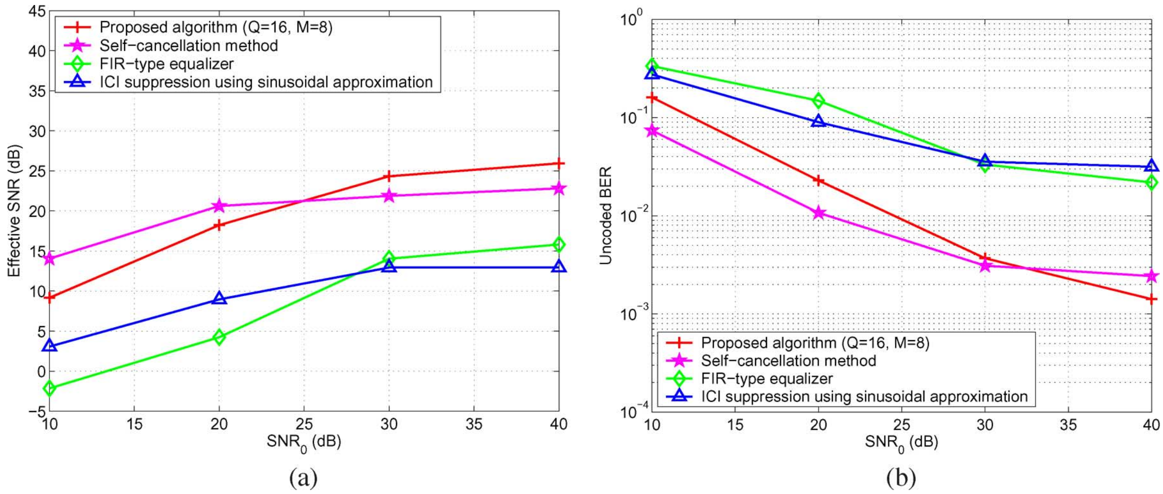


Fig. 8. Comparison of different ICI compensation methods. The phase noise is generated by a first-order PLL oscillator with $\nu = 5$ kHz and $f_L = 50$ kHz. (a) Effective SNR with perfect channel information. (b) Uncoded BER with perfect channel information.

overhead introduced by channel estimation.[3] The phase noise is generated by a first-order phase-loop locked oscillator with $\nu = 5$ kHz and $f_L = 50$ kHz. It is shown in the figure that the FIR-type equalizer[4] and the ICI suppression method using sinusoidal approximation do not perform well for this type of fast-changing phase noise because they can only compensate for the ICI from adjacent subcarriers. The self-cancellation scheme works as well as the proposed method since it uses two subcarriers to transmit one data symbol, which brings the benefit of diversity gain as in multiple-input–multiple-output (MIMO) communications. However, the self-cancellation method reduces the spectral efficiency by one half. In the simulation, each OFDM symbol can transmit $N - Q = 48$ data symbols in the proposed scheme but only $N/2 = 32$ symbols in the self-cancellation scheme, which demonstrates that the proposed method can achieve 50% more spectral efficiency than the self-cancellation method.

The channel estimation and data symbol estimation steps require the solution of two least squares problems iteratively. In the simulations, less than ten iterations are required to guarantee convergence. Solving a general least squares problem of size N has computational complexity $O(N^3)$. Thus the complexity of the proposed scheme is about $O(KN^3)$, where K denotes the number of iterations. By exploiting the circulant structure of the phase noise matrix $\mathbf{A}$ and the sparse structure of the interpolation matrix $\mathbf{P}$, the complexity can be reduced.

VI. Conclusion

In this paper, a phase noise compensation scheme is proposed for OFDM-based wireless communications. The proposed scheme consists of two phases. One phase is the joint channel and phase noise estimation, and the other phase is the joint data symbol and phase noise estimation. The simulations show that the proposed scheme can effectively improve the system performance in terms of the effective SNR and the uncoded BER. It is demonstrated that the proposed algorithm can reduce the sensitivity of OFDM receivers to phase noise by about 8 dB. Since oscillators with ultralow phase noise usually have the disadvantage of high implementation cost and high power consumption, the improvement will significantly reduce the cost and power consumption from the perspective of hardware designers.

Appendix I
Modeling of Phase Noise

There are mainly two types of oscillators used in practice, depending on whether or not they are used in a PLL [5]. The so-called free-running oscillators operate without a PLL and the generated phase noise is modeled as the accumulation of random frequency deviations and hence has unbounded variance. On the other hand, in a PLL oscillator, the closed-loop control mechanism tracks the phase variations of the carrier signal, and consequently, the generated phase noise has finite variance.

A. Free-Running Oscillator

The frequency deviation $\mu(t)$ of an oscillator is modeled as a zero-mean white Gaussian random process with single-sided PSD $N_0 = \nu/\pi$, where ν is called the oscillator linewidth. The phase noise $\phi_{\text{Free}}(t)$ generated by a free-running oscillator is modeled by integrating $\mu(t)$, i.e.,

$$\phi_{\text{Free}}(t) = 2\pi \int_0^t \mu(\lambda) d\lambda$$

which turns out to be a Wiener process. The single-sided PSD of $\phi_{\text{Free}}(t)$ is

$$S_{\phi,\text{Free}}(f) = \frac{\nu}{\pi f^2}$$

[3]The block-type pilot symbols used for channel estimation are only required at the beginning of each packet, because wireless channels are usually slowly time varying. If the packets are long enough, then the overhead caused by the block-type symbols is negligible.

[4]In the simulations, the length of the FIR-type equalizer is 9.

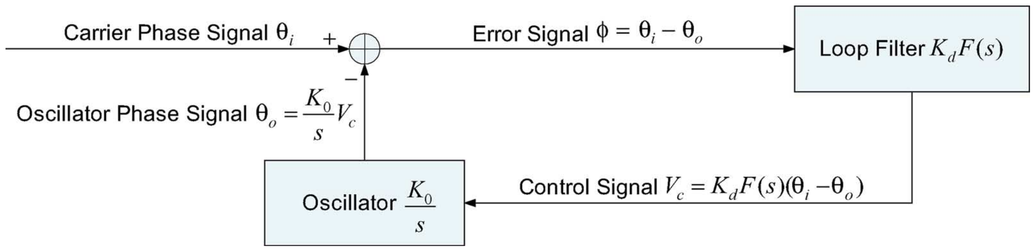


Fig. 9. Block diagram of a PLL system.

for $f \neq 0$. The carrier noise is $c_{\text{Free}}(t) = e^{j\phi_{\text{Free}}(t)}$ and its mean value satisfies

$$\mathbf{E}\{c_{\text{Free}}(t)\} \rightarrow 0$$

as $t \rightarrow \infty$. The autocorrelation function of $c_{\text{Free}}(t)$ is given by

$$R_{c,\text{Free}}(\tau) = e^{-\pi\nu|\tau|} \tag{22}$$

and the single-sided PSD of $c_{\text{Free}}(t)$ is Lorenzian

$$S_{c,\text{Free}}(f) = \frac{4\nu}{\pi(4f^2 + \nu^2)}.$$

B. Oscillator With PLLs

Fig. 9 shows the block diagram of a PLL system [24]. The phase error $\phi(t)$ between the carrier signal and the local oscillator is filtered by a low-pass loop filter that is represented by $K_d F(s)$ in Fig. 9. The resulting signal V_c is used as the input to the oscillator such that its output can actively track the phase variations of the carrier signal. It can be shown that the closed-loop transfer function between the carrier phase signal θ_i (input) and the oscillator phase signal θ_o (output) is given by

$$H(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{K_0 K_d F(s)}{s + K_0 K_d F(s)}$$

where $K_0 K_d$ is the open-loop gain. Thus, the transfer function between the input phase signal θ_i and the phase noise ϕ is

$$G(s) = \frac{\phi(s)}{\theta_i(s)} = \frac{\theta_i(s) - \theta_o(s)}{\theta_i(s)} = 1 - H(s) = \frac{s}{s + K_0 K_d F(s)}.$$

Different loop filters will generate phase noise with different PSDs. In the following, we give the models for the first- and second-order PLLs that are commonly used in practice.

1) First-Order PLL: For the first-order PLL, we have $F(s) = 1$ and the closed-loop transfer function is given by

$$H(s) = \frac{\omega_L}{s + \omega_L}$$

where $\omega_L = K_0 K_d$ is called the angular frequency of zero decibel loop gain. Since the input phase signal is generated by a free-running oscillator, we can let $\theta_i(t) = \phi_{\text{Free}}(t)$. After correction by the PLL, the single-sided PSD of the phase noise $\phi_{\text{PLL}_1}(t)$ can be expressed as

$$\begin{aligned} S_{\phi,\text{PLL}_1}(f) &= |G(j2\pi f)|^2 S_{\phi,\text{Free}}(f) \\ &= |1 - H(j2\pi f)|^2 \frac{\nu}{\pi f^2} = \frac{\nu}{\pi\left(f^2 + f_L^2\right)} \end{aligned}$$

where $f_L = \omega_L / 2\pi$ is a measure of the loop bandwidth. The variance of $\phi_{\text{PLL}_1}(t)$ is given by

$$\sigma^2_{\phi,\text{PLL}_1} = \int_0^\infty S_{\phi,\text{PLL}_1}(f) df = \frac{\nu}{2f_L} \tag{23}$$

and the autocorrelation function of $\phi_{\text{PLL}_1}(t)$ can be calculated as

$$R_{\phi,\text{PLL}_1}(\tau) = \int_{-\infty}^{\infty} \frac{\nu}{2\pi\left(f^2 + f_L^2\right)} e^{j2\pi f\tau} df. \tag{24}$$

Although there is no simple explicit expression for $R_{\phi,\text{PLL}_1}(\tau)$, expression (24) can be evaluated numerically and the result is useful in analyzing the effects of phase noise on OFDM systems as well as the performance of different compensation schemes.

The associated carrier noise is $c_{\text{PLL}_1}(t) = e^{j\phi_{\text{PLL}_1}(t)}$. Since $\phi_{\text{PLL}_1}(t)$ is Gaussian distributed with mean zero and variance $\sigma^2_{\phi,\text{PLL}_1} = \nu/(2f_L)$, the mean value of $c_{\text{PLL}_1}(t)$ is given by

$$\begin{aligned} \mathbf{E}\{c_{\text{PLL}_1}(t)\} &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2_{\phi,\text{PLL}_1}}} e^{-\frac{z^2}{2\sigma^2_{\phi,\text{PLL}_1}}} e^{jz} dz \\ &= e^{-\frac{\sigma^2_{\phi,\text{PLL}_1}}{2}} = e^{-\frac{\nu}{4f_L}}. \end{aligned}$$

By definition, the autocorrelation function of $c_{\text{PLL}_1}(t)$ is given by

$$\begin{aligned} R_{c,\text{PLL}_1}(\tau) &= \mathbf{E}\left\{c_{\text{PLL}_1}(t) c^*_{\text{PLL}_1}(t - \tau)\right\} \\ &= \mathbf{E}\left\{e^{j\left[\phi_{\text{PLL}_1}(t) - \phi_{\text{PLL}_1}(t-\tau)\right]}\right\} \end{aligned}$$

where $\phi_{\text{PLL}_1}(t) - \phi_{\text{PLL}_1}(t - \tau)$ is Gaussian distributed with mean zero and

$$\begin{aligned} \mathbf{E}\left\{\left(\phi_{\text{PLL}_1}(t) - \phi_{\text{PLL}_1}(t-\tau)\right)^2\right\} &= 2\sigma^2_{\phi,\text{PLL}_1} - 2R_{\phi,\text{PLL}_1}(\tau) \\ &= \frac{\nu}{f_L} - 2R_{\phi,\text{PLL}_1}(\tau). \end{aligned}$$

TABLE I
TABLE OF STATISTICAL MEASURES FOR DIFFERENT TYPES OF PHASE NOISE

	Free-running	1^{st}-order PLL	2^{nd}-order PLL
σ_ϕ^2	—	$\frac{\nu}{2f_L}$	$\frac{\nu}{2\sqrt{2}f_n}$
$R_\phi(\tau)$	—	$\int_{-\infty}^{\infty} \frac{\nu}{2\pi(f^2+f_L^2)} e^{j2\pi f\tau} df$	$\int_{-\infty}^{\infty} \frac{\nu f^2}{2\pi(f^4+f_n^4)} e^{j2\pi f\tau} df$
$S_\phi(f)$	$\frac{\nu}{\pi f^2}$	$\frac{\nu}{\pi(f^2+f_L^2)}$	$\frac{\nu f^2}{\pi(f^4+f_n^4)}$
$\mathbf{E}\{c(t)\}$	$\to 0$	$e^{-\frac{\nu}{4f_L}}$	$e^{-\frac{\nu}{4\sqrt{2}f_n}}$
$R_c(\tau)$	$e^{-\pi\nu\lvert\tau\rvert}$	$e^{R_{\phi,PLL_1}(\tau)-\frac{\nu}{2f_L}}$	$e^{R_{\phi,PLL_2}(\tau)-\frac{\nu}{2\sqrt{2}f_n}}$
$S_c(f)$	$\frac{4\nu}{\pi(4f^2+\nu^2)}$	$2e^{-\frac{\nu}{2f_L}} \int_{-\infty}^{\infty} e^{R_{\phi,PLL_1}(\tau)-j2\pi f\tau} d\tau$	$2e^{-\frac{\nu}{2\sqrt{2}f_n}} \int_{-\infty}^{\infty} e^{R_{\phi,PLL_2}(\tau)-j2\pi f\tau} d\tau$

Taking the expectation with respect to the distribution of $\phi_{\mathrm{PLL}_1}(t) - \phi_{\mathrm{PLL}_1}(t-\tau)$ gives

$$R_{c,\mathrm{PLL}_1}(\tau) = e^{R_{\phi,\mathrm{PLL}_1}(\tau)-\sigma^2_{\phi,\mathrm{PLL}_1}} = e^{R_{\phi,\mathrm{PLL}_1}(\tau)-\frac{\nu}{2f_L}}.$$

It then follows that the single-sided PSD of $c_{\mathrm{PLL}_1}(t)$ can be calculated by

$$\begin{aligned} S_{c,\mathrm{PLL}_1}(f) &= 2\int_{-\infty}^{\infty} R_{c,\mathrm{PLL}_1}(\tau)e^{-j2\pi f\tau}d\tau \\ &= 2e^{-\frac{\nu}{2f_L}} \int_{-\infty}^{\infty} e^{R_{\phi,\mathrm{PLL}_1}(\tau)-j2\pi f\tau}d\tau. \end{aligned}$$

2) Second-Order PLL: For the second-order PLL, the closed-loop transfer function is

$$H(s) = \frac{2\eta\omega_n s + \omega_n^2}{s^2 + 2\eta\omega_n s + \omega_n^2}$$

where ω_n is the loop natural frequency and η is the damping factor. In this paper, we assume that $\eta = 0.707$. Then, the single-sided PSD of the phase noise $\phi_{\mathrm{PLL}_2}(t)$ is given by

$$\begin{aligned} S_{\phi,\mathrm{PLL}_2}(f) &= |G(j2\pi f)|^2 S_{\phi,\mathrm{Free}}(f) \\ &= |1 - H(j2\pi f)|^2 \frac{\nu}{\pi f^2} = \frac{\nu f^2}{\pi(f^4 + f_n^4)} \end{aligned}$$

where $f_n = \omega_n/2\pi$ is a measure of the loop bandwidth. The variance of $\phi_{\mathrm{PLL}_2}(t)$ can be calculated as

$$\sigma^2_{\phi,\mathrm{PLL}_2} = \int_0^{\infty} S_{\phi,\mathrm{PLL}_2}(f)df = \frac{\nu}{2\sqrt{2}f_n}. \quad (25)$$

The autocorrelation function of $\phi_{\mathrm{PLL}_2}(t)$ can be calculated by

$$R_{\phi,\mathrm{PLL}_2}(\tau) = \int_{-\infty}^{\infty} \frac{\nu f^2}{2\pi(f^4 + f_n^4)} e^{j2\pi f\tau}df.$$

The associated carrier noise is $c_{\mathrm{PLL}_2}(t) = e^{j\phi_{\mathrm{PLL}_2}(t)}$ and its mean value is

$$\begin{aligned} \mathbf{E}\{c_{\mathrm{PLL}_2}(t)\} &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2_{\phi,\mathrm{PLL}_2}}} e^{-\frac{z^2}{2\sigma^2_{\phi,\mathrm{PLL}_2}}} e^{jz}dz \\ &= e^{-\frac{\sigma^2_{\phi,\mathrm{PLL}_2}}{2}} = e^{-\frac{\nu}{4\sqrt{2}f_n}}. \end{aligned}$$

The autocorrelation function of $c_{\mathrm{PLL}_2}(t)$ is given by

$$R_{c,\mathrm{PLL}_2}(\tau) = e^{R_{\phi,\mathrm{PLL}_2}(\tau)-\sigma^2_{\phi,\mathrm{PLL}_2}} = e^{R_{\phi,\mathrm{PLL}_2}(\tau)-\frac{\nu}{2\sqrt{2}f_n}}$$

and the single-sided PSD of $c_{\mathrm{PLL}_2}(t)$ can be calculated by

$$\begin{aligned} S_{c,\mathrm{PLL}_2}(f) &= 2\int_{-\infty}^{\infty} R_{c,\mathrm{PLL}_2}(\tau)e^{-j2\pi f\tau}d\tau \\ &= 2e^{-\frac{\nu}{2\sqrt{2}f_n}} \int_{-\infty}^{\infty} e^{R_{\phi,\mathrm{PLL}_2}(\tau)-j2\pi f\tau}d\tau. \end{aligned}$$

In Table I, we summarize the statistical measures that will be used to compute the effective SNR for different compensation schemes. For convenience of notation, if the phase noise model is not specified, we use $\phi(t)$ and $c(t)$ to represent the phase noise and carrier noise by ignoring the subscripts. It is the same for σ_ϕ^2, $R_\phi(\tau)$, $R_c(\tau)$, $S_\phi(f)$, and $S_c(f)$.

Remark: For PLL oscillators, the phase noise $\phi(t)$ is usually small. If $|\phi(t)| \ll 1$, the carrier noise $c(t)$ can be approximated by $c(t) = e^{j\phi(t)} \approx 1 + j\phi(t)$. In this case, $R_c(\tau) \approx 1 + R_\phi(\tau)$ and $S_c(f) \approx S_\phi(f)$ for $f \neq 0$.

C. Statistical Characteristics of $\alpha[k]$

Given the previous phase noise models, the mean and autocorrelation of $\alpha[k]$ can be derived from expression (2) as follows:

$$\mathbf{E}\{\alpha[k]\} = \begin{cases} \mathbf{E}\{c(t)\}, & k = 0 \\ 0, & k = 1, 2, \ldots, N-1 \end{cases} \quad (26)$$

$$\mathbf{E}\left\{\alpha[k_1](\alpha[k_2])^*\right\} = \frac{1}{N^2}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} R_c((n_1-n_2)T_s) \times e^{-j\frac{2\pi(k_1n_1-k_2n_2)}{N}} \quad (27)$$

for $k_1 = 0, 1, \ldots, N-1$ and $k_2 = 0, 1, \ldots, N-1$. In particular, the following expressions derived from (26) and (27) are useful in evaluating the performance of different phase noise compensation schemes:

$$\sigma^2_{\alpha,0} = \frac{1}{N^2}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} R_c((n_1-n_2)T_s) \quad (28)$$

$$\mathbf{E}\left\{|\alpha[0]-1|^2\right\} = \frac{1}{N^2}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} R_c((n_1-n_2)T_s) - 2\mathrm{Re}\{\mathbf{E}\{c(t)\}\} + 1 \quad (29)$$

$$\mathbf{P}'(n,m)=\begin{cases}\dfrac{a^{n-1}}{(1-b^2)b^{m-1}}-\dfrac{b^{m+1}}{(1-b^2)a^{n-1}}, & \text{if } \dfrac{(m-1)(N-1)}{M-1}\le n-1<\dfrac{m(N-1)}{M-1}\\ \dfrac{b^{m-1}}{(1-b^2)a^{n-1}}-\dfrac{a^{n-1}}{(1-b^2)b^{m-3}}, & \text{if } \dfrac{(m-2)(N-1)}{M-1}\le n-1<\dfrac{(m-1)(N-1)}{M-1}\\ 0, & \text{otherwise}\end{cases}$$

$$\sum_{k=1}^{N-1}\sigma_{\alpha,k}^2 = 1-\sigma_{\alpha,0}^2 = 1-\frac{1}{N^2}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}R_c((n_1-n_2)T_s) \quad (30)$$

where $\sigma_{\alpha,k}^2=\mathbf{E}\{|\alpha[k]|^2\}$ is the average power of $\alpha[k]$, $k=0,1,\ldots,N-1$. Here, (28) follows from (27) by setting $k_1=k_2=0$, (29) follows from (28) and (26), and (30) follows from (28) by noting that

$$\sum_{k=0}^{N-1}\sigma_{\alpha,k}^2=\sum_{k=0}^{N-1}\mathbf{E}\left\{|\alpha[k]|^2\right\}=\mathbf{E}\left\{\sum_{k=0}^{N-1}|\alpha[k]|^2\right\} =\mathbf{E}\left\{\frac{1}{N}\sum_{n=0}^{N-1}\left|e^{j\phi(nT_s)}\right|^2\right\}=1.$$

APPENDIX II
$\mathbf{P}_o \approx \mathbf{P}_L$ FOR FREE-RUNNING OSCILLATORS IF $\nu T_s \ll 1$

Proof: Recall that

$$\mathbf{c}=\begin{bmatrix} e^{j\phi_{\text{Free}}(0)}\\ e^{j\phi_{\text{Free}}(T_s)}\\ \vdots\\ e^{j\phi_{\text{Free}}((N-1)T_s)}\end{bmatrix}\quad \mathbf{c}'=\begin{bmatrix} e^{j\phi_{\text{Free}}(0)}\\ e^{j\phi_{\text{Free}}\left(\frac{(N-1)T_s}{M-1}\right)}\\ \vdots\\ e^{j\phi_{\text{Free}}((N-1)T_s)}\end{bmatrix}.$$

Let $a=e^{-\pi\nu T_s}$ and $b=e^{-(\pi\nu(N-1)T_s/(M-1))}$. It follows from expression (22) that the element of $\mathbf{R}_{\mathbf{cc}'}$ at the nth row and mth column is given by

$$\begin{aligned}\mathbf{R}_{\mathbf{cc}'}(n,m)&=\mathbf{E}\left\{\mathbf{c}(n)\mathbf{c}'(m)^*\right\}\\ &=\mathbf{E}\left\{e^{j\phi_{\text{Free}}((n-1)T_s)}e^{-j\phi_{\text{Free}}\left(\frac{(m-1)(N-1)T_s}{M-1}\right)}\right\}\\ &=e^{-\pi\nu\left|(n-1)T_s-\frac{(m-1)(N-1)T_s}{M-1}\right|}\\ &=\begin{cases}a^{n-1}b^{-(m-1)}, & \text{if } n-1\ge\frac{(m-1)(N-1)}{M-1}\\ a^{-(n-1)}b^{m-1}, & \text{if } n-1<\frac{(m-1)(N-1)}{M-1}\end{cases}\end{aligned}$$

where $n=1,2,\ldots,N$ and $m=1,2,\ldots,M$. Also, we have

$$\mathbf{R}_{\mathbf{c}'}=\mathbf{E}\{\mathbf{c}'\mathbf{c}'^*\}=\begin{bmatrix}1 & b & b^2 & \ldots & b^{M-1}\\ b & 1 & b & \ldots & b^{M-2}\\ b^2 & b & 1 & \ldots & b^{M-3}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ b^{M-1} & b^{M-2} & b^{M-3} & \ldots & 1\end{bmatrix}.$$

Consider an arbitrary integer p with $1\le p\le N$ and let q be the associated integer such that $((q-1)(N-1)/(M-1))\le p-1<(q(N-1)/(M-1))$. The pth row of $\mathbf{R}_{\mathbf{cc}'}$ is given by

$$\mathbf{r}_{\mathbf{cc}',p}=\left[a^{p-1}\ \ a^{p-1}b^{-1}\ \ \ldots\ \ a^{p-1}b^{-(q-1)}\ \ a^{-(p-1)}b^{q}\ \ a^{-(p-1)}b^{q+1}\ \ \ldots\ \ a^{-(p-1)}b^{M-1}\right].$$

Denote the qth and $(q+1)$th rows of $\mathbf{R}_{\mathbf{c}'}$ by $\mathbf{r}_{\mathbf{c}',q}$ and $\mathbf{r}_{\mathbf{c}',q+1}$, respectively, i.e.,

$$\begin{aligned}\mathbf{r}_{\mathbf{c}',q}&=[b^{q-1}\ \ b^{q-2}\ \ \ldots\ \ b\ \ 1\ \ b\ \ b^2\ \ \ldots\ \ b^{M-q}]\\ \mathbf{r}_{\mathbf{c}',q+1}&=[b^{q}\ \ b^{q-1}\ \ \ldots\ \ b^2\ \ b\ \ 1\ \ b\ \ \ldots\ \ b^{M-q-1}].\end{aligned}$$

It is easy to verify that

$$\begin{aligned}\mathbf{r}_{\mathbf{cc}',p}&=\frac{a^{p-1}}{(1-b^2)b^{q-1}}(\mathbf{r}_{\mathbf{c}',q}-b\mathbf{r}_{\mathbf{c}',q+1}) +\frac{b^q}{(1-b^2)a^{p-1}}(\mathbf{r}_{\mathbf{c}',q+1}-b\mathbf{r}_{\mathbf{c}',q})\\ &=\left[\frac{a^{p-1}}{(1-b^2)b^{q-1}}-\frac{b^{q+1}}{(1-b^2)a^{p-1}}\right]\mathbf{r}_{\mathbf{c}',q} +\left[\frac{b^q}{(1-b^2)a^{p-1}}-\frac{a^{p-1}}{(1-b^2)b^{q-2}}\right]\mathbf{r}_{\mathbf{c}',q+1}\end{aligned}\quad (31)$$

which implies that each row of $\mathbf{R}_{\mathbf{cc}'}$ can be expressed as a linear combination of two consecutive rows of $\mathbf{R}_{\mathbf{c}'}$. Hence

$$\mathbf{R}_{\mathbf{cc}'}=\mathbf{P}'\mathbf{R}_{\mathbf{c}'}$$

where the elements of $\mathbf{P}'$ can be determined from expression (31), i.e., as shown in the equation at the top of the page, for $n=1,2,\ldots,N-1$ and $m=1,2,\ldots,M$. It then follows from (14) that

$$\mathbf{P}_o=\mathbf{R}_{\mathbf{cc}'}\mathbf{R}_{\mathbf{c}'}^{-1}=\mathbf{P}'\mathbf{R}_{\mathbf{c}'}\mathbf{R}_{\mathbf{c}'}^{-1}=\mathbf{P}'.$$

In the following, we are going to show that $\mathbf{P}_o\approx\mathbf{P}_L$ if $\nu T_s\ll 1$. Using the following approximations:

$$\begin{aligned}\frac{a^{n-1}}{b^{m-1}}&=e^{-\pi\nu(n-1)T_s+\frac{\pi\nu(m-1)(N-1)T_s}{M-1}}\\ &\approx 1-\pi\nu(n-1)T_s+\frac{\pi\nu(m-1)(N-1)T_s}{M-1}\\ \frac{b^{m-1}}{a^{n-1}}&=e^{\pi\nu(n-1)T_s-\frac{\pi\nu(m-1)(N-1)T_s}{M-1}}\\ &\approx 1+\pi\nu(n-1)T_s-\frac{\pi\nu(m-1)(N-1)T_s}{M-1}\\ 1+b^2&=1+e^{-\frac{2\pi\nu(N-1)T_s}{M-1}}\approx 2\\ 1-b^2&=1-e^{-\frac{2\pi\nu(N-1)T_s}{M-1}}\approx\frac{2\pi\nu(N-1)T_s}{M-1}\end{aligned}$$

$$\mathbf{P}_o(n,m) = \mathbf{P}'(n,m) \approx \begin{cases} m - \dfrac{(n-1)(M-1)}{N-1}, & \text{if } \dfrac{(m-1)(N-1)}{M-1} \leq n-1 < \dfrac{m(N-1)}{M-1} \\ \dfrac{(n-1)(M-1)}{N-1} - (m-2), & \text{if } \dfrac{(m-2)(N-1)}{M-1} \leq n-1 < \dfrac{(m-1)(N-1)}{M-1} \\ 0, & \text{otherwise} \end{cases}$$

we then have

$$\begin{aligned} &\frac{a^{n-1}}{(1-b^2)b^{m-1}} - \frac{b^{m+1}}{(1-b^2)a^{n-1}} \\ &= \frac{1}{1-b^2} \cdot \frac{a^{n-1}}{b^{m-1}} - \frac{b^2}{1-b^2} \cdot \frac{b^{m-1}}{a^{n-1}} \\ &\approx \frac{1}{1-b^2}\left(1 - \pi\nu(n-1)T_s + \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &\quad - \frac{b^2}{1-b^2}\left(1 + \pi\nu(n-1)T_s - \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &= 1 - \frac{1+b^2}{1-b^2}\left(\pi\nu(n-1)T_s - \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &\approx 1 - \frac{2}{\frac{2\pi\nu(N-1)T_s}{M-1}}\left(\pi\nu(n-1)T_s - \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &= m - \frac{(n-1)(M-1)}{N-1} \end{aligned}$$

and

$$\begin{aligned} &\frac{b^{m-1}}{(1-b^2)a^{n-1}} - \frac{a^{n-1}}{(1-b^2)b^{m-3}} \\ &= \frac{1}{1-b^2} \cdot \frac{b^{m-1}}{a^{n-1}} - \frac{b^2}{1-b^2} \cdot \frac{a^{n-1}}{b^{m-1}} \\ &= \frac{1}{1-b^2}\left(1 + \pi\nu(n-1)T_s - \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &\quad - \frac{b^2}{1-b^2}\left(1 - \pi\nu(n-1)T_s + \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &= 1 + \frac{1+b^2}{1-b^2}\left(\pi\nu(n-1)T_s - \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &\approx 1 + \frac{2}{\frac{2\pi\nu(N-1)T_s}{M-1}}\left(\pi\nu(n-1)T_s - \frac{\pi\nu(m-1)(N-1)T_s}{M-1}\right) \\ &= \frac{(n-1)(M-1)}{N-1} - (m-2). \end{aligned}$$

Thus, the equation shown at the top of the page holds, which is identical to (15). Therefore, $\mathbf{P}_o \approx \mathbf{P}_L$. ■

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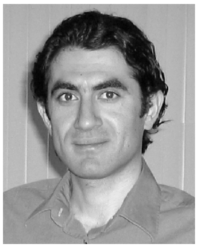

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