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Iterative learning control with advanced output data for nonlinear non-minimum phase systems

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This paper investigates iterative learning control of nonlinear discrete time non-minimum phase systems in tracking problems. The main objective of this paper is to find an input-to-output mapping in order to stabilize the non-minimum phase systems and to obtain an input update law for handling uncertain systems. In conventional approaches on the tracking of non-minimum phase systems, zero dynamics is stabilized from the system equations and the input is calculated from the state information. For the learning of uncertain systems, conventional approaches depend on the output-to-state and state-to-input mappings. In the proposed method, the inverse system is stabilized using the input-to-output mapping for nonlinear non-minimum phase systems. A new input update law is proposed based on the relative degree and the number of non-minimum phase zeros. This makes the overall proposed learning system have a simple structure as in the classical ILC.

Keywords: Iterative learning control; Nonlinear non-minimum phasing system; Advanced output data

1. Introduction

Iterative learning control (ILC) has been studied to improve tracking performance of the control system by performing the same task iteratively over a finite control horizon (Uchiyama 1978, Arimoto *et al*. 1984, Moore 1993). These conventional ILC schemes which are based on the relative degree correspond to the inverse mapping from desired output to input. However, for non-minimum phase systems, this inversion brings about instability. The input obtained by ILC may be bounded due to the finite control horizon, but may be too large for non-minimum phase systems to use in practice.

In recent years, tracking a desired trajectory for non-minimum phase systems has been widely studied. For continuous time systems, many control designs have been proposed using stable inversion (Devasia *et al*. 1996, Taylor and Li 2002). Several digital controllers have also been studied for tracking problems of discrete time non-minimum phase linear systems (Tomizuka 1987, Choi and Choi 1998). Also a discrete time version of stable inversion method has been proposed in Zeng and Hunt (2000). All these methods show reasonably good performance on tracking desired trajectories if the relevant characteristics of the given models are known.

Conventional learning approaches for non-minimum phase systems in Roh *et al*. (1996), Ghosh and Paden (2002) and Kinosita *et al*. (2000) are based on the system model and can obtain only approximate solutions. Also, the overall learning structures are more complex than the conventional ILC methods for minimum phase systems.

Classical ILC schemes have simple learning structures using input update laws. Preserving the simple structure of ILC, ILC with advanced output data was proposed for linear discrete time non-minimum phase systems in tracking a desired trajectory (Jeong and Choi 2002 a, b).

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Its results provided the condition for the stability of inverse mapping of non-minimum phase systems and an input update law which utilizes the information on the the sum of the relative degree of the system and the number of non-minimum phase zeros. This learning law can be applied to both minimum phase systems and non-minimum phase systems.

In this paper, the results in Jeong and Choi (2002b) are extended to nonlinear non-minimum phase systems. In the proposed method, the inverse system is stabilized based on the input-to-output mapping using the relative degree and the number of non-minimum phase zeros. Also, a learning scheme for a non-linear discrete time system is proposed based on the mapping. The learning structure and input update law are simple as in the classical ILC schemes. The remainder of this paper is organized as follows. In section 2, some of our preliminary results on iterative learning control with advanced output data (ADILC) for non-minimum phase systems are described briefly. In section 3, the results from section 2 are extended to nonlinear systems. The boundedness of the input obtained by the proposed method is proven. In addition, an example is presented to illustrate the need to advance the output in the input update law. The conclusion follows in section 4.

2. ILC with advanced output data for linear non-minimum phase systems

Some preliminaries and the results from Jeong and Choi (2002b) will be briefly summarized here for proving the results in section 3. An input update law which depends on the relative degree and the number of non-minimum phase zeros is considered. It can be used for non-minimum phase systems as well as for minimum phase systems. We assume that the number of non-minimum phase zeros and the relative degree are known *a priori*.

Let us consider the LTI systems described by

$$\left.\begin{aligned} x(i+1) &= Ax(i) + Bu(i), \\ y(i) &= Cx(i). \end{aligned}\right\} \tag{1}$$

where, $u \in \Re^1$, $x = [x_1, \ldots, x_n]^T \in \Re^n$, and $y \in \Re^1$ are the input, the state, and the output of the system, respectively. A, B and C are matrices of appropriate dimensions. Let $x^d(i)$, $y^d(i)$ and $u^d(i)$ represent the state, the output and the input corresponding to the desired trajectory. Also let the desired output $y^d(i), i \in [\sigma, N+\sigma-1]$ be given and $\mathbf{u}_{[i,j]} := [u(i), \ldots, u(j)]^T, \mathbf{y}_{[i,j]} := [y(i), \ldots, y(j)]^T$. Here, σ denotes the relative degree of the system. The transfer function of the system is represented as

$$G(z) = \frac{\beta_1 z^{n-1} + \cdots + \beta_n}{z^n + \alpha_1 z^{n-1} + \cdots + \alpha_n}. \tag{2}$$

Basically, the ILC based on the relative degree utilizes the following mapping:

$$\mathbf{y}_{[\sigma,\, N+\sigma-1]} = \mathbf{H}_a x(0) + \mathbf{J}_a \mathbf{u}_{[0,\, N-1]}, \tag{3}$$

where

$$\mathbf{H}_a = \begin{pmatrix} H_1 \\ H_2 \\ \cdots \\ H_N \end{pmatrix}, \quad \mathbf{J}_a = \begin{pmatrix} J_1 & 0 & \cdots & 0 \\ J_2 & J_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ J_N & J_{N-1} & \cdots & J_1 \end{pmatrix},$$

and $H_l = CA^{\sigma+l-1}$, $J_l = CA^{\sigma+l-2}B$. Since the system (1) has a relative degree σ, J_σ is non-zero and $\mathbf{J}_a$ is non-singular.

For minimum phase systems, $u^d(i)$ can be uniquely determined from $\mathbf{y}_{[\sigma,\, N+\sigma-1]}$ and $x^d(0)$. The inverse mapping from $\mathbf{y}_{[\sigma,\, N+\sigma-1]}$ to $\mathbf{u}_{[0,\, N-1]}$ is stable. However, for non-minimum phase systems, the inverse mapping from $\mathbf{y}_{[\sigma,\, N+\sigma-1]}$ to $\mathbf{u}_{[0,\, N-1]}$ is unstable.

To consider the ILC with advanced output data, we set the input horizon to $[0, N+d-1]$ with $u(N) = \cdots = u(N+d-1) = 0$ and the output horizon to $[0, N+\sigma+d-1], 0 \le d \le n-\sigma$. The desired trajectory y^d is given in $[\sigma, N+\sigma-1]$. To compare the result with that of the conventional ILC later, we set $y^d(N+\sigma), \ldots, y^d(N+\sigma+d-1)$ to some appropriate constants.

In this case, the following equation can be obtained in a manner similar to (3),

$$\mathbf{y}_{[\sigma+d,\, N+\sigma+d-1]} = \mathbf{H}_c x(0) + \mathbf{J}_c \mathbf{u}_{[0,\, N-1]}, \tag{4}$$

where

$$\mathbf{H}_c = [(H_{d+1})^T, \ldots, (H_{N+d})^T]^T,$$

$$\mathbf{J}_c = \begin{bmatrix} J_{d+1} & J_d & \cdots & 0 \\ J_{d+2} & J_{d+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ J_{N+d} & J_{N+d-1} & \cdots & J_{d+1} \end{bmatrix}.$$

The time interval for the output of interest is $[\sigma+d, N+\sigma+d-1]$ for some integer $0 \le d \le n-\sigma$ in (4) whereas it is $[\sigma, N+\sigma-1]$ in (3), but the time interval for the input of interest is $[0, N-1]$ in both (3) and (4). Let d_0 be the number of non-minimum phase zeros of $G(z)$. If we set $d = d_0$, the inverse mapping is stable for non-minimum phase systems as well as for minimum phase systems. For minimum phase systems, we can set $d = 0$, which is equivalent to the conventional ILC based on the relative degree. At every iteration, we set $x^k(0) = x^d(0)$ and $u^k(i) = u^d(i) = 0, N \le i \le N-1+d$.

To analyse the stability of the inverse mapping, some assumptions are needed.

(A1) The system is stable, controllable and observable.
(A2) The matrix A is invertible.
(A3) $\beta_n \neq 0$ in (2).
(A4) The matrix $\mathbf{J}_c$ is non-singular.

An input update law which updates the input with advanced output data is

$$\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^{k}_{[0,N-1]} + \mathbf{S}\mathbf{e}^{k}_{[\sigma+d,N+\sigma+d-1]}, \quad 0 \le d \le n-\sigma, \quad (5)$$

where $\mathbf{e}^{k}_{[\sigma+d,N+\sigma+d-1]} = \mathbf{y}^{d}_{[\sigma+d,N+\sigma+d-1]} - \mathbf{y}^{k}_{[\sigma+d,N+\sigma+d-1]}$.

From the above, the results on the stability of the inverse mapping and its convergence analysis can be derived.

Lemma 1 (Jeong and Choi 2002b)**:** *The inverse mapping from* $\mathbf{y}^{d}_{[\sigma+d,N+\sigma+d-1]}$ *to* $\mathbf{u}^{d}_{[0,N-1]}$ *is stable for* $d = d_0$ *in* (4).

Lemma 1 shows that the inverse mapping from $\mathbf{y}^{d}_{[\sigma+d,N+\sigma+d-1]}$ to $\mathbf{u}^{d}_{[0,N-1]}$ is stable for non-minimum phase systems if $d = d_0$. In the next lemma, it will be shown that the input $\mathbf{u}^{k}_{[0,N-1]}$ converges to $\mathbf{u}^{d}_{[0,N-1]}$ as $k \to \infty$ using the input update law (5). It is noted that this inverse mapping is stable.

Lemma 2 (Jeong and Choi 2002b)**:** *The uncertain system* (1) *satisfies* **(A1)**–**(A4)**. *If the condition*

$$\|I - \mathbf{S}\mathbf{J}_c\| \le \rho < 1 \quad (6)$$

holds, the input $\mathbf{u}^{k}_{[0,N-1]}$ *converge to* $\mathbf{u}^{d}_{[0,N-1]}$ *as* $k \to \infty$.

Note that the input update law (5) is a generalization of iterative learning control from the minimum phase systems to the non-minimum phase systems by setting $d = d_0$.

3. ILC with advanced output data for nonlinear non-minimum phase systems

In this section, the results presented in section 2 are extended to nonlinear systems. The basic idea is the same as in the linear systems.

Consider a class of discrete time SISO nonlinear non-minimum phase systems

$$\left.\begin{aligned} x(i+1) &= f(x(i)) + g(x(i))u(i) \\ y(i) &= h(x(i)). \end{aligned}\right\} \quad (7)$$

The functions f, g and h are $f : \Re^n \to \Re^n$, $g: \Re^n \to \Re^n$ and $h : \Re^n \to \Re^1$, analytic and $f(0) = h(0) = 0$. The relative degree r is defined for continuous time systems (Isidori 2002) and the similar notion in discrete-time systems is the characteristic number ρ (Nijmeijer and Schaft 1990). We will define the relative degree σ for discrete time systems as $\sigma = \rho + 1$. If $(\partial/\partial u)h \times (f^j(f+gu)) = 0, j < \sigma - 1$ and $(\partial/\partial u)h(f^{\sigma-1}(f+gu))$ is non-singular, the relative degree of the system is σ. This is consistent with the relative degree in linear systems which is defined as the difference between the orders of the numerator and denominator polynomials of the transfer function. It is assumed that the relative degree of the system and the number of the non-minimum phase zeros, d_0 are known *a priori*. As in the linear case, we set the input horizon to $[0, N+d-1]$ with $u(N) = \cdots = u(N+d-1) = 0$ and set the output horizon to $[0, N+\sigma+d-1]$, $0 \le d \le n-\sigma$. The desired trajectory y^d is given in $[\sigma, N+\sigma-1]$ and we set $y^d(N+\sigma), \ldots, y^d \times (N+\sigma+d-1)$ to some appropriate constants.

As in the linear case, the following mapping is considered.

$$\mathbf{y}^{\,d}_{[\sigma+d,N+\sigma+d-1]} = \mathbf{F}(x(0), \mathbf{u}^{d}_{[0,N-1]}). \quad (8)$$

Here $\mathbf{F}$ is a nonlinear mapping obtained from the system equation (7).

Linearization of (7) around $(\mathbf{x}(i), u(i)) = (0, 0)$ results in the linear system

$$\left.\begin{aligned} \mathbf{x}(i+1) &= A\mathbf{x}(i) + Bu(i) \\ y(i) &= C\mathbf{x}(i), \end{aligned}\right\} \quad (9)$$

where

$$A = \left.\frac{\partial(f(\mathbf{x}(i)) + g(\mathbf{x}(i))u(i))}{\partial \mathbf{x}(i)}\right|_{\mathbf{x}(i)=0,\,u(i)=0},$$

$$B = \left.\frac{\partial(f(\mathbf{x}(i)) + g(\mathbf{x}(i))u(i))}{\partial u(i)}\right|_{\mathbf{x}(i)=0,\,u(i)=0},$$

$$C = \left.\frac{\partial h(\mathbf{x}(i))}{\partial \mathbf{x}(i)}\right|_{\mathbf{x}(i)=0}.$$

Then (7) becomes

$$\begin{aligned} \mathbf{x}(i+1) &= A\mathbf{x}(i) + Bu(i) \\ &\quad + [f(\mathbf{x}(i)) - A\mathbf{x}(i) + (g(\mathbf{x}(i)) - B)u(i)] \\ y(i) &= C\mathbf{x}(i) + [h(\mathbf{x}(i)) - C\mathbf{x}(i)]. \end{aligned}$$

Note that for the system (9), the mapping (8) can be represented as (4).

The following assumptions are needed for further development:

(**A1′**) The system (7) is stable. Also, the relative degree of the system (17) is σ and is well defined $\forall(\mathbf{x}, u) \in \mathrm{R}^{n+1}$ with respect to $u(i)$.
(**A2′**) For the system (7), $\|\mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]}\| \le c_1, \forall N$ and $\|\mathbf{x}(0)\| \le c_2$ for some constants c_1 and c_2.
(**A3′**) The linearized system (9) is stable and satisfies the assumptions (**A1**)–(**A4**).
(**A4′**) For any realizable output trajectory $\mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]}$ that corresponds to a given initial condition $\mathbf{x}^d(0)$, $\mathbf{F}$ is a one-to-one and continuous mapping.

It is noted that $\mathbf{y}^d_{[0,\, \sigma+d-1]}$ is not taken into consideration since we consider the mapping (8). In the following lemma, the boundedness of $\mathbf{u}^d_{[0,\, N-1]}$ is shown.

Lemma 3: *Let us assume that the system* (7), *the desired trajectory and the initial condition satisfy* (**A1′**)–(**A4′**). *Let us set $d = d_0$, where d_0 is the number of non-minimum phase zeros. Then the desired trajectory $\mathbf{u}^d_{[0,\, N-1]}$ is bounded.*

Proof: From the mapping (4) and the desired trajectory $\mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]}$, the desired input $\bar{\mathbf{u}}_{[0,\, N-1]}$ for the linearized system (9) can be obtained as

$$\bar{\mathbf{u}}_{[0,\, N-1]} = \mathbf{J}_c^{-1}[\mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]} - \mathbf{H}_c\mathbf{x}(0)].$$

If the inverse mapping (4) is unstable, the magnitude of $\bar{\mathbf{u}}_{[0,\, N-1]}$ can be very large even though $\mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]}$ is very small. From (**A3′**) and Lemma 1, the inversion of (4) is stable and $\bar{\mathbf{u}}_{[0,\, N-1]}$ is bounded from above, i.e., there exists a constant c_3 such that $\|\bar{\mathbf{u}}_{[0,\, N-1]}\| \le c_3, \forall N$.

Substituting $\bar{\mathbf{u}}_{[0,\, N-1]}$ into (8), the following mapping is obtained:

$$\bar{\mathbf{y}}_{[\sigma+d,\, N+\sigma+d-1]} = \mathbf{F}(\mathbf{x}(0), \bar{\mathbf{u}}_{[0,\, N-1]}).$$

Since the system is stable from (**A1′**) and $x(0)$ and $\bar{\mathbf{u}}_{[0,\, N-1]}$ are bounded, $\bar{\mathbf{y}}_{[\sigma+d,\, N+\sigma+d-1]}$ is also bounded, i.e., $\|\bar{\mathbf{y}}_{[\sigma+d,\, N+\sigma+d-1]}\| \le c_4$ for some constant c_4. It should be noted that if $d \ne d_0$, the boundedness of $\bar{\mathbf{u}}_{[0,\, N-1]}$ and $\bar{\mathbf{y}}_{[\sigma+d,\, N+\sigma+d-1]}$ cannot be guaranteed as $N \to \infty$.

With the help of (**A2′**), the following inequality holds.

$$\begin{aligned}&\|\bar{\mathbf{y}}_{[\sigma+d,\, N+\sigma+d-1]} - \mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]}\| \\ &\quad \le \|\bar{\mathbf{y}}_{[\sigma+d,\, N+\sigma+d-1]}\| + \|\mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]}\| \\ &\quad \le c_4 + c_1.\end{aligned}$$

Since $\mathbf{F}^{-1}$ is continuous from (**A4′**), $\|\bar{\mathbf{u}}_{[0,\, N-1]} - \mathbf{u}^d_{[0,\, N-1]}\| \le c_5$ for some constant c_5 and $\mathbf{u}^d_{[0,\, N-1]}$ is bounded as $\|\mathbf{u}^d_{[0,\, N-1]}\| \le c_5 + c_3$.

Lemma 3 shows that if $d = d_0$, the boundedness of $\mathbf{u}^d_{[0,\, N-1]}$ is guaranteed under (**A1′**)–(**A4′**) even though $N \to \infty$. It should be noted that if $d \ne d_0$, the boundedness of $\mathbf{u}^d_{[0,\, N-1]}$ cannot be guaranteed. For later use, it is necessary to derive a lemma that can relate the output error with the initial state and the input.

Lemma 4 (Jeong and Choi 2002a)**:** *For the system* (7),

$$\begin{aligned}&\mathbf{y}^d_{[\sigma+d,\, N+\sigma+d-1]} - \mathbf{y}^k_{[\sigma+d,\, N+\sigma+d-1]} \\ &\quad = \mathbf{H}^k_d \delta x^k(0) + \mathbf{J}^k_d \delta\mathbf{u}^k_{[0,\, N-1]}.\end{aligned} \tag{10}$$

$$\mathbf{H}^k_d = [(H^k_{d+1})^T, \ldots, (H^k_{d+N})^T]^T,$$

$$\mathbf{J}^k_d = \begin{pmatrix} J^k_{d+1,1} & J^k_{d+1,2} & \cdots & 0 \\ J^k_{d+2,1} & J^k_{d+2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ J^k_{N+d,1} & J^k_{N+d,d+2} & \cdots & J^k_{N+d,N} \end{pmatrix},$$

$$\delta x^k(0) = x^k(0) - x^d(0),$$

$$\delta\mathbf{u}^k_{[0,\, N-1]} = \mathbf{u}^k_{[0,\, N-1]} - \mathbf{u}^d_{[0,\, N-1]}.$$

Here H^k_i and J^k_{ij}, $d+1 \le i \le N+d$, $j \le i$, are

$$\begin{aligned}H^k_i &= h'(x(i, \gamma^k(i))) \\ &\quad \times \prod_{l=0}^{i-1}\{f'(x^d(l), x^k(l)) + g'(x^d(l), x^k(l))u^d(l)\},\end{aligned}$$

$$\begin{aligned}J^k_{ij} &= h'(x(i, \gamma^k(i))) \\ &\quad \times \left[\prod_{l=j}^{i-1}\{f'(x^d(l), x^k(l)) + g'(x^d(l), x^k(l))u^d(l)\}\right] \\ &\quad \times g(x^k(j-1)),\end{aligned}$$

where

$$\left.\begin{aligned}f'(x^d(i), x^k(i)) &\equiv \begin{bmatrix} f'_1(x(i, \alpha^k_1(i))) \\ \vdots \\ f'_n(x(i, \alpha^k_n(i))) \end{bmatrix}, \\ g'(x^d(i), x^k(i)) &\equiv \begin{bmatrix} g'_1(x(i, \beta^k_1(i))) \\ \vdots \\ g'_n(x(i, \beta^k_n(i))) \end{bmatrix}.\end{aligned}\right\} \tag{11}$$

Here

$$
\begin{aligned}
f'_m(x(i,\alpha_m^k(i))) &= f'_m(\alpha_m^k(i)x^d(i) + (1-\alpha_m^k(i))x^k(i)),\\
g'_m(x(i,\beta_m^k(i))) &= g'_m(\beta_m^k(i)x^d(i) + (1-\beta_m^k(i))x^k(i)),\\
h'(x(i,\gamma^k(i))) &= h'(\gamma^k(i)x^d(i) + (1-\gamma^k(i))x^k(i)),\\
0 \le \alpha_m^k(i), &\beta_m^k(i), \gamma^k(i) \le 1, \forall i,k,m.
\end{aligned}
$$

The following input update law with advanced output data is proposed:

$$
\mathbf{u}^{k+1}_{[0,\,N-1]} = \mathbf{u}^{k}_{[0,\,N-1]} + \mathbf{S}^k \mathbf{e}^d_{[\sigma+d_0,\,N+\sigma+d_0-1]}, \tag{12}
$$

where $\mathbf{e}^k_{[\sigma+d_0,\,N+\sigma+d_0-1]} = \mathbf{y}^d_{[\sigma+d_0,\,N+\sigma+d_0-1]} - \mathbf{y}^k_{[\sigma+d_0,\,N+\sigma+d_0-1]}$, $\mathbf{S}^k$ is a learning gain matrix of dimension $N \times N$.

Theorem 1: *The system* (7) *satisfies* (**A1**$'$)–(**A4**$'$) *and the system dynamics may not be known completely. If the condition*

$$
\|I - \mathbf{S}^k \mathbf{J}^k_d\| \le \rho < 1, \quad \text{for all } k \tag{13}
$$

is satisfied, the input $\mathbf{u}^k_{[0,\,N-1]}$ *which can be computed by* (12), *converges to bounded* $\mathbf{u}^d_{[0,\,N-1]}$ *as* $k \to \infty$.

Proof: From $x^d(0) - x^k(0) = 0$ and Lemma 4,

$$
\begin{aligned}
&\mathbf{u}^d_{[0,\,N-1]} - \mathbf{u}^{k+1}_{[0,\,N-1]}\\
&\quad = \mathbf{u}^d_{[0,\,N-1]} - \mathbf{u}^k_{[0,\,N-1]} - \mathbf{S}^k \mathbf{e}^k_{[\sigma+d,\,N+\sigma+d-1]}\\
&\quad = \mathbf{u}^d_{[0,\,N-1]} - \mathbf{u}^k_{[0,\,N-1]} - \mathbf{S}^k[\mathbf{H}^k_d \delta x^k(0) + \mathbf{J}^k_d \delta \mathbf{u}^k_{[0,\,N-1]}]\\
&\quad = [I - \mathbf{S}^k \mathbf{J}^k_d][\mathbf{u}^d_{[0,\,N-1]} - \mathbf{u}^k_{[0,\,N-1]}].
\end{aligned} \tag{14}
$$

Taking norms of both sides of (14) and using (13),

$$
\|\mathbf{u}^d_{[0,\,N-1]} - \mathbf{u}^{k+1}_{[0,\,N-1]}\| \le \rho \|\mathbf{u}^d_{[0,\,N-1]} - \mathbf{u}^k_{[0,\,N-1]}\|.
$$

Hence, $\lim_{k\to\infty} \|\mathbf{u}^d_{[0,\,N-1]} - \mathbf{u}^k_{[0,\,N-1]}\| = 0$.

The boundedness of $\mathbf{u}^d_{[0,\,N-1]}$ comes from Lemma 3.

Using (12), the stabilized input to track the desired trajectory can be obtained using Lemma 3 and Theorem 1. As in the linear case the input update law (12) is a generalization from the minimum phase systems to the non-minimum phase systems if $d = d_0$.

In the well-known stable inversion approach, the input is calculated in the horizon $(-\infty, +\infty)$ and is truncated to make it causal. Also, it uses the input-to-state mapping and the state-to-output mapping. On the other hand, the proposed method utilizes the input-to-output mapping and it is possible to obtain an input that provides perfect tracking of the desired output on the output horizon except $[0, \sigma + d - 1]$.

In practice, we need to have a good model of the system in order to find the learning gain matrix $\mathbf{S}^k$ which satisfies the condition (13). □

Example: Let us consider the following nonlinear discrete time non-minimum phase system:

$$
\left.
\begin{aligned}
x_1(i+1) &= x_2(i)\\
x_2(i+1) &= x_3(i) - \sin^2(x_2(i))\\
x_3(i+1) &= 0.1x_3(i) + 2.5\sin^2(x_2(i)) + u(i)\\
y(i) &= x_1(i) + 2.5x_2(i) + x_3(i).
\end{aligned}
\right\} \tag{15}
$$

Since $\sigma = 1$, let us set $z_1 = y, z_2 = x_1, z_3 = x_2$, then $\mathbf{z}(i) = (y(i), x_1(i), x_2(i)) = \Phi(\mathbf{x}(i))$. Using this relation, the system is transformed into

$$
\begin{aligned}
\begin{bmatrix} z_1(i+1)\\ z_2(i+1)\\ z_3(i+1)\end{bmatrix} &= \begin{bmatrix} 2.6 & -2.6 & -5.5\\ 0 & 0 & 1\\ 1 & -1 & -2.5\end{bmatrix}\begin{bmatrix} z_1(i)\\ z_2(i)\\ z_3(i)\end{bmatrix}\\
&\quad + \begin{bmatrix} 0\\ 0\\ -\sin^2(z_3(i))\end{bmatrix} + \begin{bmatrix} 1\\ 0\\ 0\end{bmatrix} u(i).\\
y(i) &= z_1(i).
\end{aligned} \tag{16}
$$

The zero dynamics of the system is governed by

$$
\begin{bmatrix} z_2\\ z_3\end{bmatrix} = \begin{bmatrix} 0 & 1\\ -1 & -2.5\end{bmatrix}\begin{bmatrix} z_2\\ z_3\end{bmatrix} + \begin{bmatrix} 0\\ -\sin^2(z_3(i))\end{bmatrix} \tag{17}
$$

and it has one non-minimum phase zero and one minimum phase zero around the origin.

The desired trajectory is given as follows:

$$
y^d(i) = \begin{cases} 0, & i = 0, 1\\ 0.2\sin(0.05\pi(i-2)), & 2 \le i \le 22.\end{cases} \tag{18}
$$

Let us find the inputs that can track $y^d(i)$ by the stable inversion method and the proposed method.

Stable inversion method (Zeng and Hunt 2000)**:** Let us assume that the system is completely known. From (16), the following equation can be obtained:

$$
\begin{bmatrix} z_2(i+1)\\ z_3(i+1)\end{bmatrix} = \begin{bmatrix} 0 & 1\\ -1 & -2.5\end{bmatrix}\begin{bmatrix} z_2(i)\\ z_3(i)\end{bmatrix} + \begin{bmatrix} 0\\ y^d(i)\end{bmatrix}. \tag{19}
$$

By transforming

$$
Q = \begin{bmatrix} 0 & 1\\ -1 & -2.5\end{bmatrix}
$$

into the Jordan canonical form, i.e., $D = T^{-1}QT$, (19) becomes

$$\tilde{\eta}(i+1) = T^{-1}\mathbf{z}(i+1) = D\tilde{\eta}(i) + T^{-1}\begin{bmatrix} 0 \\ y^d(i) - \sin^2(z_3(i)) \end{bmatrix}$$

where

$$T = \begin{bmatrix} 0.8944 & -0.4472 \\ -0.4472 & 0.8944 \end{bmatrix}, \quad D = \begin{bmatrix} -0.5 & 0 \\ 0 & -2.0 \end{bmatrix}.$$

Based on the following Picard iteration, the $\tilde{\eta}$ can be obtained

$$\tilde{\eta}_0(i) = 0$$

$$\vdots$$

$$\tilde{\eta}_{m+1}(i) = \sum_{k=-\infty}^{\infty} \phi(i-k) \, T^{-1}\begin{bmatrix} 0 \\ y^d(k-1) - \sin^2(-0.4472\tilde{\eta}_1 + 0.8944\tilde{\eta}_2) \end{bmatrix}.$$

The input is calculated using $\tilde{\eta}$. Figures 1 and 2 show the output and input using the stable inversion method. In figure 3, the output error is shown. Since the input is truncated to be causal, the truncated error exists on the whole output interval.

Let us consider the learning of the non-minimum phase systems by the proposed method.

Proposed method: The exact linearized model of (15) around $(0, 0)$ is as follows:

$$\left.\begin{aligned} x_1(i+1) &= x_2(i) \\ x_2(i+1) &= x_3(i) \\ x_3(i+1) &= 0.1x_3(i) + u(i) \\ y(i) &= x_1(i) + 2.5x_2(i) + x_3(i). \end{aligned}\right\} \quad (20)$$

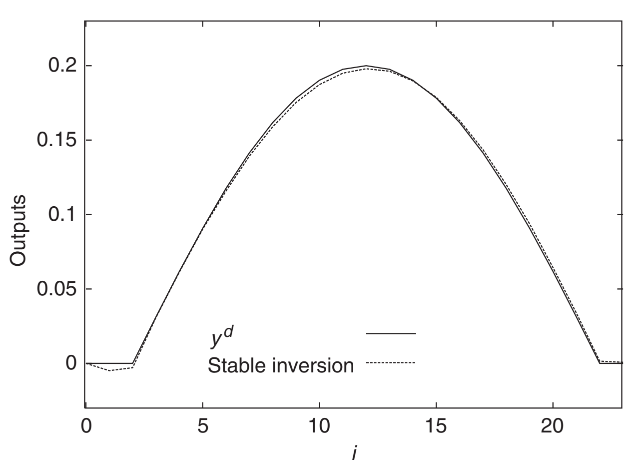

Figure 1. Output using the stable inversion.

It is difficult to have the linearized model when the system equation is unknown. It is assumed that we only have the following linear model with modelling error terms compared to (20).

$$\left.\begin{aligned} x_1(i+1) &= 1.2x_2(i) + 0.2x_3(i) \\ x_2(i+1) &= -0.1x_1(i) + x_3(i) \\ x_3(i+1) &= 0.4x_3(i) + u(i) \\ y(i) &= x_1(i) + 2.5x_2(i) + x_3(i). \end{aligned}\right\} \quad (21)$$

We want to show that the desired input can be obtained despite the modeling errors as long as we can find $\mathbf{S}^k$ which satisfies the convergence condition. We set $y^d(23) = 0$, $u(22) = 0$, $u^0(i) = 0, i = 0, \ldots, 21$. To determine $\mathbf{S}$, $\mathbf{J}_d^k$ is estimated from the above model and denote it as $\hat{\mathbf{J}}_d$. Since the model is linear, $\hat{\mathbf{J}}_d$ can be

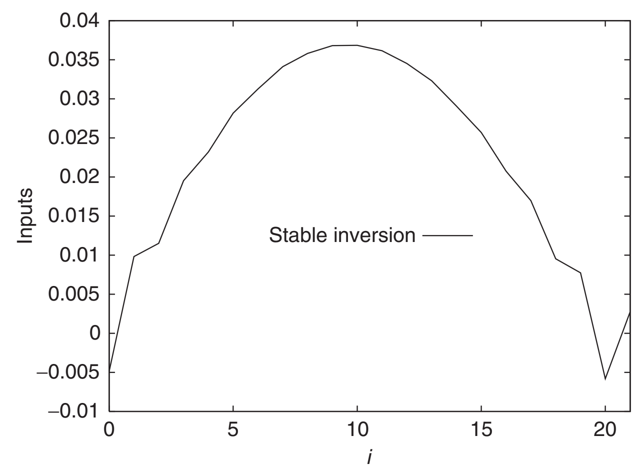

Figure 2. Input using the stable inversion.

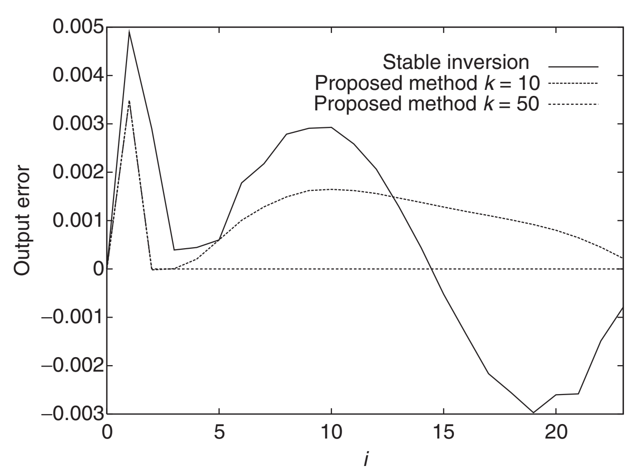

Figure 3. Output error.

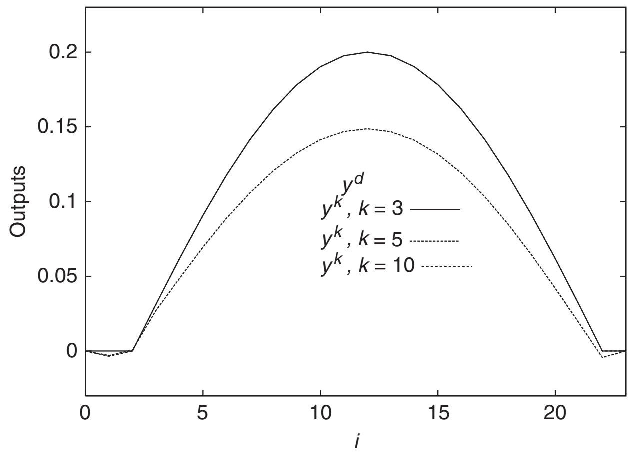

Figure 4. Outputs using the proposed method.

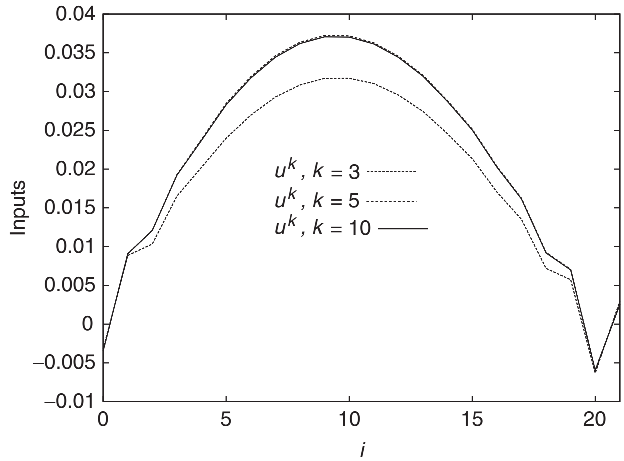

Figure 5. Inputs using the proposed method.

obtained from $\mathbf{J}_c$ in (4) with $d = 1$. We set $\mathbf{S} = 0.5 \times \hat{\mathbf{J}}_d^{-1}$ and use the input update law (12).

Figures 4 and 5 show the outputs and inputs for different values of k, respectively. The output error $\|y^d(1) - y^k(1)\|$ converges to 0.0035 and $\mathbf{y}^k_{[2,22]}$ converges to $\mathbf{y}^d_{[2,22]}$ almost perfectly for $k = 10$ with $\|\mathbf{y}^d_{[2,22]} - \mathbf{y}^{10}_{[2,22]}\|_2^2 \leq 2.6916 \times 10^{-5}$. As can be seen in figure 3, the output error $\mathbf{y}^d_{[2,22]} - \mathbf{y}^{10}_{[2,22]}$ can be made arbitrarily small by the proposed method.

Let us consider convergence condition in this example. If the system is unknown, it is impossible to check the convergence condition. Generally, the convergence condition is given by using system equation.

Likewise, we use the system equation only to check the convergence condition. However, it is difficult to calculate $\mathbf{J}_d^k$ in (10). Instead, we define

$$\bar{\mathbf{J}}_d^k = \frac{\partial \mathbf{y}^k_{[\sigma+d,\, N+\sigma+d-1]}}{\partial \mathbf{u}^k_{[0,\, N-1]}},$$

which can be computed by using (15), and approximate $\mathbf{J}_d^k$ as $\bar{\mathbf{J}}_d^k$. When $\mathbf{u}^k_{[0,N-1]}$ approaches to $\mathbf{u}^d_{[0,N-1]}$, $\bar{\mathbf{J}}_d^k$ approaches to $\mathbf{J}_d^k$. Therefore $\|I - \mathbf{S}\bar{\mathbf{J}}_d^k\|$ can be$_k$a good indicator of $\|I - \mathbf{S}\mathbf{J}_d^k\|$. We found that $\|I - \mathbf{S}\mathbf{J}_d\| \leq 0.7$ for all k and the convergence condition is more likely satisfied.

This example shows a way of obtaining the gain matrix $\mathbf{S}^k$ from a linear model even if the model has modeling error terms.

4. Conclusion

This paper investigated the iterative learning control for discrete time nonlinear non-minimum phase systems. A new tracking method based on the input to output mapping was proposed extending the results of linear discrete time non-minimum phase systems.

By properly advancing the output data, it was shown that it is possible to perfectly track the desired output except at the beginning and that the inverse mapping from the output to the input is stable. Also, a learning scheme for handling uncertain systems was proposed based on the result. Similar to the conventional ILC schemes using the relative degree, the proposed method has a simple structure based on the information of the relative degree and the number of non-minimum phase zeros. This approach is the generalization of iterative learning control from the minimum phase systems to the non-minimum phase systems. The simulation results show that the proposed method tracks the desired output better than the well-known stable inversion method.

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