An Application of Generalized Least Squares Bias Estimation For Over-The-Horizon Radar Coordinate Registration

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Abstract - *Target and sensor geometry for a particular suite of over-the-horizon radars and a single target are given. Systematic positional differences between tracks seen from two separate radar sites can be used to improve the estimation of ionospheric parameters. In this paper a description of over-the-horizon radar propagation is provided and a method, given the target/sensor geometry, is described for estimating the range and azimuth biases resulting from errors in modeling the ionospheric fluctuations. Using the Generalized Least Squares Estimation method, a quantitative analysis of the improvements in tracking target positions in regions of overlapping coverage is given* .

Keywords - Over-the-horizon radar, coordinate registration, generalized least squares bias estimation

1. Introduction

In a previous paper [4], a data set was used which consisted of two independent over-the-horizon (OTH) radar systems covering a common surveillance region at a range about 1500 nmi from both OTH radar sites. A ground-based microwave radar provided truth data in the region. During the two-hour data period, eleven ground targets were concurrently held by both OTH radar systems and by the ground-based microwave radar. The OTH radar systems, running in their standard manner, detected the targets, formed tracks in radar coordinates and identified tracks belonging to the same target. They selected and assigned ionospheric modes to be used, brought each of the radar tracks to ground coordinates using the appropriate coordinate registration tables and fused the collection into common target states (see the next section for a discussion of coordinate registration). This was done for each minute in which the OTH radar held contact on the target. Using the microwave radar to provide the true target position, the range and cross-range errors for each of the targets were calculated. The range errors and cross-range errors were plotted as a function of time and it was shown that a significant range bias was present and persisted over time. In this paper, the algorithm developed in [5] to estimate the bias in coordinate registration, is utilized for a system of two geographically distributed, high-frequency over-the-horizon radar sensors.

As described in [3], coordinate registration for high frequency (HF) over-the-horizon radar is done by ray tracing through ionospheric models which are estimated using real-time sounding observations of the ionospheric state. (For a discussion of the importance of coordinate registration, see the next section). In order to track the state of this very dynamic medium, the sounding and modeling processes are repeated every twelve minutes. This often introduces uncertainty in coordinate registration, because of the inherent variability in the ionosphere and its models. Thus biases as introduced by estimates of the ionosphere (causing uncertainties in coordinate registration), ultimately result in systematic errors in the ground positions of targets as measured by miss distances. Miss distance is defined as the distance between the radar track position estimate and the true data position at the same instant of time. It is expected that the resultant algorithmic development based on the work described in this paper will improve track positional accuracy by an additional 50%. We have also investigated the fusion of OTH radar tracks with microwave radar tracks using commercial off-the-shelf algorithms and have suggested an application to enhanced coordinate registration (cf. references [6-7]). Developments are currently underway to reference target tracks to beacon transponders. In conjunction with real-time ray tracing, these methods are expected to reduce target positional errors in areas in the vicinity of the beacon transponder locations. But many of the areas of interest for OTH radar surveillance are not convenient to operational transponders or ground-based radars, but can be covered by multiple OTH radar systems. It is in these areas that this approach promises to have applicability.

2. Detection, Tracking, CR, And Data Fusion

We consider the problem of estimating the position Z of a target from multiple measurements provided by a system of two spatially distributed OTH radar sensors. At the central tracking processor, the track plots from the multiple radars are used to update existing system tracks or initiate new system tracks as appropriate. Specifically, the central tracking processor must perform the following five functions:

1. Coordinate Registration: Transformation of the radar plots from local radar (or slant) coordinates to system coordinates, which are latitude and longitude (or ground coordinates).
2. Correlation or association of the radar plots with the appropriate system tracks.
3. Initiation of new tracks with the uncorrelated plots and rejection of clutter plots.
4. Tack filtering and track prediction.
5. Track monitoring and system track management.

Functions 2 and 4 represent the heart of the traditional data association and tracking problem. However, before either of these processes can occur successfully, function 1 must be performed; that is, the individual radar data must be expressed in a common coordinate system in which the errors due to site uncertainties,

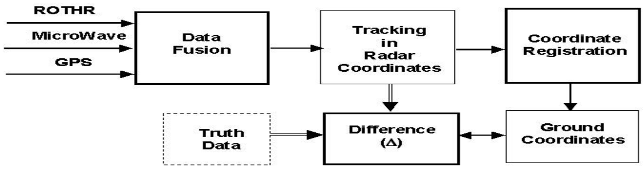


Figure 1. Integrated Data Fusion Flow for Relocatable Over-the-Horizon Radar (ROTHR).

antenna orientation, and improper calibration of range and time (usually due to ionospheric uncertainties) have been minimized so they do not cause a significant degradation of the system operation. The process of ensuring the requisite "error free" coordinate conversion of radar data is called coordinate registration (CR). Thus, CR is an absolute prerequisite for multiple radar tracking or sensor fusion in general. See figure 1 for the flow of fusion information for the Relocatable Over-the-Horizon Radar (ROTHR) system. The US Navy operates two ROTHR systems. The first one, located in Virginia, looks south over the Gulf of Mexico and the Caribbean. The second one located in south Texas, looks to the southeast over the Gulf and the Caribbean. A third system will soon be operational in Puerto Rico. The geometry of the radars and the region of overlapping coverage are shown in figure 2.

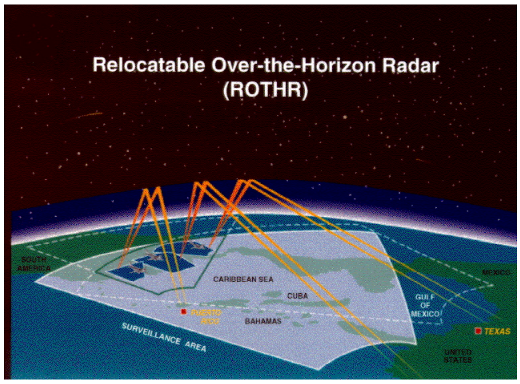


Figure 2. Multiple Relocatable Over-The-Horizon Radar Track Data Fusion.

The primary mission of the ROTHR system is to provide continuous detection and monitoring of suspected air smuggling activities in the Caribbean and Eastern Pacific Regions. A secondary activity of the program is to interact with the Australian Defence Science and Technology Organization (DSTO) in the operation and enhancement of OTH radar. The notional geometry of the ROTHR system and a single target are shown in figure 3.

3. Geometry Of OTH Radar

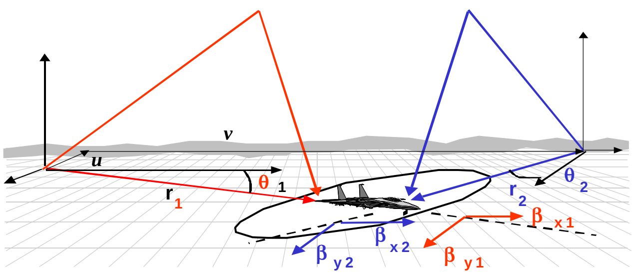
The use of ROTHR for the detection and tracking of aircraft targets raises several technical challenges (cf. reference [8]). (i) *Mode Identification*: Using the ionosphere as a means of reflecting the signal in order to peek over the horizon is of great benefit, but the ionosphere is not a simple mirror. Signals refracting through the ionosphere can use a variety of modes (i.e. paths), and a single air target can provide several tracks to the radar system. In order to place the target correctly in ground coordinates (latitude and longitude) the paths through the ionosphere must be unraveled. Errors in this process can lead to erroneous ground positions for the target. (ii) *Crossing Targets*: A second challenge arises in calculating course and speed of the target. The radial component of velocity can be very accurately derived from the Doppler shift on the received signal. but the tangential component must be calculated from the change in target azimuth with time. Because of the long ranges involved, there can be large variances in this calculation and these inaccuracies can cause significant errors in the predicted course and speed of the target. This is especially true of targets traveling near tangentially to the radar look direction. (iii) *Low Doppler Targets*: A third challenge is a consequence of the fact that ROTHR is a Doppler radar, that is the target must have enough motion relative to the radar site to make it distinguishable from the large backscattered signal from the ground. A target whose range to the radar does not change with time, is difficult to detect.

The type of measurements provided by the OTH radar systems consists of radar slant coordinates (bearing, *q*, and range, *r*, from a radar sensor to the target). As described in [3], we formulate the difference ΔP in the reported positions as a function of the set of measured variables Z (i.e., observations) and the set of bearing and range biases β (i.e., parameters) to be estimated:

$$DP = F(Z,b) \qquad (1)$$

The model parameters and the target-sensor geometry are depicted in figure 3.

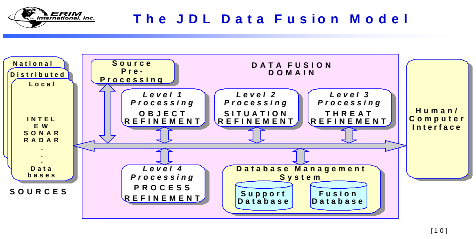
Figure 3. Geometry of OTH Radar Track Data Fusion.



4. The JDL Model For Data Fusion

The Joint Director of Labs (JDL) model comprises four levels of data fusion as shown in Figure 4.

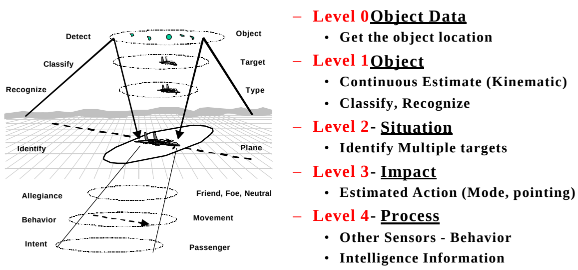
Figure 4. The JDL data fusion model.



The definitions [1] follow (cf. [2] for recent revisions).

- Level 1 Object Refinement: forms object assessments, particularly by observation to track and track to track associations from possibly different sensors. As the product of object refinement, *object assessments* are tracks, possibly classified by type and identity, with the aim of assessing one track for each object.
- Level 2 Situation Refinement: forms situation assessments by relating object assessments together or by relating object assessments with existing situation assessments. As the product of situation refinement, *situation assessments* are relational associations involving subsets of objects.
- Level 3 Threat Refinement: forms threat assessments, particularly by considering the possible consequences of situation assessments or their relation with existing threat assessments. As the product of threat refinement, *threat assessments* identify the possible roles of objects in terms of intent and capability, and the expected outcomes.
- Level 4 Process Refinement: is a metalevel process which identifies *what* is required to improve level 1, 2, and 3 assessments and *how* sensors and the sensor fusion process can be altered to obtain those improvements. As the products of process refinements, *process assessments* identify what improvements are required and *process control* identifies how the sensors and fusion process are to be adjusted to facilitate those improvements.

Figure 5. Fusion model applied to the OTHR system .



5. An Application Of GLSE To OTH Radar Multiple Sensor-Single Target Scenarios

For convenience, the discussion given in reference [3] is repeated here. Following the usual linearization technique, but with the roles of the actual values and estimators reversed, the vector equation or position difference can be transformed in the classical Gauss-Markov *generalized least squares estimation* (GLSE) model:

$$X\beta + \xi = Y \qquad (2)$$

where X is a matrix of known parameters, ξ is the vector of measurement errors, and Y is the measurement vector.

The solution of the GLSE problem is simply

$$\beta^* = \Sigma^* X^T \Sigma_{-1,\xi} Y \qquad (3)$$

where $\Sigma^* = (X^T \Sigma_{-1,\xi} X)^{-1}$ (4)

is the covariance matrix for the estimate β^* of the vector of biases β.

The GLSE approach is developed here for one range and one azimuth offset bias. To assess quantitatively the performance of the GLSE approach, the algorithm will be evaluated in detail considering both simulated and real OTHR data.

For this application, we consider the case of two overlapping OTH radars R_A located at the origin, and R_B, located at coordinates (u, v). We further assume that R_A gives biased measurements of range, while R_B gives biased measurements of target azimuth. Denote the vector of radar measurements by

$$y_k = (r_{Ak}, q_{Bk})^T \qquad (5)$$

where r_{Ak} and q_{Bk} denote the range and azimuth measurements from radar R_A and R_B, respectively, and k denotes the time index.

The generalized measurement equations from the two sensors is, as mentioned above, $DP = F(Z,b)$, based on the measurements y_k and the set of biases $b = (\Delta r_A,\ Dq_B)^T$. For this application, these measurements are

$$g_A(x(k), z_1(k), \Delta r_A) = r_{Ak} - \sqrt{x_{2,1}(k) + x_{2,2}(k)} - \Delta r_A$$
(6)

and

Erreur! (7)

Here $z_1(k)$ are range measurements from R_A at time k, $z_2(k)$ are azimuth measurements from sensor R_B at time k, Δr_A and Δq_B are the bias parameters to be estimated and $x_1(k)$ and $x_2(k)$ are target state vectors at time k. These equations relate the set β of bias parameters to be estimated from the set of measurements y_k and the vector of observations z. However this relationship is nonlinear.

To apply the theory of generalized least squares, we will need to represent the observations as a linear function of the parameters to be estimated, namely b. This can be accomplished by defining a function f as follows:

$$f(y_k, b) = [g_A(x(k), z_1(k), \Delta r_A), g_B(x(k), z_2(k), \Delta q_B)]$$
(8)

Further, let y'_k and β' denote the actual measurement sets and an initial estimate of β, respectively. Now Taylor's Theorem can be used in the usual way to approximate the function f at the true values of y_k and β in terms of the measurements y'_k and the initial estimate β'.

6. Results

These results assume sensor A is a one- or two-dimensional sensor that gives biased measurements of range while sensor B is a one- or two-dimensional sensor that gives biased measurements of azimuth. The sensor coordinates are A(0,0) and B(1400, 600). The biases used in the simulation are β_1= 20 units and β_2 = 8.6 degs with zero-mean Gaussian measurement noise. The results published here are based on the measurements g_A and g_B given in equations (6) and (7) in the previous section and using a modified version of the algorithm in reference [5], where we have assumed that sensor A gives biased estimates of range only, and sensor B gives biased estimates of target azimuth only. Figure 6 shows registered plots and unregistered plots compared to the true trajectory for a fairly tangential target track while figure 7 shows registered plots and unregistered plots compared to the true trajectory for a target track running between the sensors. The latter case might exhibit the case of a microwave radar sensor (A) and an OTH radar sensor (B). In both cases the algorithm gives good numerical results. As McMichael and Okello [5] point out, the registration algorithm works with arbitrary number of sensors in arbitrary locations, and even more importantly, arbitrary sensor types can be used. This would allow us to register and fuse data from highly disparate sensors.

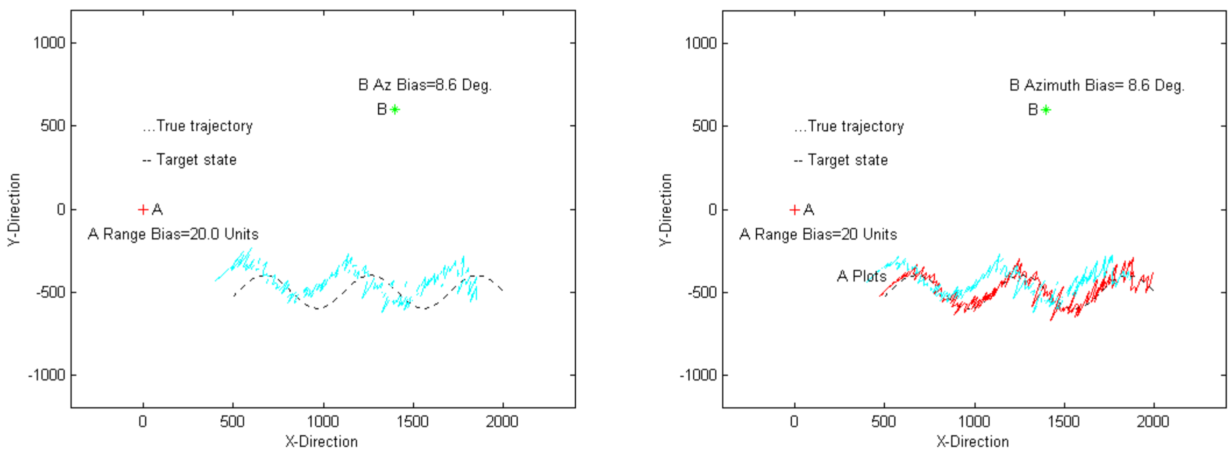


Figure 6. Tracking with range and azimuth bias registration using GLSE for bias estimation.

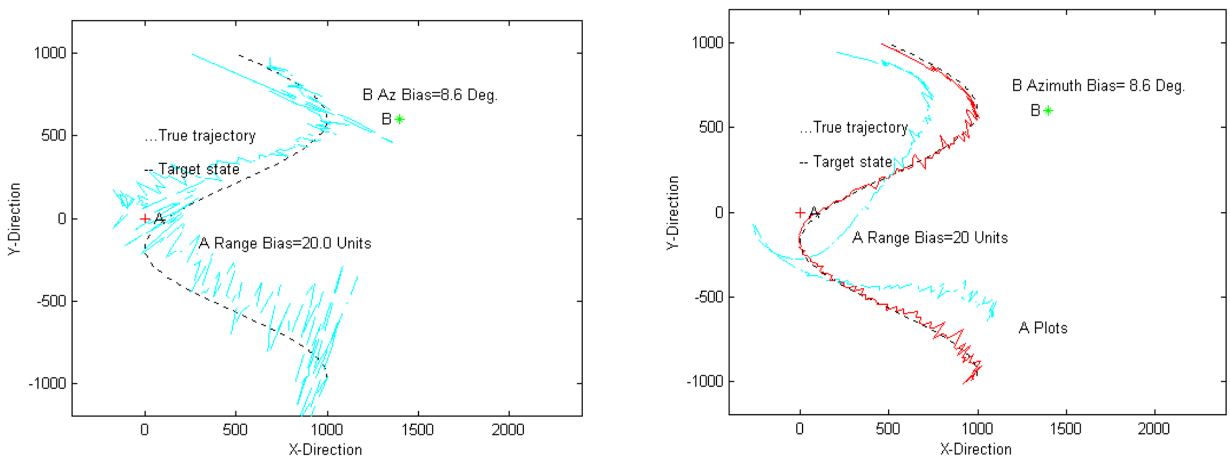


Figure 7. Tracking with range and azimuth bias registration using GLSE for bias estimation.

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