Uni-Vector-Sensor ESPRIT for Multisource Azimuth, Elevation, and Polarization Estimation

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Abstract—This paper introduces a novel eigenstructure-based algorithm *uni-vector-sensor ESPRIT* that yields closed-form direction-of-arrival (DOA) estimates and polarization estimates using one electromagnetic vector sensor. A vector sensor is composed of six spatially co-located *non*isotropic *polarization*-sensitive antennas, measuring all six electromagnetic field components of the incident wave field. Uni-vector-sensor ESPRIT is based on a matrix-pencil pair of temporally displaced data sets collected from a single electromagnetic vector sensor. The closed-form parameter estimates are obtained through a vector cross-product operation on each decoupled signal-subspace eigenvector of the data correlation matrix. This method exploits the electromagnetic sources' polarization diversity in addition to their spatial diversity, requires no *a priori* knowledge of signal frequencies, suffers no frequency-DOA ambiguity, pairs automatically the x-axis direction cosines with the y-axis direction cosines, eliminates array interelement calibration, can resolve up to five completely polarized uncorrelated monochromatic sources from near field or far field. It impressively out-performs an array of spatially displaced identically polarized antennas of comparable array-manifold size and computational load.

Index Terms—Antenna arrays, array signal processing, direction of arrival estimation, diversity, eigenvalues/eigenfunctions, frequency estimation, polarization.

I. INTRODUCTION

UNI-VECTOR-SENSOR ESPRIT is an eigenstructure-based closed-form algorithm to estimate the azimuth angles, the elevation angles, and the polarization states of multiple monochromatic incoherent incident sources from the near field or the far field using a single electromagnetic vector sensor. An electromagnetic vector sensor consists of six spatially co-located nonisotropic polarization-sensitive antennas, measuring all six electromagnetic field components of the incident wave field. Electromagnetic vector sensors are commercially available, e.g., the CART array, manufactured by Flam and Russell, Inc. of Horsham, PA [2], [6]. The CART array is composed of three orthogonally oriented short dipoles and three orthogonally oriented magnetic loops, all co-located in space. Thus, uni-vector-sensor ESPRIT exploits the incident sources' polarization diversity as well as their angular spatial diversity.

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This ESPRIT-based algorithm forms a matrix-pencil pair out of two temporally displaced sets of data collected from a single vector sensor. The data covariance matrix's signal-subspace eigenvectors can be decoupled to yield estimates of each incident signal's electromagnetic field components. (Uni-vector-sensor ESPRIT forms the matrix-pencil primarily to obtain the decoupling matrix to decouple the signal-subspace eigenvectors of the data correlation matrix into estimates of the signals' steering vectors. In the case where the data set is not decomposable into two time-delayed subsets, the eigendecomposition would only yield the signal-subspace eigenvectors but not the aforementioned decoupling matrix.) The vector cross product between each signal's electric-field vector estimate and corresponding magnetic-field vector estimate produces an estimate of the signal's normalized Poynting vector, whose components embody the source's respective three Cartesian direction cosines. From this information, each source's DOA's and polarization can be estimated. Thus, sources impinging from the same DOA can be resolved due to their distinct polarization states; and both the azimuth and elevation angles can be estimated and automatically paired with only one vector sensor.

The complicating effects of a near-field wavefront's curvature is also avoided because of the spatial co-location of the uni-vector-sensor array's constituent sensors. This novel method also does not require *a priori* information of the signals' frequencies, (but no two sources may have the same frequency) because the array manifold is independent of signal frequency due to the spatial co-location of its constituent components. Moreover, if a prefabricated vector sensor (such as the CART array) is deployed, interelement spatial calibration is eliminated.

This proposed algorithm also differs from most other direction-finding methods in how it uses the eigenvalues and the eigenvectors of the data covariance matrix. Conventional arrays of spatially displaced and identically polarized antennas typically encapsulate the DOA information in the phase offsets amongst spatially displaced scalar-sensors. However, in uni-vector-sensor ESPRIT, the DOA information is embedded in the intrinsic directionality of each constituent component.

A. Summary of Relevant Research Literature

Eigenstructure-based direction-finding methods such as *ESPRIT* [1], though suboptimal, have advantages over optimal methods such as the maximum-likelihood (ML) method because eigenstucture methods require only the second-order statistics of the additive noise and a lighter computational load while offering comparable performance in many situations.

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Arrays of diversely polarized antennas have been exploited in a number of direction-finding algorithms that extend the direction-finding approach used for spatially displaced uniformly polarized antenna arrays. Examples include Li and Compton [3], [4], [7], [8], [11], Hua [9], Swindlehurst and Viberg [12], Li [14], Li and Stoica [16], Chang and Hua [17], and Li *et al.* [24]. These works, however, use only dipole pairs or dipole triads and thus can extract information only of two or three electric field components out of all six electromagnetic field components. The first direction-finding algorithms explicitly exploiting all six electromagnetic components appear to have been developed separately by Nehorai and Paldi [5] and Li [10]. Nehorai and Paldi [5] (who coined the term "vector sensor") pioneered the simple but novel idea of using the vector cross product of the electric-field and the magnetic-field vector estimates (provided the vector-sensor outputs) to estimate directly the radial direction of a source. Their paper also proposed a scalar performance measure—the mean square angular error (MSAE)—and derived a compact expression and a bound for the asymptotic MSAE for the vector sensor. Nehorai and Paldi [5] also proposed using a single electromagnetic vector sensor to estimate the DOA's, but not by eigenstructure methods as herein proposed. Ho *et al.* [27] extended Li's result in [10] for partially polarized sources. Burgess and Van Veen [18]–[20] deployed a multiple vector-sensor array for signal detection along specific preset arrival directions and polarization states, but did not explore direction finding. Hochwald and Nehorai [21] employed vector sensors in polarimetric modeling. Electromagnetic vector-sensor identifiability and uniqueness issues have been investigated by Hatke [13], Ho *et al.* [22], Hochwald and Nehorai [23], and Tan *et al.* [25]. The vector cross-product estimator is first adapted to ESPRIT by Wong and Zoltowski in their *extended-aperture vector-sensor ESPRIT* algorithm [26], [31], which utilizes extended inter-vector-sensor spatial invariances in a multi-element planar array of vector sensors. Wong and Zoltowski [32] also presented another multiple vector-sensor direction-finding algorithm that adaptively steers null beams in polarization using a self-initiating iterative search method that requires no *a priori* source information. Wong and Zoltowski [33] further developed a closed-form direction-finding algorithm applicable to multiple arbitrarily spaced vector sensors at possibly unknown locations. However, this present algorithm represents the first *eigenstructure (subspace)* method that estimates the directions of arrival of multiple sources using only a *single* vector sensor.

II. MATHEMATICAL MODEL OF THE UNIVECTOR-SENSOR ARRAY MANIFOLD

The present signal model involves multiple uncorrelated monochromatic transverse electromagnetic waves, having traveled through a homogeneous isotropic medium, impinge upon a single vector sensor. The kth such incoming unit power completely polarized electromagnetic wavefront has the electric-field vector $\mathbf{e}_k$ and the magnetic-field vector $\mathbf{h}_k$. In spherical coordinates, $\mathbf{e}_k$ and $\mathbf{h}_k$ can be expressed as [5], [10]

$$
\begin{aligned}
\mathbf{e}_k &= \sin \gamma_k e^{j\eta_k} \vec{\mathbf{v}}_\phi + \cos \gamma_k \vec{\mathbf{v}}_\theta \\
\mathbf{h}_k &= \mathcal{Z}_\circ (\cos \gamma_k \vec{\mathbf{v}}_\phi + \sin \gamma_k e^{j\eta_k} \vec{\mathbf{v}}_\theta)
\end{aligned}
\tag{1}
$$

where $0 \leq \gamma_k < \pi/2$ is the auxiliary polarization angle and $-\pi \leq \eta_k < \pi$ is the polarization phase difference. For linearly-polarized transverse electromagnetic waves $\eta_k = 0$; for circularly-polarized waves, $\gamma_k = 45°$ and $\eta_k = \pm 90°$ (+ for left circularly polarized and − for right circularly polarized), $\mathcal{Z}_\circ$ is the transmission medium's intrinsic impedance and is real valued, $\vec{\mathbf{v}}$ is a unit vector along the subscript's coordinate. Expressed in Cartesian coordinates after normalization [5], [10]

$$
\begin{aligned}
\mathbf{e}_k &= (\sin \gamma_k \cos \theta_k \cos \phi_k e^{j\eta_k} - \cos \gamma_k \sin \phi_k)\vec{\mathbf{v}}_x \\
&\quad + (\sin \gamma_k \cos \theta_k \sin \phi_k e^{j\eta_k} + \cos \gamma_k \cos \phi_k)\vec{\mathbf{v}}_y \\
&\quad - \sin \gamma_k \sin \theta_k e^{j\eta_k} \vec{\mathbf{v}}_z \tag{2} \\
\mathbf{h}_k &= -(\cos \gamma_k \cos \theta_k \cos \phi_k + \sin \gamma_k \sin \phi_k e^{j\eta_k})\vec{\mathbf{v}}_x \\
&\quad - (\cos \gamma_k \cos \theta_k \sin \phi_k - \sin \gamma_k \cos \phi_k e^{j\eta_k})\vec{\mathbf{v}}_y \\
&\quad + \cos \gamma_k \sin \theta_k \vec{\mathbf{v}}_z \tag{3}
\end{aligned}
$$

where $0 \leq \theta_k < \pi$ the signal's elevation angle measured from the vertical z axis and $0 \leq \phi_k < 2\pi$ the azimuth angle. The above may be re-expressed in matrix form as [8], [11]

$$
\begin{aligned}
\mathbf{a}_k &\stackrel{\text{def}}{=}
\begin{bmatrix}
e_x(\theta_k, \phi_k, \gamma_k, \eta_k) \\
e_y(\theta_k, \phi_k, \gamma_k, \eta_k) \\
e_z(\theta_k, \gamma_k, \eta_k) \\
h_x(\theta_k, \phi_k, \gamma_k, \eta_k) \\
h_y(\theta_k, \phi_k, \gamma_k, \eta_k) \\
h_z(\theta_k, \gamma_k)
\end{bmatrix} \\[2mm]
&\stackrel{\text{def}}{=}
\begin{bmatrix}
\sin \gamma_k \cos \theta_k \cos \phi_k e^{j\eta_k} - \cos \gamma_k \sin \phi_k \\
\sin \gamma_k \cos \theta_k \sin \phi_k e^{j\eta_k} + \cos \gamma_k \cos \phi_k \\
- \sin \gamma_k \sin \theta_k e^{j\eta_k} \\
- \cos \gamma_k \cos \theta_k \cos \phi_k - \sin \gamma_k \sin \phi_k e^{j\eta_k} \\
- \cos \gamma_k \cos \theta_k \sin \phi_k + \sin \gamma_k \cos \phi_k e^{j\eta_k} \\
\cos \gamma_k \sin \theta_k
\end{bmatrix} \tag{4} \\[2mm]
&= \underbrace{
\begin{bmatrix}
\cos \theta_k \cos \phi_k & - \sin \phi_k \\
\cos \theta_k \sin \phi_k & \cos \phi_k \\
- \sin \theta_k & 0 \\
- \sin \phi_k & - \cos \theta_k \cos \phi_k \\
\cos \phi_k & - \cos \theta_k \sin \phi_k \\
0 & \sin \theta_k
\end{bmatrix}}_{\Theta_k(\theta_k, \phi_k)}
\underbrace{
\begin{bmatrix}
\sin \gamma_k e^{j\eta_k} \\
\cos \gamma_k
\end{bmatrix}}_{\mathbf{g}_k(\gamma_k, \eta_k)} .
\end{aligned}
$$

$$\tag{5}$$

While the above vector-sensor model has not accounted for mutual coupling among the vector sensor's six component antennas, this model has been reported by Flam and Russel, Inc., Horsham, PA, to be a very good approximation of their CART array implementation of the vector-sensor concept.[1]

There are several essential observations about this vector-sensor array-manifold. First, one single vector-sensor measurement yields a 6×1 steering vector. Thus, a single vector sensor effectively embodies a six-element array in and of itself. Second, these vector-sensor array-manifolds contain no time-delay phase factors; that is, the vector-sensor array manifolds, unlike those of spatially displaced arrays, are

[1] "··· the patterns of the loops and dipoles (of the CART array) are *extremely* close to the theoretical patterns, indicating very good isolation and balance among the elements," private correspondence from Mr. Richard Flam of Flam and Russel to the first author, January 15, 1997.

*in*dependent of the impinging signals' frequency spectra. This frequency-independence is due to the spatial co-location of the six component-sensors that comprise the vector sensor. Third, the electromagnetic vector-sensor array manifold is polarization sensitive; that is, it is a function of $\{\gamma_k, \eta_k\}$. This means that signals having the same DOA's but different polarizations will have different array manifolds and are thus distinguishable based on their polarization diversity. Fourth, any broad-band or narrow-band electromagnetic source's $\mathbf{e}_k$ and $\mathbf{h}_k^*$ are orthogonal to each other and to the electromagnetic source's normalized Poynting vector $\mathbf{p}_k$, whose components are simply the three direction cosines along the three Cartesian coordinates [5]

$$\mathbf{p}_k \stackrel{\text{def}}{=} \begin{bmatrix} p_x(\theta, \phi) \\ p_y(\theta, \phi) \\ p_z(\theta) \end{bmatrix}$$

$$= \mathbf{e}(\theta_k, \phi_k, \gamma_k, \eta_k) \times \mathbf{h}^*(\theta_k, \phi_k, \gamma_k, \eta_k) \quad (6)$$

$$= \begin{bmatrix} u(\theta_k, \phi_k) \\ v(\theta_k, \phi_k) \\ w(\theta_k) \end{bmatrix} \stackrel{\text{def}}{=} \begin{bmatrix} \sin\theta_k \cos\phi_k \\ \sin\theta_k \sin\phi_k \\ \cos\theta_k \end{bmatrix} \quad (7)$$

$$\|\mathbf{p}_k\| = 1 \quad (8)$$

where $*$ denotes complex conjugation. Note that this normalized Poynting vector uniquely determines the source's DOA. After that, it would also be possible in the electromagnetic case to estimate the signals' polarization states; the details of this will be presented in Section III-B. Thus, if the array-manifolds of all impinging sources can be estimated from the received data, then the signal-of-interests' DOA's can be estimated by performing the above vector cross product.

III. ESTIMATION OF AZIMUTH ANGLES, ELEVATION ANGLES, AND POLARIZATIONS

A. Uni-Vector-Sensor ESPRIT Data Model

Uni-vector-sensor ESPRIT forms a temporal invariance utilizing two time-delayed sets of data collected from one vector sensor; that is, the kth monochromatic signal impinging upon the vector sensor would contribute toward two $6 \times N$ data sets

$$\mathbf{a}_k s(t_n, f_k) \qquad t_n, n = 1, \cdots, N$$
$$\mathbf{a}_k s(t_n + \triangle_T, f_k) = \mathbf{a}_k s(t_n, f_k) e^{j2\pi f_k \triangle_T}$$

where the vector-sensor manifold $\mathbf{a}_k$ is defined as in (4) in the electromagnetic case and

$$s(t_n, f_k) \stackrel{\text{def}}{=} \sqrt{\mathcal{P}_k} e^{j(2\pi f_k t_n + \varphi_k)} \qquad k = 1, \cdots, K \quad (9)$$

where $\mathcal{P}_k$ is the kth source's energy, f_k is the kth signal's frequency, φ_k is the kth signal's uniformly distributed random phase, $\triangle_T$ is the constant time delay between the two sets of time samples. Note that the invariance $e^{j(2\pi f_k \triangle_T)}$ does *not* depend on either the DOA nor the polarization state, but only on the signal frequency and the time delay $\triangle_T$.

With a total of $K \leq 5$ impinging signals and additive zero-mean white Gaussian noise (AWGN) at each constituent-sensor of the vector sensor

$$\mathbf{z}(t_n) \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \mathbf{s}(t_n) + \mathbf{n}(t_n)$$

$$= \sum_{k=1}^{K} \begin{bmatrix} \mathbf{a}_k s(t_n, f_k) \\ \mathbf{a}_k e^{j2\pi f_k \triangle_T} s(t_n, f_k) \end{bmatrix} + \mathbf{n}(t_n). \quad (10)$$

The entire $12 \times N$ data set is

$$\mathbf{Z} \stackrel{\text{def}}{=} [\mathbf{z}(t_1) \quad \cdots \quad \mathbf{z}(t_N)] = \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix} \quad (11)$$

where $\mathbf{Z}_1$ represents the $6 \times N$ data set sampled at $\{t_1, \cdots, t_N\}$ and $\mathbf{Z}_2$ represents the $6 \times N$ data set sampled at $\{t_1 + \triangle_T, \cdots, t_N + \triangle_T\}$ (note that these two sets of sampling times may overlap). Furthermore,

$$\mathbf{A}_1 \stackrel{\text{def}}{=} [\mathbf{a}_1, \cdots, \mathbf{a}_K] \quad (12)$$

$$\mathbf{A}_2 \stackrel{\text{def}}{=} [\mathbf{a}_1 e^{j2\pi f_1 \triangle_T}, \cdots, \mathbf{a}_K e^{j2\pi f_K \triangle_T}] \quad (13)$$

$$\mathbf{s}(t_n) \stackrel{\text{def}}{=} \begin{bmatrix} s(t_n, f_1) \\ \vdots \\ s(t_n, f_K) \end{bmatrix}; \mathbf{n}(t_n) \stackrel{\text{def}}{=} \begin{bmatrix} n_1(t_n) \\ \vdots \\ n_{12}(t_n) \end{bmatrix};$$

$$\mathbf{\Phi} \stackrel{\text{def}}{=} \begin{bmatrix} e^{j2\pi f_1 \triangle_T} & & \\ & \ddots & \\ & & e^{j2\pi f_K \triangle_T} \end{bmatrix}. \quad (14)$$

The present direction-finding problem[2] is to determine $\{\theta_k, \phi_k, k = 1, \cdots, K\}$ from the $12 \times N$ data set above with *a priori* knowledge only of $\triangle_T$ but not of $\{f_k, k = 1, \cdots, K\}$. For beamforming purposes, it may also be useful to subsequently estimate the corresponding polarizations states $\{\gamma_k, \eta_k, k = 1, \cdots, K\}$.

B. Adopting ESPRIT to Uni-Vector-Sensor Array

Let $\mathbf{E}_1$ denote the $6 \times K$ signal-subspace eigenvector matrix whose K columns are the 6×1 signal-subspace eigenvectors associated with the K largest eigenvalues of $\mathbf{Z}_1 \mathbf{Z}_1^H$. Let $\mathbf{E}_2$ denote the corresponding signal-subspace eigenvector matrix for $\mathbf{Z}_2 \mathbf{Z}_2^H$. For analytical purposes, first consider the noiseless case. $\mathbf{E}_1$ and $\mathbf{E}_2$ span the respective K-dimensional signal-subspaces of $\mathbf{Z}_1$ and $\mathbf{Z}_2$, which are, in fact, equivalent. Thus, a unique $K \times K$ nonsingular matrix $\mathbf{T}$ exists such that

$$\mathbf{E}_1 = \mathbf{A}_1 \mathbf{T}$$

and

$$\mathbf{E}_2 = \mathbf{A}_2 \mathbf{T} = \mathbf{A}_1 \mathbf{\Phi} \mathbf{T}. \quad (15)$$

Because both $\mathbf{E}_1$ and $\mathbf{E}_2$ are full-rank, a unique nonsingular $K \times K$ matrix $\mathbf{\Psi}$ exists such that

$$\mathbf{E}_1 \mathbf{\Psi} = \mathbf{E}_2 \Rightarrow \mathbf{A}_1 \mathbf{T} \mathbf{\Psi} = \mathbf{A}_1 \mathbf{\Phi} \mathbf{T} \quad (16)$$

$$\Rightarrow \mathbf{\Psi} = (\mathbf{E}_1^H \mathbf{E}_1)^{-1} (\mathbf{E}_1^H \mathbf{E}_2) = \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T} \quad (17)$$

$$\Rightarrow \mathbf{\Phi} = \mathbf{T} \mathbf{\Psi} \mathbf{T}^{-1}. \quad (18)$$

[2]Although the proposed algorithm will be presented in the batch processing mode, real-time adaptive implementations of this present algorithm may be readily realized for nonstationary environments using fast recursive eigendecomposition updating methods such as that in [15].

The last equality holds because both $\mathbf{A}_1$ and $\mathbf{\Phi}$ are full-ranked. (It is assumed that $\{\theta_i, \phi_i, \gamma_i, \eta_i\} \neq \{\theta_j, \phi_j, \gamma_j, \eta_j\}$ for all $i \neq j$ and $i, j \in \{1, \cdots, K\}$.) Consequentially, $\mathbf{\Psi}$'s eigenvalues equal $\{[\mathbf{\Phi}]_{kk} = e^{j2\pi f_k \triangle_T}, k = 1, \cdots, K\}$ and $\mathbf{\Psi}$'s right eigenvectors constitute the columns of $\mathbf{T}$. Thus, the array manifolds may each be estimated as

$$\hat{\mathbf{A}}_1 = [\hat{\mathbf{a}}_k, \cdots, \hat{\mathbf{a}}_k] = \mathbf{E}_1 \mathbf{T}^{-1} = \mathbf{E}_2 \mathbf{T}^{-1} \mathbf{\Phi}^{-1}$$
$$= \tfrac{1}{2} \{\mathbf{E}_1 \mathbf{T}^{-1} + \mathbf{E}_2 \mathbf{T}^{-1} \mathbf{\Phi}^{-1}\}. \qquad (19)$$

The $\mathbf{\Phi}^{-1}$ factor in above expression ensures *coherent* addition of the two sets of signal-subspace eigenvectors and is pivotal to achieving the best possible estimation performance. With noise, the above estimation becomes only approximate.

From the array manifold estimates of (19)

$$\hat{\mathbf{a}}_k = \begin{bmatrix} \hat{\mathbf{e}}_1 \\ \hat{\mathbf{h}}_1 \end{bmatrix}. \qquad (20)$$

Thus, a vector cross product between each signals' electric-field estimate and magnetic-field estimate straightforwardly estimates the direction cosines $\{u_k(\theta_k, \phi_k), v_k(\theta_k, \phi_k), w_k(\theta_k)\}$

$$\begin{bmatrix} \hat{u}_k \\ \hat{v}_k \\ \hat{w}_k \end{bmatrix} = \hat{\mathbf{p}}_k = \begin{bmatrix} \hat{p}_{x_k} \\ \hat{p}_{y_k} \\ \hat{p}_{z_k} \end{bmatrix} = \frac{\hat{\mathbf{e}}_k}{\|\hat{\mathbf{e}}_k\|} \times \frac{\hat{\mathbf{h}}_k^*}{\|\hat{\mathbf{h}}_k\|}. \qquad (21)$$

The kth signal's DOA can then be estimated as

$$\hat{\theta}_k = \arcsin\left(\sqrt{\hat{u}_k^2 + \hat{v}_k^2}\right) = \arccos(\hat{w}_k) \qquad (22)$$

$$\hat{\phi}_k = \arctan\left(\frac{\hat{v}_k}{\hat{u}_k}\right). \qquad (23)$$

The polarization parameters of the signals of interest can then be estimated as

$$\hat{\gamma}_k = \sum_{n=1}^{N} \arctan \frac{\hat{g}_{k_1}}{\hat{g}_{k_2}} \qquad (24)$$

$$\hat{\eta}_k = \sum_{n=1}^{N} \angle \hat{g}_{k_1}(t_n) \qquad (25)$$

where

$$\hat{\mathbf{g}}_k = \begin{bmatrix} \hat{g}_{k_1} \\ \hat{g}_{k_2} \end{bmatrix}$$
$$= [\mathbf{\Theta}_k^H(\hat{\theta}_k, \hat{\phi}_k) \mathbf{\Theta}_k(\hat{\theta}_k, \hat{\phi}_k)]^{-1} \mathbf{\Theta}_k^H(\hat{\theta}_k, \hat{\phi}_k) \hat{\mathbf{a}}_k. \qquad (26)$$

Note that two- (angular-) dimensional azimuth-elevation direction finding has been performed with*out* any *a priori* knowledge of the signal frequencies while using just one solitary vector sensor and no planar arrays. The azimuth angle estimates and the elevation angle estimates and the polarization estimates are all automatically matched without any additional processing.

Note also that $\triangle_T$ can be completely arbitrary (and *not* constricted by the Nyquist sampling rate to be twice the highest signal frequency) so long as a set of distinct phase offsets $\{e^{2\pi f_k \triangle_T}, k = 1, \cdots, K\}$ are preserved. This would guarantee the diagonality of $\mathbf{\Phi}$ and would ensure a distinct

invariance for each source. This flexibility compares favorably with phased-array ESPRIT, which requires the interelement spacing to be half wavelength or less to avoid ambiguity in the phases of ESPRIT's eigenvalues.

In fact, the values of $\{f_k, k = 1, \cdots, K\}$ need not be known *a priori* for uni-vector-sensor ESPRIT, which also incurs no frequency-DOA ambiguity as would a phased array of antennas. Such phased arrays estimate the DOA's through the phase factors $e^{(2\pi\Delta/\lambda_k)\cos\theta_k \sin\phi_k}$ and $e^{(2\pi\Delta/\lambda_k)\cos\theta_k \cos\phi_k}$. Thus, λ_k (or equivalently, f_k) must be precisely known *a priori* or otherwise estimated through extra computation to estimate the DOA's *un*ambiguously. In contrast, uni-vector-sensor ESPRIT estimates the DOA's by performing a vector cross product or normalization on the frequency-independent array manifolds and thus suffers no frequency DOA ambiguity.

Moreover, this present delay-sampling construction of a temporal invariance would not be useful with a general array of spatially displaced but identically polarized antennas. Such an array's array-manifolds can still be estimated, but there would *generally* exist no *closed*-form DOA estimation solution for such identically polarized antenna array-manifold estimates. The iterative searches over the array manifold would become necessary, resulting in much heavier computational costs.

Uni-vector-sensor ESPRIT needs all six components of the electromagnetic vector-sensor array manifold to estimate the Poynting vector. If, instead, only a three-component electromagnetic vector sensor measuring the three electric fields or the three magnetic fields is used, the vector cross product then cannot be performed.

Lastly, Hochwald and Nehorai [23] discovered that, in general, two is the maximum number of arbitrary electromagnetic sources uniquely identifiable by one vector sensor. Ho *et al*. [22] and Tan *et al*. [25] further determined that five represents the maximum number of distinguishable sources if certain restrictions are imposed on the sources' DOA's and polarizations. These two findings do not contradict the earlier assertion in this paper that this uni-vector-sensor ESPRIT algorithm can resolve up to any five arbitrary sources provided that their $e^{j2\pi f_k \Delta_T}$ terms are all distinct. There is no contradiction because the present algorithm makes the additional restriction of monochromatic signals and the further requirement of two time-delayed data sets. In other words, this algorithm has presumed a certain observable temporal structure in the data set—an assumption not made in [22], [23] and [25].

IV. SIMULATIONS

Simulation results presented in Figs. 1 and 2 verify the efficacy of the proposed electromagnetic uni-vector-sensor ESPRIT algorithm to resolve five uncorrelated monochromatic electromagnetic sources. Simulation results in Figs. 3–6 demonstrate the superior performance of uni-vector-sensor ESPRIT over ESPRIT applied to a customary array of spatially displaced but uniformly polarized antennas of comparable computational complexity. In all these simulations, the total-least-squares variance of ESPRIT (TLS-ESPRIT) [1] is used. In this simulation, each "identically polarized" antenna's mea-
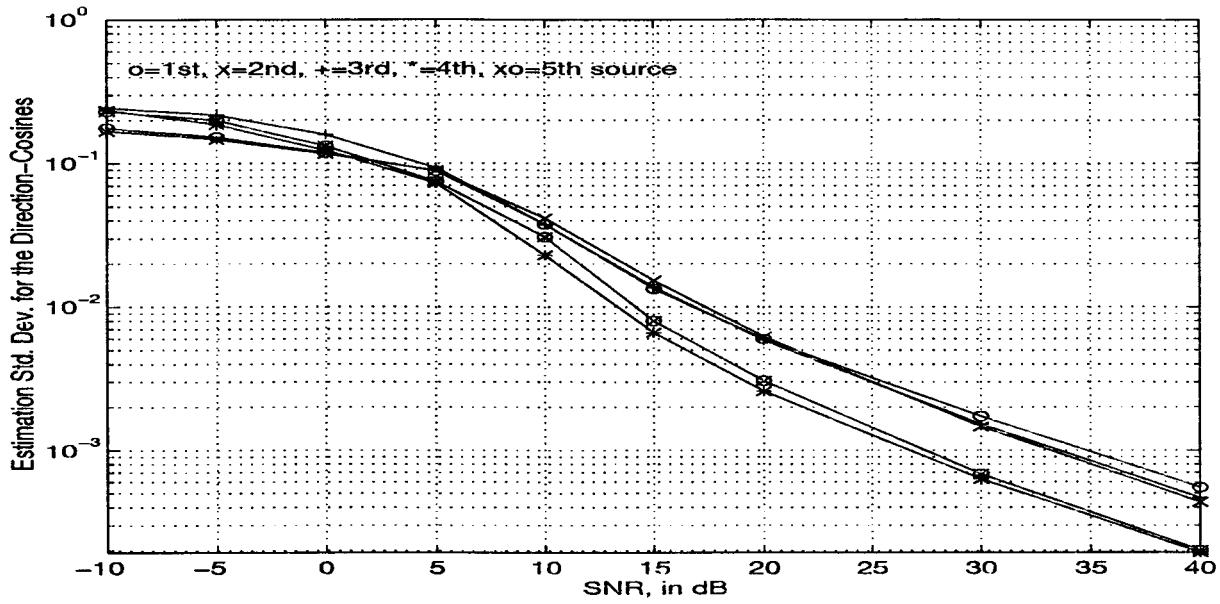
Fig. 1. Electromagnetic uni-vector-sensor ESPRIT's RMS estimation standard deviations ($\hat{u}_k$, $\hat{v}_k$, $k = 1, 2, 3, 4, 5$) at various SNR's: five monochromatic uncorrelated sources $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\} = (58.6°, 26.7°, 51.4°, 30.0°, 48.6°)$, $\{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5\} = -57.4°, -69.1°, 13.3°, 106.3°, -170.8°$, $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\} = \{0, \pi/2, 3\pi/4, \pi/4, \pi/4\}$, $\{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5\} = \{0, 0, 0, \pi/2, \pi/2\}$, $\mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}_3 = \mathcal{P}_4 = \mathcal{P}_5 = 1$, digital frequencies $\{f_1^f, f_2^f, f_3^f, f_4^f, f_5^f\} = \{0.75, 0.55, 0.65, 0.85, 0.45\}$ impinge upon a vector sensor, SNR is relative to unity signal power, 200 snapshots with uniform sampling rate of 0.1 in each of 500 independent experiments.
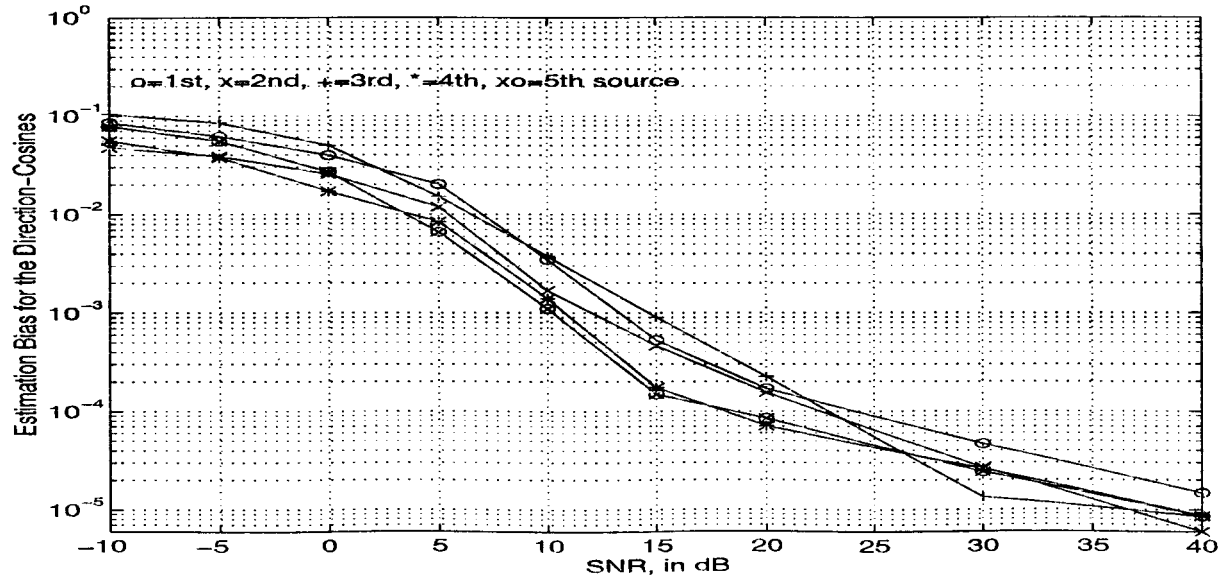

Fig. 2. Electromagnetic uni-vector-sensor ESPRIT's rms estimation bias of $\{\hat{u}_k, \hat{v}_k, k = 1, 2, 3, 4, 5\}$ at various SNR's. Same settings same as in Fig. 1.

surement of an impinging signal is taken to equal the Frobenius norm of that signal's Poynting vector.

Fig. 1 plots the standard deviations and Fig. 2 plots the biases of uni-vector-sensor ESPRIT's direction-cosine estimates in a scenario involving five uncorrelated monochromatic electromagnetic sources impinging upon a single electromagnetic vector sensor. The electromagnetic sources' parameters are given in the caption of Fig. 1. Their polarization states are, respectively, horizontally linear, vertically linear, 45° linear, left circular, and right circular. Their direction cosines are $u_1 = 0.46$, $u_2 = 0.16$, $u_3 = 0.76$, $u_4 = -0.14$, $u_5 = -0.74$, $v_1 = -0.72$, $v_2 = -0.42$, $v_3 = 0.18$, $v_4 = 0.48$, $V_5 = -0.12$.

With the smallest difference among the u_k's and among the v_k's being 0.30, the proposed algorithm successfully resolves all five electromagnetic sources with high probability at SNR's above 0 dB.

Figs. 3 and 4 compare uni-vector-sensor ESPRIT performance with that of ESPRIT for an array of spatially displaced but identically polarized antennas in a scenario with two equal-powered uncorrelated monochromatic electromagnetic sources (one left-circularly polarized and the other right-circularly polarized). An identically polarized L-shaped half-wavelength uniformly spaced array with six antennas on each leg is used for comparison. This uniformly polarized and
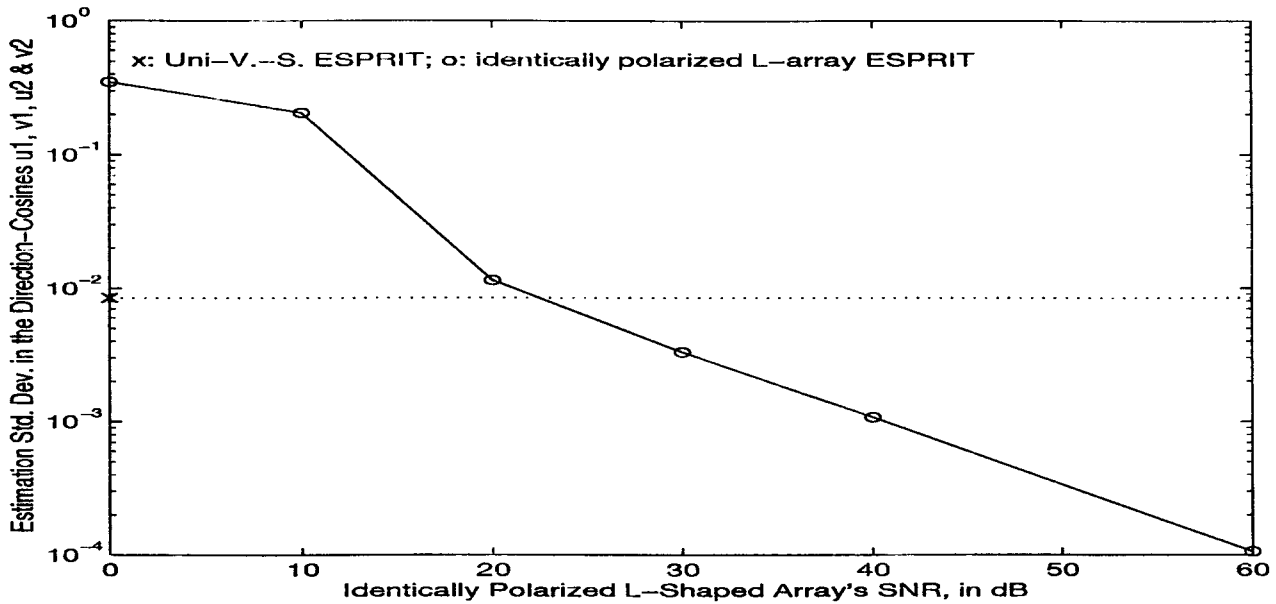
Fig. 3. Comparing electromagnetic uni-vector-sensor array to an L-shaped array of six uniformly polarized antennas on each leg: rms standard deviation of $\{\hat{u}_1, \hat{v}_1, \hat{u}_2, \hat{v}_2\}$ versus signal-to-noise ratio (SNR) for two closely spaced narrowband uncorrelated sources with $\{\theta_1, \theta_2\} = \{42°, 44°\}$, $\{\phi_1, \phi_2\} = \{89°, 87°\}$, $\mathcal{P}_1 = \mathcal{P}_2 = 1$, digital frequency's $\{f_1, f_2\} = \{0.95, 0.70\}$, 200 snapshots in each of 500 independent experiments.
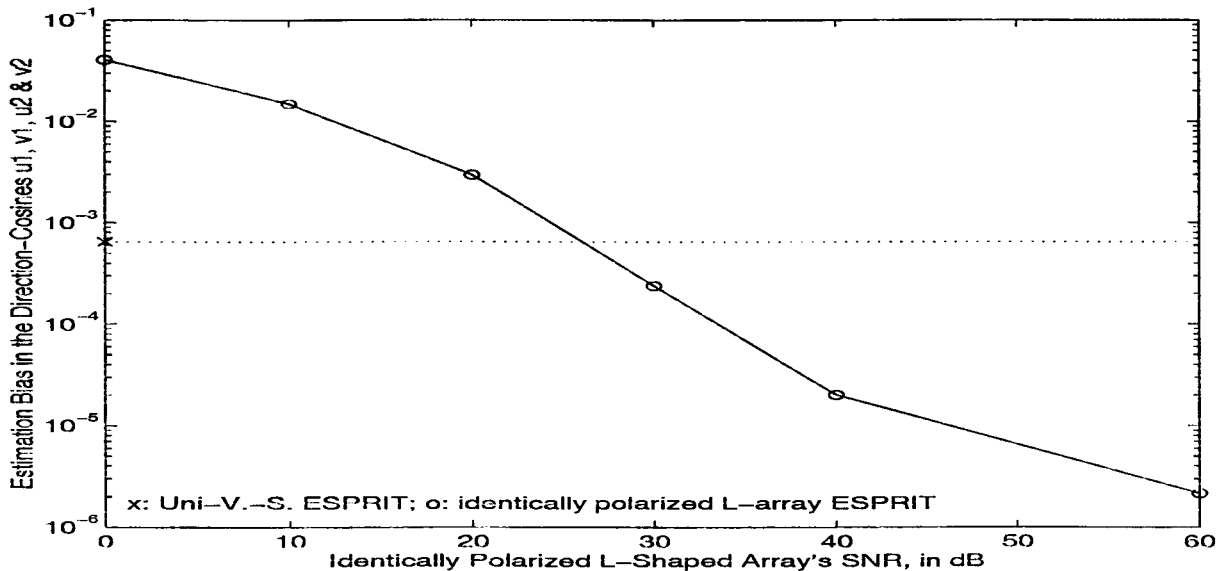

Fig. 4. Comparing electromagnetic uni-vector-sensor array to an L-shaped array of six uniformly polarized antennas on each leg: rms bias of $\{\hat{u}_1, \hat{v}_1, \hat{u}_2, \hat{v}_2\}$ versus SNR for same setup as in Fig. 3.

spatially displaced array has a data correlation matrix $\mathbf{ZZ}^H$ of identical size as that of the uni-vector-sensor array; the computational load is comparable in the two cases. Fig. 3 shows that only at 22 dB SNR does the spatially displaced and uniformly polarized scalar-sensor array provide a standard deviation comparable to that by uni-vector-sensor ESPRIT at 0 dB. This represents an astounding 22-dB performance gain by uni-vector-sensor ESPRIT's exploitation of the sources' polarization diversity.

In Figs. 5 and 6, the number of elements is varied on each leg of the L-shaped identically polarized and spatially displaced array, while the SNR is constant at 0 dB for both arrays. In order to match uni-vector-sensor ESPRIT's standard deviation performance, the uniformly polarized and spatially displaced array would need 25 antennas on *each* leg; that is 49 antennas altogether. This implies an impressive $8\frac{1}{3}$-fold hardware and computation cost saving by the proposed algorithm.

V. CONCLUSION

This paper introduces a novel ESPRIT-based azimuth-elevation two- (angular-) dimensional direction finding and polarization-estimation algorithm. It also exploits the sources' polarization diversity in addition to the sources' spatial diversity. It adapts the vector cross-product relation between an electromagnetic source's electric field, magnetic field,
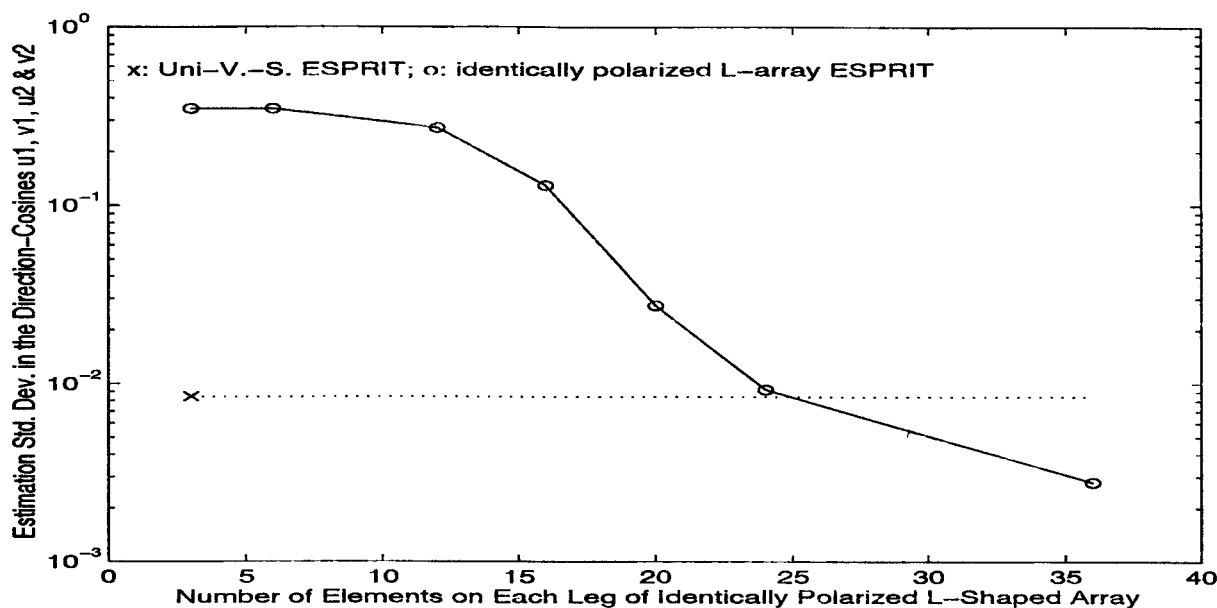
Fig. 5. Comparing electromagnetic uni-vector-sensor array to an L-shaped array of uniformly polarized antennas: rms standard deviation of $\{\hat{u}_1, \hat{v}_1, \hat{u}_2, \hat{v}_2\}$ versus number of elements on each leg. (SNR = 0 dB; all other settings same as in Fig. 3.)
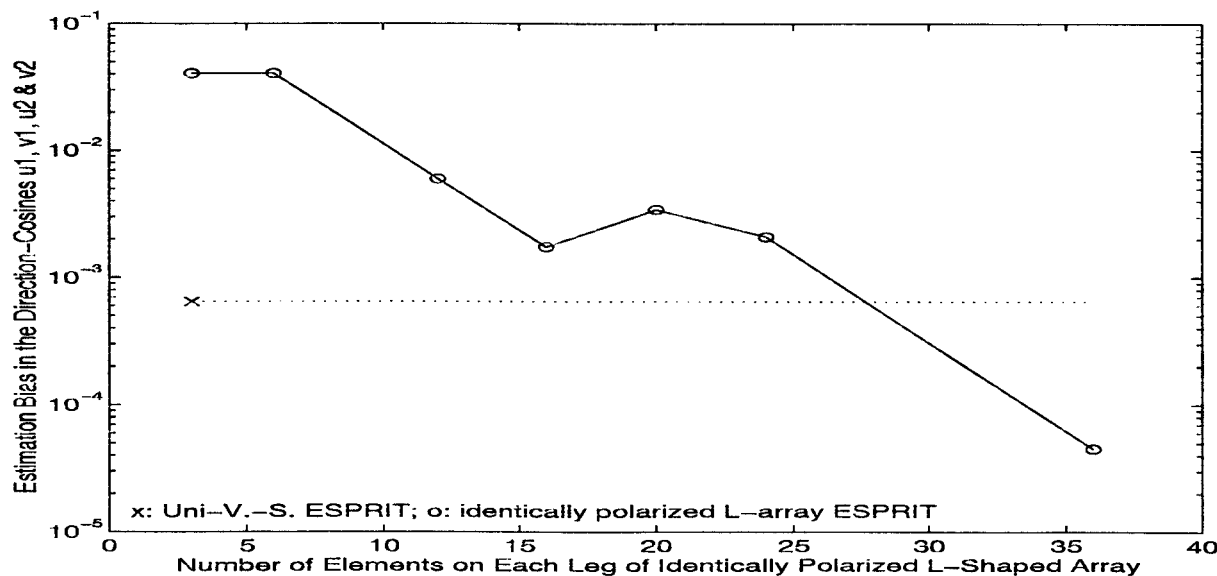

Fig. 6. Comparing electromagnetic uni-vector-sensor array to an L-shaped array of uniformly polarized antennas: rms bias of $\{\hat{u}_1, \hat{v}_1, \hat{u}_2, \hat{v}_2\}$ versus number of elements on each leg. (All settings same as in Fig. 5.)

and Poynting vector to the electromagnetic eigenstructure (subspace) direction-finding context. This innovative approach requires no *a priori* information of signal frequencies, automatically pairs its estimates of the direction cosines along the x-axis with those along the y-axis, simplifies array calibration, but uses only a solitary vector sensor sampled at two time-delayed sets of sampling times, and can resolve up to five uncorrelated monochromatic electromagnetic sources. It easily out performs a customary array spatially displaced identically polarized antennas of comparable computational load. Although the proposed algorithm will be presented in the batch-processing mode; however, real-time adaptive implementations of this present algorithm may be readily realized for nonstationary environments using the fast

recursive eigendecomposition updating methods such as that in [15]. An underwater analog of the present algorithm is presented in [29], using three orthogonally oriented acoustic particle velocity hydrophones plus an optional pressure hydrophone, all co-located in space.

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