Interpolation Based Unitary Precoding for Spatial Multiplexing MIMO-OFDM With Limited Feedback

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***Abstract*—Spatial multiplexing with linear precoding is a simple technique for achieving high spectral efficiency in multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. Linear precoding requires channel state information for each OFDM subcarrier, which can be achieved using feedback. To reduce the amount of feedback, this paper proposes a limited feedback architecture that combines precoder quantization with a special matrix interpolator. In the proposed system, the receiver sends information about a fraction of the precoding matrices to the transmitter and the transmitter reconstructs the precoding matrices for all the subcarriers. A new interpolator is proposed inspired by spherical interpolation that respects the orthogonal columns of the precoding matrices and the performance invariance to right multiplication by a unitary matrix. The interpolator is parameterized by a set of unitary matrices; a construction of a suitable set is briefly described. Simulations illustrate the performance of limited feedback precoding with coding, estimation or prediction error, and time variation for bit error rate (BER), mutual information, and mean squared error (MSE).**

***Index Terms*—Interpolation, multiple-input multiple-output (MIMO), spatial multiplexing.**

I. Introduction

SPATIAL multiplexing is a method to exploit the high spectral efficiency offered by multiple-input multiple-output (MIMO) systems [1], [2]. Spatial multiplexing provides higher multiplexing gain than traditional space-time codes [3], [4], but is sensitive to ill-conditioning of the channel matrix deficiencies in MIMO channels [5]–[8]. One way to improve the robustness of spatial multiplexing to rank deficiencies in the channel is to use linear precoding, where the data streams are premultiplied by a matrix chosen based on channel state information (CSI). Precoding has been proposed based on perfect CSI [9], [10] or perfect knowledge of the first/second-order channel statistics [11]–[17].

When perfect CSI is not readily available at the transmitter, it is often possible to use a low-rate feedback channel to provide quantized channel state information to the transmitter [18]. In the case of a frequency selective MIMO link, full CSI may be represented in the form of a $M_r \times M_t \times L$ matrix with complex entries for every realization of the channel, where M_r, M_t is the number of receive and transmit antennas and, L is the number of resolvable multipath components. In the case of a frequency flat MIMO channel ($L = 1$), instead of quantizing the channel matrix [19], [20], alternative approaches have been proposed that involve quantizing an orthogonal precoding matrix of dimension $M_t \times M_s$ corresponding to every channel realization, where M_s is the number of multiplexed streams and $M_s \leq \min(M_t, M_r)$ [21], [22].[1] It has been pointed out that, in contrast to the channel matrix, the precoding matrix does not contain power allocation information but provides most of the gains of full CSI [23]. Further, quantization of the precoding matrix is more efficient than that of the channel matrix, because of smaller dimension and the orthogonality constraint [24]. Other approaches include quantizing the transmit covariance matrix [25] and antenna subset selection [5], [7], [26]–[29], which can be viewed as a special case of quantized precoding.

The problem of quantization of CSI for the case of frequency selective channels for linearly precoded MIMO systems has received little attention in the literature. Prior work in [30] deals with adaptive two-dimensional (2-D) beamforming ($M_s = 2$) based on channel statistics as opposed to quantized instantaneous CSI. In our previous work, we considered quantized beamforming [31] which made connections to spherical interpolation [32] and could be considered as a special case of precoding with one substream ($M_s = 1$).

In this paper, a MIMO-OFDM signaling scheme (combining orthogonal frequency division multiplexing with MIMO) is chosen that converts a frequency selective channel into a series of narrowband MIMO channels called tones or subcarriers and enables efficient equalization [30], [33]–[35]. A primary reason for choosing MIMO-OFDM is the fact that the precoding techniques developed for frequency-flat MIMO channels can be applied to MIMO-OFDM by treating each subcarrier as a narrowband MIMO channel.

The focus of the present paper is to develop a CSI quantization strategy for a MIMO-OFDM system that optimizes a system performance metric, for example, mean squared error (MSE) with a linear receiver, mutual information, etc. For this

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[1]In the remaining of the paper, precoding matrix (or precoder) will always be orthogonal and of dimension $M_t \times M_s$.

purpose, we assume perfect CSI at the receiver and a zero-delay error free feedback link (later simulations are provided that relaxes these constraints). A simple strategy is to feedback the time-domain channel taps. This involves quantization and feedback of the $M_r \times M_t \times L$ complex coefficients and may be used for precoding, as well as power allocation for each tone at the transmitter.[2] An alternative approach is to perform the quantization in the frequency domain where the CSI can be modeled as a matrix of $M_r \times M_t \times N$ complex parameters, where N is the number of narrowband subcarriers (see, for example, [36]). It is not established conclusively, which approach is more bit-efficient but in this paper we pursue the frequency-domain approach due to the following reasons.

- The techniques of compressing CSI used for narrowband channels can be applied to each of the subcarriers in frequency-domain. This implies that for each subcarrier, a $M_t \times M_s$ orthogonal precoder matrix may be used for quantization. This forms a trivial extension of the strategies used for quantizing precoding matrices in narrowband channels [23] at the cost of a N-fold increase in the feedback load.
- The $M_r \times M_t \times N$ complex parameters described above are, in general, strongly correlated and provides the scope of utilizing smart quantization techniques that reduces the feedback overhead substantially without significantly compromising the performance.

For simplicity, we also consider uniform power allocation across subcarriers. It may be noted that adaptive modulation and power control across different subcarriers and streams can be included in the proposed framework but is relegated to future research. In this paper, we restrict ourselves to the class of optimal precoding matrices designed with perfect CSI that are *orthonormal* [37]. It is also observed that if we constrain the quantized precoding matrices to be orthonormal, then the quantization effectively occurs in a lower dimensional subset (a Grassmann manifold) of the complex Euclidean space $\mathbb{C}^{M_t \times M_s}$ and can lead to an efficient quantization [24]. With this motivation, we constrain the quantized precoding matrices to be orthonormal.

In this paper, we propose clustering and interpolation as methods to exploit the redundancy of the N precoding matrices, one for each subcarrier. The optimal strategy would be to jointly quantize the N precoder matrices but due to unrealistic computational complexity and quantizer design and storage limitations, we resort to suboptimal techniques. In the clustering algorithm, adjacent tones are grouped into clusters and a representative quantized precoder is chosen for each cluster. Precoders of different clusters are quantized independently. This reduces the computational complexity at the cost of optimality.

The idea of interpolation is to quantize the precoding matrices for select subcarriers and reconstruct the other precoding matrices using a special nonlinear interpolator. It has been observed using simulations that preserving the orthonormal structure of the precoders after interpolation provides better performance than simply normalizing the columns of the precoding matrix to reflect the power constraint at the transmitter. With this premise, we propose a nonlinear interpolation algorithm along the same lines of [31].

[2]It may be recalled that the efficiency of quantization with a fixed number of bits depends on the number of parameters as well as the joint entropy of the parameters.

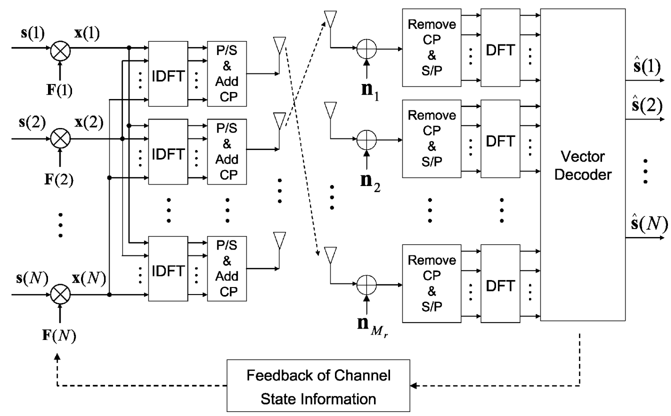

Fig. 1. Spatial multiplexing MIMO-OFDM system with linear precoding using M_t transmit antennas, M_r receive antennas, M_s substreams, and N subcarriers. $\mathbf{s}(k)$ is a M_s-dimensional vector, $\mathbf{x}(k)$ is a M_t-dimensional vector, $\mathbf{F}(k)$ is $M_t \times M_s$.

The optimal solution of the problem of compressing channel information for a frequency selective MIMO channel is an open problem. Here, we consider a MIMO-OFDM system and propose a suboptimal quantization strategy based on a simple clustering technique and a nonlinear interpolation as a way to exploit the correlation between the different tones. It is worth mentioning that present and upcoming wireless broadband standards including IEEE 802.16e [38] and IEEE 802.11n [39] include the possibility of limited feedback. In this paper, simulation results with a IEEE 802.11n channel model and a minimum mean square error (MMSE) receiver illustrate the performance of our approach with channel coding, channel estimation or prediction error using the performance metric of bit error rate (BER).

This paper is organized as follows. In Section II we present the overall system model and summarize the principles of narrowband quantized precoding. In Section III we present the details of our interpolation algorithm including the numerical results on precoder correlation, the proposed interpolator, optimization of the proposed interpolator, and a discussion on the derotation codebook. Finally, we present simulation results in Section IV and wrap up with conclusions in Section V.

II. System Description and Background

In this section, we review the MIMO-OFDM system with linear precoding under consideration. Then we provide background on the cost functions that are used to determine the precoding matrices. Finally, we encapsulate quantized precoding for narrowband spatial multiplexing systems, which we will also employ in our system.

A. System Overview

A spatial multiplexing MIMO-OFDM system with linear precoding using M_t transmit antennas, M_r receive antennas, and N subcarriers is illustrated in Fig. 1. At the transmitter, the kth subcarrier transmits a M_s-dimensional vector[3] $\mathbf{s}(k) = [s_1(k),\ s_2(k),\ldots,\ s_{M_s}(k)]^T$, where $M_s < M_t$, $M_s \leq M_r$, and $s_i(k)$ is the ith data substream. The vector $\mathbf{s}(k)$ is multiplied by an $M_t \times M_s$ precoding matrix $\mathbf{F}(k)$ yielding $\mathbf{x}(k) = \mathbf{F}(k)\mathbf{s}(k)$. We assume that the transmit power is identically assigned to all subcarriers and that $E[\mathbf{s}(k)\mathbf{s}^H(k)] = (\mathcal{E}_s/M_s)\mathbf{I}_{M_s}$. We consider precoding matrices with orthonormal columns [37], thus $\mathbf{F}^H(k)\mathbf{F}(k) = \mathbf{I}_{M_s}$. We assume that the sampled impulse response of the channel is shorter than the cyclic prefix. Then the channel for the kth subcarrier after the discrete Fourier transform (DFT) can be described by a $M_r \times M_t$ channel matrix $\mathbf{H}(k)$ whose entries represent the channel gains experienced by subcarrier k. After the DFT, the received signal at the kth subcarrier yields a M_r-dimensional vector $\mathbf{y}(k)$, which is expressed as

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{F}(k)\mathbf{s}(k) + \mathbf{n}(k) \tag{1}$$

where $\mathbf{n}(k)$ is a noise vector whose entries have the i.i.d. complex Gaussian distribution with zero mean and variance N_0.

The signal model in (1) is identical to that of a narrowband MIMO system with linear precoding, thus in (1), $\mathbf{F}(k)$ can be chosen by using the criteria proposed for narrowband MIMO systems with linear precoding [9]–[11], [40]. The received symbol vector $\mathbf{y}(k)$ is decoded by a vector decoder assuming perfect knowledge of $\mathbf{H}(k)\mathbf{F}(k)$.

B. Optimal Precoders With Perfect CSI

Assuming perfect channel knowledge and an appropriate objective function, the optimal precoding matrix can often be derived. In the case of a maximum likelihood (ML) receiver, however, a closed form expression for the precoding matrix is not available, partly due to the coupling between the constellation and the channel realization in the BER expressions [41]. Since the complexity of the ML receiver increases exponentially in the number of substreams, zero-forcing (ZF) and MMSE receivers become of interest. In this case, a matrix $\mathbf{G}(k)$ is found under the desired criteria, applied to the received symbol vector, and followed by independent detection of each substream

$$\hat{\mathbf{s}}(k) = \mathrm{decision}[\mathbf{G}(k)\mathbf{y}(k)] \tag{2}$$

where $\mathrm{decision}[\cdot]$ means the slicing. For the MMSE criterion

$$\mathbf{G}(k) = \left[\frac{M_s N_0 \mathbf{I}_{M_s}}{\mathcal{E}_s} + \mathbf{F}^H(k)\mathbf{H}^H(k)\mathbf{H}(k)\mathbf{F}(k)\right]^{-1}\mathbf{F}^H(k)\mathbf{H}^H(k). \tag{3}$$

In the case of a ZF or MMSE receiver, it is shown in [7] that a bound on the average probability of a vector symbol error is minimized by maximizing a bound on the minimum substream signal-to-noise ratio (SNR). Based on this fact, the optimal precoder with orthonormal columns (say $\mathbf{F}_{\mathrm{opt}}(k)$) is derived in [22]. Let us denote the singular value decomposition of $\mathbf{H}(k)$ as

$$\mathbf{H}(k) = \mathbf{V}_L(k)\Sigma(k)\mathbf{V}_R^*(k) \tag{4}$$

where $\mathbf{V}_L(k) \in \mathcal{U}(M_r, M_r)$, $\mathbf{V}_R(k) \in \mathcal{U}(M_t, M_t)$, and $\Sigma(k)$ is an $M_r \times M_t$ diagonal matrix. Then the optimal precoder denoted by $\mathbf{F}_{\mathrm{opt}}(k) = \bar{\mathbf{V}}_R(k)$ where $\bar{\mathbf{V}}_R(k)$ is a submatrix consisting of M_s columns of $V_R(k)$ corresponding to the M_s largest singular values of $\mathbf{H}(k)$. Further, the solution $\mathbf{F}_{\mathrm{opt}}(k)$ is not unique since $\mathbf{F}_{\mathrm{opt}}(k)\mathbf{Q}$ for any $\mathbf{Q} \in \mathcal{U}(M_s, M_s)$ is also a solution.

Another criterion that is used to evaluate linear precoding followed by MMSE reception is the MSE. In [10] the optimal precoder is derived by minimizing either the trace or determinant of the MSE matrix

$$\mathrm{MSE}(\mathbf{F}(k)) = \frac{\mathcal{E}_s}{M_s}\left\{\mathbf{I}_{M_s} + \frac{\mathcal{E}_s}{M_s N_0}\mathbf{F}^H(k)\mathbf{H}^H(k)\mathbf{H}(k)\mathbf{F}(k)\right\}^{-1}. \tag{5}$$

It follows that the optimal precoder, restricted to the feasible set $\mathcal{U}(M_t, M_s)$, is again given by $\mathbf{F}_{\mathrm{opt}}(k) = \bar{\mathbf{V}}_R(k)$ and the unitary invariance property of solution is also retained [10], [22] since both $\mathrm{trace}(\mathrm{MSE}(\mathbf{F}(k))$ or $\det(\mathrm{MSE}(\mathbf{F}(k)))$ are invariant to the right multiplication of $\mathbf{F}(k)$ by a unitary matrix.

In systems with near capacity achieving codes, another objective function of interest is the mutual information (or capacity). The mutual information has been used to formulate precoder selection criteria [5], [42]. Let us express the mutual information for a given channel $\mathbf{H}(k)$ and precoder $\mathbf{F}(k)$ as

$$I(\mathbf{F}(k)) = \log_2 \det\left(\mathbf{I}_{M_s} + \frac{\mathcal{E}_s}{M_s N_0}\mathbf{F}^H(k)\mathbf{H}^H(k)\mathbf{H}(k)\mathbf{F}(k)\right). \tag{6}$$

The optimal precoder that maximizes the mutual information, restricted to the feasible set $\mathcal{U}(M_t, M_s)$, is given by $\mathbf{F}_{\mathrm{opt}}(k) = \bar{\mathbf{V}}_R(k)$. Note that in (6) we have not performed waterfilling over the eigenmodes thus this is not the true capacity with channel knowledge. As with the other objective functions, the optimal solution $\mathbf{F}_{\mathrm{opt}}(k)$ is nonunique and the objective function is invariant to a unitary transformation of $\mathbf{F}_{\mathrm{opt}}(k)$.

C. Background on Narrowband Precoder Quantization

In this section, consider a narrowband MIMO channel, for example the kth subcarrier of a MIMO-OFDM system. Assume that CSI is not available at the transmitter and there exists a low-rate feedback channel to convey the channel information. Also consider that a pre-determined set of precoding matrices, called codebook, denoted by $\mathcal{F} = \{\mathbf{F}_1, \ldots, \mathbf{F}_K\}$ is known a priori to both the transmitter and the receiver. Based on the knowledge of the channel realization $\mathbf{H}(k)$, the receiver chooses an element of $\mathcal{F}$, termed as the codeword, via the codeword selection rule and the particular index is conveyed to the transmitter through the feedback channel. Therefore, the feedback data-rate is determined by the cardinality of $\mathcal{F}$, specifically $\log_2 \mathrm{card}(\mathcal{F})$ bits per update. The cardinality of $\mathcal{F}$ is small for practical purposes and this allows us to search for the optimal codeword by evaluating the objective function (or a simplified version of it) for all codewords.

[3]We use ^{T} to denote transposition, ^{H} to denote conjugate transposition, $^{-1}$ to denote matrix inversion, † the pseudoinverse, $\mathbf{I}_M$ to denote a $M \times M$ identity matrix, $\|\cdot\|_2$ to denote the matrix 2-norm, $\|\cdot\|_F$ to denote the Frobenius norm, $|\cdot|$ to denote absolute value, $\mathrm{trace}(\cdot)$ to denote the trace of a matrix, $\det(\cdot)$ to denote the determinant of a matrix, $\mathbb{C}^M$ to denote the M dimensional complex vector space, $\mathcal{CN}(0, \sigma^2)$ to denote complex normal distribution with independent real and imaginary parts distributed according to $\mathcal{N}(0,1)$, $\mathcal{U}(M_t, M)$ to denote the set of $M_t \times M$ matrices with orthonormal columns, $\lambda_i\{\mathbf{A}\}$ denotes the ith largest singular value of $\mathbf{A}$ and $\mathrm{card}(\cdot)$ to denote the cardinality of a set.

The codeword selection rule depends on the performance objective, as described in Section II-B. It has been established in [22] that maximizing the minimum substream SNR for a linear receiver (ZF or MMSE) leads to a selection rule given by

$$\mathbf{F}(k) = \arg\max_{\mathbf{F}_i \in \mathcal{F}} \lambda_{\min}\{\mathbf{H}(k)\mathbf{F}_i\} \tag{7}$$

where $\lambda_{\min}\{\mathbf{H}(k)\mathbf{F}_i\}$ denotes the minimum eigenvalue of $\mathbf{H}(k)\mathbf{F}_i$. The selection rules for minimizing the MSE in (5) with the trace function or maximizing the mutual information as in (6) are straightforward and are given by

$$\mathbf{F}(k) = \arg\min_{\mathbf{F}_i \in \mathcal{F}} \operatorname{trace}[\mathrm{MSE}(\mathbf{F}_i)] \tag{8}$$

$$\mathbf{F}(k) = \arg\max_{\mathbf{F}_i \in \mathcal{F}} I(\mathbf{F}_i). \tag{9}$$

It may be mentioned that in the cases of QPSK and square QAM constellations, quantized precoders that are designed and chosen based on a BER criterion may result in a slightly better BER performance. The issues of computational complexity requirements and the structure of an optimal quantized precoder are unclear and we choose to use MSE as a simpler design criterion.

The performance of quantized precoding depends on the design of the codebook $\mathcal{F}$. Based on bounds on the average distortion, defined relative to the performance objective under consideration, for Rayleigh fading channels it was found that codebooks should be designed from packings on the Grassmann manifold [22]. The set of all M_s-dimensional subspaces spanned by the matrices in $\mathcal{U}(M_t, M_s)$ is the complex Grassmann manifold, denoted as $\mathcal{G}(M_t, M_s)$. Thus, if $\mathbf{F}_1, \mathbf{F}_2 \in \mathcal{U}(M_t, M_s)$, then the column spaces of $\mathbf{F}_1$ and $\mathbf{F}_2$, denoted by $\mathcal{P}_{\mathbf{F}_1}$ and $\mathcal{P}_{\mathbf{F}_2}$ are elements of $\mathcal{G}(M_t, M_s)$. Several distance metrics can be defined on $\mathcal{G}(M_t, M_s)$ [43] including, the projection two-norm

$$d_{\mathrm{proj}}(\mathbf{F}_1, \mathbf{F}_2) = \|\mathbf{F}_1\mathbf{F}_1^H - \mathbf{F}_2\mathbf{F}_2^H\|_2 = \sqrt{1 - \lambda_{\min}^2\{\mathbf{F}_1^H\mathbf{F}_2\}} \tag{10}$$

and the Fubini-Study distance

$$d_{FS}(\mathbf{F}_1, \mathbf{F}_2) = \arccos|\det(\mathbf{F}_1^H\mathbf{F}_2)|. \tag{11}$$

The projection two-norm distance depends only on the minimum eigenvalue of $\mathbf{F}_1^H\mathbf{F}_2$, while the Fubini-Study distance depends on the product of the eigenvalues of $\mathbf{F}_1^H\mathbf{F}_2$.

It is shown in [22] that the objectives of maximizing minimum substream SNR and minimizing the MSE lead to the codebook design criterion of maximizing the minimum projection two-norm distance between any pair of codewords belonging to $\mathcal{F}$. In contrast, maximizing the mutual information lead to the design criterion of maximizing the minimum Fubini-Study distance between any pair of codewords. Thus, the codebooks designed for different objectives result in different packings of subspaces in $\mathcal{G}(M_t, M_s)$. These Grassmannian codebooks naturally incorporate the unitary invariance property of $\mathbf{F}_{\mathrm{opt}}$ and perform well as compared to the case of perfect channel knowledge at the transmitter (unquantized $\mathbf{F}_{\mathrm{opt}}$) with a few bits of feedback [22]. Consequently, our solution to the limited feedback problem for MIMO-OFDM will leverage Grassmannian codebooks but with an additional twist to reduce feedback due to adjacent subcarrier correlation.

III. Proposed Interpolation Based Precoding

In this section, we propose an interpolation algorithm that allows us to exploit the coherence of adjacent subcarriers and quantize only a fraction of the precoding matrices. First, we observe that the channel coherence is related to the precoder coherence. Then we present two methods for improving performance: one is based on simple clustering and the other is on clustering followed by interpolation. Our interpolation algorithm is the natural extension of the spherical interpolation algorithm proposed in [31] but needs a unitary derotation matrix $\mathbf{Q}$ associated with each precoding matrix to take the unitary invariance property into account. We discuss optimization of the interpolation by choosing a good derotation matrix and then some approaches for quantizing $\mathbf{Q}$.

A. Correlation Between Precoding Matrices

Adjacent subchannels in OFDM systems are correlated, by virtue of the number of subcarriers being much greater than the length of the cyclic prefix. Since the precoders for each subcarrier are determined from the channel matrices, precoders for adjacent subcarriers are also correlated and we further assume that the correlation between the precoding matrices is similar to that between the channel matrices. It may be noted that an optimal precoder $\mathbf{F}_{\mathrm{opt}}(k)$ is derived from the right singular vectors of the channel matrix $\mathbf{H}(k)$. In the case of an uncorrelated Rayleigh fading channel model for $\mathbf{H}(k)$, the joint distribution of the right singular vectors of the channel matrix is independent of the number of receive antennas [44]. This observation implies that the the number of receive antennas does not significantly affect correlation and we estimate the coherence of the precoding matrices by the coherence bandwidth of the vector channel (with one receive antenna) which essentially provides a conventional measure of coherence bandwidth.

B. Clustered Linear Precoding

As aforementioned, due to correlation among the subcarriers, the optimal quantization policy for the precoding matrices is the joint quantization of all the precoding matrices. This is clearly infeasible due to computation and storage. A simple approach to take advantage of the correlation in the frequency domain is to cluster adjacent subcarriers and use the precoder for the middle subcarrier for the whole cluster. Clustering has been proposed in various contexts in multicarrier modulation including efficient adaptive modulation in OFDM [45] and simplified transmit beamforming in MIMO-OFDM [31].

With clustering, the N subcarriers are divided into clusters of size K. Specifically, we consider that the kth cluster contains the carriers from $kK+1$ to $kK+K$ and $0 \le k \le N/K - 1$. Then with clustered precoding, $\mathbf{F}_{\mathrm{opt}}(kK + \lfloor K/2 \rfloor)$ (corresponding to the channel $\mathbf{H}(kK + \lfloor K/2 \rfloor)$) is considered to be the optimal precoding matrix for the kth cluster and this is quantized to $\mathbf{F}(kK + \lfloor K/2 \rfloor)$ using criteria (7), (8) or (9) as appropriate. The receiver sends the indices of $\{\mathbf{F}(kK + \lfloor K/2 \rfloor)\}_{k=0}^{N/K-1}$ back to the transmitter and the transmitter uses this information for clustered linear precoding. Specifically, $\mathbf{F}(kK + \lfloor K/2 \rfloor)$ is used as the precoding matrix for all the K subcarriers in cluster k. The feedback requirements are $(N/K)\lceil\log_2|\mathcal{F}|\rceil$ and grow as the cluster shrinks. We use clustering to provide a baseline for subsequent comparisons. More sophisticated forms of clustering can be devised, however, this is beyond the scope of this paper.

C. Proposed Linear Precoding Scheme

Clustering is an efficient quantization strategy that exploits correlation to reduce feedback requirements. It suffers from mismatch at the edges of the cluster boundaries because the subcarriers near the cluster boundary are more likely to experience mismatch. As a cure, we consider interpolation to improve the smoothness of the precoder reconstruction at the transmitter. Given $\{\mathbf{F}(kK+\lfloor K/2\rfloor)\}_{k=0}^{N/K-1}$ derived according to (7), (8) or (9) we propose to find $\{\mathbf{F}(k)\}_{k=1}^{N}$ by interpolating between adjacent clusters.

The basic idea of interpolation is not new, e.g., low complexity channel estimators in OFDM based on pilot tones often use linear interpolation. In the context of linear precoding, the interpolation problem is more challenging since, based on empirical knowledge, we constrain the interpolated precoding matrices such that they retain their orthogonal structure.

Our approach is to use the fact that if a function is smooth in the Euclidean space it remains smooth on the Stiefel manifold [46]. We perform the interpolation in Euclidean space then project the result back to the Stiefel manifold. This approach has been employed for spherical interpolation (see, e.g., [32]) where the interpolated points are projected back to the surface of the sphere as well as for solving optimization problems on manifolds (see, e.g., [47]) where the tangent space is projected onto the Stiefel or Grassmann manifold.

Previously, we used a similar approach based on spherical interpolation to solve the special case of beamformer interpolation in [31]. In the case of beamforming, $\mathbf{F}(k)$ are unit-norm vectors and the unitary invariance amounts to a complex phase $e^{j\theta}$. The spherical interpolator in [31] takes the weighted sum of beamforming vectors and normalizes the result to have it the unit norm. For example, for vectors in the first cluster it computes

$$\mathbf{z}(m) = (1-c_m)\mathbf{f}(1) + c_m e^{j\theta}\mathbf{f}(K+1) \quad (12)$$

$$\mathbf{f}(m) = \mathbf{z}(m)\{\mathbf{z}^H(m)\mathbf{z}(m)\}^{-1/2} \quad (13)$$

where $\mathbf{f}(m)$ are the beamforming vectors (we use lower case notation to emphasize they are vectors), $c_m = (m-1)/K$, and $1 < m \leq K$. The first step in (12) is simply a linear interpolation. The second step in (13) forces the output of the interpolator to be normalized. The phase rotation parameter θ in (12) is used to remove the distortion caused by the unitary invariance of the optimal beamforming vector, i.e., the optimal beamforming vector can be multiplied by an arbitrary phase rotation. The phase rotation is optimized, quantized, and conveyed back to the transmitter.

In the absence of a straightforward algorithm for interpolation on the Grassmann manifold we propose a simple extension of the beamformer interpolator as in (12), (13) [31]. In essence, we perform a linear interpolation with an invariance parameter and then solve a matrix nearness problem to find the closest precoder with orthonormal columns. The proposed interpolation strategy has not been proved to be optimal but provides a practical way to improve the precoding performance with limited feedback as shown using simulations in Section IV.

1) Linear Interpolation: To formulate the appropriate linear interpolation problem, recall the MSE and capacity criteria in (5) and (6). Given a $M_s \times M_s$ unitary matrix $\mathbf{Q}$, both cost functions satisfy $\mathrm{MSE}(\mathbf{F}(k)) = \mathrm{MSE}(\mathbf{F}(k)\mathbf{Q})$ and $C(\mathbf{F}(k)) = C(\mathbf{F}(k)\mathbf{Q})$. In other words, the optimal precoder is not unique under the criteria, but denoted as a set of matrices. Because the precoding matrix is calculated independently for each subcarrier, the matrix $\mathbf{Q}$ is arbitrarily determined without considering interpolation. The matrix $\mathbf{Q}$, however, has a substantial influence on the performance of an interpolator. Based on this observation, we propose to interpolate using a simple weighted average, with an additional unitary matrix $\mathbf{Q}_l$ as a free parameter for interpolation

$$\mathbf{Z}(lK+m) = (1-c_m)\mathbf{F}(lK+1) + c_m\mathbf{Q}_l\mathbf{F}((l+1)K+1) \quad (14)$$

where $1 \leq m \leq K$ and $0 \leq l \leq N/K-1$. We only need to consider a unitary multiplication of the second term because we can always factor out the rotation of the first term. We elaborate on how to choose $\mathbf{Q}_l$ in Section III-D.

2) Nearest Precoder: After interpolation, our goal is to find the nearest candidate precoding matrix to (14). This is an example of a matrix nearness problem [48]–[50], and this solution is well known. We summarize the relevant result in the following.

Theorem 1: Consider a $M_t \times M$ matrix $\mathbf{Z}$ where $M_t \geq M$ and $\mathbf{Z}$ has singular value decomposition $\mathbf{U}\Sigma\mathbf{V}^H$. Then

$$\arg\min_{\mathbf{F}\in\mathcal{U}(M_t,M)} \|\mathbf{Z}-\mathbf{F}\|_F^2 = \mathbf{U}\mathbf{V}^H.$$

Further, $\mathbf{U}\mathbf{V}^H$ can be computed using $\mathbf{Z}(\mathbf{Z}^H\mathbf{Z})^{-1/2}$.

Proof: See [50, Section III-F] or [51, pp. 431,432]. □

Equipped with Theorem 1 we propose to solve for the interpolated precoders using

$$\mathbf{F}(lK+m) = \mathbf{Z}(lK+m)\{\mathbf{Z}^H(lK+m)\mathbf{Z}(lK+m)\}^{-1/2} \quad (15)$$

where $\mathbf{Z}(lK+m)$ is the output of (14). Note that efficient algorithms are available for computing the polar factors in [52] and may be preferred in implementation over (15). It may be remarked that (14) and (15) provide a simple and bit-efficient way of customizing the linear interpolator for the orthogonality constraint using a $\mathbf{Q}$ matrix per cluster. Depending on the complexity and compression requirements, however, more parameters and better parameterizations may be used, which are currently under investigation.

3) Proposed Interpolator: To summarize, our proposed interpolator uses $\{\mathbf{F}(kK+\lfloor K/2\rfloor)\}_{k=0}^{N/K-1}$ to compute $\{\mathbf{F}(k)\}_{k=1}^{N}$ by solving the linear interpolation in (14) followed by the projection in (15) to enforce $\mathbf{F}(k)\in\mathcal{U}(M_t,M_s)$. The parameters of the interpolator are the set of derotation matrices $\{\mathbf{Q}_l\}_{l=0}^{N/K-1}$, which serve to account for the unitary invariance of the precoder. For the best reconstruction, the derotation parameters should be optimized based on the same performance measure used to evaluate precoding.

D. Optimization of the Proposed Interpolator

In the proposed interpolator, the matrix $\mathbf{Q}_l$ only contributes to the calculation of the precoders for a single cluster $\{\mathbf{F}(lK+m), 1 \leq m \leq K\}$. This allows us to optimize each $\mathbf{Q}_l$ separately, reducing the complexity of the optimization. There are many possible criteria for formulating the optimization; for simplicity we focus on improving the performance of the worst subcarrier.

Considering that the quantization criterion is (8), we find the $\mathbf{Q}_l$ minimizing the maximum MSE as follows:

$$\mathbf{Q}_l = \arg \min_{\mathbf{Q}\in\mathcal{U}(M_s,M_s)} \max_{1\leq m\leq K} \mathrm{trace}\left[\mathrm{MSE}(\mathbf{F}(lK+m))\right] \tag{16}$$

where $0 \leq l \leq N/K-1$ and $1 \leq m \leq K$. Note that $F(lK+m)$ depends on $\mathbf{Q}_l$ through $\mathbf{Z}(lK+m)$ in (14). The optimization problem for $\mathbf{Q}_l$ can be formulated similarly for the quantization criteria (7).

Because of the projection operation in (15), a closed-form solution to (16) is difficult. Fortunately, we are quantizing $\mathbf{Q}_l$ for limited feedback, thus we can numerically find the best $\mathbf{Q}$ by selecting a matrix from a codebook $\mathcal{Q}$ of unitary matrices. We comment on quantization of $\mathbf{Q}$ in Section III-E. Given a codebook $\mathcal{Q}$, we can modify (16) to

$$\mathbf{Q}_l = \arg \min_{\mathbf{Q}\in\mathcal{Q}} \max_{1\leq m\leq K} \mathrm{trace}\left[\mathrm{MSE}(\mathbf{F}(lK+m))\right]. \tag{17}$$

Evaluating this optimization requires a search over the unitary matrices in (17). The performance and complexity are dependent on $\mathrm{card}(\mathcal{Q})$.

As an alternative to the MSE, we can quantize $\mathbf{F}(lK+1)$ as well as optimize $\mathbf{Q}_l$ based on the capacity criterion. In this case, we solve for $\mathbf{F}(lK+1)$ using (9). Then we find the best $\mathbf{Q}_l$ by maximizing the sum rate of all OFDM subcarriers in the cluster under the equal power assumption

$$\mathbf{Q}_l = \arg \max_{\mathbf{Q}\in\mathcal{Q}} \sum_{m=1}^{K} I(\mathbf{F}(lK+m)). \tag{18}$$

1) Feedback Requirements: The proposed approach requires $(N/K)(\log_2 \mathrm{card}(\mathcal{F}) + \log_2 \mathrm{card}(\mathcal{Q}))$ feedback bits. Simple clustering and antenna subset selection [7] necessitate $(N/K)\log_2 \mathrm{card}(\mathcal{F})$ and $N \log_2 \lceil M_t!/(M_t-M_s)!M_s! \rceil$, respectively. Optimal precoding that sends a quantized version of each precoder to the transmitter requires $N \log_2 \mathrm{card}(\mathcal{F})$ feedback bits.

E. Designing Codebooks of Unitary Matrices $\mathcal{Q}$

For limited feedback implementation, we need quantize the derotation matrices. Unfortunately, our previous work on Grassmannian quantization only addresses quantizing tall matrices with orthonormal columns and specifically does not provide a solution when the matrices are square. Consequently, in this section, we describe methods of designing the codebook of unitary matrices $\mathcal{Q}$. We do not present optimal designs; finding optimal sets of matrices is a topic for future work.

From a vector quantization perspective, designing $\mathcal{Q}$ requires the knowledge of the probability distribution of the solution of (16). In the absence of this distribution, we assume that the solution of (16) is isotropically random. This is reasonable following the isotropic invariance of the Rayleigh fading channel. Under a uniformity assumption the goal of the codebook design is to choose $\mathrm{card}(\mathcal{Q})$ unitary matrices that are maximally spaced in some sense.

TABLE I
CODEBOOK $\mathcal{Q}$ FOR $M_s = 2$, AND $\mathrm{card}(\mathcal{Q}) = 3$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i \\ -\frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}} \end{bmatrix}$

One approach is to treat $\mathcal{Q}$ as a set of orthonormal bases (ONB) and to maximize the minimum correlation between any pair of basis. It is known that for the optimal solution, the maximal correlation between elements of different ONBs is equal to $1/M_s$. To make this more formal, two (or more) ONBs $\{\mathbf{x}_i\}_{i=1}^{M_s}$ and $\{\mathbf{y}_i\}_{i=1}^{M_s}$ are said to be mutually unbiased bases (MUB) if they satisfy [53], [54]

$$|\mathbf{x}_i^H \mathbf{y}_j| = \frac{1}{M_s}, \qquad i = 1,\ldots,M_s; j = 1,\ldots,M_s. \tag{19}$$

Each ONB represents a unitary matrix, an element of $\mathcal{Q}$. Constructions of MUBs have been dealt with in [54]–[56]. The existence of M_s+1 MUBs in $\mathbb{C}^{M_s}$ where $M_s = 2^k$, $k > 0$ is an integer has been shown in [57]. In this paper, we use the construction provided in [54] that leads to an algebraic solution for $\mathcal{Q}$ for the particular case of $\mathrm{card}(\mathcal{Q}) = M_s+1$ given in Table I.

While the algebraic construction of MUBs is appealing, if the feedback is binary than this construction may not make full use of all the available feedback bits. Consequently, we also consider the design of $\mathcal{Q}$ for arbitrary M_s and cardinality. The strategy is to parameterize the unitary matrices and then quantize the parameter space. We consider the parameterization obtained by decomposing a unitary matrix into Givens rotations.

Proposition 1: (Theorem 1 in [58]) A unitary matrix $\mathbf{Q} \in \mathbb{C}^{M_s\times M_s}$ can be decomposed as

$$\mathbf{Q} = \left[\prod_{k=1}^{M_s} \mathbf{D}_k(\phi_{k,k},\ldots,\phi_{k,M_s}) \prod_{l=1}^{M_s-k} \mathbf{G}_{M_s-l,M_s-l+1}(\theta_{k,l})\right] \mathbf{I}_{M_s} \tag{20}$$

where the M_s dimensional diagonal matrix

$$\mathbf{D}_k(\phi_{k,k},\ldots,\phi_{k,M_s}) = \mathrm{diag}(1_{k-1}, e^{j\phi_{k,k}},\ldots,e^{j\phi_{k,M_s}}) \tag{21}$$

1_{k-1} is $(k-1)$ 1's, $\mathbf{G}_{p-1,p}(\theta)$ is the Givens matrix which operates in the $(p-1,p)$ coordinate plane of the form

$$\mathbf{G}_{p-1,p}(\theta) = \begin{bmatrix} \mathbf{I}_{p-2} & & & \\ & \cos(\theta) & -\sin(\theta) & \\ & \sin(\theta) & \cos(\theta) & \\ & & & \mathbf{I}_{M_s-p} \end{bmatrix}. \tag{22}$$

Proposition 1 parameterizes a unitary matrix $\mathbf{Q} \in \mathbb{C}^{M_s\times M_s}$ into M_s^2 parameters $\{\phi_{k,j}, \theta_{k,l}\}$.

A method to quantize the set of unitary matrices is to use standard algorithms for vector quantizer design, for example, the Lloyd algorithm [59] to jointly determine the reconstruction points for all the parameters $\left\{\widehat{\phi}_{k,j}, \widehat{\theta}_{k,l}\right\}$. Then, the codebook of unitary matrices, $\mathcal{Q}$ is designed by reconstructing unitary ma-

TABLE II
CODEBOOK $\mathcal{Q}$ FOR $M_s = 2$, AND $\mathrm{card}(\mathcal{Q}) = 4$ (2 BITS)

$$\begin{bmatrix} 0.7069 + 0.6600i & -0.2415 - 0.0803i \\ 0.2314 - 0.1061i & 0.6311 - 0.7328i \end{bmatrix}$$

$$\begin{bmatrix} 0.7076 - 0.6593i & -0.2414 + 0.0802i \\ 0.2309 + 0.1066i & 0.6298 + 0.7339i \end{bmatrix}$$

$$\begin{bmatrix} 0.7318 + 0.6411i & -0.0877 - 0.2141i \\ 0.2070 + 0.1033i & 0.5854 + 0.7770i \end{bmatrix}$$

$$\begin{bmatrix} 0.7312 - 0.6419i & -0.0877 + 0.2136i \\ 0.2066 - 0.1032i & 0.5865 - 0.7763i \end{bmatrix}$$

trices from the quantized parameters $\left\{\widehat{\phi}_{k,j}, \widehat{\theta}_{k,l}\right\}$. We used the Linde–Buzo–Gray (LBG) algorithm for vector quantizer design [60] with a squared error distortion and an example codebook designed in the case of $M_s = 2$, $\mathrm{card}(\mathcal{Q}) = 4$ is illustrated in Table II. We show in the simulations that both codebook designs perform well.

IV. SIMULATION RESULTS

In the simulation, we considered a MIMO-OFDM system with $M_t = 4$, $M_r = 2$, $M_s = 2$, $N = 64$, and cyclic prefix length of 16. We used the (correlated) MIMO channel model provided by the IEEE 802.11 TGn [39] assuming the following parameters: channel model B for downlink and non line-of-sight where the channel length in time is 9, antenna spacings at the transmitter and receiver are 4λ and λ, respectively, where λ is the carrier wavelength, and a sampling rate of 20 MHz. We assumed that the channel was fixed during a data burst and randomly changed between data bursts; $\mathbf{n}(k)$ was i.i.d. zero mean complex Gaussian; the feedback channel had no delay and no transmission error; quadrature phase shift keying (QPSK) modulation and MMSE detection were used for BER simulations; and the transmit power was uniformly distributed to all subcarriers. Every point of the simulation results was obtained by averaging over more than 500 independent realizations of the channel and noise.

A. Performance Evaluation for Clustering and Interpolation

Given a limit on the total number of feedback bits, we find the optimal values for the codebook sizes $\mathrm{card}(\mathcal{F})$, $\mathrm{card}(\mathcal{Q})$, and the cluster size K through BER simulations based on clustering and interpolation, respectively. Then we present the performance of clustering and interpolation as a function of total feedback bits. To quantize the precoding matrices, we used the codebook in [22] (see the authors' web site to obtain a copy of the codebooks). We designed the codebook for quantization of the derotation matrix $\mathbf{Q}_l$ using the LBG algorithm addressed in Section III-E. For convenience, we define a new notation (m, n, K); $n = 0$ implies the clustering method with cluster size K and m-bit feedback for precoder quantization; $n \geq 1$ implies the proposed interpolation scheme with cluster size K using m-bit feedback for precoder quantization and n-bit feedback for quantization of the derotation matrix $\mathbf{Q}_l$. Accordingly, we use codebooks with $\mathrm{card}(\mathcal{F}) = 2^m$ and $\mathrm{card}(\mathcal{Q}) = 2^n$, respectively, and the total number of feedback bits becomes $(m+n)N/K$.

TABLE III
OPTIMAL CHOICE OF PARAMETERS FOR CLUSTERING AND REQUIRED E_b/N_0 TO ACHIEVE $\mathrm{BER} = 10^{-5}$ WHEN $M_t = 4$, $M_r = 2$, $M_s = 2$, AND $N = 64$

Feedback Bits	Candidates of Parameters	Optimal Choice	Required E_b/N_0 (dB)
16	(1,0,4), (2,0,8), (4,0,16)	(2,0,8)	27.0
24	(1.5,0,4), (3,0,8), (6,0,16)	(3,0,8)	24.8
32	(1,0,2), (2,0,4), (4,0,8) (8,0,16)	(4,0,8)	23.1
48	(1.5,0,2),(3,0,4),(6,0,8)	(3,0,4)	20.3
64	(1,0,1),(2,0,2),(4,0,4) (8,0,8)	(4,0,4)	17.9
128	(2,0,1),(4,0,2),(8,0,4)	(4,0,2)	15.7
256	(4,0,1),(8,0,2)	(4,0,1)	14.7
∞	–	perfect CSI	14.3

When the total number of feedback bits is fixed, we can make several combinations of triple (m, n, K) satisfying the feedback requirements. Table III shows possible combinations of parameters for the clustering algorithm ($n = 0$). In the table, $(1.5, 0, K)$ means that half tones used $(1, 0, K)$ and the other half utilized $(2, 0, K)$. The table also denotes the optimal choice of parameters and the corresponding E_b/N_0 to achieve $\mathrm{BER} = 10^{-5}$ which were found by numerical simulations based on the BER criterion. As expected, the required E_b/N_0 for clustering decreased as the total number of feedback bits increased. Given an m-bit codebook $\mathcal{F}$, the BER improvement by reducing the cluster size rapidly decreases when K is smaller than the channel coherence bandwidth. This results in a tradeoff between m and K. Note that the optimal pairs of m and K are highly dependent on the number of transmit antennas and the channel length in time. In a manner similar to Table III, we found the optimal parameters for the proposed interpolator as shown in Table IV. For the proposed interpolator, $\mathbf{Q}_l$ was determined by (17). Comparing to Table III, the optimal cluster size for interpolation was equal to or slightly greater than that for clustering. The interpolation scheme required less E_b/N_0 than the clustering method when the number of feedback bits was greater than 24. When the number of feedback bits is small, the precoding gain is significantly reduced by the feedback overhead for $\mathbf{Q}_l$. Specifically, the interpolation approach presented 2.0 dB loss over clustering for the total 16-bit feedback.

Fig. 2 compares the BER performance of the interpolator with that of clustering, when the total number of feedback bits is 32, 48, and 64. $\mathbf{Q}_l$ for interpolation was determined by (17). This figure shows the performance gain of interpolation over clustering with the same feedback bits. As the number of feedback bits increased, the BER curves of clustering and interpolation approached that of the precoder with perfect CSI which attained $\mathrm{BER} = 10^{-5}$ at $E_b/N_0 = 14.3$ dB.

B. Comparison With Existing Precoding Methods

We performed numerical simulations and compared the proposed interpolation method with the clustering method and other existing methods (using typical parameters) such as antenna subset selection [5], [7] and, MMSE detection with optimal precoding. To quantize the precoding matrices, we

TABLE IV
OPTIMAL CHOICE OF PARAMETERS FOR PROPOSED INTERPOLATOR AND REQUIRED E_b/N_0 TO ACHIEVE BER $= 10^{-5}$ WHEN $M_t = 4$, $M_r = 2$, $M_s = 2$, AND $N = 64$

Feedback Bits	Candidates of Parameters	Optimal Choice	Required E_b/N_0 (dB)
16	(1,1,8),(3,1,16)	(3,1,16)	29.0
24	(2,1,8),(1,2,8),(5,1,16) (4,2,16),(3,3,16),(2,4,16)	(2,1,8)	24.9
32	(1,1,4),(3,1,8),(2,2,8) (1,3,8),(7,1,16),(6,2,16) (5,3,16),(4,4,16),(3,5,16)	(3,1,8)	22.3
48	(2,1,4),(1,2,4),(5,1,8) (4,2,8),(3,3,8),(2,4,8)	(4,2,8)	19.2
64	(1,1,2),(3,1,4),(2,2,4) (1,3,4),(7,1,8),(6,2,8) (5,3,8),(4,4,8),(3,5,8)	(3,1,4)	17.2
128	(3,1,2),(2,2,2),(1,3,2) (7,1,4),(6,2,4),(5,3,4) (4,4,4),(3,5,4)	(6,2,4)	15.2
256	(7,1,2),(6,2,2),(5,3,2) (4,4,2),(3,5,2)	(6,2,2)	14.5
∞	–	perfect CSI	14.3

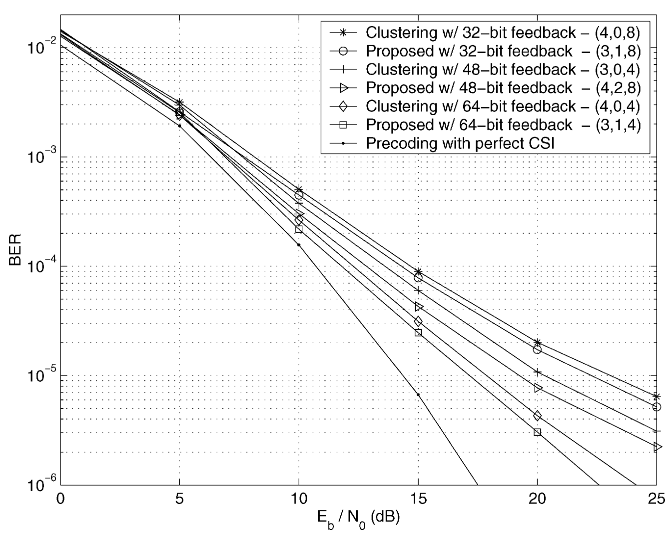

Fig. 2. BER performance of clustering and proposed interpolation with various feedback bits when $M_t = 4, M_r = 2, M_s = 2$, and $N = 64$. For the proposed interpolator, $\mathbf{Q}_l$ was determined by (17).

used the same type of codebooks used in Section IV-A. We set the total amount of feedback bits to 48 and selected the optimal parameters – (3, 0, 4) for clustering and (4, 2, 8) for interpolation. For comparison purposes, we also considered antenna subset selection where the best 2 of 4 antennas are selected per subcarrier according to the same design criterion as that used for precoder quantization. In this case we required a total of 192 feedback bits. We used 256 bits for the optimal quantized precoder with feedback for all subcarriers. As a bound on achievable performance, we also considered the optimal precoder without quantization.

1) BER Comparison: The BER performance is illustrated in Fig. 3. The maximum minimum singular value criterion [7] was used for antenna subset selection and $\mathbf{Q}_l$ for the proposed

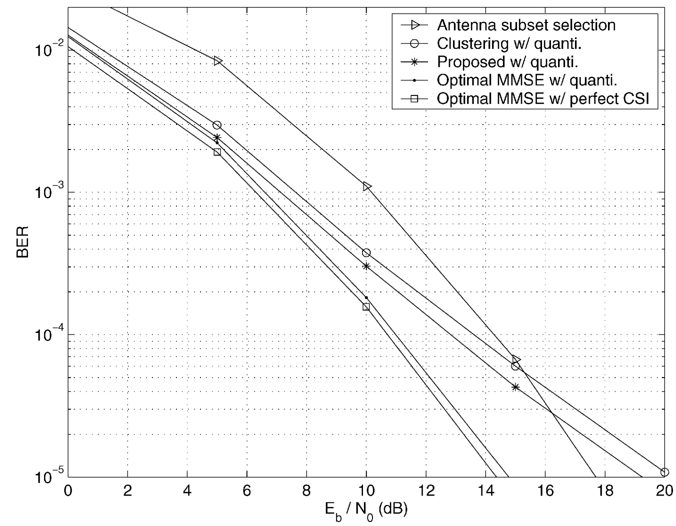

Fig. 3. Comparison of BER performance when $M_t = 4, M_r = 2, M_s = 2$, $N = 64$, and $K = 8$. For the proposed interpolator, $\mathbf{Q}_l$ was determined by (17).

scheme was determined by (17). The proposed method performed better than clustering, yet exhibited some loss in diversity order compared to MMSE precoding with quantization. The reason is that interpolation introduces mismatch with the optimal precoder. Antenna subset selection, as is typical when comparing with optimal diversity methods, exhibits array gain loss. Antenna subset selection, however, is applied to every subcarrier and thus achieves the full diversity order as MMSE precoding since there is no interpolation loss (see [61] for example for a discussion of linear receiver performance with transmit correlation). As a consequence, the proposed performed better than antenna subset selection in the low E_b/N_0 region while the BER performance was reversed in high E_b/N_0 region. *Note that the proposed scheme requires only 48 feedback bits, which is just* 25% *of the feedback for antenna subset selection.*

2) BER Comparison Under Channel Mismatch: In real systems, in addition to quantization error, there are also errors due to channel estimation errors at the receiver and/or delays in the feedback channel. To study this effect, we employ the first-order autoregressive model given by

$$\tilde{h}_{i,j}(k) = \gamma h_{i,j}(k) + \sqrt{1-\gamma^2}u_{i,j}(k) \tag{23}$$

where $\tilde{h}_{i,j}(k)$ is the predicted channel at the transmitter, $h_{i,j}(k)$ is the (i,j)th element of $\mathbf{H}(k)$, $u_{i,j}(k)$ is i.i.d. complex Gaussian noise with zero mean and unit variance, and $0 \leq \gamma \leq 1$. Then the MSE between $\tilde{h}_{i,j}(k)$ and $h_{i,j}(k)$ is denoted as

$$E[|\tilde{h}_{i,j}(k) - h_{i,j}(k)|^2] = (1-\gamma)^2 + 1 - \gamma^2 = 2 - 2\gamma. \tag{24}$$

With suitable choice of parameters this model accounts for channel estimation error or delay in the feedback channel. Fig. 4 illustrates the BER degradation caused by mismatch for $E_b/N_0 = 12$ dB as a function of γ. As follows from Fig. 3, the proposed method performed better than clustering and antenna subset selection, and the BER loss over the optimal MMSE decreases with MSE of the predictor. Note that clustering and

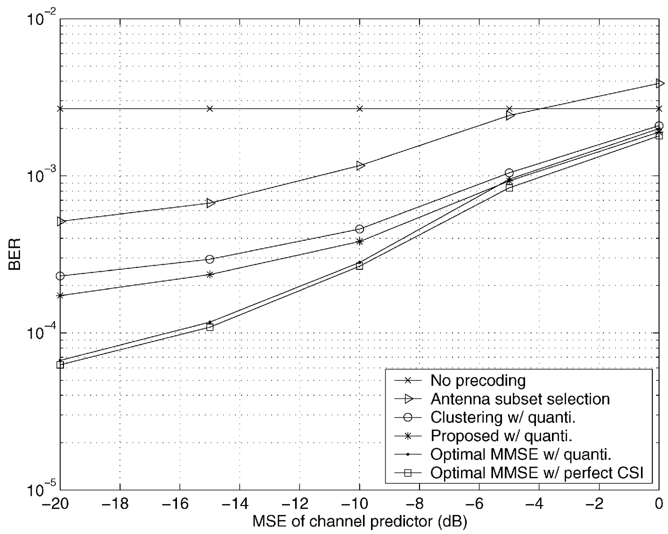

Fig. 4. BER degradation by channel prediction error when $M_t = 4, M_r = 2, M_s = 2, N = 64$, and $K = 8$. For the proposed interpolator, $\mathbf{Q}_l$ was determined by (17).

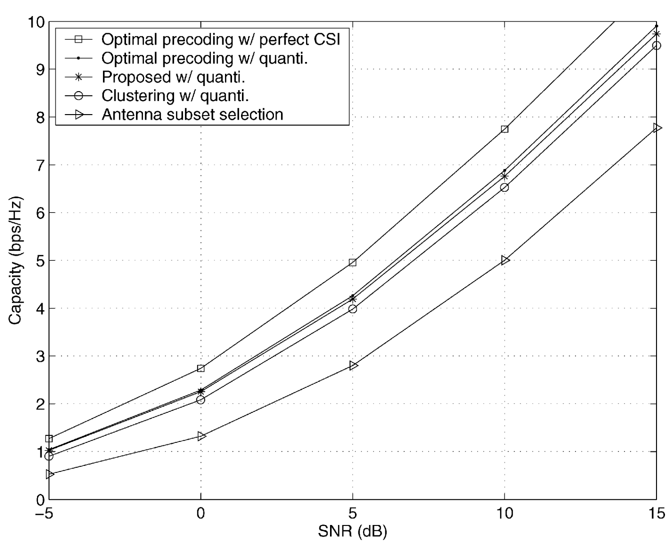

Fig. 5. Comparison of capacity when $M_t = 4, M_r = 2, M_s = 2, N = 64$, and $K = 8$. For the proposed interpolator, $\mathbf{Q}_l$ was determined by (18).

the proposed method performs better than spatial multiplexing without precoding even when the MSE approaches 0 dB. This means precoding has advantages over non-precoding spatial multiplexing for a substantial amount of channel mismatch.

3) Capacity Comparison: Fig. 5 compares the average channel capacity. Maximum capacity antenna subset selection was used [5] and the matrix $\mathbf{Q}_l$ for the proposed scheme was determined by (18). The proposed method outperformed clustering and antenna subset selection with performance that is almost comparable to the optimal precoding. The proposed exhibited 1.5 dB loss at 6 bps/Hz compared to optimal precoding without quantization, which is the upper bound. We have found that this gap can be reduced through better quantization of the precoding matrices.

4) Uncoded BER vs f_dT_s in Time-Varying Channels: The uncoded BER performance in time-varying channels is shown

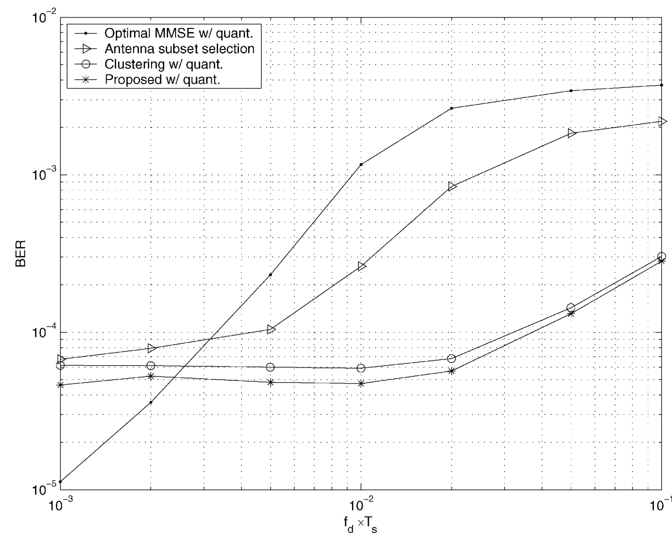

Fig. 6. Uncoded BER performance when $E_b/N_0 = 15$ dB, $M_t = 4, M_r = 2, M_s = 2, N = 64$, and $K = 8$. Channel was time-varying within a frame and precoding matrices were periodically updated through feedback. The proposed interpolator was the same in Fig. 3.

in Fig. 6 where f_dT_s denotes the Doppler frequency normalized by the OFDM symbol duration T_s. In the figure, it was assumed that all the precoding methods use the same feedback ratio. To make this comparison fair, consequently, we had to use different frame lengths for each method. Optimal precoding, for example, uses a very long frame length since it has more bits to send while clustering can use a very short frame length and thus update the precoders more often. Taking into account the feedback requirements discussed in Section III-C, the frame length was set as 30, 30, 120, 160 symbols for clustering, the proposed method, antenna subset selection, and the optimal MMSE, respectively. The proposed method had the lowest BER when $f_dT_s \geq 2.5 \times 10^{-3}$. The BER performance of the optimal MMSE increases rapidly with f_dT_s because it has the longest precoder update period.

5) Coded BER in Time-Varying Channels: The coded BER values in time-varying channels with $f_dT_s = 0.01$ are presented in Fig. 7. For channel coding, we used a convolutional code with generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$ with coding rate 1/2, along with the interleaver and deinterleaver defined in [62], and soft Viterbi decoding. The feedback ratio was fixed and thus the frame length in this simulation was different for each method as in the previous case according to Fig. 6. Each OFDM symbol transmitted 128 bits which is a half of the uncoded case. The proposed scheme outperformed antenna subset selection and the optimal MMSE precoding, and exhibited 0.9 dB gain over clustering at $\mathrm{BER} = 10^{-5}$.

V. Conclusion

We proposed a limited feedback implementation for spatial multiplexing MIMO-OFDM systems with linear precoding. By combining quantized precoding with a structured interpolation, we were able to substantially reduce the amount of feedback required versus quantizing the precoder for every subcarrier. Our interpolation introduces a derotation parameter, which is optimized according to the desired performance metric, e.g.,

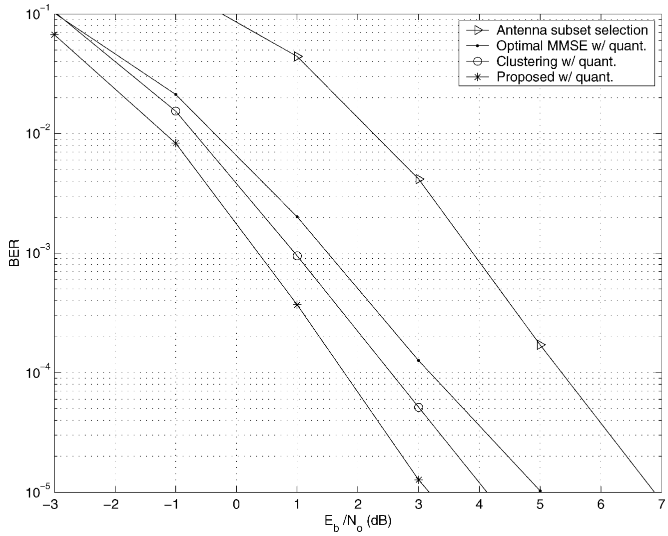

Fig. 7. Coded BER performance when $M_t = 4, M_r = 2, M_s = 2, N = 64$, and $K = 8$. Channel was time-varying within a frame with $f_d T_s = 0.01$. A convolutional code with rate 1/2 was used along with interleaving.

MSE or mutual information. Our Monte Carlo simulations illustrated the reductions in feedback in a variety of circumstances, overall resiliance to delay in the feedback channel, and ability to cope better in moderately fading channels when the feedback capacity is fixed.

The proposed interpolator was derived by performing a linear interpolation in Euclidean space then projecting the results back to the Stiefel manifold. An alternative way to perform interpolation is to work directly in the Grassmann manifold. This would eliminate the need for a derotation matrix and thus would further reduce the feedback required. We are currently investigating solutions along these lines.

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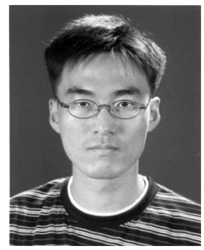

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