Iterative Decision Feedback Equalization and Decoding for Rotated Multidimensional Constellations in Block Fading Channels

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***Abstract*— It is known that rotated multidimensional constellations can be used effectively to achieve full-rate and full-diversity transmission in block fading channels. However, the optimal decoding complexity is exponential with the number of fading blocks (or degrees of freedom). In this paper, we propose a reduced-complexity iterative receiver structure operating on a block basis for coded modulation schemes with rotated constellations. The proposed detector is based on iterative forward and backward filtering followed by a channel decoder that uses a priori log-likelihood ratios (LLR) of coded symbols. Forward and feedback filters are jointly optimized according to the minimum mean square error (MMSE) criterion to minimize the spatial interference induced by rotation. It is observed that the proposed structure achieves full diversity and performance close to outage probability for rotated inputs even with simple Discrete Fourier Transform (DFT) rotations.**

***Index Terms*— Rotated constellations, block-fading channel, decision feedback equalization, diversity, outage, singleton bound, coded modulation, iterative decoding, soft feedback**

I. Introduction

Rotated multidimensional constellations with uncoded modulation has been studied and shown to be an effective way to attain full-rate and full-diversity transmission in fading channels [1], [2], [3]. Even random multidimensional rotations are shown to exhibit good diversity distributions to combat channel fading for uncoded transmission in [3]. The problem of constructing general coded modulation schemes over multi-dimensional signal sets obtained by rotating classical complex-plane signal constellations has recently been studied in [4] for block fading channels with B fading blocks.

Despite the benefits of rotation over B fading blocks, they induce large decoding complexity due to the inter-symbol interference (ISI) caused by rotated constellations. A problem here is related to the complexity of optimum decoding, i.e., maximum likelihood (ML) receiver interfaces exhibit a complexity that grows exponentially with the modulation size and the dimension of rotation (B), and becomes quickly unpractical as either parameter is large. In [3], a suboptimal MMSE equalizer with decision feedback is proposed and it is shown to achieve good performance without destroying the high diversity order in the rotated constellation. In [5], the sphere decoding is employed to avoid exhaustive search over all candidate points. However, the structures in [3] and [5] were proposed for uncoded rotations. When coded modulation is used, the code trellis structure has to be incorporated and soft information should be provided to the decoder, which further complicates the problem. As a remedy to this problem, in [4], the use of rotations with dimension smaller than the number of fading blocks was considered. The intuition behind this idea is that the channel code itself can help to achieve full diversity and sometimes rotations of smaller dimension might be sufficient. However, for some rate values and constellation sizes, using rotations with small dimensions may not be sufficient to achieve optimal rate-diversity tradeoff, i.e., the rotations of large dimensions might be necessary to attain full diversity order and the decoding complexity has still exponential dependence on the dimension of rotation. Soft-output sphere decoding technique for rotated constellation was proposed in [6], but it still shows some undesirable limitations in practice.

In this paper, we propose an iterative receiver structure with reasonable complexity for coded modulation schemes with rotated constellations. The proposed detector is based on iterative forward and backward filtering followed by a channel decoder that works by using preliminary soft values of the coded symbols. Since the reliability of coded symbols from the decoding process are used in deriving the jointly optimal forward and backward filters, the filters employed in this work have a different structure from that of previous interference-cancellation based turbo equalizers, such as [7], [8], [9]. It has been observed that the proposed scheme yields a very close performance to the outage probability with reasonable complexity for rotated constellations. The benefits that rotation brings in terms of diversity exponent is justified without compromising the decoding complexity when compared to the optimal ML based structures with exponential complexity.

This paper is organized as follows. In Section II, the system model is described. In Section III, iterative decision feedback equalization technique for decoding of rotated constellations is discussed in detail. In Section IV, iterative decoding structure concatenated to equalization stage is explained. Finally, simulation results and concluding remarks are presented in Section V and Section VI respectively.

II. System Model

The following notation is used throughout the paper. Bold-face upper-case letters denote matrices and scalars are denoted

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by plain lower-case letters. The superscript $(\cdot)^*$ denotes the complex conjugate for scalars and $(\cdot)^H$ denotes the conjugate transpose for vectors and matrices. The $n \times n$ identity matrix is shown with $\mathbf{I}_n$. The autocorrelation matrix for a random vector $\mathbf{a}$ is $\mathbf{R_a} = E\{\mathbf{aa}^H\}$ where $E\{\cdot\}$ stands for the expected value operator. The $(i,j)^{th}$ element of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}(i,j)$ and the i^{th} element of a vector $\mathbf{a}$ is denoted by a^i.

This paper considers block based transmission as in [10], [11], [8]. During the transmission of one block, the channel is assumed to be constant and it changes independently from block to block. Without dealing with the channel estimation problem, the channel is assumed to be perfectly known at each block transmission.

Assuming symbol rate sampling, the discrete time baseband equivalent model of the point-to-point single-input single-output block fading channel with B fading blocks can be written as [12],

$$\mathbf{y}_k = \mathbf{D}\mathbf{a}_k + \mathbf{n}_k, \qquad k = 0, 1, \ldots, N-1, \tag{1}$$

where N is the codeword length (block length) and $\mathbf{D}$ is a diagonal $B \times B$ matrix with main entries, $d_i,\ i = 1, \ldots, B$, i.e., $\mathbf{D} = \text{diag}(d_1, \ldots, d_B)$. $\mathbf{a}_k = [a_k^1, \ldots, a_k^B]^T$ is the portion of the transmitted codeword at time k and $\mathbf{y}_k = [y_k^1, \ldots, y_k^B]^T$ is the corresponding received vector at time k. Main diagonal entries of $\mathbf{D}$, d_i's are the fading coefficients which are independent zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance 1. Block fading model is considered and thus the channel matrices are assumed to be constant during a coherence interval significantly larger than a duration needed for the transmission of one block [13] and channel state information at transmitter (CSIT) is not available. Noise vectors $\mathbf{n}_k$ are also taken as ZMCSCG white (spatially and temporally) noise with variance N_0.

We consider that $\mathbf{a}_k$'s are obtained via the rotation of the symbols, i.e.,

$$\mathbf{a}_k = \mathbf{V}\mathbf{x}_k, \qquad k = 0, 1, \ldots, N-1, \tag{2}$$

where $\mathbf{x}_k = [x_k^1, \ldots, x_k^B]^T$ is the vector of complex-plane signal constellation symbols that is rotated by the $B \times B$ rotation matrix $\mathbf{V}$. The rotation matrix is unitary, i.e., $\mathbf{V}\mathbf{V}^H = \mathbf{I}_B$ and applied uniformly throughout transmitted block.

The codewords $\mathbf{X} = [\mathbf{x}_0, \ldots, \mathbf{x}_{N-1}]$ form a coded modulation scheme $\chi \subset \mathbb{C}^{B \times N}$. In particular, we consider that χ is obtained as the concatenation of a binary code of rate r and a modulation over the signal constellation $S \in \mathbb{C}$ with $M = \log_2 |S|$. The rate in bits per channel use of this scheme is $R = rM$. After the transmitted signal block has been rotated, one can get the equivalent channel from (1) and (2) as

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k, \qquad k = 0, 1, \ldots, N-1, \tag{3}$$

where $\mathbf{H} = \mathbf{DV}$. This form resembles to the baseband equivalent form of MIMO channel. Therefore, we will call our structure as space-time decoder hereafter and construct our receiver based on (3) in Section III.

When no rotations are used, the optimal diversity reliability exponent is given by the Singleton bound for a given rate R (bits per channel use) and signal constellation M as

$$d_\chi^* = 1 + \left\lfloor B\left(1 - \frac{R}{M}\right)\right\rfloor \tag{4}$$

for B Rayleigh faded blocks. This value is an upper bound for the block-diversity of any coded modulation scheme $\chi \subset \mathbb{C}^{B \times N}$ with rate R and constellation $S \in \mathbb{C}$ with $M = \log_2 |S|$. A code is block-wise maximum-distance separable (MDS) if it achieves the maximum diversity order given in (4) [12], [4].

It was shown in [4] that the optimal diversity reliability exponent achieved by random Gaussian codes can also be achieved by random coded modulation schemes concatenated with a full-diversity rotation of dimension B when $R < M$. In this case, the optimal reliability exponent is given by

$$d^* = B \tag{5}$$

which is the available degrees of freedom in the channel. The rotation of dimension B takes care of achieving full diversity while the coding gain is left to the outer coded modulation scheme over S and, so for rotated schemes, the MDS constraint on the code is relaxed [4].

As it will be seen in Section V, simple rotations like DFT which is not full-diversity rotation may be sufficient to reach optimal diversity order B in coded schemes, since the code itself help to achieve maximum reliability exponent. In other words, the optimal diversity order in (5) is achieved by both coded modulation and the rotation in this case.

III. Iterative Decision Feedback Equalization (DFE) for Rotated Constellations

We consider iterative space-time decoder with soft decision feedback in this paper. Since both equalization and decoding processes can be performed in each iteration, turbo principle can be applied as done in [7], [8], [9]. In Fig. 1, an exemplary receiver structure is shown for iterative decision feedback equalizer (DFE).

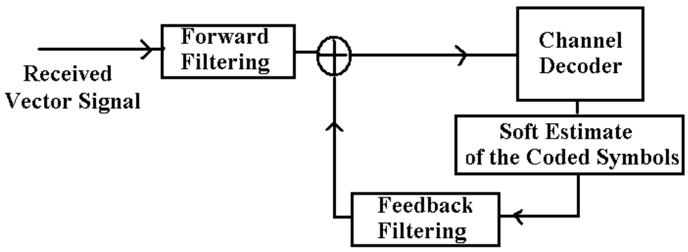

Fig. 1. Iterative Decision Feedback Equalization (DFE) and decoding for rotated constellations

One can write the output from the DFE for the k^{th} vector in the block in the i^{th} iteration as

$$\tilde{\mathbf{x}}_k^{(i)} = (\mathbf{W}^{(i)})^H \mathbf{y}_k - (\mathbf{F}^{(i)})^H \hat{\mathbf{x}}_k^{(i-1)} \tag{6}$$

for $k = 0, \ldots, N-1$. $\mathbf{W}^{(i)}$'s and $\mathbf{F}^{(i)}$'s are forward and feedback filters with sizes $B \times B$ and $\hat{\mathbf{x}}_k^{(i-1)}$'s are soft decisions from the previous iteration. When the filters are designed based on the MMSE criterion and the information bearing signals are Gaussian, this structure is information theoretically optimum as stated in [14]. The first term in (6) is actually the feedforward estimate of the k^{th} transmitted vector. In (6), $\hat{\mathbf{x}}_k^{(i-1)}$'s are the soft feedback decisions from the previous iteration and they are utilized at the feedback filtering process to improve the estimate of $\mathbf{x}_k$. The forward and backward filter matrices are jointly optimized and found according to the MMSE criterion given by $E\left\{\sum_{k=0}^{N-1} ||\tilde{\mathbf{x}}_k^{(i)} - \mathbf{x}_k||^2\right\}$ presented in [10], [11].

The n^{th} component of the estimation is not used in the feedback equalization of the n^{th} component of the received vector, and so we impose the following condition on the feedback filter

$$\mathbf{F}^{(i)}(n,n) = 0,\ n = 1,\ldots,B \tag{7}$$

since, by imposing this constraint, one can avoid self-subtraction of the desired symbol by its previous estimate.

The Lagrange multiplier method can be used to obtain optimal filter coefficients. Lagrangian vectors and the corresponding scalar constraints (Lagrangian function) can be written as

$$\mathbf{\Gamma}^{(i)} = \text{diag}\left[\Gamma_1^{(i)},\ldots,\Gamma_B^{(i)}\right]_{(B\times B)},$$
$$Lagrangian(\mathbf{\Gamma}^{(i)}) = \sum_{n=1}^{B} (\mathbf{F}^{(i)}(n,n))^* \Gamma_n^{(i)} \tag{8}$$

Due to an interleaving operation both in time and space, we can assume that,

$$E\{\mathbf{x}_k(\mathbf{x}_l)^H\} = E_s \mathbf{I}_{n_t}\delta_{kl},\ \text{for}\ k,l = 0,\ldots,N-1, \tag{9}$$

where δ_{kl} is the delta function which is 0 for all k but $k = l$. Some important correlation matrices used by the forward and feedback filters are defined for the i^{th} iteration as

$$\mathbf{P}^{(i)} = E\{\mathbf{x}_k(\hat{\mathbf{x}}_k^{(i-1)})^H\}, \quad \mathbf{B}^{(i)} = E\{\hat{\mathbf{x}}_k^{(i-1)}(\hat{\mathbf{x}}_k^{(i-1)})^H\} \tag{10}$$

for $k = 0,\ldots,N-1$. To simplify the computation of the filter coefficients, feedback decisions are assumed to be independent. Furthermore, due to interleaving operation of the coded symbols, feedback decisions are assumed to be uncorrelated with the symbols transmitted at different block or symbol time. It is further assumed that the reliability matrices of the decision feedback are the same for all k, i.e.,

$$E\{\mathbf{x}_k(\hat{\mathbf{x}}_l^{(i-1)})^H\} = \mathbf{0}, \quad E\{\hat{\mathbf{x}}_k^{(i-1)}(\hat{\mathbf{x}}_l^{(i-1)})^H\} = \mathbf{0},\ \text{for}\ k \neq l \tag{11}$$

$$E\{x_k^m(\hat{x}_k^n)^*\} = \rho_m\delta_{mn},\ E\{\hat{x}_k^m(\hat{x}_k^n)^*\} = \beta_m\delta_{mn} \tag{12}$$

for $m,n = 1,\ldots,B$ and the expectations are independent of symbol index k. Then, we can write

$$\mathbf{P}^{(i)} = \text{diag}\left[\rho_1,\ldots,\rho_B\right],\ \mathbf{B}^{(i)} = \text{diag}\left[\beta_1,\ldots,\beta_B\right]. \tag{13}$$

This assumption makes the forward and backward filters independent of time index k and, so the block processing on each received signal can be implemented effectively. This can be achieved by simply averaging the correlations of soft feedback decisions from the previous iteration as will be done in Section IV. These are standard and reasonable assumptions as stated in [10], [7], [8] since the average symbol error probability is approximately the same for each symbol in a large block with quasi-static fading. Calculation of the correlation matrices $\mathbf{P}^{(i)}$ and $\mathbf{B}^{(i)}$ will be done in Section IV.

After taking the gradient of the MMSE cost function and the Lagrangian with respect to the rows of $(\mathbf{W}^{(i)})^H$ and $(\mathbf{F}^{(i)})^H$, equating the gradients to the zero vector, taking expectations and combining vectors into single matrix equations for $n = 1,\ldots,B$, one can obtain the following matrix equations giving the optimal forward and backward filter matrices

$$\mathbf{R_y}\mathbf{W}^{(i)} = \mathbf{H}\left[E_s\mathbf{I}_B + \mathbf{P}^{(i)}\mathbf{F}^{(i)}\right] \tag{14}$$

$$\mathbf{B}^{(i)}\mathbf{F}^{(i)} = (\mathbf{P}^{(i)})^H\left[\mathbf{H}^H\mathbf{W}^{(i)} - \mathbf{I}_B\right] - \mathbf{\Gamma}^{(i)} \tag{15}$$

where

$$\mathbf{R_y} = E\{\mathbf{y}_k(\mathbf{y}_k)^H\} = \left(\mathbf{H}\mathbf{H}^H E_s + N_0\mathbf{I}_B\right) \tag{16}$$

and $\mathbf{\Gamma}^{(i)}$ can be obtained from the constraint in (7). By substituting $\mathbf{W}^{(i)}$ into (15) and using the constraint, the Lagrangian terms given in (8) and backward filter matrices can be readily found after some calculations as,

$$\Gamma_n^{(i)} = \frac{\left[\mathbf{A}^{(i)}(n,:)\mathbf{D}^{(i)}(:,n)\right]}{\mathbf{A}^{(i)}(n,n)},\ n = 1,\ldots,B \tag{17}$$

$$\mathbf{F}^{(i)} = \mathbf{A}^{(i)}\left[\mathbf{D}^{(i)} - \mathbf{\Gamma}^{(i)}\right], \tag{18}$$

where

$$\mathbf{A}^{(i)} = \left[\mathbf{B}^{(i)} - (\mathbf{P}^{(i)})^H\mathbf{H}^H\mathbf{R}_\mathbf{y}^{-1}\mathbf{H}\mathbf{P}^{(i)}\right]^{-1}, \tag{19}$$

$$\mathbf{D}^{(i)} = (\mathbf{P}^{(i)})^H\mathbf{H}^H\mathbf{R}_\mathbf{y}^{-1}\mathbf{H}E_s - (\mathbf{P}^{(i)})^H, \tag{20}$$

$\mathbf{A}^{(i)}(n,:)$ is the $n-$th row of $\mathbf{A}^{(i)}$, $\mathbf{D}^{(i)}(:,n)$ is the $n-$th column of $\mathbf{D}^{(i)}$ and forward filter $\mathbf{W}^{(i)}$ can be obtained from (14).

IV. Iterative Decoding

In this section, we will calculate the log-likelihood ratios (LLR) and soft decisions of the coded symbols to be used in decision feedback. BPSK modulation is assumed for simplicity, but the extension to other M-ary or M-PSK modulations is straightforward. At each iteration, extrinsic information is extracted from detection and decoding stages and is then used as a priori information in the next iteration, just as in turbo decoding. The soft output from the DFE in the i^{th} iteration after (6) can be written as,

$$\tilde{x}_k^{m\ (i)} = \mu_m^{(i)} x_k^m + \eta_k^{m\ (i)} \tag{21}$$

for $k = 0,\ldots,N-1$ and $m = 1,\ldots,B$. In this case, the equalized channel in (21) can be considered as a quasi-parallelized channel and the LLR for the m^{th} component of the k^{th} transmitted symbol can be written as

$$\lambda_k^{m\ (e)} = \log_e \frac{P(\tilde{x}_k^{m\ (i)}|x_k^m = +1)}{P(\tilde{x}_k^{m\ (i)}|x_k^m = -1)}. \tag{22}$$

The LLR term $\lambda_k^{m\ (e)}$ is the extrinsic information that can be obtained from the equalizer output. An a priori probability ratio $\lambda_k^{m\,(p)}$ $(\log_e \frac{P(x_k^m=+1)}{P(x_k^m=-1)})$ is given by the decoder as the intrinsic information obtained from the previous iteration [10], [7] and used to construct a soft estimate of the coded symbol x_k^m. The extrinsic information given in (22) can be expressed as,

$$\lambda_k^{m\,(e)} = \frac{4\mathrm{Re}\{(\mu_m^{(i)})^* \tilde{x}_k^{m\,(i)}\}}{E\{|\eta_k^m|^2\}} \tag{23}$$

by using the equivalent complex amplitude, $\mu_m^{(i)}$ of x_k^m at the output of the equalizer and the residual interference power, $E\{|\eta_k^{m\,(i)}|^2\}$. These values can be easily found in terms of channel matrices, forward and backward filter coefficients and correlation matrices as done for the SISO systems in [10], [7]. While computing the LLRs, we resort to simplification of the decoding algorithm by neglecting the correlation existing between the residual noise terms, i.e., the η_k^m's are taken as uncorrelated for $m = 1, \ldots, B$ as done in the decoding stage of [8] for flat fading MIMO channel and the residual interference is further approximated by a Gaussian distribution as in [10], [7]. It can be shown that $\mu_m^{(i)}$ and $E\{|\eta_k^{m\,(i)}|^2\}$ values do not depend on symbol time index k, so these values are calculated only once for the decoding of one block in each iteration, which reduces the complexity significantly.

Soft feedback decisions, $\hat{x}_k^m$ for the DFE can be taken as $\tanh\left(\frac{1}{2}\lambda_k^{m\,(p)}\right)$ for $E_s = 1$, $m = 1, \ldots, B$ and $k = 0, \ldots, N-1$ as done in [7], [8], [10]. The non-zero diagonal entries of the correlation matrices $\mathbf{P}^{(i)}$ and $\mathbf{B}^{(i)}$ in (10) used by the forward and backward filters can be calculated by using the following approximation,

$$\rho_{k,m} \triangleq E\{x_k^m (\hat{x}_k^m)^*\} = E\{E\{x_k^m\}(\hat{x}_k^m)^*\} = |\hat{x}_k^m|^2 \tag{24}$$

$$\rho_m = \beta_m = \frac{1}{N}\sum_{k=0}^{N-1} \rho_{k,m} \tag{25}$$

$E\{x_k^m\}$ was taken as $\hat{x}_k^m$ and this is a common assumption in various turbo detection techniques as done in [10], [7] and [15].

Correct estimation of $\mathbf{P}^{(i)}$ and $\mathbf{B}^{(i)}$'s are important since our proposed DFE takes into account the reliability of the feedback decisions and therefore alleviates the error propagation problem different than the original DFE studies assuming perfect feedback decisions. In the first iteration, $\mathbf{P}^{(i)}$ and $\mathbf{B}^{(i)}$ can be taken as $\mathbf{0}_B$, i.e, reliable feedback decisions are not available. As the number of iterations incresases, both metrics approach the asymptotic value: $E_s\mathbf{I}_B$.

V. Simulation Results

A. Outage Probability Calculations

For sufficiently large block length N, the packet error probability of any coding scheme is lower bounded by the information outage probability [13]. In this section, we will compare the performance of our proposed decoding structure with the corresponding constrained outage probability of rotated and unrotated schemes. The constrained capacity can be found for the system model in (3) given the complex vector set χ of cardinality $|S|^B = (2^M)^B$ (e.g., M-ary or M-PSK modulations) similar to the derivations for block fading channels in [12] and rotated schemes in [4] as

$$C_{rotated}^{\chi} = \frac{1}{N}\sum_{k=0}^{N-1}\frac{1}{B} I(\mathbf{x}_k;\mathbf{y}_k|\mathbf{H}) = \log_2|S| - \frac{1}{B}E_{\mathbf{n}}\{\sum_{\mathbf{x}_k\in\chi}\frac{1}{|S|^B}\log_2\sum_{\mathbf{x}_i\in\chi} \exp\left(\frac{-\|\mathbf{H}(\mathbf{x}_k-\mathbf{x}_i)+\mathbf{n}\|^2+\|\mathbf{n}\|^2}{N_0}\right)\} \tag{26}$$

where $\mathbf{n}$ is ZMCSCG vector and the corresponding outage probability can be written as

$$P_{out}^{rotated,\,\chi}(R) = \mathbb{P}\{C_{rotated}^{\chi} < R\}\,. \tag{27}$$

Constrained outage probabilities will be used for performance evaluation in the next part.

B. Performance Results

In Fig. 2, simulation results are depicted for block fading channels with 3 fading blocks. Each block is Rayleigh faded with unity power. The error probability of rotated and unrotated systems with QPSK modulation and their corresponding outage probabilities are shown. A full block diversity attaining blockwise concatenated convolutional code (BCCC) is used for encoding for both rotated and unrotated cases as adapted from [12]. The outer code is a rate-$\frac{1}{2}$ convolutional code and the inner codes are 3 trivial rate-1 accumulators. The information block length, i.e., the information bits entering the outer encoder is taken as 148 per frame and the rate in bits per channel use of this scheme is $R = rM = \frac{1}{2}2 = 1$. A DFT matrix with size 3 is used to rotate discrete QPSK inputs. Number of iterations inside the Turbo BCCC decoder is set to 10 and the number of equalizer iterations at which the forward and backward filters are updated by using the reliability matrices is taken as 3.

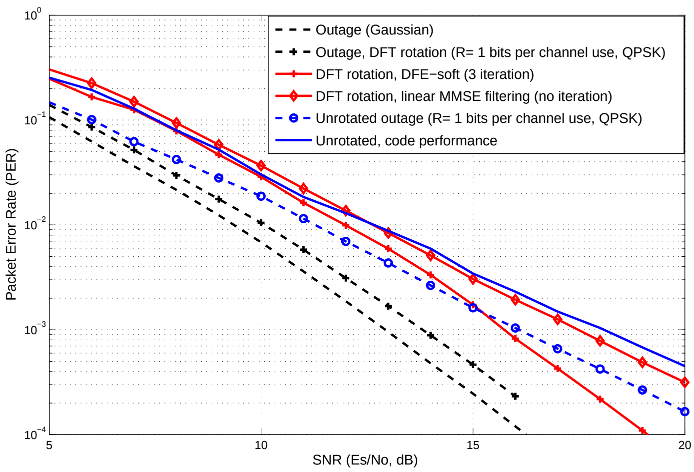

Fig. 2. Performance comparison of iterative DFE and outage for rotated and unrotated constellations, $B = 3$

As it is seen from the outage probabilities, rotation enables to capture largest possible reliability exponent achieved by Gaussian inputs, namely $d^* = B = 3$, while unrotated inputs have $d_\chi^* = 2$. It has been observed that there is approximately

2 dB difference between the outage probability with rotated inputs and the performance of decision feedback equalizer (DFE) with 3 iterations. This gap from the outage is similar to the gap between the outage and code performance of unrotated inputs. Then, one can say that the spatial interference and the error propagation problem inherent in decision feedback are almost eliminated and it is possible to attain optimal diversity of the block fading channel by using the proposed space-time equalizer. These results show that the theoretical benefit of rotation can be materialized by the proposed practical decoding structure with significantly reduced complexity. Moreover, it is seen that the simple DFT rotation is sufficient to attain optimal diversity order in coded schemes since the code itself helps achieve full diversity different than the uncoded rotations in which the full diversity rotations are necessary to get the optimal exponent.

Furthermore, it is interesting to note that the performance improvement of the iterative DFE with soft feedback over the linear MMSE filtering without decision feedback is about 3 dB at PER=0.0001. There is also a loss in diversity as observed in the reduced PER slope without decision feedback. The suboptimality of linear equalizer prevents the system achieving high diversity orders. One can say that the proposed equalizer gains more diversity in comparison to the linear forward MMSE filtering by a careful design of both the forward and backward filters.

In Fig. 3, simulations are repeated for 6 fading blocks and outage probabilities are constructed for Gaussian inputs, BPSK inputs and rotated BPSK inputs with DFT rotation of size 6. The same BCCC structure is used with rate-$\frac{1}{2}$ outer convolutional encoder and 6 inner rate-1 accumulators. The information block length is taken as 238. Similar results are obtained as in Fig. 2 and the optimal reliability exponent $d^* = 6$ is achieved by coded modulation scheme with simple DFT rotation, while unrotated inputs have $d^*_\chi = 4$ from the singleton bound. The optimal diversity order and a close performance to outage probability of rotated scheme at rate $R = 0.5$ bits per channel use within 2 dB are achieved by our practical decoding structure.

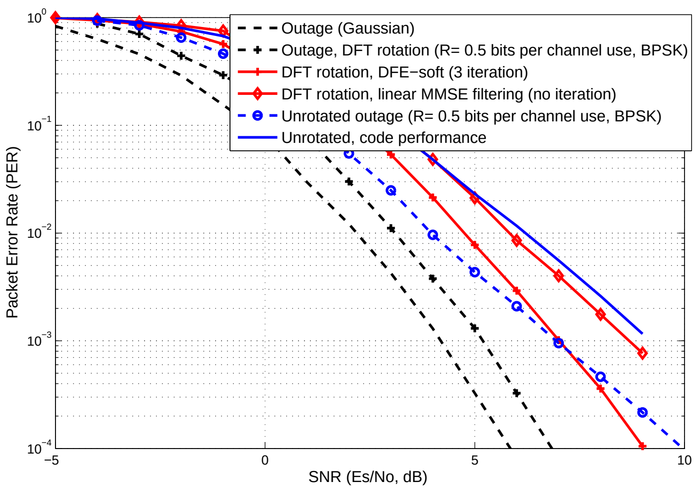

Fig. 3. Performance comparison of iterative DFE and outage for rotated and unrotated constellations, $B = 6$

Fig. 4 shows the benefits of rotations by comparing the performance of the proposed iterative DFE for rotated QPSK inputs and unrotated code performances for 8 fading blocks. DFT rotation and BCCC structure with rate-$\frac{1}{2}$ outer convolutional encoder and 8 inner rate-1 accumulators are used. The information block length is taken as 318. The maximum diversity order, namely $d^* = 8$ is achieved by the iterative DFE with soft feedback since the performance of iterative DFE shows the same slope as outage with Gaussian inputs, while the code performances with unrotated inputs can get $d^*_\chi = 5$. However, the gap between rotated and unrotated schemes may not be so significant at moderate PER values and even performance of the rotated scheme with the use of suboptimal non-iterative MMSE equalizer is below the performance of unrotated schemes. Therefore, for channels with large diversity order, one may not observe a considerable benefit of rotated constellations over some PER values. One has to be careful while choosing decoding architecture, since the use of non-iterative suboptimal structures may destroy the high diversity benefits induced by rotated constellations due to residual spatial interference.

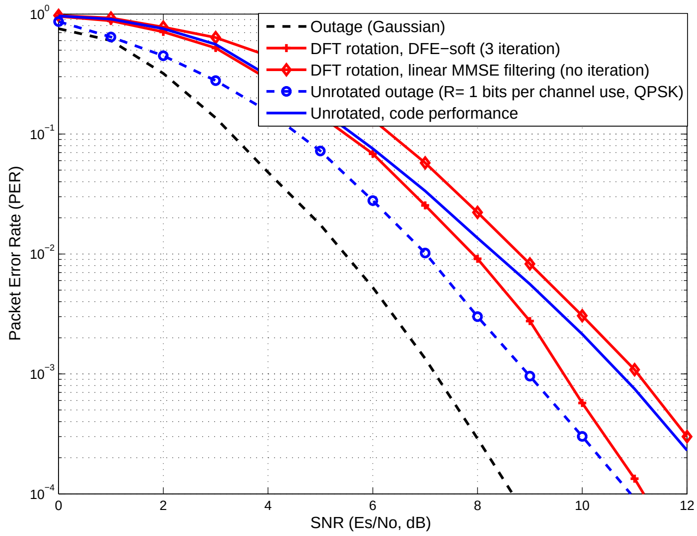

Fig. 4. Performance comparison of iterative DFE and outage for rotated and unrotated constellations, $B = 8$

VI. Conclusion

We have studied the block-fading channels with rotated signal constellations. Although rotated schemes can provide large diversity to combat fading, demodulation is prohibitive for large number of fading blocks and combined with coded modulations. We have proposed an iterative MMSE type decoding structure based on soft decision feedback in this paper. The proposed architecture shows a very close performance to the outage probability with rotated inputs and achieves the optimal diversity order attained by Gaussian inputs. Therefore, the theoretical benefit of rotated constellations is captured by the proposed structure with significantly reduced complexity.

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