REVIEW

A review of the integrate-and-fire neuron model: II. Inhomogeneous synaptic input and network properties

A. N. Burkitt

Abstract The integrate-and-fire neuron model describes the state of a neuron in terms of its membrane potential, which is determined by the synaptic inputs and the injected current that the neuron receives. When the membrane potential reaches a threshold, an action potential (spike) is generated. This review considers the model in which the synaptic input varies periodically and is described by an inhomogeneous Poisson process, with both current and conductance synapses. The focus is on the mathematical methods that allow the output spike distribution to be analyzed, including first passage time methods and the Fokker–Planck equation. Recent interest in the response of neurons to periodic input has in part arisen from the study of stochastic resonance, which is the noise-induced enhancement of the signal-to-noise ratio. Networks of integrate-and-fire neurons behave in a wide variety of ways and have been used to model a variety of neural, physiological, and psychological phenomena. The properties of the integrate-and-fire neuron model with synaptic input described as a temporally homogeneous Poisson process are reviewed in an accompanying paper (Burkitt in Biol Cybern, 2006).

A. N. Burkitt (✉)
The Bionic Ear Institute, 384–388 Albert Street,
East Melbourne, VIC 3002, Australia
e-mail: aburkitt@bionicear.org

A. N. Burkitt
Department of Electrical and Electronic Engineering,
The University of Melbourne,
Melbourne, VIC 3010, Australia

1 Introduction

The integrate-and-fire neuron model has become widely accepted as one of the canonical models for the study of neural systems. The model provides a good description of the subthreshold integration of synaptic inputs, which occurs on a timescale that is slow in comparison to the rapid spike generation. Until relatively recently most of the stochastic models studied analytically in theoretical neuroscience considered only the case of time-homogeneous synaptic input, for which there are a number of well developed mathematical techniques, reviewed in Burkitt (2006). A number of recent developments have promoted interest in the analysis of the model with periodic (i.e., inhomogeneous) synaptic input. The most important has been the interest in stochastic resonance, which is the enhancement of the output signal-to-noise ratio in a system by the addition of noise (Gammaitoni et al. 1998). The integrate-and-fire neuron model has played an important role in exploring this phenomena in neural systems. Other important developments have arisen from modeling of auditory neural processing, where neural responses show activity that is phase-locked to periodic acoustical signals (Weiss 1966; Siebert 1970; Johnson 1980; Gummer 1991), and the study of neuronal rhythms and oscillations (Singer 1993). Such oscillations occur naturally in networks of interconnected neurons (Strogatz 2000). This review focusses on the analytical methods that are used to study inhomogeneous periodic synaptic input. These methods represent significant extensions to the techniques used for the study of the model with homogeneous synaptic input, but they provide only a partial description of the behavior of the model. There is considerable scope for

the continuing development of methods to analyze the model with inhomogeneous synaptic input.

The integrate-and-fire neuron model is introduced in Sect. 2, giving the membrane potential for injected current and details of the synaptic input for current synapses and conductance synapses. Stochastic models are discussed in Sect. 3, including the Ornstein–Uhlenbeck model, the Gaussian approximation, and the Fokker–Planck equation. Techniques for analyzing the model with inhomogeneous Poisson input and both current and conductance synapses are presented. The spiking rate and interspike interval distribution are presented in Sect. 3.1, which involves the solution for the first passage time problem using the renewal equation with appropriate averaging over the initial phases from which the integration starts. A perturbative approach using the Fokker–Planck formalism for homogeneous background synaptic input together with small a amplitude periodic synaptic input is outlined in Sect. 3.2. The application of the integrate-and-fire neuron model in two domains of neural information processing are reviewed: stochastic resonance in Sect. 4 and the behavior of networks of neurons in Sect. 5.

2 The integrate-and-fire neuron model

The integrate-and-fire neuron model is described by the dynamics of the neuron's membrane potential, $v(t)$,

$$C_{\mathrm{m}} \frac{\mathrm{d}v(t)}{\mathrm{d}t} = -\frac{C_{\mathrm{m}}}{\tau_{\mathrm{m}}} [v(t) - V_0] + I_{\mathrm{s}}(t) + I_{\mathrm{inj}}(t) , \qquad (1)$$

where C_{m} is the membrane capacitance, V_0 the resting potential, τ_{m} the passive membrane time constant, $I_{\mathrm{s}}(t)$ a current describing the effect of synaptic input to the neuron, and $I_{\mathrm{inj}}(t)$ a current injected into the neuron (by an intracellular electrode). The first term on the right is the current due to the passive leak of the membrane, and the passive membrane time constant is related to the neuron's capacitance and leak resistance R_{m} of the membrane potential by $\tau_{\mathrm{m}} = R_{\mathrm{m}} C_{\mathrm{m}}$. The notation used throughout follows that in the accompanying review of the integrate-and-fire neuron model with homogeneous synaptic input (Burkitt 2006).

For subthreshold potentials, the response of the model to periodic deterministic input is (Tuckwell 1988a)

$$v(t) = V_0 + e^{-t/\tau_{\mathrm{m}}} \int_{t_0}^{t} \frac{I_{\mathrm{inj}}(t')}{C_{\mathrm{m}}} e^{t'/\tau_{\mathrm{m}}} \mathrm{d}t' , \qquad (2)$$

where it is assumed that the membrane potential at the initial time t_0 is at the resting potential $v(t_0) = V_0$. When the membrane potential reaches the threshold V_{th} a spike is generated and the membrane potential is reset to its initial value V_{reset}. A number of authors have examined the response of the model to periodic deterministic input (Knight 1972a), including an examination of the relationship between the phase locking of the output spikes with the periodicity of the input (Scharstein 1979; Keener et al. 1981).

The main interest in the integrate-and fire neuron model is with stochastic synaptic input. In the case of current synapses, the synaptic current is described by

$$I_{\mathrm{s}}(t) = C_{\mathrm{m}} \sum_{k=1}^{N_{\mathrm{E}}} a_{\mathrm{E},k} S_{\mathrm{E},k}(t) + C_{\mathrm{m}} \sum_{k=1}^{N_I} a_{\mathrm{I},k} S_{\mathrm{I},k}(t) . \quad (3)$$

As in the case of homogeneous synaptic input, the excitatory and inhibitory synaptic inputs, $S_{\mathrm{E},k}(t)$ and $S_{\mathrm{I},k}(t)$, are described as a series of δ-function inputs to each synapse

$$S_{\mathrm{E},k}(t) = \sum_{t_{\mathrm{E},k}} \delta(t - t_{\mathrm{E},k}) , \quad S_{\mathrm{I},k}(t) = \sum_{t_{\mathrm{I},k}} \delta(t - t_{\mathrm{I},k}) , \quad (4)$$

where $t_{\mathrm{E},k}$ and $t_{\mathrm{I},k}$ are the times of the synaptic input spikes for the excitatory and inhibitory synapses, respectively. The input spiking rates are modeled as inhomogeneous Poisson processes with time-dependent intensities for the excitatory and inhibitory synaptic inputs $\gamma_{\mathrm{E},k}(t)$ and $\gamma_{\mathrm{I},k}(t)$, respectively. The pooled Poisson processes associated with the N_{E} excitatory and N_I inhibitory synaptic inputs are denoted by $S_{\mathrm{E}}(t)$ and $S_{\mathrm{I}}(t)$,

$$S_{\mathrm{E}}(t) = \sum_{k} S_{\mathrm{E},k}(t) , \quad S_{\mathrm{I}}(t) = \sum_{k} S_{\mathrm{I},k}(t) , \qquad (5)$$

with time-dependent spiking rates $\lambda_{\mathrm{E}}(t)$ and $\lambda_{\mathrm{I}}(t)$, respectively. These equations implicitly take into account the *pooling property*, also called the superposition property, whereby the statistical properties of the combined process are "locally random", in the sense that it appears random over periods of time that are small compared to the individual mean interspike times (Cox and Smith 1954).

Most studies have typically examined the situation in which the inputs on each synapse are in phase. These methods are also applicable when the individual synaptic inputs have the same frequency of oscillation but a non-zero phase relationship. This occurs naturally, e.g. in various nuclei of the auditory pathway due to the traveling-wave of the basilar membrane and the tonotopic organization of the auditory nerve, resulting in the convergence of synaptic inputs with varying phase relationships (Kuhlmann et al. 2002).

In the case of conductance synapses, the synaptic current is described by (Tuckwell 1979, 1988b)

$$I_s(t) = C_m \left[V_E - v(t) \right] \sum_{k=1}^{N_E} g_{E,k} \, S_{E,k}(t)$$

$$+ \, C_m \left[V_I - v(t) \right] \sum_{k=1}^{N_I} g_{I,k} \, S_{I,k}(t) \quad , \tag{6}$$

where the potentials V_E and V_I are the (constant) reversal potentials ($V_I \leq V_{reset} < V_{th} < V_E$). The reversal potentials, which introduce a nonlinearity into the summation of the individual synaptic inputs, arise from the equilibrium potentials of the ion channels. They are so named because the direction of associated current flow reverses when the membrane potential passes through the corresponding reversal potential. The parameters $g_{E,k}$, $g_{I,k} > 0$ represent the integrated inhibitory and excitatory conductances over the time course of the synaptic event divided by the neural capacitance and are thus dimensionless (they are non-negative and for convenience are taken here to be identical for all excitatory and inhibitory inputs respectively, so that the subscripts k in the above equation are subsequently dropped).

3 Stochastic models

As for the case with homogeneous synaptic input, the above mentioned pooling property Eq. 5 of independent renewal processes provides an excellent description of the combined input of inhomogeneous input to a large number of synapses. The Ornstein–Uhlenbeck model (Uhlenbeck and Ornstein 1930) provides a good description of inhomogeneous stochastic synaptic input (Tuckwell 1989),

$$\tau \, \frac{dv(t)}{dt} = -[v(t) - V_0] + \mu(t) + \sigma(t) \sqrt{2\tau} \, \xi(t) \, , \tag{7}$$

where $\xi(t)$ is Gaussian white noise, and the drift $\mu(t)$ and variance $\sigma(t)$ of the input can both have a dependence upon time. For a description of the derivation of the this equation and the range over which it is accurate, see Burkitt (2006). For simplicity, it will be assumed throughout that the membrane potential at the initial time t_0 is at the resting potential V_0 and that this is also the reset potential: $v(t_0) \equiv v_0 = V_0 = V_{reset}$.

The time-dependent first and second moments, $\mu(t; v_0)$ and $\sigma^2(t; v_0)$, of the free or unrestricted membrane potential (i.e., neglecting the spiking threshold) are defined by

$$\mu(t; v_0) \equiv \mathrm{E}[v(t) | v_0, t_0 = 0] \tag{8}$$
$$\sigma^2(t; v_0) \equiv \mathrm{Var}[v(t) | v_0, t_0 = 0] \, .$$

Their evaluation requires the expectation value of a Poisson process $S(t)$ with intensity $\lambda(t)$ over the time interval Δt, and is given by

$$E[S(t + \Delta t)] = E\left[S^2(t + \Delta t) \right] = \lambda(t) \, \Delta t + o(\Delta t) \, . \tag{9}$$

An exposition of the properties of inhomogeneous Poisson processes is given in (Kempter et al. 1998, App. A).

Different neuron models have been considered, depending upon such issues as whether the periodic component is *endogenous* (i.e., the periodic input is reset to the same initial phase after each spike) or *exogenous* (i.e., without reset of the periodic input) (Lánský 1997), and whether the noise term is constant or contains a periodic component (Hohn and Burkitt 2001; Lindner and Schimansky-Geier 2001). The case of endogenous input is mathematically simpler to deal with, since it allows a straightforward application of the renewal theory (Bulsara et al. 1996; Plesser and Tanaka 1997; Shimokawa et al. 1999b; Plesser and Geisel 1999), but such a phase resetting is unrealistic for biological neural systems (apart from some special cases involving sensory systems) and will not be considered further here (for a review, see (Plesser and Geisel 2001)).

Consider the Ornstein–Uhlenbeck process, Eq. 7, with input $\mu(t)$ that is a periodic function with frequency ω given by

$$\mu(t) = \mu_0 + \mu_1 \, \cos(\omega \, t + \phi_0) \, , \tag{10}$$

where ϕ_0 is the phase of the input from which the synaptic integration commences. In the absence of noise ($\sigma = 0$), spikes will only be generated if

$$v_\infty = \lim_{t \to \infty} v(t)$$

$$= V_0 + \tau_m \left(\mu_0 + \frac{\mu_1}{\sqrt{1 + \omega^2 \, \tau_m^2}} \right) > V_{th} \, , \tag{11}$$

where the $t \to \infty$ limit is taken over the local maxima of $v(t)$. Inputs for which $v_\infty \leq V_{th}$ are called sub-threshold or otherwise are called supra-threshold (although strictly speaking they are only sub-threshold in the noiseless limit).

The correspondence between the Ornstein–Uhlenbeck diffusion equation and the discontinuous Stein model, defined by Eqs. 1 and 3, with periodic synaptic input proceeds in the same way as the case with homogeneous Poisson synaptic input (Burkitt 2006). The drift and diffusion coefficients $\mu(t)$ and $\sigma(t)$ of the Ornstein–Uhlenbeck diffusion equation are identified as the first and second moments of the free membrane potential in the large time limit (Lánský 1997), since in the diffusion limit the higher moments vanish. In the case of current

synapses, the conditional mean and variance of the free membrane potential, Eq. 8, are given by

$$\mu(t; v_0) = v_0\, e^{-t/\tau_{\mathrm{m}}} + \int_0^t \mathrm{d}t'\, e^{(t'-t)/\tau_{\mathrm{m}}}$$

$$\times \left[\sum_{k=1}^{N_{\mathrm{E}}} a_{\mathrm{E},k}\, \gamma_{\mathrm{E},k}(t) - \sum_{k=1}^{N_{\mathrm{I}}} a_{\mathrm{I},k}\, \gamma_{\mathrm{I},k}(t) \right], \quad (12)$$

and

$$\sigma^2(t; v_0) = \int_0^t \mathrm{d}t'\, e^{2(t'-t)/\tau_{\mathrm{m}}}$$

$$\times \left[\sum_{k=1}^{N_{\mathrm{E}}} a_{\mathrm{E},k}^2\, \gamma_{\mathrm{E},k}(t) + \sum_{k=1}^{N_{\mathrm{I}}} a_{\mathrm{I},k}^2\, \gamma_{\mathrm{I},k}(t) \right]. \quad (13)$$

An alternative formulation uses the Gaussian approximation, which proceeds by considering the evolution of the free membrane potential (Burkitt and Clark 2001). The conditional probability density of the free membrane potential $p(v, t \mid v_0, t_0)$, which is the probability that it has the value v at time t given that it had the value v_0 at some earlier time t_0, is parameterized as (Burkitt and Clark 2000)

$$p(v, t \mid v_0, 0)$$
$$= \frac{1}{\sqrt{2\pi\, \sigma^2(t; v_0)}}\, \exp\left\{ -\frac{[v - \mu(t; v_0)]^2}{2\, \sigma^2(t; v_0)} \right\}. \quad (14)$$

This approach allows $\mu(t; v_0)$ and $\sigma^2(t; v_0)$ to be evaluated using a self-consistent formalism, which yields the expressions in Eqs. 12 and 13. The conditional dependence of $p(v, t \mid v_0, t_0)$ upon the periodicity, ω, and the initial phase, ϕ, has been suppressed in order to avoid making the notation cumbersome.

In order to model biologically realistic neurons with a large number of small amplitude synaptic inputs with current-based synapses, each with (roughly) the same amplitude, the following distribution of current synapses is considered (Lánský 1997). The N_{E} excitatory synapses are partitioned into a set of N_{B} background synapses and a smaller set of N_{F} foreground synapses, with $N_{\mathrm{E}} = N_{\mathrm{B}} + N_{\mathrm{F}}$, $N_{\mathrm{B}} \sim N_{\mathrm{I}}$, and $N_{\mathrm{F}} \sim \sqrt{N_{\mathrm{B}}}$ (Note: this relationship between the number of foreground and background synapses ensures the existence of the Ornstein–Uhlenbeck limit of the jump process in the diffusion limit). The synaptic inputs, membrane potential, and spike outputs are illustrated in Fig. 1 for an integrate-and-fire neuron with current synapses receiving both foreground periodic and background stationary synaptic input. The figure shows that both the membrane potential and its variance have a periodic component arising from the foreground synaptic input. The inhibitory synapses and the background excitatory synapses correspond to the
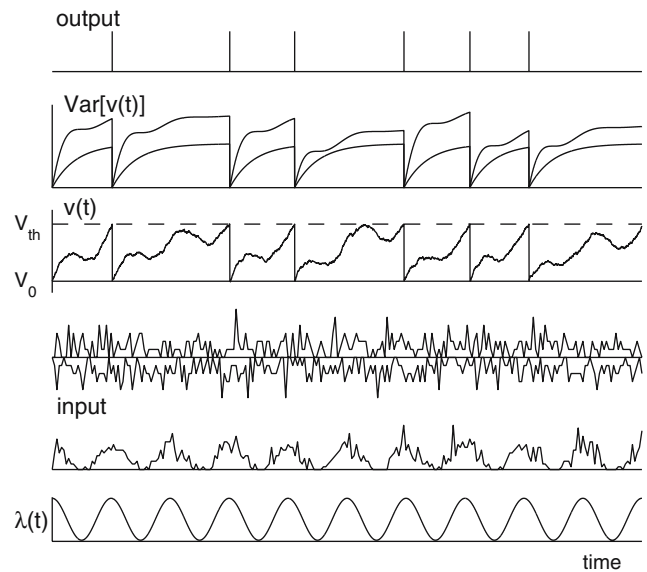

Fig. 1 Illustration of the integrate-and-fire neuron with current synapses receiving inhomogeneous input. The synaptic input consists of background input that is both excitatory (*above the time axis*) and inhibitory (*below*), as well as foreground periodic excitatory input, with inhomogeneous Poisson rate $\lambda(t)$. The membrane potential, $v(t)$, is plotted, showing the output spikes generated (*top plot*) when the potential reaches threshold, V_{th}, and the subsequent reset, $V_{\mathrm{r}} = V_0$. Also plotted is the theoretical value for the variance of the membrane potential – the lower line is the contribution arising from the background synaptic inputs (no periodicity) and the upper line is the total variance (that includes the foreground periodic component)

spontaneous activity and their spiking-rates are taken to be homogeneous Poisson processes whose average contributions cancel each other. Consequently they do not contribute to the drift $\mu(t; v_0)$ in Eq. 12, but their contributions summate in the diffusion term $\sigma(t; v_0)$ in Eq. 13. The remaining foreground synapses are responsible for the drift, $\mu(t)$, as well as contributing to the diffusion term, $\sigma(t)$, of the Ornstein–Uhlenbeck process.

3.1 The spiking rate and interspike interval distribution

The first passage time problem for the membrane potential has a conditional probability distribution, $f_\theta(t; \omega, \phi)$, that depends upon both the frequency, ω, of the input and its phase, ϕ, at the beginning of the interspike interval (Plesser and Geisel 1999). This conditional first passage time density obeys the renewal equation (Plesser and Tanaka 1997; Burkitt and Clark 1999)

$$p(V_{\mathrm{th}}, t \mid v_0, t_0) = \int_{t_0}^t \mathrm{d}t'\, f_\theta(t'; \omega, \phi)\, p(V_{\mathrm{th}}, t \mid V_{\mathrm{th}}, t'). \quad (15)$$

This formulation of the first passage time problem, due to Schrödinger (1915), involves splitting the trajectory of the freely evolving membrane potential into two sec-
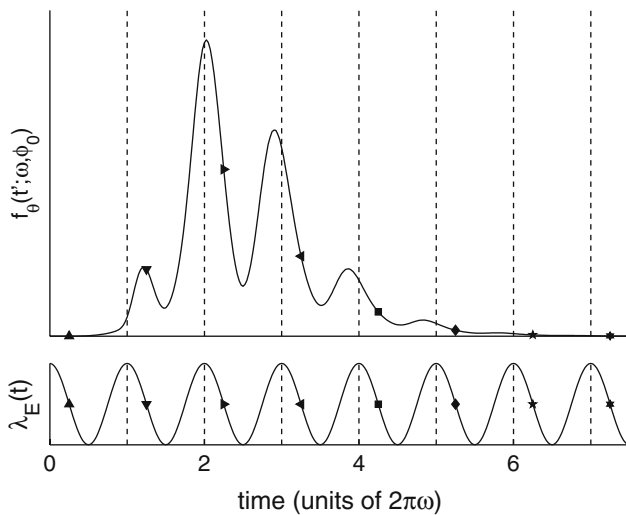
Fig. 2 Illustrative plot of the conditional first passage time distribution $f_\theta(t'; \omega, \phi_0)$ (*upper plot*) and the input spiking-rate $\lambda_E(t)$ (*lower plot*). The *vertical dashed lines* indicate multiples of the period of the input. The initial phase is $\phi_0 = 0$, and the *small symbols* indicate the contributions that sum in Fig. 3 at the successive phases of the input $\phi' = \pi/2$. (Taken from Burkitt and van Hemmen 2003)

Fig. 3 Illustrative plot of one column (*solid line*) of the phase transition density matrix G_{ϕ, ϕ_0} resulting from the first passage time density in Fig. 2 with $\phi_0 = 0$. The contributions from each individual period (*dashed lines*) are summed to give the total (*solid line*). The symbols correspond to the contributions at phase $\phi = \pi/2$, as illustrated in Fig. 2, and sum to give the value described by the *open circle*. (Taken from Burkitt and van Hemmen 2003)

tions as described in relation to homogeneous synaptic input in (Burkitt 2006, Sect. 4). In the situation here with inhomogeneous synaptic input, both the conditional first passage time density, $f_\theta(t'; \omega, \phi)$, and the conditional probability density, $p(v, t | v_0, t_0)$, have a dependence upon the periodicity, ω, and the initial phase, ϕ, of the periodic input. There are few results that can be derived completely analytically for this case with arbitrarily large amplitude of the periodic foreground synaptic input, apart from those discussed in the following section for the perfect integrator (a perturbative approach using the Fokker–Planck formalism for small amplitude periodic input is outlined in Sect. 3.2). Most results require the first passage time to be evaluated numerically from the renewal equation, which is a Volterra integral equation (Plesser and Tanaka 1997; Burkitt and Clark 2001). An illustrative plot of the resulting conditional first passage time distribution is shown in Fig. 2 together with the input spiking rate, $\lambda_E(t)$ (in this example there is no inhibition, and hence no background neural activity).

The interspike interval distribution, $\rho(t; \omega)$, that is generated by the inhomogeneous Poisson inputs is evaluated by taking the appropriate average over the initial phases, ϕ_0, of the conditional first passage time density

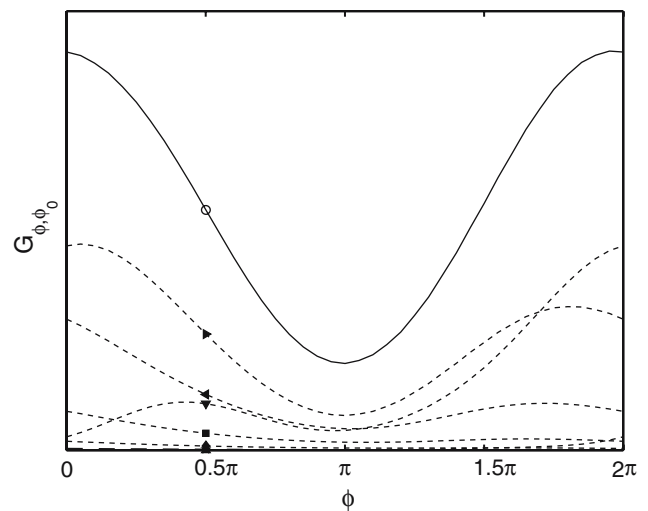$$\rho(t; \omega) = \int_0^{2\pi} \frac{d\phi_0}{2\pi} f_\theta(t; \omega, \phi_0) \chi^{(s)}(\phi_0) , \qquad (16)$$

where $\chi^{(s)}(\phi)$ is the stationary distribution of phases, evaluated as follows. A phase transition density may be defined by

$$G(\phi, \phi_0) := \int_0^\infty dt' f_\theta(t'; \omega, \phi_0) \delta([\omega t' + \phi_0]_{\bmod 2\pi} - \phi) ,$$

$$(17)$$

which gives the probability density for output spikes with phase ϕ when the initial phase is ϕ_0. This is illustrated in Fig. 3, which shows one column of the matrix, G_{ϕ, ϕ_0}, (obtained from the transition matrix by discretizing the phase) namely, for $\phi_0 = 0$, constructed using the first passage time density of Fig. 2. The small symbols in this plot show the contributions from $\phi = \pi/2$, which arise from the summation of successive periods, as illustrated by the corresponding symbols in Fig. 2.

This provides a spike phase representation of successive output spikes. The phase of the $(n + 1)^{\text{th}}$ spike is given by

$$\chi^{(n+1)}(\phi') = \int_0^{2\pi} \frac{d\phi}{2\pi} G(\phi', \phi) \chi^{(n)}(\phi) , \qquad (18)$$

where $\chi^{(n)}(\phi)$ to be the output spike phase density for the nth spike. This expression is essentially a recurrence relation between spike phases that describes the

successive phases of the spike times in terms of a Markov process. Consequently, the stationary spike phase density, $\chi^{(s)}(\phi)$, is given by the (nontrivial) solution of

$$\chi^{(s)}(\phi') = \int_0^{2\pi} \frac{d\phi}{2\pi} \, G(\phi', \phi) \, \chi^{(s)}(\phi) \, . \tag{19}$$

The stationary spike phase distribution vector, $\chi_\phi^{(s)}$, (in matrix notation) is the eigenvector corresponding to the unique eigenvalue 1 (Plesser and Geisel 1999). Alternatively, the stationary spike phase distribution may be obtained by iterating the recurrence relation Eq. 18 to its stable fixed-point (Shimokawa et al. 1999b).

The average output spiking rate, ν_{out}, is determined from the average interspike interval of the output spikes

$$\nu_{\text{out}} = \left[\tau_{\text{r}} + \int_0^{\infty} dt \, t \, \rho(t; \omega) \right]^{-1}$$
$$= \left[\tau_{\text{r}} + \int_0^{\infty} dt \int_0^{2\pi} \frac{d\phi_0}{2\pi} \, t \, f_\theta(t; \omega, \phi_0) \, \chi^{(s)}(\phi_0) \right]^{-1}. \tag{20}$$

Consequently, the time or ensemble averaged output spiking rate may be described as an inhomogeneous Poisson process with a periodic spiking rate given by the product of two terms,

$$\lambda_{\text{out}}(t) = \nu_{\text{out}} \, \chi^{(s)}(\phi) \, , \tag{21}$$

where $\lambda_{\text{out}}(t)$ is a periodic function with period $2\pi/\omega$ and $\phi = [\omega t]_{\text{mod } 2\pi}$. Consequently, ν_{out} gives the phase-averaged rate and $\chi^{(s)}(\phi)$ gives the phase dependence of the output spike distribution.

The degree of phase locking (or synchronization) of the neural response to periodic synaptic input is measured by the vector strength r (Goldberg and Brown 1969), also known as the synchronization index (Anderson 1973; Johnson 1980),

$$r = \left(r_S^2 + r_C^2 \right)^{1/2}$$
$$r_S = \int_0^{2\pi} d\phi \, \chi^{(s)}(\phi) \, \sin \phi \tag{22}$$
$$r_C = \int_0^{2\pi} d\phi \, \chi^{(s)}(\phi) \, \cos \phi \, .$$

This has been examined in the leaky integrate-and-fire neuron model with current synapses using a combination of both numerical simulations and analytical techniques (Kempter et al. 1998; Burkitt and Clark 2001). The

application of this formalism to particular models, namely both the perfect integrator and the leaky integrate-and-fire neuron, now follows.

3.1.1 The perfect integrator neuron model:

The solution for the perfect integrator neuron model with excitatory synaptic input described by an inhomogeneous Poisson process is derived from the solution for the homogeneous case, reviewed in (Burkitt 2006, Sect. 4.1). It follows by noting that an inhomogeneous Poisson process with a time-dependent spiking rate, $\lambda_{\text{E}}(t)$, can be converted to a standard temporally homogeneous Poisson process by the change in time scale (Tuckwell 1989)

$$\hat{t} = \int_0^t \lambda_{\text{E}}(t') \, dt' \equiv \Lambda(t) \, . \tag{23}$$

This transformation explicitly gives the conditional output spike density at time t

$$f_\theta(t; \omega, \phi_0) = \lambda_{\text{E}}(t) \, \hat{f}_\theta(\Lambda(t))$$
$$= \frac{\theta \, \lambda_{\text{E}}(t)}{\sqrt{2\pi \, a_{\text{E}}^2 \, \Lambda^3(t)}} \exp \left\{ -\frac{(\theta - a_{\text{E}} \, \Lambda(t))^2}{2 \, a_{\text{E}}^2 \, \Lambda(t)} \right\}, \tag{24}$$

where $\theta = V_{\text{th}} - V_{\text{reset}}$. This expression is valid for any inhomogeneous Poisson process with spiking rate $\lambda_{\text{E}}(t) > 0$, taken here to be periodic with frequency, ω, and initial phase, ϕ_0. It is straightforward to extend this result to include inhibitory synaptic inputs, so long as the inhibition has the same spiking-rate function, $\lambda_{\text{I}}(t) = \lambda_{\text{E}}(t)$, by the substitution of $a_{\text{E}}^2 \rightarrow (a_{\text{E}}^2 + a_{\text{I}}^2)$ in both denominators and $a_{\text{E}} \rightarrow (a_{\text{E}} - a_{\text{I}})$ in the numerator of the exponential.

The interspike interval distribution, $\rho(t; \omega)$, is obtained by the appropriate average over the initial phase using the stationary spike phase density of the phase transition matrix, as described in Eqs. 16–19. Note that for the perfect integrator model the synchronization index of the output spikes is exactly equal to the synchronization index of the inputs.

3.1.2 The leaky integrate-and-fire neuron model:

In the case of the leaky integrate-and-fire neuron model with inhomogeneous Poisson synaptic input and current synapses, the drift $\mu(t)$ and diffusion terms $\sigma(t)$ of the Ornstein–Uhlenbeck process have the values (cf. Eqs. 12, 13), (Lánský and Sato 1999; Burkitt and Clark 2001)

$$\mu(t) = \int_0^t dt' \, e^{(t'-t)/\tau_m} a_E \, \lambda_F(t)$$
$$\sigma^2(t) = \int_0^t dt' \, e^{2(t'-t)/\tau_m} \left[a_E^2 \left(\lambda_F(t) + \lambda_B \right) + a_I^2 \, \lambda_I \right], \quad (25)$$

where $\lambda_F(t)$, λ_B are the pooled Poisson spiking rates of the excitatory foreground and background synapses respectively, $\lambda_F(t)$ is the time-dependent spiking rate of the excitatory inhomogeneous Poisson synaptic inputs (i.e., periodic with angular frequency ω: $\lambda_F(t + 2\pi/\omega) = \lambda_F(t)$), and λ_I is the pooled spiking rate of the inhibitory synaptic inputs, typically taken to be described by a homogeneous Poisson process (see discussion following Eq. 14). Note that for inhomogeneous Poisson synaptic input this gives a time-dependent diffusion coefficient $\sigma(t)$ in the Ornstein–Uhlenbeck equation, Eq. 7.

The leaky integrate-and-fire neuron with current synapses has been investigated using the above methods (Burkitt and Clark 2001; Hohn and Burkitt 2001). In the situation where there are only excitatory inputs, the results provide the relationship between the input and output spiking rates, as well as the input and output synchronization, as measured by the vector strength, Eq. 22, (Burkitt and Clark 2001). The model in which there is an excitatory foreground input with spontaneous background (excitatory and inhibitory) activity has been studied in the context of stochastic resonance (Plesser and Geisel 1999; Hohn and Burkitt 2001) and the results are discussed in Sect. 4.

An approximation that is frequently used is to take the diffusion term to be constant (i.e. its average value). The rationale for this approximation is that the major contribution to the variance is from the background synapses (since $N_F \sim \sqrt{N_B} \ll N_B$), and the background input is constant, as illustrated in Fig.1. While this is often a good approximation, a number of studies have examined the model with time-dependent diffusion, using the Gaussian approximation (Burkitt and Clark 2001), the Ornstein–Uhlenbeck approach (Hohn and Burkitt 2001), and the Fokker–Planck formalism (Lindner and Schimansky-Geier 2001; Giraudo and Sacerdote 2005), and found that there are important differences. The modulation of the diffusion has been shown, for example, to provide better signal processing capabilities in a study of stochastic resonance (Hohn and Burkitt 2001), as discussed in the following section.

The integrate-and-fire neuron model with conductance synapses for inhomogeneous synaptic inputs can be analyzed in the Gaussian approximation using the approach outlined above (Burkitt and van Hemmen 2003). After an output spike is generated, the membrane potential approaches a (time-dependent) equilibrium value, $\mu_O(t)$, about which it fluctuates with variance $\sigma_O^2(t)$. The membrane potential approaches $\mu_O(t)$ with an effective membrane time constant, τ_O, that is different from the passive membrane time constant due to the effect of the synaptic conductances. In the case of homogeneous δ-function synaptic inputs, the time-independent values of μ_O, σ_O^2, and τ_O are (Hanson and Tuckwell 1983; Burkitt 2001)

$$\mu_O = \tau_O \left(\frac{V_p}{\tau_p} + r_{11} \right)$$
$$\sigma_O^2 = \frac{\mu_O^2 r_{20} - 2\mu_O r_{21} + r_{22}}{2/\tau_O - r_{20}} \quad (26)$$
$$\frac{1}{\tau_O} = \frac{1}{\tau_p} + r_{10} ,$$
$$r_{kl} := \lambda_I \, g_I^k \, V_I^l + \lambda_E \, g_E^k \, V_E^l , \quad (27)$$

where $(kl) = \{(10), (11), (20), (21), (22)\}$. The parameter μ_O represents the average equilibrium value of the membrane potential in the absence of spike generation. It arises naturally from a balance of the net input (i.e., sum of the excitatory and inhibitory inputs) and decays with time (i.e., leakage with time constant τ_O).

With inhomogeneous synaptic input the constants, r_{kl}, become time-dependent functions, $r_{kl}(t)$, defined as

$$r_{kl}(t) = \lambda_I \, g_I^k \, V_I^l + (\lambda_F(t) + \lambda_B) \, g_E^k \, V_E^l , \quad (28)$$

where $\lambda_F(t)$, λ_B, and λ_I are as defined following Eq. 25. The solutions to these equations in the diffusion approximation are (Burkitt and van Hemmen, 2003)

$$\mu(t; v_0, \phi_0)$$
$$= e^{-t/\tau} \left\{ v_0 + \int_0^t dt' \left[\frac{V_p}{\tau_p} + r_{11}(t') \right] e^{t'/\tau} \right\}$$
$$\sigma^2(t; v_0, \phi_0) \quad (29)$$
$$= e^{-2t/\tau} \int_0^t dt' \Big[r_{20}(t') \, \mu^2(t'; v_0, \phi_0)$$
$$- 2r_{21}(t') \, \mu(t'; v_0, \phi_0) + r_{22}(t') \Big] e^{2t'/\tau},$$

where τ is the effective time constant, τ_O. The dependence upon ϕ_0 arises because this is the phase of the periodic synaptic input, $\lambda_E(t)$, at the start of the integration time. The analysis then proceeds exactly as for current synapses.

3.2 Fokker–planck formalism

An alternative formalism is the Fokker–Planck equation, which describes the time evolution of the probability density $P(v, t)$ of the membrane potential (van Kampen 1992; Risken 1996)

$$\frac{\partial}{\partial t} P(v, t) = \left[-\frac{\partial}{\partial v} A(v, t) + \frac{1}{2} \frac{\partial^2}{\partial v^2} B(v, t) \right] P(v, t), \quad (30)$$

where $A(v, t)$ is the drift function and $B(v, t)$ is the diffusion function, which are the first two moments of the distribution of the independent jumps in the membrane potential due to the stochastic synaptic input. For the leaky integrate-and-fire neuron model with current synapses, Eq. 7, these functions are given by (Brunel and Hakim 1999)

$$A(v, t) = -\frac{1}{\tau}(v - V_0 - \mu(t)), \quad B(v, t) = \frac{2\sigma^2(t)}{\tau}, \quad (31)$$

where $\tau = \tau_{\mathrm{m}}$ and $\mu(t)$, $\sigma(t)$ are given in Eq. 25. The diffusion term, σ, is typically taken to be constant by assigning its average value, as discussed in Sect. 3.1.2. Brunel and colleagues have developed a perturbative method based upon the Fokker–Planck equation (Brunel and Hakim 1999; Brunel et al. 2001, 2003; Fourcaud and Brunel 2002). They consider an oscillating input of the form

$$\mu(t) = \mu \{1 + \epsilon \, r(\omega) \cos[\omega t + \Phi(\omega)]\}, \quad (32)$$

where ϵ is the small parameter of the perturbation expansion, denoting the ratio of the stationary and oscillating contributions. The drift coefficient μ and diffusion coefficient σ are related to the foreground and background spiking rates by

$$\mu = \tilde{\mu} \, \tau_{\mathrm{m}}, \quad \tilde{\mu} = a_{\mathrm{E}} \, \overline{\lambda}_{\mathrm{F}}$$
$$\sigma^2 = \tilde{\sigma}^2 \, \tau_{\mathrm{m}}, \quad \tilde{\sigma}^2 = \frac{1}{2} \left[a_{\mathrm{E}}^2 \, (\overline{\lambda}_{\mathrm{F}} + \lambda_{\mathrm{B}}) + a_{\mathrm{I}}^2 \, \lambda_{\mathrm{I}} \right], \quad (33)$$

where $\overline{\lambda}_{\mathrm{F}}$ is the time average of $\lambda_{\mathrm{F}}(t)$. The Fokker–Planck equation can be split into a continuity equation for the probability density

$$\frac{\partial}{\partial t} P(v, t) = -\frac{\partial J(v, t)}{\partial v}, \quad (34)$$

and a constitutive equation for the probability flux $J(v, t)$

$$J(v, t) = A(v, t) \, P(v, t) - \frac{1}{2} \frac{\partial}{\partial v} B(v) \, P(v, t)$$
$$= -\frac{1}{\tau}(v - \mu(t)) \, P(v, t) - \frac{\sigma^2}{\tau} \frac{\partial}{\partial v} P(v, t), \quad (35)$$

which gives the probability current through v at time t (Risken 1996). The instantaneous spiking rate is given by the flux through the threshold

$$\lambda_{\mathrm{out}}(t) = J(V_{\mathrm{th}}, t). \quad (36)$$

The boundary conditions need to be specified; at the lower boundary the conditions are

$$\lim_{v \to -\infty} P(v, t) = 0, \quad \lim_{v \to -\infty} v P(v, t) = 0, \quad (37)$$

at the threshold, V_{th}, there is an absorbing boundary condition

$$P(V_{\mathrm{th}}, t) = 0, \quad \frac{\partial}{\partial v} P(V_{\mathrm{th}}, t) = -\frac{\lambda_{\mathrm{out}}(t) \, \tau}{\sigma^2}, \quad (38)$$

and at the reset potential, V_{reset}, the flux from the threshold flows in as a result of the reset mechanism

$$J(V_{\mathrm{reset}}^+, t) - J(V_{\mathrm{reset}}^-, t) = \lambda_{\mathrm{out}}(t - \tau_{\mathrm{r}}). \quad (39)$$

The probability distribution also has to satisfy the normalization condition

$$\int_{-\infty}^{V_{\mathrm{th}}} \mathrm{d}v \, P(v, t) + p_{\mathrm{r}}(t) = 1, \quad (40)$$

where $p_{\mathrm{r}}(t) = \int_{t-\tau_{\mathrm{r}}}^{t} \mathrm{d}u \, \lambda_{\mathrm{out}}(u)$ is the probability of the neuron being refractory at time t. The strategy is to solve the Fokker–Planck equation, Eq. 30, with these boundary conditions to first order in the parameter ϵ (using complex input notation for simplicity) (Brunel 2000; Fourcaud and Brunel 2002)

$$\lambda_{\mathrm{out}}(t) = \lambda_0 \left[1 + \epsilon \, \hat{n}_0(\omega) \, \mathrm{e}^{i\omega t} + O(\epsilon^2) \right]$$
$$P_0(v, t) = P_0(v, t) + \epsilon \, \mathrm{e}^{i\omega t} \, \hat{P}_0(v, \omega) + O(\epsilon^2), \quad (41)$$

where $\hat{n}_0(\omega) = r_0(\omega) \exp[i\Phi(\omega)]$ and $\hat{P}_0(v, \omega)$ are complex quantities describing the oscillatory component with frequency ω of the instantaneous spiking rate of the synaptic input and the voltage probability density, respectively.

For the perfect integrator with Gaussian white noise (i.e., δ-function synaptic inputs) the drift and diffusion functions are given by $\tilde{\mu}$ and $\tilde{\sigma}$, as defined in Eq. 33. The expressions for $r(\omega)$ and $\hat{n}_0(\omega)$ are given by (Abbott and van Vreeswijk 1993; Fourcaud and Brunel 2002)

$$r(\omega) = \frac{1 + \sqrt{1 + 2i\tau_{\mathrm{e}}\omega}}{2}, \quad \hat{n}_0(\omega) = \frac{\sqrt{1 + 2i\tau_{\mathrm{e}}\omega} - 1}{i\tau_{\mathrm{e}}\omega}, \quad (42)$$

where $\tau_{\mathrm{e}} = 2\tilde{\sigma}^2/\tilde{\mu}^2$ [see Fourcaud and Brunel (2002) for the expression for $\hat{P}_0(v, \omega)$]. Consequently, the neuron attenuates inputs at high frequencies by a factor $\sqrt{\frac{2}{\tau_{\mathrm{e}}\omega}}$, with a phase lag that tends to $-\frac{\pi}{4}$. At frequencies lower than $\frac{1}{\tau_{\mathrm{e}}}$ there is little attenuation and only a small phase lag.

For the leaky integrate-and-fire neuron model with current synapses and Gaussian white noise, the full expression for $\hat{n}_0(\omega)$ has been evaluated (Brunel and Hakim 1999; Brunel et al. 2001; Lindner and Schimansky-Geier 2001; Fourcaud and Brunel 2002), and in the high-frequency limit behaves as $\hat{n}_0(\omega) \sim \sqrt{\frac{2}{i\tau_{\mathrm{e}}\omega}}$, which is the same behavior as for the perfect integrator. This analysis using the Fokker–Planck equation and an ϵ expansion

for small amplitude modulations of a current synaptic input has been extended to examine the model with finite synaptic time constants (i.e., colored noise) (Fourcaud and Brunel 2002). The analysis is carried out using an expansion in small value of the ratio $k = \sqrt{\tau_s/\tau_m}$, where τ_s is the synaptic time constant. The results indicate that the spiking-rate modulation due to an oscillatory input remains finite in the high-frequency limit with no phase lag.

The results of these studies in relation to stochastic resonance are discussed in the following section. A general theory for deriving the moments of the first passage time for the Ornstein–Uhlenbeck process in the presence of a weak time-dependent drift using a Fokker–Planck approach has been developed (Lindner 2004). While most studies involve the use of either a weak stimulus, thus enabling the use of linear response theory, or weak noise, a recently proposed method that is based upon discrete state Markovian modeling provides results that show good agreement over a wide parameter range that extends beyond the weak driving limit (Schindler et al. 2004, 2005).

4 Stochastic resonance

Recent interest in the response of the integrate-and-fire neuron model to noisy periodic synaptic input has in part been motivated by the existence of stochastic resonance in these systems. Stochastic resonance occurs when the detection of a subthreshold periodic stimulus is enhanced by the presence of noise (Fauve and Heslot 1983). Although originally proposed as an explanation for ice-ages (Benzi et al. 1981), it has been found to play a role in the sensory pathways of various organisms (Douglas et al. 1993; Braun et al. 1994). Stochastic resonance has been extensively studied by analyzing the effect of a noisy weak periodic stimulus upon a system in a double-well potential, which has two stable solutions in the absence of both noise and external forcing. If the periodic external forcing (signal) is not sufficiently large to overcome the potential barrier then the system remains in its initial state. In the presence of noise, the system may jump between the two states at random times. Such transitions are found to be correlated with the (subthreshold) signal and cause a peak in the power spectrum at the frequency of the signal. A review of stochastic resonance is given in Gammaitoni et al. (1998).

In neural systems stochastic resonance is typically quantified by the improvement that it causes in measures of information transmission such as the signal-to-noise ratio (Chapeau-Blondeau et al. 1996), cross-correlations (Heneghan et al. 1996), or mutual information between the input and output (Bulsara and Zador 1996). The phenomena of stochastic resonance was demonstrated in neural systems by using external noise applied to crayfish mechanoreceptor cells (Douglas et al. 1993), and this sparked its discovery and examination in numerous other neural systems (Lindner et al. 2004). Consequently this generated considerable interest in the analysis of stochastic resonance in neural models, including the perfect integrator (Bulsara et al. 1994), the Fitzhugh–Nagomo model (Longtin 1993; Chialvo et al. 1997), a bistable neuron model (Longtin et al. 1994), and the leaky integrate-and-fire neuron model (Bulsara et al. 1996; Stemmler 1996). These studies typically involved both analytical and numerical investigations of an Ornstein–Uhlenbeck process with a drift function consisting of a constant contribution and a much weaker periodic contribution, and a noise of constant amplitude. However, much of the early work involved the analysis of the more mathematically tractable situation in which the phase of the input stimulus is reset after each output spike is generated (Plesser and Tanaka 1997), a situation which is implausible in the neurophysiological context. The method for solving this shortcoming, which involves the appropriate phase averaging, is outlined in Sect. 3 (Plesser and Geisel 1999; Shimokawa et al. 1999b).

While most studies of stochastic resonance have focussed upon the response of single neurons, there have been a number of studies of neuronal populations (Shimokawa et al. 1999a; Spiridon and Gerstner 1999; Lindner and Schimansky-Geier 2001), and these studies show that the population can reliably transmit fast periodic signals with a resolution that is better than an individual neuron. This increase in the temporal resolution at the population level is largely determined by the total rate of synaptic input. However, this population code can be enhanced by strong inhibitory couplings, which can considerably reduce the noise level in certain frequency bands and increase the temporal resolution (Spiridon and Gerstner 1999).

Stochastic resonance has also been studied with synaptic input that generates colored noise in the membrane potential, such as arises with finite synaptic time constants and temporal correlations in the synaptic input (Brunel et al. 2001; Fourcaud and Brunel 2002). Colored noise considerably alters the dynamics and expression of stochastic resonance in the integrate-and-fire neuron model. Using a Fokker–Planck formalism, these studies showed that the amplitude of the modulation of the spiking rate remains finite even in the large frequency limit, whereas it vanishes in this limit with instantaneous synaptic dynamics (i.e., Gaussian white noise synaptic input). Moreover, the phase lag of the modulated response is $\frac{\pi}{4}$ for instantaneous synaptic dynamics,

whereas it vanishes for realistic synaptic dynamics. Colored noise has subsequently also been investigated in a study of information processing in which the ability of a neuron to detect transient inputs was examined (Wenning et al. 2005), and the results likewise indicate that stochastic resonance is enhanced. In order to take account of the spatial extent of the dendritic tree a two-compartment leaky integrate-and-fire neuron model, comprising a dendritic compartment and trigger zone, has been investigated (Rodriguez and Lánský 2000). This study of the phase locking properties of the model with stochastic sub-threshold periodic stimulation found that stochastic resonance is enhanced, both in terms of the amplitude of the response and the range of values of the noise over which it is observed. The effect of refractory and adaptation have also been examined (Gedeon and Holzer 2004), and the phase-locking behavior has been shown to be robust when these effects are included.

The traditional treatment of the diffusion approach with the Ornstein–Uhlenbeck process uses a noise component that is constant in time. However, in a neuron with a finite number of synapses the variance of the membrane potential is modulated by the input signal. The effect of such signal-dependent noise has been investigated in the leaky integrate-and-fire neuron model with current synapses and is found to enhance stochastic resonance and the signal processing capabilities (Hohn and Burkitt 2001; Lindner and Schimansky-Geier 2001; Giraudo and Sacerdote 2005). A phenomena related to, but distinct from, stochastic resonance, namely a sub-threshold and spiking-rate resonance, has been found in a generalized leaky integrate-and-fire neural model (Brunel et al. 2003).

There is considerable ongoing interest in the function of stochastic resonance in neural systems. One viewpoint is that stochastic resonance is simply a passive means by which organisms can extend the sensitivity of their sensory neural mechanisms. However, it is also possible that stochastic resonance is used actively, i.e., there may be mechanisms to adjust the amount of noise in the neural system so that the sensitivity of the neural output is optimally adjusted (Wenning and Obermayer 2003).

5 Networks of integrate-and-fire neurons

This review has focussed on analytical methods for understanding the properties of a single integrate-and-fire neuron. However, the behavior of such neurons within a network may have a much richer structure. It is clearly beyond the scope of this review to provide a comprehensive overview of the diverse literature on the network behavior of neurons. Nevertheless it is possible to outline some of the main areas of network behavior in which the study of integrate-and-fire neurons have played an important role.

Early studies of the network behavior of integrate-and-fire neurons (Amit and Tsodyks 1991a,b; Gerstner and van Hemmen 1992) analyzed their behavior in terms of their suitability as models of associative memory (Hopfield 1982). In these models, the spatial patterns of self-sustained spiking through recurrent connections represent stored memories that the neuronal dynamics retrieves. A number of studies, some of which used a discrete-time formalism, examined the general nature of the patterns of spiking behavior that such networks produce (van Vreeswijk and Abbott 1993; Usher et al. 1993; Golomb and Rinzel 1993; Usher et al. 1994). Using a combination of analytical techniques (such as the mean-field approach) and computer simulations, they found that the network behavior is characterized by stable spiking patterns.

5.1 Synchronization and oscillations within a network

Biological neural systems may exhibit coherent oscillations, such as the theta rhythm of the hippocampus and the various EEG rhythms of the cortex, and these have been postulated to play an important role in neural information processing (Singer 1993). These large scale oscillations are generally regarded as being due to the interaction of excitatory and inhibitory neurons, and detailed neuronal models have been constructed for both the theta rhythm (Tsodyks et al. 1996) and the EEG signal (Rennie et al. 2002). The global synchronization of coupled oscillators (Winfree 1967) was solved analytically by Kuramoto, who showed that global synchronization occurs in the limit of a large number of identical oscillators with small couplings and subject to small noise (Kuramoto 1984, 1991) [for a review see (Strogatz 2000)]. These results were extended to integrate-and-fire neurons in the case of identical neurons, where it was shown that perfect synchronization occurs (i.e., zero phase differences) in a finite time (Mirollo and Strogatz 1990).

Gerstner and co-workers provided a general unified theory of phase-locking in a globally coupled network of integrate-and-fire neurons, incorporating transmission delays and using the spike-response model. The dynamics was solved exactly in the limit of a large number of units (Gerstner and van Hemmen 1993; Ritz et al. 1994; Gerstner 1995). Their results established that for a spatially homogeneous network of neurons to exhibit coherent oscillations that are asymptotically stable, the postsynaptic potential must increase in time

as the neurons fire. Conversely, if the postsynaptic potential is decreasing, then the oscillations are unstable (Gerstner et al. 1996b). The reliability with which a population of spiking neurons can transmit a continuous-time signal was investigated by examining the noise spectrum using both analytical methods and numerical simulations (Spiridon and Gerstner 1999). The time evolution of a population of integrate-and-fire neurons was analyzed using an integral equation approach, similar to that of earlier studies (Knight 1972a; Wilson and Cowan 1972), that is valid for both weak and strong coupling. In the low-noise regime, it was found that transitions between the asynchronous and the synchronized state occur almost immediately, which indicates that a population of neurons can transmit information fast and reliably (Gerstner 2000). The oscillation frequency of a randomly connected network of integrate-and-fire neurons with realistic synaptic dynamics has been derived (Brunel and Wang 2003).

The results on the synchronization of network activity by Mirollo and Strogatz (1990) can be extended to take into account such effects as synaptic filtering (Hansel et al. 1995), axonal transmission delays (Ernst et al. 1995), and different intrinsic frequencies and thresholds of the neurons, as well as varying the couplings (but so long as they remain positive) (Senn and Urbanczik 2000). A network of noisy integrate-and-fire neurons has recently been studied using numerical simulations of the Fokker–Planck equation, where a propagating pulse state appears (Sakaguchi 2004).

Another method employed to investigate the behavior of a network of neurons is the population density approach, originally proposed for the study of non-interacting populations of neurons (Knight 1972a,b), and recently extended to the study of interacting linear integrate-and-fire neurons (Fusi and Mattia 1999) using a mean-field approach based upon the Fokker–Planck formalism (Nykamp and Trachina 2000; Mattia and Del Giudice 2002, 2004).

5.2 Traveling pulses of activity within a network

Experimental studies in slices of neural tissue have observed the propagation of synaptically generated waves of activity [see Bressloff (1999) and references therein]. Models of the propagation of this activity in networks of coupled integrate-and-fire neurons have been developed and the velocity of the waves as a function of the parameters of the model analyzed (Bressloff and Coombes 1998; Ermentrout 1998; Kistler and van Hemmen 1998; Golomb and Ermentrout 1999). A stability analysis of these waves in a spatially distributed network of integrate-and-fire neurons has shown that the fast wave, even with small axonal delays, are stable, whereas the slow waves are unstable as a result of a Hopf bifurcation in the firing times (Bressloff 1999). A dynamical theory of this behavior in networks of integrate-and-fire neurons has been developed, which allows the study of the stability of phase-locked solutions in both the weak and strong coupling regimes (Bressloff and Coombes 2000; Bressloff 2000). The results obtained using the integrate-and-fire neuron model and using rate-based models have been shown to be closely related (Cremers and Herz 2002).

5.3 Recurrently connected networks

In order to understand the properties of a network of randomly interconnected excitatory and inhibitory neurons, Amit and Brunel (1997a,b) developed a self-consistent analysis to determine the spiking rates in the stationary states of the network dynamics. Subsequent work, using a self-consistent analysis of a sparsely connected inhibitory network of integrate-and-fire neurons based upon the Fokker–Planck formalism and a linear stability analysis of the stationary states, found a Hopf bifurcation line that separated regions with stationary and oscillatory global activity, where the neurons are weakly synchronized (Brunel and Hakim 1999). The oscillatory regime corresponds to strong inhibitory feedback, and the period of the global oscillations is mainly controlled by the synaptic time constants. The same methods were used to study a sparsely connected network of excitatory and inhibitory leaky integrate-and-fire neurons with current synapses (Brunel 2000). This study found a rich structure of dynamical states of the network, including both synchronous and asynchronous states as the external input and the ratio of excitation to inhibition was varied. This study was extended to a sparsely connected network with conductance synapses (Meffin et al. 2004), which gave results consistent with both experimental data and numerical simulations of a Hodgkin–Huxley model for quantities characterizing in vivo data (Destexhe and Paré 1999; Destexhe et al. 2001). Recent studies comparing the response of neocortical pyramidal neurons with integrate-and-fire neurons have likewise concluded that the model provides a good description of observed responses (Rauch et al. 2003; Giugliano et al. 2004).

5.4 Layered networks

One of the central questions concerning the behavior of networks of neurons is under what conditions the stable propagation of spike activity through successive layers of neurons is possible. This question has

been examined in a number of different contexts. The propagation of pulses of spike activity in a layered network has been examined using the spike-response model (but with a spiking probability that depends linearly upon the membrane potential, rather than an integrate-and-fire mechanism for spiking) to determine the conditions upon the feed-forward synaptic strengths that allow a spike packet with constant waveform to propagate (Kistler and Gerstner 2002). A study of the propagation of neural activity through a feed-forward network of integrate-and-fire neurons with conductance synapses and in the presence of spontaneous background spiking activity found that spiking-rate modulations are transmitted linearly through many layers with little distortion (van Rossum et al. 2002). Consequently such a network is capable of fast, robust, and accurate computation with a population code based upon spiking rates (van Rossum and Renart 2004). However, a subsequent study found that under realistic cortical conditions the neurons within a layer tend to synchronize and that a transmission of a rate code through many layers is highly unlikely (Litvak et al. 2003). This within-layer synchronization has also been observed in in vitro studies of propagation through multi-layered systems (Reyes 2003). There is clearly much to be understood about the properties of layered networks of neurons, which may require the development of new analytical techniques.

5.5 Learning with the integrate-and-fire neuron model

The traditional learning paradigm in biological neural systems is Hebbian rate-based learning, in which the synaptic modification is based upon correlations between input and output rates. Such models have been widely used to understand neural information processing, including models of memory (associative and working memory), sensory perception, motor control, and classical conditioning. One such model is the Hopfield model of associative memory (Hopfield 1982), which has been subsequently considerably elaborated as a model of working memory (Compte et al. 2000; Amit and Mongillo 2003; Mongillo and Amit 2003; Giudice et al. 2003; Mongillo and Amit 2005). In these models there is an interplay between the dynamics of the neurons (on a faster time scale) and the dynamics of the synaptic weights (on a slower time scale). The models contain a background state that is spontaneously active and that represents a global attractor of the neuronal dynamics, as well as a set of attractor states of the neuronal dynamics, corresponding to specific memories. This learning paradigm requires that these input-selective attractor states of the network dynamics (the "memories") are generated by the learning dynamics of the

weights. There has been considerable interest in such models to incorporate as much biological detail as possible, including the use of the spiking rate that results from the analysis of the integrate-and-fire neuron model (Amit and Tsodyks 1991a,b; Amit and Brunel 1997a,b). Likewise, it is possible to use the spiking rate of the integrate-and-fire neuron model in studies of artificial neural systems using supervised learning algorithms (Feng et al. 2003).

There is considerable experimental evidence that synaptic modification can depend upon the correlations in timing of the pre- and postsynaptic spikes [for a review of the experimental evidence see Bi and Poo (2001)]. The study of spike timing dependent plasticity (STDP), originally introduced in the context of sound localization in auditory processing (Gerstner et al. 1996a), requires an explicit neuron model for the relationship between synaptic input and spike output. Many of the advances have been made using the linear Poisson neuron model (Gerstner et al. 1996a; Kempter et al. 1999a), in which the probability of spike generation is described by an inhomogeneous Poisson process (van Hemmen 2001). However, the same methods used in some of these STDP analyzes of Poisson neurons have also been applied to the leaky integrate-and-fire neuron (van Rossum et al. 2000; Burkitt and van Hemmen 2003; Burkitt et al. 2004).

5.6 Numerical studies of integrate-and-fire neurons

Although this review focusses on analytical techniques for the integrate-and-fire neuron model, numerical simulations have also played an important role in their study, amongst the earliest of which is Segundo (1968). The numerical method of Hansel (1998), which consists of second-order Runge–Kutta time-stepping, a linear interpolation to accurately determine the spike times, and then using the spike times to recalibrate the post-spike potential, has been shown to be second-order in the time step size, but can be modified to obtain fourth-order accuracy (Shelley and Tao 2001). A particularly efficient way to simulate a large-scale recurrently connected network of neurons is to update the variables only when a synaptic input arrives. Such an event-driven simulation procedure explicitly uses the deterministic evolution of the membrane potential between successive inputs (Mattia and Del Giudice 2000). Population density methods provide an efficient alternative for the simulation of network dynamics (Knight et al. 1996, 2000; Omurtag et al. 2000; Nykamp and Trachina 2000; Knight 2000; Haskell et al. 2001. Simulations of the Ornstein–Uhlenbeck equation, Eq. 7, have also played an important role in understanding

the integrate-and-fire neuron model. Tuckwell and Lánský (1997) examine and compare simulations of this stochastic differential equation using the weak Euler scheme and normal pseudorandom numbers with the strong Euler scheme with Bernoulli pseudorandom numbers.

6 Conclusions

Because the integrate-and-fire neuron model is sufficiently simple to allow mathematical analysis, it is likely to play an important future role in understanding a number of mechanisms in neural information processing. This review has highlighted the role the model has played in understanding the response properties of neurons to inhomogeneous Poisson inputs and the behavior of multi-layered networks of neurons with recurrent connections. As is clearly apparent from this review, both of these areas still have a number of important open questions that can be addressed with this neuron model.

Another aspect of neural processing for which the analysis of integrate-and-fire neuron models may become more important is the analysis of temporal responses. To date, the analysis of temporal properties has largely been limited to synchronization (Maršálek et al. 1997; Burkitt and Clark 1999; Diesmann et al. 1999), coincidence detection (Abeles 1982; König et al. 1996; Joris et al. 1998; Kempter et al. 1999b), onset properties (Smith 1996; van Rossum 2001; Tiesinga and Sejnowski 2001), phase locking (Burkitt and Clark 2001; Tiesinga 2002), and the response to periodic stimuli (such as occurs in relation to phase locking to sensory stimuli (Cariani 1995) and in stochastic resonance, as discussed in Sect. 4 above). However, there are many other possible types of temporal behavior that may play a role in neural processing (Theunissen and Miller 1995; Cariani 2001). These include the study of the non-equilibrium dynamics of networks (Prete and Coolen 2004), dynamic attractors of the sort hypothesized to provide a model of olfactory processing (Laurent 2002), and synfire chains (Abeles 1991; Diesmann et al. 1996; Câteau and Fukai 2001). It is very possible that the analysis of integrate-and-fire neuron models may play an important role in understanding these processes.

References

Abbott LF, van Vreeswijk C (1993) Asynchronous states in networks of pulse-coupled oscillators. Phys Rev E 48:1483–1490

Abeles M (1982) Role of the cortical neuron: integrator or coincidence detector? Isr J Med Sci 18:83–92

Abeles M (1991) Corticonics: Neural circuits of the cerebral cortex. Cambridge University Press, New York

Amit DJ, Brunel N (1997a) Dynamics of a recurrent network of spiking neurons before and following learning. Netw Comput Neur Sys 8:373–404

Amit DJ, Brunel N (1997b) Model of global spontaneous activity and local structured activity during delay periods in the cerebral cortex. Cerebral Cortex 7:237–252

Amit DJ, Mongillo G (2003) Spike-driven synaptic dynamics generating working memory states. Neural Comput 15:565–596

Amit DJ, Tsodyks MV (1991a) Quantitative study of attractor neural network retrieving at low spike rates: I. Substrate - spikes, rates and neuronal gain. Netw Comput Neur Sys 2:259–273

Amit DJ, Tsodyks MV (1991b) Quantitative study of attractor neural network retrieving at low spike rates: II. Low-rate retrieval in symmetric networks. Netw Comput Neur Sys 2:275–294

Anderson DJ (1973) Quantitative model for the effects of stimulus frequency upon synchronization of auditory nerve discharges. J Acoust Soc Am 54:361–364

Benzi R, Sutera A, Vulpiani A (1981) The mechanism of stochastic resonance. J Phys A 14:L453–L457

Bi G-Q, Poo M-M (2001) Synaptic modification by correlated activity: Hebb's postulate revisited. Ann Rev Neurosci 24:139–166

Braun HA, Wissing H, Schaefer K, Hirsch MC (1994) Oscillation and noise determine signal transduction in shark multimodal sensory cells. Nature 367:270–273

Bressloff PC (1999) Synaptically generated wave propagation in excitable neural media. Phys Rev Lett 82:2979–2982

Bressloff PC (2000) Traveling waves and pulses in a one-dimensional networks of excitable integrate-and-fire neurons. J Math Biol 40:169–198

Bressloff PC, Coombes S (1998) Desynchronization, mode locking, and bursting in strongly coupled integrate-and-fire oscillators. Phys Rev Lett 81:2168–2171

Bressloff PC, Coombes S (2000) A dynamical theory of spike train transitions in networks of integrate-and-fire oscillators. SIAM J Appl Math 60:820–841

Brunel N (2000) Dynamics of sparsely connected networks of excitatory and inhibitory spiking neurons. J Comput Neurosci 8: 183–208

Brunel N, Hakim V (1999) Fast global oscillations in networks of integrate-and-fire neurons with low firing rates. Neural Comput 11: 1621–1671

Brunel N, Wang X-J (2003) What determines the frequency of fast network oscillations with irregular neural discharges? I. Synaptic dynamics and excitation-inhibition balance. J Neurophysiol 90:415–430

Brunel N, Chance FS, Fourcaud N, Abbott LF (2001) Effects of synaptic noise and filtering on the frequency response of spiking neurons. Phys Rev Lett 86:2186–2189

Brunel N, Hakim V, Richardson MJE (2003) Firing-rate resonance in a generalized integrate-and-fire neuron with subthreshold resonance. Phys Rev E 67:051916

Bulsara AR, Zador A (1996) Threshold detection of wideband signals: a noise-induced maximum in the mutual information. Phys Rev E 54:R2185–2188

Bulsara AR, Lowen SB, Rees CD (1994) Cooperative behavior in the periodically modulated Wiener process: noise-induced complexity in a model neuron. Phys Rev E 49:4989–5000

Bulsara AR, Elston TC, Doering CR, Lowen SB, Lindenberg K (1996) Cooperative behavior in periodically driven noisy integrate-fire models of neuronal dynamics. Phys Rev E 53:3958–3969

Burkitt AN (2001) Balanced neurons: analysis of leaky integrate-and-fire neurons with reversal potentials. Biol Cybern 85: 247–255

Burkitt AN (2006) A review of the integrate-and-fire neuron model: I. Homogeneous synaptic input. Biol Cybern DOI: 10.1007/s00422-006-0068-6

Burkitt AN, Clark GM (1999) Analysis of integrate-and-fire neurons: Synchronization of synaptic input and spike output in neural systems. Neural Comput 11:871–901

Burkitt AN, Clark GM (2000) Calculation of interspike intervals for integrate-and-fire neurons with Poisson distribution of synaptic inputs. Neural Comput 12:1789–1820

Burkitt AN, Clark GM (2001) Synchronization of the neural response to noisy periodic synaptic input. Neural Comput 13:2639–2672

Burkitt AN, van Hemmen JL (2003) How synapses in the auditory system wax and wane: Theoretical perspectives. Biol Cybern 89:318–332

Burkitt AN, Meffin H, Grayden DB (2004) Spike timing-dependent plasticity: The relationship to rate-based learning for models with weight dynamics determined by a stable fixed-point. Neural Comput 16:885–940

Cariani PA (1995) As if time really mattered: Temporal strategies for neural coding of sensory information. Commun Cog-Art Intell 12:157–219

Cariani PA (2001) Neural timing nets. Neural Netw 14:737–753

Câteau H, Fukai T (2001) Fokker–Planck approach to the pulse packet propagation in synfire chain. Neural Netw 14:675–685

Chapeau-Blondeau F, Godivier X, Chambet N (1996) Stochastic resonance in a neuron model that transmits spike trains. Phys Rev E 53:1273–1275

Chialvo DR, Longtin A, Müller-Gerking J (1997) Stochastic resonance in models of neuronal ensembles. Phys Rev E 55:1798–1808

Compte A, Brunel N, Goldman-Rakic PS, Wang X-J (2000) Synaptic mechanisms and network dynamics underlying spatial working memory in a cortical network model. Cereb Cortex 10:910–923

Cox DR, Smith WL (1954) On the superposition of renewal processes. Biometrika 41:91–99

Cremers D, Herz AVM (2002) Traveling waves of excitation in neural field models: equivalence of rate descriptions and integrate-and-fire dynamics. Neural Comput 14:1651–1667

Destexhe A, Paré D (1999) Impact of network activity on the integrative properties of neocortical neurons in vivo. J Neurophysiol 81:1531–1547

Destexhe A, Rudolph M, Fellous J-M, Sejnowski TJ (2001) Fluctuating synaptic conductances recreate in-vivo-like activity in neocortical neurons. Neuroscience 107:13–24

Diesmann M, Gewaltig MO, Aertsen A (1996) Characterization of synfire activity by propagating 'pulse packets'. In: Bower J (ed), Computational neuroscience: trends in research. Academic, San Diego pp 59–64

Diesmann M, Gewaltig MO, Aertsen A (1999) Stable propagation of synchronous spiking in cortical neural networks. Nature 402:529–533

Douglas JK, Wilkens L, Pantazelou E, Moss F (1993) Noise enhancement of information transfer in crayfish mechanoreceptors by stochastic resonance. Nature 365:337–340

Ermentrout GB (1998) Linearization of F-I curves by adaptation. Neural Comput 10:1721–1729

Ernst U, Pawelzik K, Geisel T (1995) Synchronization induced by temporal delays in pulse-coupled oscillators. Phys Rev Lett 74:1570–1573

Fauve S, Heslot F (1983) Stochastic resonance in a bistable system. Phys Lett 97A:5–7

Feng J, Sun Y, Buxton H, Wei G (2003) Training integrate-and-fire neurons with the informax principle II. IEEE Trans Neural Netw 14:326–336

Fourcaud N, Brunel N (2002) Dynamics of the firing probability of noisy integrate-and-fire neurons. Neural Comput 14:2057–2110

Fusi S, Mattia M (1999) Collective behavior of networks of linear (VLSI) integrate-and-fire neurons. Neural Comput 11:633–652

Gammaitoni L, Hänggi P, Jung P, Marchesoni F (1998) Stochastic resonance. Rev Mod Phys 70:223–287

Gedeon T, Holzer M (2004) Phase locking in integrate-and-fire models with refractory periods and modulation. J Math Biol 49:577–603

Gerstner W (1995) Time structure of the activity in neural network models. Phys Rev E 51:738–758

Gerstner W (2000) Population dynamics of spiking neurons: Fast transients, asynchronous states, and locking. Neural Comput 12:43–89

Gerstner W, van Hemmen JL (1992) Associative memory in a network of 'spiking' neurons. Netw Comput Neural Sys 3:139–164

Gerstner W, van Hemmen JL (1993) Coherence and incoherence in a globally coupled ensemble of pulse-emitting units. Phys Rev Lett 71:312–315

Gerstner W, Kempter R, van Hemmen JL, Wagner H (1996a) A neuronal learning rule for sub-millisecond temporal coding. Nature 383:76–78

Gerstner W, van Hemmen JL, Cowan JD (1996b) What matters in neuronal locking? Neural Comput 8:1653–1676

Giraudo MT, Sacerdote L (2005) Effect of periodic stimuluson a neuronal diffusion model with signal-dependent noise. BioSys 79:73–81

Giudice P Del, Fusi S, Mattia M (2003) Modelling the formation of working memory with networks of integrate-and-fire neurons connected by plastic synapses. J Physiol (Paris) 97:659–681

Giugliano M, Darbon P, Arsiero M, Lüscher HR, Streit J (2004) Single-neuron discharge properties and network activity in dissociated cultures of neocortex. J Neurophysiol 92:977–996

Goldberg JM, Brown PB (1969) Response of binaural neurons of dog superior olivary complex to dichotic tonal stimuli: some physiological mechanisms of sound localization. J Neurophysiol 32:613–636

Golomb D, Ermentrout GB (1999) Continuous and lurching traveling pulses in neuronal networks with delay and spatially decaying connectivity. Proc Natl Acad Sci 96:13480–13485

Golomb D, Rinzel J (1993) Dynamics of globally coupled inhibitory neurons with heterogeneity. Phys Rev E 48:4810–4814

Gummer AW (1991) Probability density function of successive intervals of a nonhomogeneous Poisson process under low-frequency conditions. Biol Cybern 65: 23–30

Hansel D, Mato G, Meunier C (1995) Synchrony in excitatory neural networks. Neural Comput 7:307–337

Hansel D, Mato G, Meunier C, Neltner L (1998) On numerical simulations of integrate-and-fire neural networks. Neural Comput 10:467–483

Hanson FB, Tuckwell HC (1983) Diffusion approximations for neuronal activity including synaptic reversal potentials. J Theor Neurobiol 2:127–153

Haskell E, Nykamp DQ, Trachina D (2001) Population density methods for large-scale modelling of neuronal networks with

realistic synaptic kinetics: cutting the dimension down to size. Netw Comput Neural Syst 12:141–174

Heneghan C, Chow CC, Collins JJ, Imhoff TT, Lowen SB, Teich MC (1996) Information measures quantifying aperiodic stochastic resonance. Phys Rev E 54:R2228–2231

Hohn N, Burkitt AN (2001) Shot noise in the leaky integrate-and-fire neuron. Phys Rev E 63:031902

Hopfield JJ (1982) Neural networks and physical systems with emergent collective computational abilities. Proc Natl Acad Sci USA 79:2554–2558

Johnson DH (1980) The relationship between spike rate and synchrony in responses of auditory-nerve fibers to single tones. J Acoust Soc Am 68:1115–1122

Joris PX, Smith PH, Yin TCT (1998) Coincidence detection in the auditory system: 50 years after Jeffress. Neuron 21:1235–1238

Keener JP, Hoppensteadt FC, Rinzel J (1981) Integrate-and-fire models of nerve membrane response to oscillatory input. SIAM J Appl Math 41:503–517

Kempter R, Gerstner W, van Hemmen JL, Wagner H (1998) Extracting oscillations: neuronal coincidence detection with noisy periodic spike input. neural Comput 10:1987–2017

Kempter R, Gerstner W, van Hemmen JL (1999a) Hebbian learning and spiking neurons. Phys Rev E 59:4498–4514

Kempter R, Gerstner W, van Hemmen JL, Wagner H (1999b) Quality of coincidence detection and ITD tuning: a theoretical framework. In: Dau T, Hohmann V, Kollmeier B (eds), Psychophysics, physiology and models of hearing. World Scientific, Singapore pp 185–194

Kistler WM, Gerstner W (2002) Stable propagation of activity pulses in populations of spiking neurons. Neural Comput 14:987–997

Kistler WM, van Hemmen JL (1998) Modeling collective excitations in cortical tissue. Physica D 114:273–295

Knight BW (1972a) Dynamics of encoding in a population of neurons. J Gen Physiol 59:734–766

Knight BW (1972b) The relationship between the firing rate of a single neuron and the level of activity in a population of neurons. J Gen Physiol 59:767–778

Knight BW (2000) Dynamics of encoding in neuron populations: Some general mathematical features. Neural Comput 12:473–518

Knight BW, Manin D, Sirovich L (1996) Dynamical models of interacting neuron populations. In: Symposium on robotics and cybernetics: computational engineering in systems applications. Lille, Cite Scientifique, France

Knight BW, Omurtag A, Sirovich L (2000) The approach of a neuron population firing rate to a new equilibrium: an exact theoretical result. Neural Comput 12:1045–1055

König P, Engel AE, Singer W (1996) Integrator or coincidence detector? The role of the cortical neuron revisited. TINS 19:130–137

Kuhlmann L, Burkitt AN, Paolini A, Clark GM (2002) Summation of spatiotemporal input patterns in leaky integrate-and-fire neurons: Application to neurons in the cochlear nucleus receiving converging auditory nerve fiber input. J Comput Neurosci 12:55–73

Kuramoto Y (1984) Chemical oscillations, waves and turbulence. Springer, Berlin Heidelberg New York

Kuramoto Y (1991) Collective synchronization of pulse-coupled oscillators and excitable units. Physica D 50:15–30

Lánský P (1997) Sources of periodical force in noisy integrate-and-fire models of neuronal dynamics. Phys Rev E 55:2040–2043

Lánský P, Sato S (1999) The stochastic diffusion models of nerve membrane depolarization and interspike interval generation. J Periph Nerv Syst 4:27–42

Laurent G (2002) Olfactory network dynamics and the coding of multidimensional signals. Nat Rev 3:884–895

Lindner B (2004) Moments of the first passage time under external driving. J Stat Phys 117:703–737

Lindner B, Schimansky-Geier L (2001) Transmission of noise coded versus additive signals through a neuronal ensemble. Phys Rev Lett 86:2934–2937

Lindner B, Garcia-Ojalvo J, Neiman A, Schimansky-Geier L (2004) Effects of noise in excitable systems. Phys Rep 392:321–424

Litvak V, Sompolinsky H, Segev I, Abeles M (2003) On the transmission of rate code in long feedforward networks with excitatory-inhibitory balance. J Neurosci 23:3006–3015

Longtin A (1993) Stochastic resonance in neuron models. J Stat Phys 70:309–327

Longtin A, Bulsara A, Pierson D, Moss F (1994) Bistability and the dynamics of periodically forced sensory neurons. Biol Cybern 70:569–578

Maršálek P, Koch C, Maunsell J (1997) On the relationship between synaptic input and spike output jitter in individual neurons. Proc Natl Acad Sci USA 94:735–740

Mattia M, Del Giudice P (2000) Efficient event-driven simulation of large networks of spiking neurons and dynamical synapses. Neural Comput 12:2305–2329

Mattia M, Del Giudice P (2002) Population dynamics of interacting spiking neurons. Phys Rev E 66:051917

Mattia M, Del Giudice P (2004) Finite-size dynamics of inhibitory and excitatory Interacting spiking neurons. Phys Rev E 70:052903

Meffin H, Burkitt AN, Grayden DB (2004) An analytical model for the 'large, fluctuating conductance state' typical of neocortical neurons in vivo. J Comput Neurosci 16:159–175

Mirollo RE, Strogatz SH (1990) Synchronization of pulse-coupled biological oscillators. SIAM J Appl Math 50:1645–1662

Mongillo G, Amit DJ (2003) Retrospective and prospective persistent activity induced by Hebbian learning in a recurrent cortical network. Eur J Neurosci 18:2011–2024

Mongillo G, Amit DJ (2005) Learning in realistic networks of spiking neurons and spike-driven plastic synapses. Eur J Neurosci 21:3143–3160

Nykamp DQ, Trachina D (2000) A population density approach that facilitates large-scale modeling of neural networks: analysis and an application to orientation tuning. J Comput Neurosci 8:19–50

Omurtag A, Knight BW, Sirovich L (2000) On the simulation of large populations of neurons. J Comput Neurosci 8:51–63

Plesser HE, Geisel T (1999) Markov analysis of stochastic resonance in a periodically driven integrate-fire neuron. Phys Rev E 59:7008–7017

Plesser HE, Geisel T (2001) Stochastic resonance in model neurons: Endogenous stimulation revisited. Phys Rev E 63:031916

Plesser HE, Tanaka S (1997) Stochastic resonance in a model neuron with reset. Phys Lett A 225:228–234

Prete VD, Coolen ACC (2004) Non-equilibrium statistical mechanics of recurrent networks with realistic neurons. Neurocomputing 58–60:239–244

Rauch A, La Camera G, Lüscher HR, Senn W, Fusi S (2003) Neocortical pyramidal cells respond as integrate-and-fire neurons to in vivo-like input currents. J Neurophysiol 90:1598–1612

Rennie CJ, Robinson PA, Wright JJ (2002) Unified neurophysical model of EEG spectra and evoked potentials. Biol Cybern 86:457–471

Reyes AD (2003) Synchrony-dependent propagation of firing rate in iteratively constructed networks in vitro. Nat Neurosci 6:593–599

Risken H (1996) The Fokker–Planck equation 3rd edn. Springer, Berlin Heidelberg New York

Ritz R, Gerstner W, Fuentes U, van Hemmen JL (1994) A biologically motivated and analytically soluble model of collective oscillations in the cortex II. Application to binding and pattern segmentation. Biol Cybern 71:349–358

Rodriguez R, Lánský P (2000) Effect of spatial extension on noise-enhanced phase locking in a leaky integrate-and-fire model of a neuron. Phys Rev E 62:1–11

Sakaguchi H (2004) Oscillatory phase transition and pulse propagation in noisy integrate-and-fire neurons. Phys Rev E 70:022901

Scharstein H (1979) Input–output relationship of the leaky-integrator neuron model. J Math Biol 8:403–420

Schindler M, Talkner P, Hänggi P (2004) Firing time statistics for driven neuron models: analytic expressions versus numerics. Phys Rev Lett 93:048102

Schindler M, Talkner P, Hänggi P (2005) Escape rates in periodically driven Markov processes. Physica A 351:40–50

Schrödinger E (1915) Zur Theorie der Fall- und Steigversuche an Teilchen mit Brownscher Bewegung. Phys Zeitschr 16:289–295

Segundo JP, Perkel DH, Wyman J, Hegsted H, Moore GP (1968) Input-output relations in computer simulated nerve cells. Kybernetik 4:157–171

Senn W, Urbanczik R (2000) Similar non-leaky integrate-and-fire neurons with instantaneous couplings always synchronize. SIAM J Appl Math 61:1143–1155

Shelley MJ, Tao L (2001) Efficient and accurate time-stepping schemes for integrate-and-fire neuronal networks. J Comput Neurosci 11:111–119

Shimokawa T, Rogel A, Pakdaman K, Sato S (1999a) Stochastic resonance and spike-timing precision in an ensemble of leaky integrate and fire neuron models. Phys Rev E 59:3461–3470

Shimokawa T, Pakdaman K, Sato S (1999b) Time-scale matching in the response of a leaky integrate-and-fire neuron model to periodic stimulus with additive noise. Phys Rev E 59:3427–3443

Siebert WM (1970) Frequency discrimination in the auditory system: place or periodicity mechanisms? Proc IEEE 58:723–30

Singer W (1993) Synchronization of cortical activity and its putative role in information processing and learning. Annu Rev Physiol 55:349–374

Smith LS (1996) Onset-based sound segmentation. In: Touretzky DS, Mozer MC, Haselmo ME (eds), Advances in neural information processing systems, vol 8. MIT Press, Cambridge

Spiridon M, Gerstner W (1999) Noise spectrum and signal transmission through a population of spiking neurons. Netw Comput Neural Syst 10:257–272

Stemmler M (1996) A single spike suffices: the simplest form of stochastic resonance in model neurons. Netw Comput Neural Syst 7:687–716

Strogatz SH (2000) From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators. Physica D 143:1–20

Theunissen F, Miller JP (1995) Temporal encoding in nervous systems: A rigorous definition. J Comput Neurosci 2:149–162

Tiesinga PHE (2002) Precision and reliability of periodically and quasiperiodically driven integrate-and-fire neurons. Phys Rev E 65:041913

Tiesinga PHE, Sejnowski TJ (2001) Precision of pulse-coupled networks of integrate-and-fire neurons. Netw Comput Neural Syst 12:215–233

Tsodyks MV, Skaggs WE, Sejnowski TJ, McNaughton BL (1996) Population dynamics and theta rhythm phase precession of hippocampal place cell firing: a spiking neuron model. Hippocampus 6:271–280

Tuckwell HC (1979) Synaptic transmission in a model for stochastic neural activity. J Theor Biol 77:65–81

Tuckwell HC (1988a) Introduction to theoretical neurobiology: linear cable theory and dendritic structure vol 1. Cambridge University Press, Cambridge

Tuckwell HC (1988b) Introduction to theoretical neurobiology: nonlinear and stochastic theories vol 2. Cambridge University Press, Cambridge

Tuckwell HC (1989) Stochastic processes in the neurosciences. Society for Industrial and Applied Mathematics, Philadelphia

Tuckwell HC, Lánský P (1997) On the simulation of biological diffusion processes. Comput Biol Med 27:1–7

Uhlenbeck GE, Ornstein LS (1930) On the theory of Brownian motion. Phys Rev 36:823–841

Usher M, Schuster HG, Niebur E (1993) Dynamics of populations of integrate-and-fire neurons, partial synchronizaton and memory. Neural Comput 5:570–586

Usher M, Stemmler M, Koch C, Olami Z (1994) Network amplification of local fluctuations causes high spike rate variability, fractal firing patterns and oscillatory local field potentials. Neural Comput 6:795–836

van Hemmen JL (2001) Theory of synaptic plasticity. In: Moss F, Gielen S (eds), Handbook of biological physics: neuroinformatics and neural modelling, vol 4. Elsevier, Amsterdam pp 771–823

van Kampen NG (1992) Stochastic processes in physics and chemistry. North-Holland, Amsterdam

van Rossum MCW (2001) The transient precision of integrate-and-fire neurons: Effect of background activity and noise. J Comput Neurosci 10:303–311

van Rossum MCW, Renart A (2004) Computation with populations codes in layered networks of integrate-and-fire neurons. Neurocomputing 58–60:265–270

van Rossum MCW, Bi GQ, Turrigiano GG (2000) Stable Hebbian learning from spike timing-dependent plasticity. J Neurosci 20:8812–8821

van Rossum MCW, Turrigiano GG, Nelson SB (2002) Fast propagation of firing rates through layered networks of noisy neurons. J Neurosci 22:1956–1966

van Vreeswijk C, Abbott LF (1993) Self-sustained firing in populations of integrate-and-fire neurons. SIAM J Appl Math 53:253–264

Weiss TF (1966) A model of the peripheral auditory system. Kybernetik 3:153–175

Wenning G, Obermayer K (2003) Activity driven adaptive stochastic resonance. Phys Rev Lett 90:120602

Wenning G, Hoch T, Obermayer K (2005) Detection of pulses in a colored noise setting. Phys Rev E 71:021902

Wilson HR, Cowan JD (1972) Excitatory and inhibitory interactions in localized populations of model neurons. Biophys J 12:1–24

Winfree AT (1967) Biological rhythms and the behavior of populations of coupled oscillators. J Theor Biol 16:14–42