Survey of Robust Control for Rigid Robots

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This survey discusses current approaches to the robust control of the motion of rigid robots and summarizes the available literature on the subject. The five major designs discussed are the "Linear-Multivariable" approach, the "Passivity" approach, the "Variable-Structure" approach, the "Saturation" approach and the "Robust-Adaptive" approach.

Introduction

There are basically two underlying philosophies to the control of uncertain systems: the adaptive control philosophy, and the robust control philosophy. In the adaptive approach, one designs a controller which attempts to "learn" the uncertain parameters of the particular system and, if properly designed will eventually be a "best" controller for the system in question. In the robust approach, the controller has a fixed-structure which yields "acceptable" performance for a given plant-uncertainty set. In general, the adaptive approach is applicable to a wider range of uncertainties , but robust controllers are simpler to implement and no time is required to "tune" the controller to the plant variations. More recently, researchers have attempted to "robustify" certain adaptive controllers in order to combine the advantages of both approaches.

We review here different robust control designs used in controlling the motion of robots. A discussion of adaptive controllers in robotics may be found in [1]. A comprehensive survey of robust control theory is available in [3],[18]. The techniques discussed in this survey belong to one of five categories. The first is the linear-multivariable or feedback-linearization approach [2] where the inverse dynamics of the robot are used in order to globally linearize and decouple the robot's equations. Since one does not have access to the exact inverse dynamics, the linearization and the decoupling will not be exact. This will be manifested by uncertain feedback terms that may be handled using multivariable linear robust control techniques [3]. The methods based on computed-torque such as those of [4]-[11] fall under this heading.

The second category contains methods that exploit the passive nature of the robot [12]. These techniques try to maintain the passivity of the closed-loop robot/controller system despite uncertain knowledge of the robot's parameters. Although not as transparent to linear control techniques as the computed-torque approach is, passivity-based methods can nonetheless guarantee the robust stability of the closed-loop robot/controller system. The works described in [13],[14],[40] fall under this category and will be discussed in this paper.

Next, we group methods that are for the most part Lyapunov-based nonlinear control schemes. These include variable-structure and saturation controllers which attempt to robustly control a rigid robot. Some of these techniques may actually rely on the feedback-linearizability or the passivity of the robot dynamics and may have been included in those approaches.

Finally, we briefly survey approaches that combine robust and adaptive techniques. It should be noted that other classifications of robust controllers in robotics are possible and that this survey reflects our own philosophy rather than a universally accepted division.

Let the rigid robot dynamics be given in joint-space by the Lagrange-Euler equations [19] where q is an n vector of generalized coordinates representing the joints positions, and τ is the generalized n torque input vector. The matrix $D(q)$ is an $n{\times}n$ symmetric positive-definite inertia matrix and $h(q,\dot{q})$ is an n vector containing the Coriolis, centrifugal, and gravity terms.

$$D(q)\ddot{q} + h(q,\dot{q}) = \tau. \qquad (1)$$

In general, (1) arises as a solution to the Lagrange equations of motion for natural systems [20]. In this paper, we survey methods which deal primarily with designing controllers that will make q and $\dot{q}$ track some desired q_d and $\dot{q}_d$ when some entries of $D(q)$ and $h(q,\dot{q})$ are uncertain. This survey is by no means exhaustive and will also exclude the important case when the robot comes in contact with the environment.

Linear-Multivariable Approach

In this section we review different designs which use linear multivariable techniques to obtain robust robot controllers. In the early days of robot control, the idea of linearizing the nonlinear robot dynamics about their desired trajectory (using a Taylor series expansion for example) was popular, and many controllers were designed that way [21],[23],[42],[43]. Later however, the physics and special structure of equation (1), coupled with the fact that the control τ provides an independent input for each degree of freedom [2],[12], led to the "global" linearization of the nonlinear robotic system. It is this later approach that is stressed in this section. For an excellent description of the exact linearization of robots see [2]. By defining the trajectory error vector, $e_1 = q - q_d$, $e_2 = \dot{e}_1$, one is able to globally linearize the nonlinear error system, to the following:

$$\begin{aligned} \dot{e} &= A\,e + B\,v \\ A &= \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \\ B &= \begin{bmatrix} 0 \\ I \end{bmatrix} \\ e &= \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\ v &= D(q)^{-1}[\,\tau - h(q,\dot{q})] - \ddot{q}_d. \end{aligned} \qquad (2)$$

The problem is then reduced to finding a linear control v which will achieve a desired closed-loop performance, i.e. find F,G,H and J in

$$\dot{z} = F\,z + G\,e,$$
$$v = H\,z + J\,e,$$

or

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$$v(t) = [(sI - F)^{-1} G + J] e(t) \equiv C(s) e(t) \quad (3)$$

Note that the above notation indicates that *v(t)* is the output of a system *C(s)* when an input *e(t)* is applied. The following static stat-feedback controller is often used:

$$\begin{aligned} &F = G = H = 0, \\ &J = -K \\ &v = -K_1 e_1 - K_2 e_2 \equiv -K e \end{aligned} \quad (4)$$

leading to the nonlinear controller

$$\tau = D(q)\,[\ddot{q}_d + v] + h(q, \dot{q}) \quad (5)$$

which, due to the invertibility of *D(q)* gives the following closed-loop system:

$$\ddot{e}_1 + K_2 \dot{e}_1 + K_1 e_1 = 0. \quad (6)$$

Unfortunately, the control law (5) can not usually be implemented due to its complexity or to uncertainties present in *D(q)* and $h(q,\dot{q})$. Instead, one applies τ in (7) where $\hat{D}$ and $\hat{h}$ are estimates of *D* and *h*:

$$\tau = \hat{D}\,[\ddot{q}_d + v] + \hat{h}\,. \quad (7)$$

This in turn leads to (Fig. 1):

$$\begin{aligned} &\dot{e} = A e + B(v + \eta) \\ &\eta = E(v + \ddot{q}_d) + D^{-1} \Delta h \\ &E = D^{-1} \hat{D} - I_n, \\ &\Delta h = \hat{h} - h\,. \end{aligned} \quad (8)$$

The vector η is a nonlinear function of both *e* and *v* and can not be treated as an external disturbance. It represents a disturbance of the globally linearized error dynamics which is caused by modeling uncertainties, parameter variations, external disturbances and maybe even noisy measurements [4]. The linear multivariable approaches then revolve around the design of linear controllers *C(s)* (which may be dynamical), such that the complete closed-loop system (Fig. 1) is stable in some suitable sense, e.g. uniformly ultimately bounded [38], globally asymptotically stable, etc. for a given class of nonlinear perturbation η. In other words, choose *C(s)* in (3) such that the error *e(t)* in (9) is stable,

$$\dot{e} = A e + B(v + \eta) \quad (9)$$

$$v(t) = C(s) e(t)\,. \quad (10)$$

The reasonable assumptions (11)-(13) below are often made for revolute-joint robots when using this approach [24]. In the following, d_1, d_2, α, β_0, β_1, and β_2 are nonnegative finite constants which depend on the size of the uncertainties:

$$(d_2)^{-1} I_n \le \| D^{-1} \| \le (d_1)^{-1} I_n \quad (11)$$

$$\| E \| \le \alpha \quad (12)$$

$$\| \Delta h \| \le \beta_0 + \beta_1 \| e \| + \beta_2 \| e \|^2 \quad (13)$$

Note that assumption (13) must be modified for robots with prismatic joints.

In general, the small-gain theorem [25], the passivity theorem [25], or the total stability theorem [26] are invoked to find *C(s)*. The most general of these controllers have been designed using Youla parametrization and H^∞ control theory [27],[28] and will be discussed first.

Spong and Vidyasagar [4] used the factorization approach [27] to design a class of linear compensators *C(s)*, parametrized by a stable transfer matrix *R(s)*, which guarantee that the solution *e(t)* to the linear system (8) has a bounded L_∞ norm. The authors actually assumed that the bound on Δh is linear, i.e. $\beta_2=0$ in (13) and found the family of all L_∞ stabilizing compensators of the nominal plant. A particular compensator may then be obtained by choosing the parameter *R(s)* to satisfy other design criteria such as surpressing the effects of η. As was discussed in [24], including the more reasonable quadratic bound will not destroy the L_∞ stability result, but will exclude any L_2 results unless the problem is reformulated and more assumptions are made. In particular, noisy measurements are no longer tolerated. Craig [29] discussed the L_2 problem in a similar setting, and under certain conditions, was able to show the boundedness of the error signals.

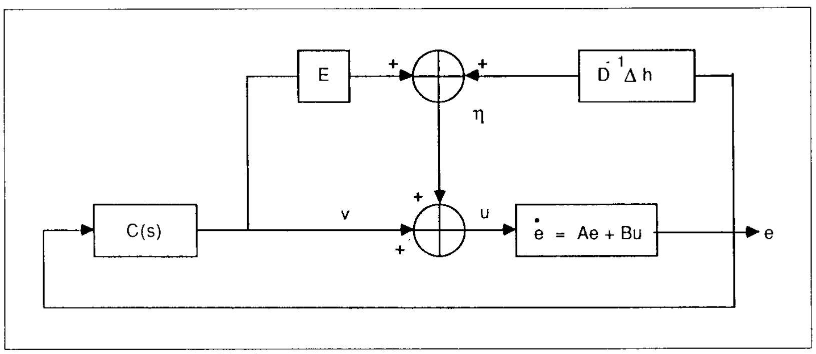

Fig. 1. Linear multivariable design.

Static feedback compensators such as the ones given in (4) have also been used extensively starting with the works of Freund [30], and Tarn *et al.* [6], where

$$v = C(s) e = -K e \quad (14)$$

such that

$$\begin{aligned} \dot{e} &= A e + B(v + \eta) \\ &= (A - BK) e + B\eta = A_c e + B\eta\,. \end{aligned} \quad (15)$$

In these papers, the authors use state feedback to either place the poles sufficiently far in the left-half-plane [9], therefore guaranteeing stability in the presence of η (by the total stability theorem for example), or an extra control loop [6] to correct for the effects of η. In [40], the state-feedback controller was used to define an appropriate output *Ke* such that the input-output closed-loop linear systems $K(sI-A+BK)^{-1}B$ is Strictly-Positive-Real (SPR). The closed-loop stability was then assured for all η resulting from a passive nonlinear system by using the passivity theorem [25]. In Kuo and Wang [31], the internal model principle developed by Francis and Wonham [32] is used to design a linear controller which minimizes the effects of the disturbance term η. However, since η is a nonlinear function of *e* and *v*, minimizing its effects does not necessarily guarantee closed-loop stability. In Gilbert and Ha [10], Proportional-Integral-Derivative control is applied in order to obtain some sensitivity improvements. Cai and Goldenberg [33] use Proportional-Integral control to improve the robustness properties of the controller. Arimoto and Miyazaki [34] use Proportional-Integral-Derivative feedback control to robustly stabilize robot manipulators.

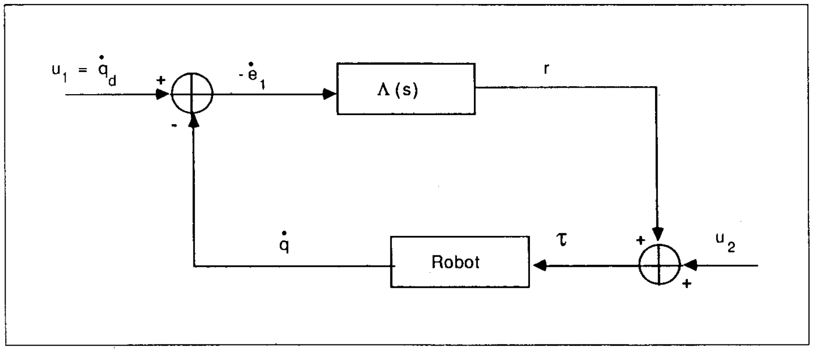

Fig. 2. Passive controller design.

The feedback-linearization approach has been popular (under different names) in the robotics field. Its main advantage is obviously the wealth of linear techniques which may be used in the linear outer loop. In the presence of contact forces however, this approach becomes much more involved as was discussed in [14]. In addition, many controllers designed using this approach are not practical because they require a large control effort.

In some cases, the previously mentioned local linearization approach was combined with other techniques in order to guarantee robust stability [21],[23],[42]. In particular, Desa and Roth [23] used the internal model principle to minimize the effects of disturbances for a robot model linearized over segments of the total operating time. Here also, closed-loop stability is not guaranteed.

Passivity-Based Approach

In this section, we review approaches which rely on the passive structure of rigid robots as described in equation (16) where $h(q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q)$, and $\dot{D}(q) - 2C(q,\dot{q})$ is skew-symmetric by an appropriate choice of $C(q,\dot{q})$ [12]:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau. \quad (16)$$

As a result of the skew-symmetry of $\dot{D}$–2C, the following theorem is obtained.

Theorem [1]: The Lagrange-Euler dynamical equations of a rigid robot (1) define a passive mapping from τ to $\dot{q}$, i.e. for some $\beta>0$ and all T finite, the following inequality holds:

$$\langle \dot{q}, \tau \rangle_T \equiv \int_0^T \dot{q}^T \tau \, dt \geq -\beta. \quad (17)$$

Based on this theorem, if one can close the loop from $\dot{q}$ to τ with a passive system (along with l_2 bounded inputs) as in Fig. 2, the closed-loop system will be asymptotically stable using the passivity theorem [25]. This however, will only show the asymptotic stability of $\dot{e}_1$ and not of e_1. On the other hand, if one can show the passivity of the system which maps τ to a new vector r which is a filtered version of e_1, then a controller which closes the loop between $-r$ and τ will guarantee the asymptotic stability of both e_1 and $\dot{e}_1$. This indirect use of the passivity property was illustrated in [1],[55] and will be discussed next. Let the controller be given by (18)-(21) where $F(s)$ is a strictly proper, stable, rational function and K_r is a positive definite matrix,

$$\tau = D(q)a + C(q,\dot{q})v + G(q) - K_r(\dot{q} - v) \quad (18)$$

$$v = \dot{q} - r \quad (19)$$

$$r = -\left[sI + \frac{K(s)}{s} \right] e_1 = -F(s)^{-1} e_1 \quad (20)$$

$$a = \dot{v}. \quad (21)$$

Then it may be shown that both e_1 and $\dot{e}_1$, are asymptotically stable. This approach was used in the adaptive control literature to design passive controllers [1] but its modification in the design of robust controllers when D, C and G are not exactly known is not obvious.

On the other hand, consider the control law (22) where $\Lambda(s)$ is an SPR transfer function,

$$\tau = -\Lambda(s)\dot{e}_1 + u_2. \quad (22)$$

The external input u_2 has to be bounded in the L_2 norm. Unfortunately, the inclusion of an integrator which reconstructs the error e_1 will destroy the SPR condition. Substituting the above control law into equation (16), one gets from Fig. 2:

$$r = -\Lambda(s)\dot{e}_1. \quad (22)$$

By an appropriate choice of $\Lambda(s)$ and u_2, one can apply the passivity theorem and deduce that $\dot{e}_1$ and r are bounded in the L_2 norm, and since $\Lambda(s)^{-1}$ is SPR (being the inverse of an SPR function), one deduces that $\dot{e}_1$, is asymptotically stable because

$$\dot{e}_1 = -\Lambda(s)^{-1} r. \quad (23)$$

Unfortunately, as discussed above, this will only imply that the position error e_1 is bounded but not its asymptotic stability in the case of time-varying trajectories $[q_d^T \;\; \dot{q}_d^T]\, T$. In the set-point tracking case however, and with gravity precompensation, the asymptotic stability of e_1 may be deduced using LaSalle's theorem [19]. The robustness of the controller (22) is guaranteed as long as $\Lambda(s)$ is SPR and that u_2 is L_2 bounded, regardless of the exact values of the robot's parameters. Note that the controller (22) may be deduced from (5) by choosing the nonlinear controller

$$\begin{aligned} \tau &= D[\ddot{q} + v] + C\dot{q} + g \\ v &= -D^{-1}[\Lambda(s)\dot{e}_1 + C\dot{q} + g] \\ u_2 &= D\ddot{q}_d. \end{aligned} \quad (24)$$

The passivity approach in (22) is then a modified version of the feedback-linearization approaches. In [13],[14], however, Anderson demonstrated using network-theoretic concepts, that even in the absence of contact forces, a feedback-linearization-based controller is not passive and may therefore cause instabilities in the presence of uncertainties. His solution to the problem consisted of using Proportional-Derivative (PD) controllers with variable gains $K_1(q)$ and $K_2(q)$ which depend on the inertia matrix $D(q)$, i.e.

$$\tau = -K_1(q)e_1 - K_2(q)e_2 + g. \quad (25)$$

Even though $D(q)$ is not exactly known, the stability of the closed-loop error is guaranteed by the passivity of the robot and the feedback law. The advantage of this approach is that contact forces and larger uncertainties may now be accommodated. Its main disadvantage is that although robust stability is guaranteed, the closed-loop performance depends on the knowledge of $D(q)$ whose singular values are needed in order to find K_1 and K_2.

Variable-Structure Controllers

In this section, we group designs that use variable-structure controllers [15]. The VSS theory has been applied to the control of many nonlinear processes [63]. One of the main features of this approach is that one only needs to drive the error to a "switching surface," after which the system is in "sliding mode" and will not be affected by any modeling uncertainties and/or disturbances [15],[16]. The first application of this theory to robot control seems to be in the work of Young [16] where the set point regulation problem ($\dot{q}_d$=0) was solved using the following controller:

$$\tau_i = \begin{cases} \tau_i^+, & \text{if } s_i(e_{1i}, \dot{q}_i) > 0 \\ \tau_i^-, & \text{if } s_i(e_{1i}, a_i) < 0 \end{cases} \qquad (26)$$

where i=1,...,n for an n - link robot, and s_i are the switching planes,

$$s_i(e_{1i}, \dot{q}_i) = c_i e_{1i} + \dot{q}_i, \quad c_i > 0. \qquad (27)$$

It is then shown using the hierarchy of the sliding surfaces $s_1,s_2,...,s_n$ and given bounds on the uncertainties in the manipulators model, that one can find τ^+ and τ^- in order to drive the error signal to the intersection of the sliding surfaces after which the error will "slide" to zero. This controller eliminates the nonlinear coupling of the joints by forcing the system into the sliding mode. In [58], a modification of the Young controller was presented. Other VSS robot controllers may be found in [53],[59],[60]. Unfortunately, for most of these schemes, the control effort as seen from (26) is discontinuous along s_i = 0 and will therefore create "chattering" which may excite unmodeled high-frequency dynamics.

To address this problem, Slotine modified the original VSS controllers using the so-called "suction control" [17],[41]. In this approach, the sliding surface s is allowed to be time-varying and the control procedure consists of two steps. In the first, the control law forces the trajectory towards the sliding surface while in the second step, the controller is smoothed inside a possibly time-varying boundary layer. This will achieve optimal trade-off between control bandwidth and tracking precision, therefore eliminating chattering and the sensitivity of the controller to high-frequency unmodeled dynamics. The controller structure in this case is given by (28) where Λ is a diagonal matrix of positive elements λ_i (which may be time-varying) and $\phi(.)$ is a nonlinear term determined by the extent of the parametric uncertainties and the suction control modifications [17],

$$\begin{aligned} \tau &= \hat{D}\,[\,\ddot{q}_d - K_2\dot{e}_1 - K_1^2 e_1 - \phi(q,\dot{q},t)\,] + \hat{h} \\ K_1 &= \Lambda^2 \\ K_2 &= 2\Lambda \end{aligned} \qquad (28)$$

More recently, in [61],[62], VSS controllers which avoided the inversion of the inertia matrix were introduced. The VSS approach although theoretically appealing, does not fully exploit the physics of the robots. In addition, in practice and to avoid chattering, the asymptotic stability of the error is sacrificed.

Robust Saturation Approach

In this section, we review the research that utilizes an auxiliary saturating controller to compensate for the uncertainty present in the robot dynamics as given by (29) where $D(q)$ and $C(q,\dot{q})$ are defined in (16), and $Z(q,\dot{q})$ is an n- vector representing friction, gravity and bounded torque disturbances:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + Z(q,\dot{q}) = \tau\,. \qquad (29)$$

The controllers introduced in this section are robust due to the fact they only depend on uncertainty bounds rather than on the actual values of the parameters. The following bounds are needed and may be physically justified. The d_i's and ζ_i's in (30) and (31) are positive scalar constants and the trajectory error e is defined before:

$$d_1 I_n = D(q)\, d_2 I_n \qquad (30)$$

$$\begin{aligned} &\| C(q,\dot{q})\dot{q} + Z(q,\dot{q}) \| \\ &\quad \le \zeta_0 + \zeta_1 \| e \| + \zeta_2 \| e \|^2. \end{aligned} \qquad (31)$$

Note the similarity between (11)-(13) and (30) and (31).

Based on (30),(31), Spong [8] used Lyapunov stability theory to guarantee the ultimate boundedness of e, a concept defined in [38] for example. The control strategy is actually based on the works of Cvetkovic [35] and the linear high-gain theory of Barmish [36], Gutman [37] and Corless [38]. Spong's controller is representative of this class and is given as follows:

$$\begin{aligned} \tau &= (2 d_1 d_2)(d_1 + d_2)^{-1} \\ &\cdot [\ddot{q}_d - K_2 e_2 - K_1 e_1 - v_r] \\ &+ \hat{C}(q,\dot{q}) + \hat{Z}(q,\dot{q}) \end{aligned} \qquad (32)$$

where

$$v_r = \begin{cases} (B^T P e)\,(\| B^T P e \|)^{-1}\,\rho; \\ \qquad \text{if } \| B^T P e \| > \varepsilon \\ (B^T P e)\,\varepsilon^{-1}\rho; \\ \qquad \text{if } \| B^T P e \| \le \varepsilon \end{cases} \qquad (33)$$

and

$$\begin{aligned} \rho = (1-\alpha)^{-1} [\, &\alpha \| \ddot{q}_d \| + \| K_1 \| \\ &\cdot \| e_1 \| + \| K_2 \| \cdot \| e_2 \| + (d_1)^{-1}\phi \end{aligned} \qquad (34)$$

$$\phi = \beta_0 + \beta_1 \| e \| + \beta_2 \| e \|^2 \qquad (35)$$

$$\alpha = (d_2 - d_1)\,(d_2 + d_1)^{-1}\,. \qquad (36)$$

Note that in the equations above, the matrix B is defined as in (2), the β_i's are defined as in (13), and the matrix P is the symmetric, positive-definite solution of the Lyapunov equation (37), where Q is symmetric and positive-definite matrix and A_c is given in (15):

$$A_c^T P + P A_c = -Q\,. \qquad (37)$$

Upon closer examination of Spong's controller (32)-(36), it becomes clear that v_r depends on the servo gains K_1 and K_2 through ρ. This might obscure the effect of adjusting the servo gains and may be avoided as described in [45]. In fact, let the controller be given by

$$\tau = -K_2 e_2 - K_1 e_1 - v_r(\rho, e_1, e_2, \varepsilon) \qquad (38)$$

where v_r is given as in (33) and

$$\rho = \delta_0 + \delta_1 \| e \| + \delta_2 \| e \|^2 \qquad (39)$$

where δ_i's are positive scalars. Note that ρ no longer contains the servo gains and as such, one may adjust K_1 and K_2 without tampering with the auxiliary control v_r. As was also shown in [45], if the initial error $e(0)$=0 and by choosing $K_2 = 2K_1 = k_v I_n$, the tracking error may be bounded by the following which shows the direct effect of the control parameters on the tracking error,

$$\| e \| \le \left[4\left(2k_v + \frac{3d_2}{2}\right)\varepsilon\,(k_v d_1)^{-1} \right]^{1/2}. \qquad (40)$$

In [44], Corless presented a simulation of a similar controller using a Manutec R3 robot. A similar control scheme was given by Chen in [39]. Chen's controller however, requires acceleration measurements. In [5], Gilbert and Ha used a saturating-type feedback derived from Lyapunov-stability theory in order to guarantee the ultimate boundedness of the tracking error. Similarly in [11], Samson derived a "high-gain" controller which guarantees the ultimate boundedness of the error.

Robust Adaptive Approach

In this section, we briefly review some approaches that combine adaptive and robust control concepts. Since so much work has been done in the field of adaptive control of robotic manipulators [1], we only concentrate on schemes that are robust in addition to being adaptive. Let us first review one of the most commonly used robot adaptive controllers. This scheme was derived by Slotine [46] and a simplified version is given by the following where $\hat{\phi}$ is an r vector of the estimated parameters, and $Y(.)$ is an $n \times r$ regression matrix of known time functions:

$$\tau = \tau_a = Y(\,.\,)\,\hat{\phi} - K_2 e_2 - K_1 e_1 \qquad (41)$$

$$\frac{d\hat{\phi}}{dt} = -Y^T(\,.\,)\,[\,e_1 + e_2\,]\ . \qquad (42)$$

If there are no disturbances in the model (16), the tracking error is shown to be asymptotically stable with the above controller. However, the parameter estimate $\hat{\phi}$ in (42) may become unbounded in the presence of a bounded disturbance T_d, or unmodeled dynamics [29],[54]. Robust-Adaptive controllers have attempted to robustify adaptive schemes against such uncertainties.

In [47], Slotine showed that the parameter estimates remain bounded if one uses

$$\tau = \tau_a + k_d \ \mathrm{sgn}\ (\,e_1 + e_2\,) \qquad (43)$$

where τ_a is given in (41) and k_d is a positive scalar constant satisfying

$$k_d > \|\,T_d\,\|\ . \qquad (44)$$

More recently [48], Reed introduced the σ-modification method originated by Ioannou [49] in order to compensate for both unmodeled dynamics and bounded disturbances. The control law is now given by

$$\tau = \tau_a = Y(\,.\,)\,\hat{\phi} - K_2 e_2 - K_1 e_1 \qquad (45)$$

$$\frac{d\hat{\phi}}{dt} = -Y^T(\,.\,)\ [\,e_1 + e_2\,] - \sigma\,\hat{\phi} \qquad (46)$$

where

$$\sigma = \begin{cases} 0, & \text{if } \|\hat{\phi}\| < \phi_0 \\ \|\hat{\phi}\|\,(\phi_0)^{-1} - 1, & \text{if } \phi_0 < \|\hat{\phi}\| < 2\phi_0 \\ 1, & \text{if } \|\hat{\phi}\| > 2\phi_0 \end{cases} \qquad (47)$$

and

$$\phi_0 > \|\,\phi\,\|\ . \qquad (48)$$

Using this controller, Reed was able to show that the tracking error and all closed-loop signals are bounded.

Another approach in this section is that of Singh [50] which combines Spong's controller in (32) with adaptive techniques to estimate the uncertainty terms β_0, β_1, and β_2 in (35). Therefore, no prior knowledge about the exact size of the uncertainties is needed.

In [64], Spong and Ghorbel addressed certain instability mechanisms in the adaptive control of robots. A composite control law was used to damp out the fast dynamics, then a slow adaptive control law based on the algorithm of Slotine and Li [47] was robustified using the σ– modification [65]. Unfortunately, asymptotic stability is then lost if tracking a time-varying trajectory is desired. The algorithm is modified again using the switching σ–modification to ensure the asymptotic stability to a class of time-varying trajectories.

Conclusions

The robust motion control of rigid robot was reviewed. Five main areas were identified and explained. All controllers were robust with respect to a range of uncertain parameters although some of them could only guarantee the boundedness of the position-tracking error rather than its asymptotic convergence. In the last section, we also included adaptive controllers that are also robust. The question of which robust control method to choose is difficult to answer analytically but the following guidelines are suggested. The linear-multivariable approach is useful when linear performance specifications (Percent overshoot, Damping ratio, etc.) are available. This approach may however result in high-gain control laws in the attempt to achieve robustness. The passive controllers are easy to implement but do not provide easily quantifiable performance measures. The robust version of these controllers does not exploit the physics of the robot as their adaptive versions do. The variable-structure controllers should not be used when the flexibilities of the links are considerable for fear of exciting their high frequency dynamics. The saturation controllers, are most useful when a short transient error can be tolerated but ultimately, the error will have to be bounded. The robust adaptive controllers require more computing power and an adaptation time. On the other hand, they are most useful when repetitive or long duration tasks are performed. Their performance actually improves with time and they should be used when a high degree of performance is required. It is useful to note, that although the robot's dynamics are highly nonlinear, most successful controllers have exploited their physics and their very special structure [55],[56]. This observation should be useful as we try to include force control, and flexibility effects in the current and future robotics research.

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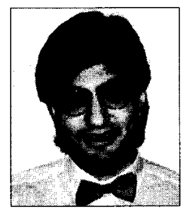

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