Parallel Spatial Processing: A Cure for Signal Cancellation in Adaptive Arrays

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Abstract—**A spatial processing algorithm with parallel structure is presented for the prevention of signal cancellation phenomena in conventional adaptive arrays. This algorithm basically uses a parallel structure with a spatial averaging effect to combat coherent jamming. It results in a spatially smoothed maximum-likelihood estimate of the desired signal when the adaptive beamformer converges. Simulations have been conducted which verify the effectiveness of the proposed structure.**

I. Introduction

SIGNAL CANCELLATION phenomena existing in many conventional adaptive arrays, such as adaptive sidelobe cancellers [1]–[3] and linearly constrained adaptive beamformers [4]–[6], have recently been studied and explored by Widrow *et al.* [7] and Duvall [8]. These effects seriously degrade the performance of the adaptive beamformer, and can cause signal loss in the case of narrow-band signals, or significant signal distortion in the case of wideband signals. Additionally, they may result in the adaptive beamformer forming a false null in a direction other than that of the jammer. For example, Fig. 1 shows a converged beampattern of a Frost [6] linearly constrained beamformer in an environment consisting of a desired signal S plus a single coherent jamming signal J. The desired signal is in the constrained look direction, whereas the jammer is in an off-look direction. Coherence between the desired and jamming signals may imply, for instance, sinusoids with the same frequency and fixed phase shift. One should note in Fig. 1 that the adaptive beamformer has a null in an incorrect direction; the null does not correspond to the direction of the jammer. Besides, this null is not very deep. Also notice that the time-domain output of the beamformer, as shown in Fig. 2, is forced to zero, even though the linear constraints in the desired look direction have been imposed. Such signal cancellation effects and false nulling can lead to serious performance deterioration in adaptive beamforming or in direction finding applications. These effects can also be caused by high adaptation rate as well as by high correlation between the desired and jamming signals. Such a scenario is probable in many applications that are characterized by multipath and "smart-jamming."

Methods for preventing signal cancellation have been suggested [7]–[10]. Duvall [8] used a master-slave version of the linearly constrained Frost beamformer to subtract out the

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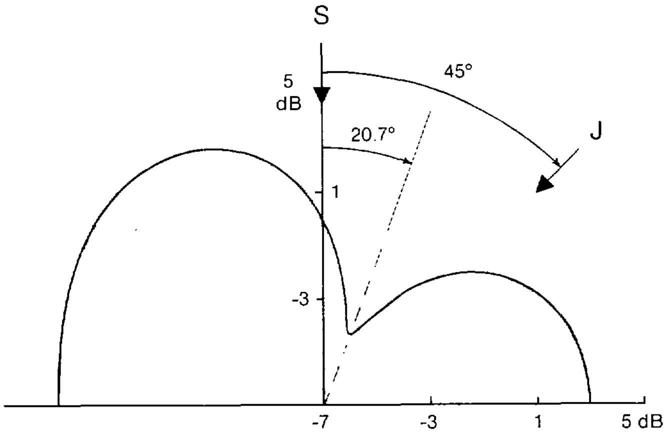

Fig. 1. A converged beampattern of a two-element Frost beamformer in a coherent jamming environment.

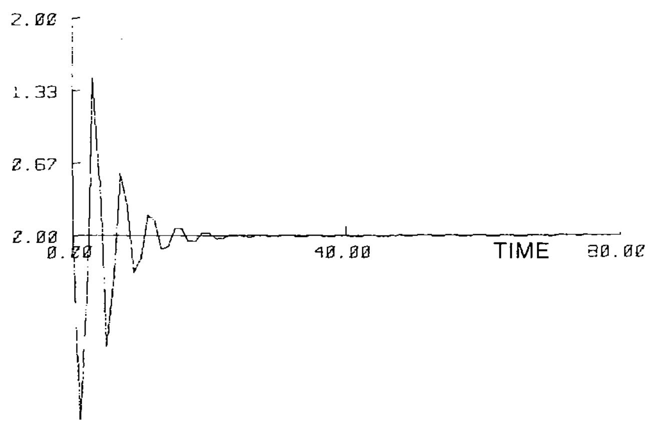

Fig. 2. The output of a Frost beamformer when signal cancellation occurs.

desired signal from the adaptive process. Widrow [7] and Gabriel [9] also suggested certain forms of spatial dither that modulate the coherent jammer in the off-look direction to destroy the correlation. Another approach has recently been suggested by Shan and Kailath [10], based on eigenvector methods, in which spatial smoothing is incorporated so that the sample covariance matrix preserves full rank of the signal space when a coherent jamming environment exists. This method is found effective in applications of direction finding and adaptive beamforming. For each snapshot, it requires a considerable amount of computation to achieve spatial smoothing. The recovered signal, however, is still sensitive to the adaptation rate, and another form of signal distortion can result from using a high adaptation rate. For many signal

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cancellation problems, however, the array output signal, rather than the output power, is of great concern.

The purpose of this paper is twofold: firstly, to address signal cancellation effects and false nulling problems from various perspectives (e.g., phasor plots); and secondly, to introduce an approach, namely the "parallel spatial processing" algorithm, for adaptive beamforming to combat signal cancellation and to recover the desired signal with less distortion at the receiving site. The new approach has several advantages. Since it is of parallel structure, computational speed can be high. Furthermore, when the adaptive process converges, the system output signal will be a maximum-likelihood estimate of the desired signal in a spatial averaging sense, even if a high adaptation rate is used. Structually, a number of subbeamformers are needed in the algorithm. Every subbeamformer is identical in configuration, and each simultaneously uses the same set of adaptive weights. Only one set of adaptive weights is thus required for all the subbeamformers.

II. Signal Cancellation in Frost Beamformers

To demonstrate how signal cancellation can occur, consider a simple two-element Frost maximum-likelihood beamformer, as shown in Fig. 3. A typical Frost beamformer constraint is one that forces the beamformer to form a unit gain and zero phase over a certain frequency band in the desired look direction. The desired look direction can be preselected by time-delay steering of the array elements. The Frost beamformer is then adapted to minimize its own output power subject to the constraint. Suppose a sinusoidal desired signal is arriving from the look direction, and a jammer at the same frequency as the desired signal and with fixed phase shift is arriving from an off-look direction. Let the desired signal S and the jammer J be the following:

$$S = Ae^{j\omega t}, \quad J = Be^{j\omega t + j\phi}, \tag{1}$$

where A and B are the corresponding amplitude of the signal and the jammer, ϕ is a constant phase difference between S and J, and ω is the angular frequency. In Fig. 3, the receiving element 1 receives both the desired signal and jammer as

$$X_1 = Ae^{j\omega t} + Be^{j\omega t + j\phi}, \tag{2}$$

and element 2 receives the same signal plus the delayed jammer as

$$X_2 = Ae^{j\omega t} + Be^{j\omega t + j\phi - j\omega\Delta}, \tag{3}$$

where

- Δ = $d \sin \theta / c$
- d the interelement distance
- c the speed of propagation
- θ the jammer's incident angle from broadside.

Denote the weight vector and the received signal vector as

$$W = [W_1 W_2]^T, \quad X = [X_1 X_2]^T. \tag{4}$$

The beamformer output is thus given by

$$y = W^T X = X^T W = W_1 X_1 + W_2 X_2. \tag{5}$$

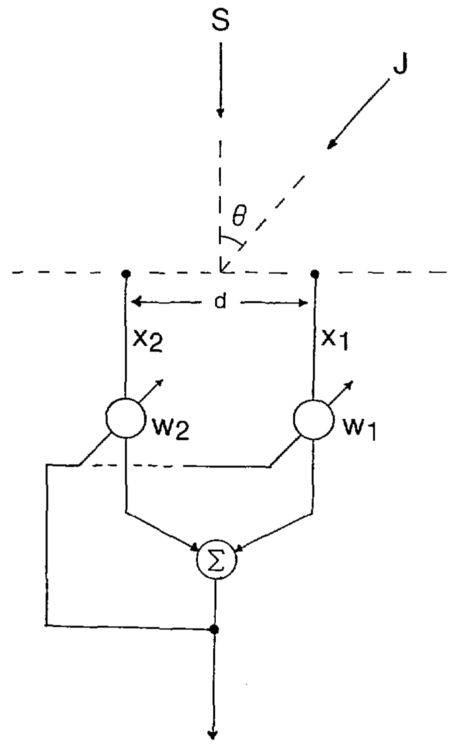

Fig. 3. The structure of a simple two-element Frost beamformer.

The adaptive weights are complex, and the complex algorithm for linearly constrained adaptive beamformers [11] is used. The constraint in the look direction is set to cause the receiving array to have a unit gain and zero phase from zero frequency to half the sampling rate. Thus the Frost algorithm can be expressed as the following:

$$\min_W |y|^2, \qquad \text{subject to } W_1 + W_2 = 1, \tag{6}$$

or is equivalently given by

$$\min_W |W_1 X_1 + W_2 X_2|, \qquad \text{subject to } W_1 + W_2 = 1. \tag{7}$$

Substituting X_1, X_2 and $W_1 = 1 - W_2$ into (7) yields an unconstrained minimization as follows:

$$\min_{W_2} |e^{j\omega t}| \cdot |A + Be^{j\phi}[1 - W_2 + W_2 e^{-j\omega\Delta}]|. \tag{8}$$

Solving (8), one easily finds that the optimal weight W_2^* is

$$W_2^* = \frac{1}{1 - e^{-j\omega\Delta}} + \frac{Ae^{-j\phi}}{B(1 - e^{-j\omega\Delta})}. \tag{9a}$$

The optimal weight W_1^* is obtained from (7) and (9a) as

$$W_1^* = 1 - W_2^*. \tag{9b}$$

Note that the optimal solution results in a zero output when the adaptive processor reaches steady state, i.e.,

$$\lim_{t\to\infty} \mathbf{y}_{\min}(t) = W^{*T} X \to 0.$$

Of course this is undesirable. Ideally the output should contain the desired signal only, with no added coherent jammer. By this criterion, the optimal solution $W_{2\text{opt}}$ should be

$$W_{2\text{ opt}} = \frac{1}{1 - e^{-j\omega\Delta}}. \tag{10}$$

Comparing (9) and (10), we may see that there are two ways to force the weights to the optimal solution in a coherent jamming environment. The first one would set A to zero, or eliminate the desired signal in the adaptive processor. The second one would make $A \ll B$, which means the signal power would be much smaller than the jammer power.

In his master-slave beamformer, Duvall [8] applied the first idea to remove the desired signal from the adaptation process. Since no desired signal is involved due to interelement subtraction in Duvall's beamformer, the influence over weight settings will be dominated by the jammers. The adaptive weights therefore reach an optimal solution which cancels the jammers only.

Now consider the case when $A \ll B$, which means a very strong jammer is present, then $W_2^* \cong W_{2\text{opt}}$. Note that the output y equals W^TX. This still results in a zero output, even though the weights come very close to the optimal solution. This is better explained from the perspective of covariance space. Shan [10] has shown that in a coherent signaling environment the array sample covariance matrix becomes singular. Minimization with respect to the weights will thus steer the weight vector to align with the eigenvectors corresponding to the zero eigenvalues. The output of the beamformer hence falls to zero.

To understand the false nulling phenomenon of the beam pattern, it is helpful to consider a phasor diagram as shown in Fig. 4. In this phasor diagram, $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ are the jammer components received by the elements 1 and 2, respectively. The angle $\angle QOR$ represents the phase delay $\omega\Delta$ between the jammer components at elements 1 and 2. $\overrightarrow{PO}$ is the desired signal received by both elements. An ideal adaptive beamformer should form a null in a direction such that the phase delay is $\omega\Delta$.

Let the length represent the amplitude. For a far-field planewave jammer, each element receives equal jammer amplitude, namely $|\overrightarrow{OQ}| = |\overrightarrow{OR}|$. The received amplitude may vary from element to element for a near-field jammer. Without loss of generality, suppose both the jammer and the desired signal have equal power intensity. In other words, $|\overrightarrow{PO}| = |\overrightarrow{OQ}| = |\overrightarrow{OR}|$. For the coherent jamming situation, both the jammer and the desired signal have the same frequency ω. The relative phase difference ϕ between signal and jammer is a fixed constant. In the phasor diagram, this means $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ are rotating about point O with angular speed ω, and $\overrightarrow{PO}$ is rotating about point P with the same angular speed. The relative phase difference ϕ between signal and jammer should not be confused with the phase delay $\omega\Delta$ between the jammer components at elements 1 and 2.

One can easily see that $\overrightarrow{PQ}$ is the phasor superposition of $\overrightarrow{PO}$ and $\overrightarrow{OQ}$, whereas $\overrightarrow{PR}$ is the phasor superposition of $\overrightarrow{PO}$ and $\overrightarrow{OR}$. In other words, $|\overrightarrow{PQ}|$ represents X_1 as received at element 1, and $|\overrightarrow{PR}|$ represents X_2 as received at element 2. Since the signal and the jammer have the same frequency ω,

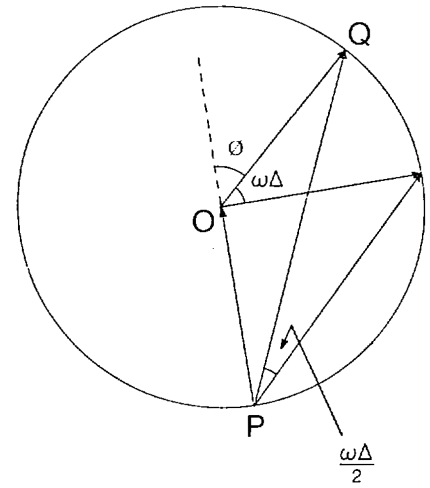

Fig. 4. The phasor diagram corresponding to the two-element Frost beamformer.

both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are then rotating about point P with the same angular speed ω. Notice that the phase delay $\angle QPR$ between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ has been fixed and it is easy to verify that $\angle QPR = \omega\Delta/2$ by geometrical identities. This means that the phase delay between antenna element 1 and antenna element 2 is changed to another fixed value, $\omega\Delta/2$ instead of $\omega\Delta$. Note that this is virtually equivalent to the following scenario: a jammer, with no desired signal, arrives in a direction for which the inter-element phase delay is $\omega\Delta/2$, and is received by the antenna elements with different attenuations. The adaptive beamformer, subject to the minimization algorithm, still adapts to minimize the beamformer output power. Forming a null in a wrong direction with phase delay $\omega\Delta/2$, rather than $\omega\Delta$, will achieve the power minimization. This accounts for the false nulling by the adaptive array which results in the desired signal being completely cancelled.

Simulations with the Frost beamformer, shown in Fig. 3, were conducted to verify the above argument. The interelement distance was half a wavelength. A coherent jammer as well as a desired signal were received by the adaptive beamformer. Both had equal power intensities of one. The desired signal arrived broadside to the array and the jammer arrived at 45° from broadside. The converged beam pattern is shown in Fig. 1, where the null is in a direction 20.7° from broadside. By the above false nulling argument, one could verify that

$$\omega \sin 20.7° = \frac{\omega \sin 45°}{2}. \tag{11}$$

The beamformer output signal is shown in Fig. 2. It shows that forming a null at 20.7° from broadside can minimize array output power, reducing it to a lower level than can be achieved by forming a null at 45° from broadside. This false nulling is not easily seen in the converged beampattern when the signal power and the jammer power are not of the same order of magnitude, especially when the jammer power is much stronger than the signal power. In such a situation, the fixed phase delay $\angle QPR$ between X_1 and X_2 will be very close to

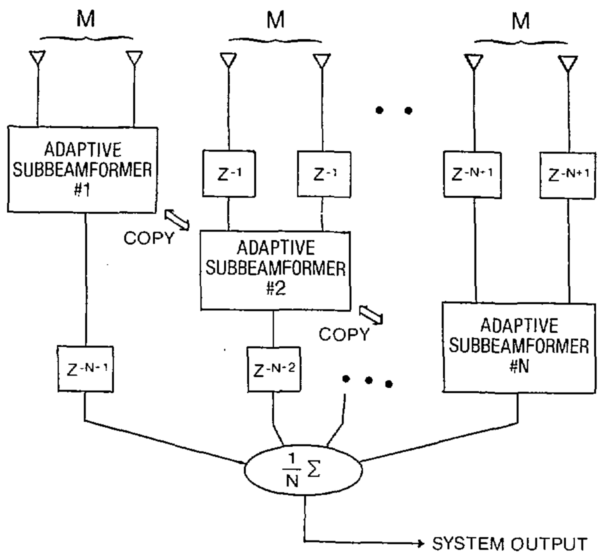

Fig. 5. A general block diagram of the parallel spatial processing algorithm.

$\omega\Delta$. The null will be formed in a direction close to that of the jammer. Even though the beam pattern looks correct in this case, the desired signal component at the output still falls to zero.

III. Parallel Spatial Processing Algorithm

The spatial smoothing technique can be another alternative for breaking up the signal/jammer correlation. Widrow [7], Shan [10], and Evan [12] have proposed different ways of spatial smoothing. The idea in general is to spatially smooth in directions orthogonal to the desired look direction of the beamformer.

In this section we present an approach called the "parallel spatial processing algorithm" to combat signal cancellation. With this algorithm, a number of subbeamformers having the same structures as conventional beamformers are used. These subbeamformers are arranged in a parallel way. Fig. 5 illustrates a general block diagram of the algorithm. It consists of a linear array with L equal-distance elements. These L elements are partitioned into N groups, where N is the number of subbeamformers. Each subbeamformer has M input elements. The input elements of adjacent subbeamformers could be partially overlapping. If the adjacent subbeamformers have overlapping input elements, every subbeamformer should do the overlapping in the same way. This implies that the total number of elements in the linear array should be less than or equal to $M \cdot N$. Since every subbeamformer has the same structure, each one can share the same set of weights.

The parallel spatial processing algorithm is given as follows: for the first time instant, we use the first subbeamformer to update the weights and then copy the weights into the rest of the subbeamformers. For the second time instant, we use the second subbeamformer to update the weights and then copy the weights into the rest of the subbeamformers. So the adaptation process sequentially propagates one by one along the subbeamformers. After the adaptation reaches the last subbeamformer, it restarts at the first one. Meanwhile, for each time instant, every subbeamformer uses the same set of weights to yield its own output. The system output is then generated by averaging the various delayed outputs of all these subbeamformers. In other words, the set of weights is spatially propagated and updated along the subbeamformers, and the received signals are processed together to produce the system output. The weight propagation from one subbeamformer to another will incorporate spatial smoothing as well as time averaging of the sample covariance matrix. It thus performs similar spatial smoothing to Shan's beamformer, and restores the rank of the sample covariance matrix when coherent jamming situations take place.

Analysis will show that the algorithm results in a maximum likelihood estimate of the desired signal in a spatial averaging sense. In addition, the algorithm only takes one adaptation to generate a system output sample. Due to its parallel structure, the adaptive algorithm requires only the same computation power for each time instant as would any conventional adaptive beamformer. This contrasts to N adaptations in Shan's beamformer.

In order to have greater insight into this algorithm, let us take an example with N Frost subbeamformers. Each subbeamformer has M input elements. The adjacent subbeamformers have $M - 1$ overlapping elements. This means, there are total $M + N - 1$ elements in the linear array. Suppose the desired signal and the jammer are impinging on the array; the signal is from the look direction and the jammer is from an off-look direction. Since the elements of the linear array are equally spaced, each element receives

$$X_m(k) = Ae^{j\omega kT} + Be^{j\omega kT + j\phi + j(m-1)\omega\Delta}, \quad m = 1, 2, \cdots, M+N-1. \tag{12}$$

Denote the signal vector received at the nth Frost subbeamformer by

$$Z_n(k) \triangleq [X_m(k-n+1) \cdot X_{m+1}(k-n+1) \cdots X_{m+M-1}(k-n+1)]^T, \tag{13}$$

where m is the labeling number of the first element of the nth subbeamformer. Mathematically, the algorithm can be expressed as the following:

$$\mathbf{y}_n(k) = W^T(k) Z_n(k)$$

$$W(k+1) = P[W(k) + \mu \mathbf{y}_n(k) \bar{Z}_n(k)] + F, \tag{14}$$

where

k	the discrete time index
n	mod $(k, N) + 1$
$\mathbf{y}_n(k)$	the output of the nth Frost subbeamformer
$\bar{Z}_n(k)$	the complex conjugate of $Z_n(k)$
P, F	the constant vectors of the Frost algorithm.

The system output is generated by averaging the various delayed outputs of each of the subbeamformers and is given by

$$\mathbf{y}(k) = \frac{1}{N} (\mathbf{y}_N(k) + \mathbf{y}_{N-1}(k-1) + \cdots + \mathbf{y}_1(k-N+1)). \tag{15}$$

The linear constraint in the weights is expressed as follows:

$$\sum_{i=1}^{N} W_i(k)=1, \qquad \text{for any } k. \tag{16}$$

It is easy to find from (13), (14) that the output of each subbeamformer is

$$\mathbf{y}_n(k)=\sum_{i=1}^{M} W_i(k)X_{i+n-1}(k-n+1), \qquad n=1,\ 2,\ \cdots,\ N. \tag{17}$$

For the $k + n - 1^{th}$ time instant, one may have

$$\mathbf{y}_n(k+n-1)=\sum_{i=1}^{M} W_i(k+n-1)X_{i+n-1}(k), \qquad n=1,\ 2,\ \cdots,\ N. \tag{18}$$

The overall system output $y(k + N - 1)$ is

$$\begin{aligned}\mathbf{y}(k+N-1)&=\frac{1}{N}\left[\mathbf{y}_1(k)+\mathbf{y}_2(k+1)+\cdots+\mathbf{y}_N(k+N-1)\right]\\ &=\frac{1}{N}\sum_{n=1}^{N}\mathbf{y}_n(k+n-1).\end{aligned} \tag{19}$$

Substituting (12) and (18) into (19), one has

$$\begin{aligned}\mathbf{y}(k+N-1)&=Ae^{j\omega kT}+Be^{j\omega kT+j\phi}\\ &\quad\cdot\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{M} W_i(k+n-1)e^{j(i+n-2)\omega\Delta}\\ &=Ae^{j\omega kT}+Be^{j\omega kT+j\phi}\cdot\frac{1}{N}\sum_{n=1}^{N}\alpha(n+k)e^{j(n-1)\omega\Delta},\end{aligned} \tag{20a}$$

where

$$\alpha(n+k)=\sum_{i=1}^{M} W_i(k+n-1)e^{j(i-1)\omega\Delta}. \tag{20b}$$

Notice that (20) can be a "quality" measure of the system output. According to this equation, the system output contains the desired signal plus a coherent jammer which is multiplied by a spatial averaging term. This spatial averaging term determines the extent to which the whole adaptive beamformer recovers the desired signal.

Another interesting point is that the weights are modulated by the spatial frequency $e^{j(i-1)\omega\Delta}$ as shown in (20b). This modulated term $\alpha(n + k)$ shown in (20b) is a function of the time index k. As the adaptive process reaches the minimum of the performance surface, it is very likely that

$$W_i(k+n-1) \cong W_i(k+N-1), \qquad n=1,\ 2,\ \cdots,\ N-1.$$

Thus, it is easy to obtain the following:

$$\mathbf{y}(k+N-1)=Ae^{j\omega kT}+Be^{j\omega kt+j\phi}\cdot\alpha(k+N)\cdot\left[\frac{1}{N}\sum_{n=1}^{N}e^{j(n-1)\omega\Delta}\right]. \tag{21}$$

There are two factors in (21) which can modify the jammer. The first factor is a function of time as shown in (20b), and is subject to the least mean square criterion and the linear constraint. The second term is given as

$$\frac{1}{N}\sum_{n=1}^{N}e^{j(n-1)\omega\Delta},$$

which is the summation of N uniformly spaced terms on the unit circle. Notice that this results in a very small value, close to zero, and it also asymptotically approaches zero as N goes to infinity. When the adaptive process reaches steady state, the coherent jamming effect will be greatly reduced by such a modification. Therefore, if a large number of subbeamformers are used, it is easy to get

$$\lim_{N\to\infty}\mathbf{y}(k+N-1)=Ae^{j\omega kT}. \tag{22}$$

If the desired signal is stationary, the expected value of the system output will be a minimum-variance estimate of the signal. Capon *et al.* [13] showed that a constrained minimum-variance estimate is equivalent to the maximum-likelihood estimate. Since the spatial summation factor asymptotically approaches zero, the system output is a maximum-likelihood estimate of the desired signal in a spatial averaging sense. To make the spatial summation factor close to zero, the number of subbeamformers, N, should be large enough so that the terms $e^{j(n-1)\omega\Delta}$ span the unit circle. This implies that if the incident angle of the jammer from broadside is very small, then a large number of subbeamformers are required. Finally, the desired signal appears at the system output with a delay of $N - 1$ sampling periods.

Although the analysis is based on the Frost linearly constrained beamformer, any other known adaptive beamformer can be used as the subbeamformer of the parallel spatial processing algorithm. The spatial averaging effect on the jammer from an off-look direction can still be achieved.

IV. Simulation Results

Experiments were conducted for the parallel spatial processing algorithm. The structure in Fig. 5 with four Frost subbeamformers was simulated in a coherent signaling environment. Each subbeamformer had three elements. The adjacent subbeamformers had two overlapping elements. In other words, the linear array had a total of six elements. Each element was assumed omnidirectional, and the interelement distance was one-half wavelength. The ambient white noise was assumed negligible. The constraint was set up to be unit gain and zero phase over the frequency band from zero to half the sampling rate in the desired direction. The initial quiescent beampattern of the proposed scheme is shown in Fig. 6. In

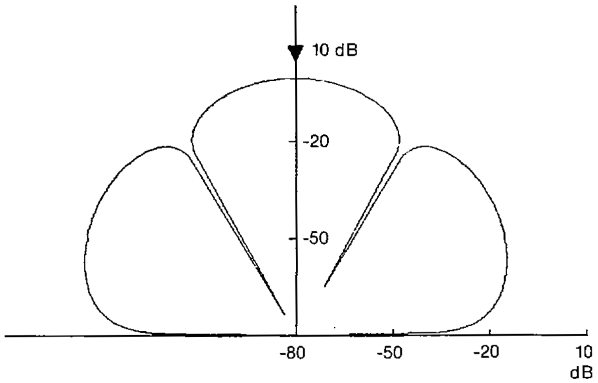

Fig. 6. A quiescent beampattern for the parallel spatial processing algorithm.

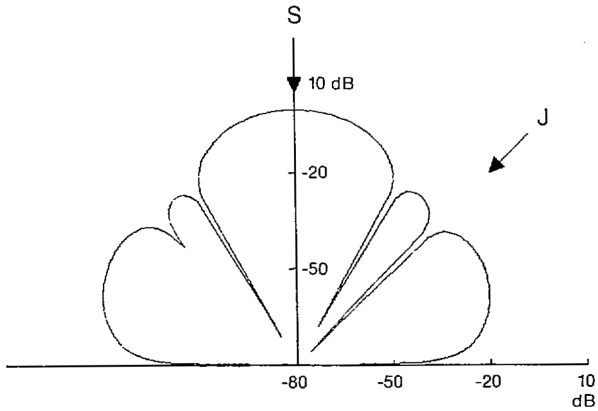

Fig. 7. A converged beampattern for the parallel spatial processing algorithm in a coherent jamming environment.

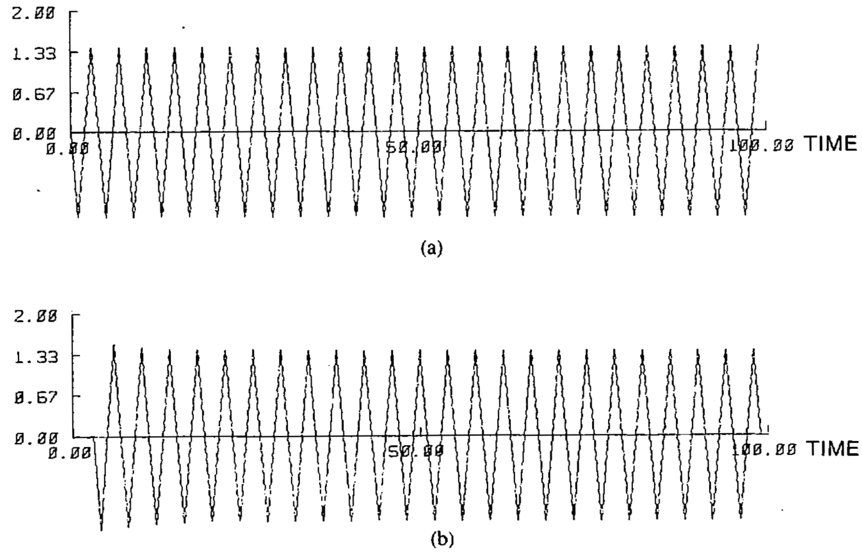

Fig. 8. (a) The time wavefront of the desired signal. (b) The system output of the parallel spatial processing algorithm.

this quiescent beampattern, some inherent nulls exist in the off-look direction, and sometimes these nulls are referred to as grating nulls. The constraints in the look direction were still preserved.

Now suppose a desired sinusoidal signal arrived from the look direction, and a coherent jammer arrived 45° off the look direction. Both the signal and the jammer had equal power intensity of one. Fig. 7 shows a beampattern of the proposed adaptive beamformer when the adaptation process converged. A sharp null with a depth of nearly −70 dB was formed in the incoming direction of the jammer. The linear constraint in the look direction was still preserved at unity. The beampattern resulted as desired. In comparison with Fig. 1, the false nulling effects were essentially eliminated. The system output of the whole structure is shown in Fig. 8, where the desired signal appears clearly and steadily. Notice that the amplitude

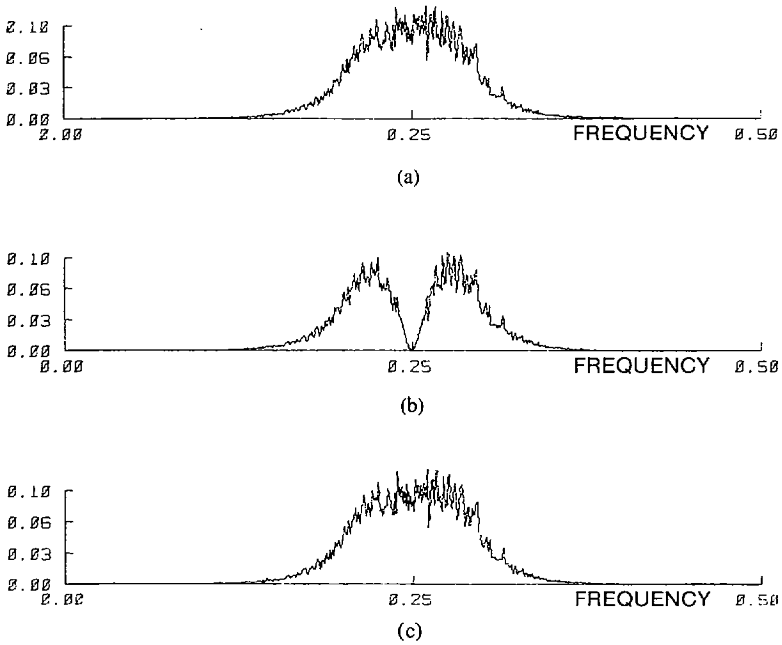

Fig. 9. (a) The power spectrum of a wide-band desired signal. (b) The output power spectrum when signal cancellation occurs. (c) The output power spectrum for the parallel spatial processing algorithm.

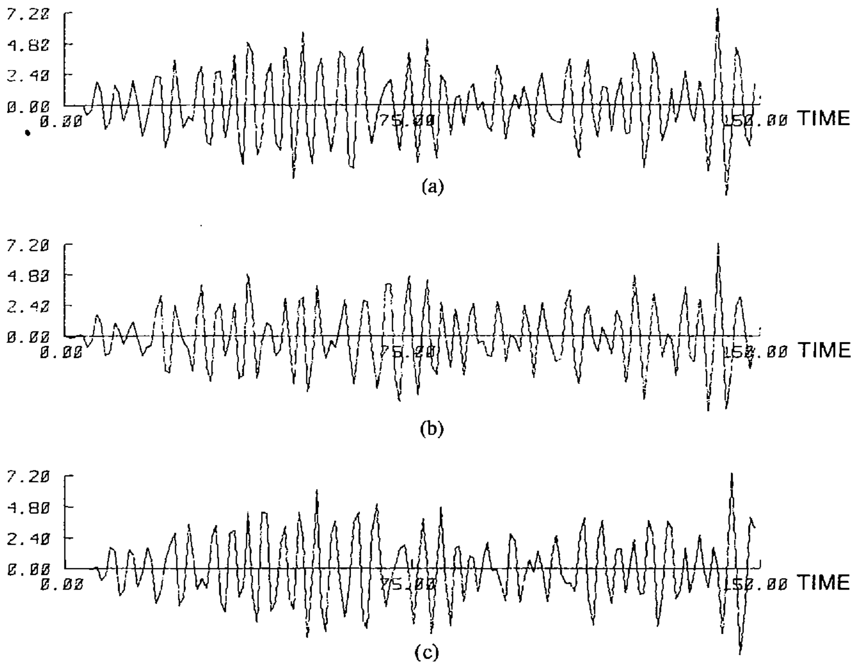

Fig. 10. (a) The time waveform of a wide-band desired signal. (b) The output time waveform when signal cancellation occurs. (c) The output time waveform for the parallel spatial processing algorithm.

corresponds to the desired signal power of one. Also notice that the desired signal is delayed by a few sampling periods while passing through the parallel processing structure. This is in contrast to the output in Fig. 2, where the desired signal was cancelled by the coherent jammer.

The next experiment was similar to the first one except that the desired signal was a wide-band signal and the jammer was still a sinusoid at the center frequency of the signal band. Fig. 9(a) shows the power spectrum of the desired signal. In general, the output spectrum of the Frost beamformer is as shown in Fig. 9(b), where signal cancellation occured in the jamming frequency band. In contrast, the signal-cancellation-free output spectrum for the parallel processing structure is shown in Fig. 9(c). One can easily see that the original signal spectrum was recovered without any signal cancellation effect. Fig. 10 shows the corresponding output time waveforms for the system of Fig. 9. The proposed scheme obviously resulted in a better replica of the desired signal than the conventional Frost beamformer. Note that the output of this parallel spatial processing algorithm was delayed for several sampling periods relative to the desired signal. Convergence of the adaptive process took place after about 60 adaptations.

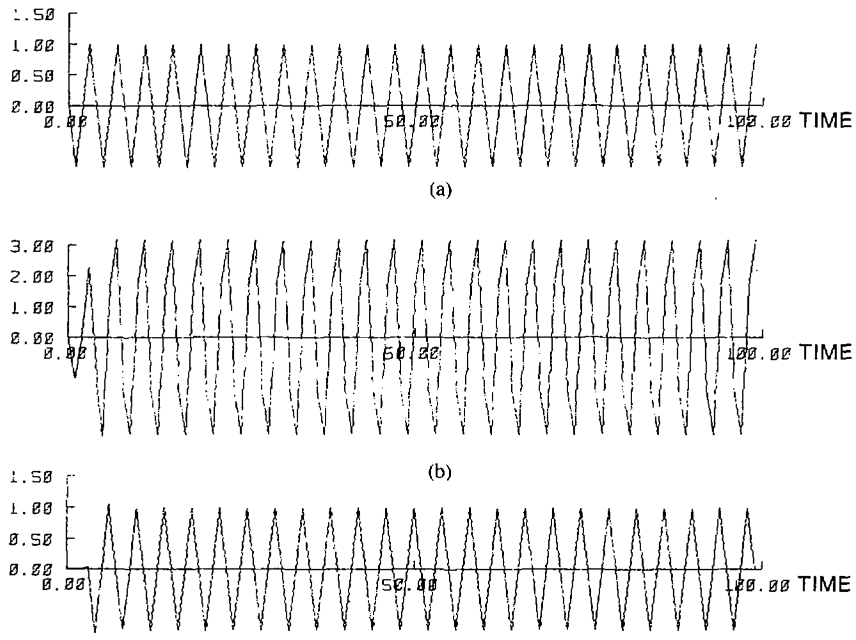

Fig. 11. (a) The time waveform of a desired sinusoidal signal. (b) The output time waveform when using Shan's spatial smoothing algorithm. (c) The output time waveform by using the parallel spatial processing algorithm.

The final experiment compared the output qualities for the proposed method and the spatial smoothing method in [15]. The desired signal, shown in Fig. 11(a), was set to be of unit amplitude. A strong, coherent jammer arrived off the look direction. Both methods were tested by using the same Frost subbeamformers running at a high adaptation rate. The beamformer output of Shan's beamformer is shown in Fig. 11(b), and the output of the proposed method is shown in Fig. 11(c). Apparently, Shan's beamformer introduced some amplitude and phase distortions. For the proposed method, the desired signal was recovered without any distortion, but with a delay of several sampling periods.

V. Conclusion

The "parallel spatial processing" algorithm for adaptive arrays is proposed herein to combat signal cancellation effects in correlated jamming environments. The effectiveness of this algorithm was verified by several computer simulations. The algorithm requires the same computation power as conventional adaptive arrays, although it also requires additional array sensing elements. Analysis shows that the system output results in a maximum-likelihood estimate of the desired signal in a spatial averaging sense, even in the presence of coherent jamming.

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Wide-Band Adaptive Array Processing Using Pole-Zero Digital Filters

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Abstract—**Conventional broad-band array processing is accomplished by linearly combining the outputs of tapped-delay lines attached to each sensor of an array. This type of processing can be interpreted as using an all-zero digital filter at each sensor to generate a frequency-dependent magnitude and phase shift (weighting) over the array operating bandwidth. A new array processing method is presented which uses digital filters having both poles and zeros to perform the frequency-dependent array weighting. Several algorithms for adapting the pole-zero array filters are introduced. Computer simulations are then presented demonstrating the potential for substantial improvement in broad-band interference nulling provided by pole-zero array processing.**

I. Introduction

CONVENTIONAL broad-band array processing is accomplished by linearly combining the outputs of tapped-delay lines attached to each sensor of an array [1]. This type of processing can be interpreted as using an all-zero (or feedforward) digital filter to generate a frequency-dependent magnitude and phase shift over the array operating bandwidth. It is well known that as the operating bandwidth increases, so does the number of taps (i.e., zeros) required to achieve a given level of interference rejection [2], [3]. For relative operating bandwidths above 50 percent, the number of adjustable weights becomes overwhelmingly large.

In this paper, we present a new linearly constrained array processing structure that is able to achieve the same level of broad-band interference rejection as a tapped-delay-line processor but with a considerably reduced number of variable weights. The new structure uses both feedforward and feedback digital filtering to perform the frequency-dependent array weighting. The discrete-time transfer function of this type of filter consists of the ratio of two polynomials and is thus called a pole-zero digital filter. Consequently, the overall array structure is referred to as a *pole-zero array processor*. The use of pole-zero filtering in adaptive arrays has apparently not been previously studied, although the original idea was presented by Gooch in 1981 [4], [5]. For fixed (nonadaptive) array processing, Leung and Barnes have recently proposed the use of infinite-impulse response (IIR) filters for performing the fractional-delay interpolation needed to steer a beam in a given direction [6]. They conclude that IIR filter implementations can be more efficient than conventional finite-impulse response (FIR) implementations for approximating the high transition ratios required for signals with frequencies approaching one-half the sampling rate. In this paper, this improvement in efficiency is also shown to be true for the case of linearly constrained adaptive array processing.

Section II of this paper motivates the application of pole-zero filtering to array processing. It begins by briefly

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