Upper Bound for the Loss of Energy Detection of Signals in Multipath Fading Channels

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***Abstract*—The performance of energy detection under multipath fading is analyzed and compared with locally optimal detection using Pitman's asymptotic relative efficiency. Under the L-tap finite impulse response channel model with zero-mean independent and identically distributed tap coefficients, it is shown that the average performance loss of energy detection is no greater than 50% in sample size for the same performance compared with locally optimal detection exploiting signal correlation. Also, an algorithm exploiting signal correlation and improving the detection performance is proposed based on the estimation of signal correlation. Numerical results show that the proposed algorithm almost achieves the performance of locally optimal detection.**

***Index Terms*—Asymptotic relative efficiency, cognitive radios, energy detecton, multipath channel.**

I. Introduction

ENERGY detection has widely been used to detect unknown signals in noise. It has gained renewed interest recently in cognitive radio communications [1]–[3]. In cognitive radio networks, secondary users opportunistically access the spectrum allocated to a primary user based on channel sensing. Typically, secondary users sense the channel to detect the transmission of the primary user for further access. Since the signature of primary user's signal is not known in most cases, simple energy detection is a reasonable choice for channel sensing in cognitive radio communications [2], [3]. The performance limit of energy detection was considered under noise variance uncertainty in [2]. In this letter, we focus on the performance of energy detection under multipath fading in the context of cognitive radio, and investigate the performance loss of energy detection caused by neglecting the signal correlation induced by multipath fading[1]. Since the exact error probability is not known in general correlated cases, we approach the problem using Pitman's asymptotic relative efficiency (ARE), and show that under the L-tap finite impulse response (FIR) channel model with zero-mean independent and identically distributed (i.i.d.) tap coefficients the average performance loss by energy detection is no greater than 1/2 compared with locally optimal detection exploiting the signal correlation. We also propose an algorithm to improve the detection performance based on the estimation of signal correlation.

The rest of the paper is organized as follows. System model and optimal sensing scheme is provided in Section II. The performance loss of energy detection is analyzed in Section III, and an algorithm exploiting signal correlation is proposed in Section IV, followed by a conclusion in Section V.

II. Data Model and Optimal Sensing

We assume that the signal of the primary user is generated according to general digital modulation. Specifically, a sequence $s[i]$ of transmission symbols is filtered by a pulse shaping filter $p(t)$, and transmitted[2]. We assume that the transmitted signal propagates through FIR channels to secondary users. At the receiver of a secondary user, the received signal is corrupted by additive white Gaussian noise (AWGN), filtered by $p^*(-t)$, and sampled at the symbol rate of the primary user signal which is assumed to be known to the secondary user. Here, $(\cdot)^*$ denotes the complex conjugate. Thus, the discrete-time received signal $y[i]$ at the secondary user is given, as shown in Fig. 1, by

$$\begin{aligned} H_0 &: y[i] = w[i], \quad i = 1, 2, \ldots, n \\ H_1 &: y[i] = \sqrt{\theta} r[i] + w[i], \quad i = 1, 2, \ldots, n \end{aligned} \tag{1}$$

where the alternative hypothesis H_1 represents the case that the primary user transmits and the null hypothesis H_0 represents the case of noise only. Here, $w[i]$ is i.i.d. proper complex zero-mean Gaussian noise with a known variance σ^2, which is assumed to be independent of the signal part $r[i]$, and the signal part is given by

$$r[i] = \sum_{l=0}^{L-1} h[l] s[i-l], \quad i = 1, 2, \ldots, n \tag{2}$$

where $\{h[l], l = 0, 1, \ldots, L-1 : \mathbb{E}\{\sum_l |h[l]|^2\} = 1\}$ are the normalized FIR channel response between the primary user and the secondary user, and θ is the unknown average received energy parameter.[3] We assume that $h[l]$ is drawn i.i.d. from a distribution with zero mean and does not vary over one sensing period. To simplify the analysis, we further assume that the primary symbol sequence $\{s[i]\}$ is an i.i.d. zero-mean complex Gaussian process with unit variance, which is independent of the channel coefficients. Note that the received signal process $r[i]$ is wide-sense stationary but not i.i.d. because of the memory effect of the FIR channel although $s[i]$ is assumed to be i.i.d., which is valid for most coded transmissions. The autocorrelation function of $r[i]$ conditioned on the channel realization $\mathbf{h} = [h[0], h[1], \ldots, h[L-1]]^T$ is given by

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[1]Some part of the work is presented in [4].

[2]The upconversion step is not important since we can adopt the baseband equivalent model with the knowledge of the primary carrier frequency.

[3]Let $\mathbf{h}_t = [h[1], \ldots, h[L]]^T$ be the true propagation channel. Then, $\theta = \mathbb{E}\|\mathbf{h}_t\|^2$ and $\mathbf{h} = \mathbf{h}_t/\sqrt{\mathbb{E}\|\mathbf{h}_t\|^2}$.

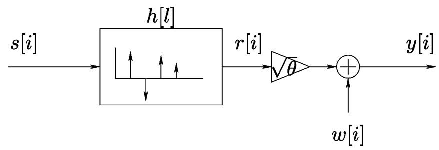

Fig. 1. Discrete-time system model.

$$\gamma_m = \mathbb{E}\{r[i]r^*[i-m]\}, \\ = \begin{cases} \sum_{k=m+1}^{L} h[k]h^*[k-m], & -L+1 \le m \le L-1. \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The SNR under the alternative hypothesis in (1) is given by $\mathrm{SNR} = \theta/\sigma^2$ since $\mathbb{E}\{r^2[i]\} = \mathbb{E}\{|s[i]|^2\}\mathbb{E}\left\{\sum_{l=0}^{L-1}|h[l]|^2\right\} = 1$.

A. Preliminary: Locally Optimal Sensing

First, assume that the channel realization $\mathbf{h}$ and the average signal energy θ are known to the secondary user. In this case, the optimal detection is given under the Gaussian signal assumption by a likelihood ratio detector [5, pp. 72–76]

$$T_{LRT}(\mathbf{y}_n) = \mathbf{y}_n^H \mathbf{\Sigma}_r \left(\sigma^{-2}\mathbf{I} + \theta\mathbf{\Sigma}_r\right)^{-1} \mathbf{y}_n \underset{H_1}{\overset{H_0}{\lessgtr}} \tau_1 \tag{4}$$

where $\mathbf{y}_n = [y[1], y[2], \ldots, y[n]]^T$ and $\mathbf{\Sigma}_r$ is an $n \times n$ signal covariance matrix which is Hermitian and Toeplitz with $[\gamma_0, \gamma_{-1}, \ldots, \gamma_{-L+1}, 0, \ldots, 0]$ as the first row. Here, the detection threshold τ_1 is determined to satisfy a size constraint, $\Pr\{T_{LRT}(\mathbf{y}_n) \ge \tau_1\} = P_F$, where P_F is a desired false alarm probability.

In cognitive radio context, however, it is difficult for the secondary user to acquire the channel information $\{h[l]\}$ and average signal energy θ. When the signal power is unknown as in this case, the sensing problem can be formulated as a composite hypothesis test in which the null and alternative hypotheses are given by

$$\begin{cases} H_0 : \theta = 0 \\ H_1 : \theta > 0 \end{cases} \tag{5}$$

respectively, for the same signal model (1). For the test of (5) with signal model (1, 2), no uniformly most powerful (UMP) detector exists [5, p. 36]. Instead, the detection (5) can be focused on a local setup in which the alternative hypothesis arbitrarily close to the null hypothesis $\theta = 0$ is considered with large sample size, and the locally most powerful (LMP) detection criterion is used. In this local setup, the criterion mainly focuses on the low SNR range since the parameter θ represents the average signal energy, and this is especially suitable to cognitive radio communications in which the strength of primary user's signal is very weak. For the local setup, the locally optimal or LMP detector for (1, 5) is given by the score test [5, p. 80,], [6, p. 220]

$$T_{LMP}(\mathbf{y}_n) = \frac{1}{n}\mathbf{y}_n^H \mathbf{\Sigma}_r \mathbf{y}_n \underset{H_1}{\overset{H_0}{\lessgtr}} \tau_2 \tag{6}$$

where the threshold τ_2 is determined to satisfy the size constraint as in (4). Due to the banded structure of $\mathbf{\Sigma}_r$, $T_{LMP}(\mathbf{y}_n)$ can easily be computed by exploiting this structure and is given by

$$\begin{aligned} T_{LMP}(\mathbf{y}_n) &= \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} y^*[i]y[j]\gamma_{i-j} \\ &= \gamma_0\hat{\gamma}_0 + 2\sum_{m=1}^{L-1} Re\{\gamma_m\hat{\gamma}_m\} \end{aligned} \tag{7}$$

where $\{\gamma_m : -L+1 \le m \le L+1\}$ are the true autocorrelation parameters and $\hat{\gamma}_m$ is the sample autocorrelation given by

$$\hat{\gamma}_m = \frac{1}{n}\sum_{i=1}^{n-m} y^*[i]y[i+m], \quad m = 0, 1, \ldots, L-1. \tag{8}$$

Note that the computational complexity of the LMP detection is $O(Ln)$, which has the same order $O(n)$ as the simple energy detector.

III. Suboptimal Energy Detection and its Performance Limit

For the LMP detection in the previous section, the knowledge of signal correlation γ_m is required. Since the autocorrelation coefficient γ_m is a function of channel coefficients, this requires the knowledge of channels, which is difficult to obtain in cognitive radio communications. To circumvent this difficulty, simple energy detection is commonly adopted and given by

$$T_{\mathrm{EN}}(\mathbf{y}_n) = \frac{1}{n}\gamma_0\mathbf{y}_n^H\mathbf{y}_n = \gamma_0\hat{\gamma}_0. \tag{9}$$

Note that $\mathbf{\Sigma}_r = \gamma_0\mathbf{I}$ and the LMP detection reduces to the energy detection if the signal process $r[i]$ is i.i.d. with flat fading. In case of multipath fading, however, delay spread induces signal correlation and the energy detection yields performance degradation. Since the exact error probability of the LMP detection and energy detection under signal correlation is not available, we adopt Pitman's ARE to evaluate the performance degradation of the energy detection compared with the LMP detection [5, p. 91]. The ARE $\eta_{\mathrm{EN,LMP}}$ of the suboptimal energy detection to the LMP detection is defined as the ratio of the number n_{EN} of samples for the energy detection to the number n_{LMP} of samples for the LMP detection to achieve the same miss probability under the same false alarm probability as $n_{\mathrm{EN}} \to \infty$, i.e., [5, p. 91], [6, p. 195], [7]

$$\eta_{\mathrm{EN,LMP}} \triangleq \lim_{n_{\mathrm{EN}}\to\infty} \frac{n_{LMP}}{n_{\mathrm{EN}}}. \tag{10}$$

Thus, the ARE is a good measure for the relative performance between the two detectors. For example, $\eta_{\mathrm{EN,LMP}} = 1/2$ means that the energy detection requires twice more samples than the LMP detection for the same performance. Under some regularity conditions the ARE is given by the asymptotic ratio of generalized SNR [8]

$$\eta_{\mathrm{EN,LMP}} = \frac{S(T_{\mathrm{EN},\infty})}{S(T_{LMP,\infty})} \tag{11}$$

where

$$S(T_{\mathrm{EN},\infty}) = \lim_{n\to\infty} \frac{S(T_{\mathrm{EN}}(\mathbf{y}_n))}{n}$$

$$S(T_{LMP,\infty}) = \lim_{n\to\infty} \frac{S(T_{LMP}(\mathbf{y}_n))}{n} \tag{12}$$

and for detector X

$$S(T_X(\mathbf{y}_n)) \triangleq \frac{(\mathbb{E}_1\{T_X(\mathbf{y}_n)\} - \mathbb{E}_0\{T_X(\mathbf{y}_n)\})^2}{\mathrm{Var}_0\{T_X(\mathbf{y}_n)\}}. \tag{13}$$

Here, $\mathbb{E}_j\{\cdot\}$ represents expectation under H_j $(j = 0, 1)$, $\mathrm{Var}_0\{\cdot\}$ denotes variance under H_0, and $T_X(\mathbf{y}_n)$ are given in (6) and (9). The ARE of the energy detection to the LMP detection is obtained using the Toeplitz distribution theorem [9] and given by [8]

$$\eta_{\mathrm{EN,LMP}} = \frac{\left(\frac{1}{2\pi}\int_0^{2\pi} f_r(\omega)d\omega\right)^2}{\frac{1}{2\pi}\int_0^{2\pi} f_r^2(\omega)d\omega} \tag{14}$$

where $f_r(\omega)$ is the spectrum of the signal process $r[i]$, given by

$$\begin{aligned} f_r(\omega) &= (2\pi)^{-1}\sum_{m=-\infty}^{\infty} \gamma_m e^{-jm\omega} \\ &= (2\pi)^{-1}\left(\gamma_0 + 2\sum_{m=1}^{L-1}\mathrm{Re}\{\gamma_m e^{-jm\omega}\}\right). \end{aligned} \tag{15}$$

Note from (14) that $\eta_{\mathrm{EN,LMP}} \le 1$ by the Cauchy-Schwarz inequality unless the signal spectrum $f_r(\omega)$ is flat, i.e., $L = 1$. Thus, the energy detection always yields worse performance than the LMP detection in multipath fading channels. By direct computation we have

$$\int_0^{2\pi} f_s(\omega)d\omega = \gamma_0, \tag{16}$$

$$\int_0^{2\pi} f_s^2(\omega)d\omega = (2\pi)^{-1}\left(\gamma_0^2 + 2\sum_{m=1}^{L-1}|\gamma_m|^2\right) \tag{17}$$

and the ARE is given by[4]

$$\eta_{\mathrm{EN,LMP}}(\boldsymbol{\gamma}(\mathbf{h})) = \frac{\gamma_0^2}{\gamma_0^2 + 2\sum_{m=1}^{L-1}|\gamma_m|^2}. \tag{18}$$

Note that the ARE is a function of γ_m, which is a function of the channel coefficients $\mathbf{h}$ in turn through (3). [This dependency is explicitly shown in the left-handed side of (18).] Since $\eta_{\mathrm{EN,LMP}}(\boldsymbol{\gamma}(\mathbf{h}))$ in (18) depends on the channel realization $\mathbf{h}$, it is random. To remove this instantaneous channel dependency and derive the loss factor depending on channel statistics only, we define the average ARE as

$$\overline{\eta}_{\mathrm{EN,LMP}} \triangleq \mathbb{E}_{\mathbf{h}}\{\eta_{\mathrm{EN,LMP}}(\boldsymbol{\gamma}(\mathbf{h}))\} \tag{19}$$

where the expectation is taken over the channel distribution.

We shall now provide a lower bound on the average ARE of the energy detection to the LMP detection based on (18).

[4]Note that for the calculation of (13) only the first and second order properties of distribution is required. Thus, (18) is valid for any second-order stationary signals and the Gaussian signal assumption is not required. However, the test (6) is LMP when the signal is Gaussian. For general stationary signals the test (6) is the maximum-deflection detector.

Lemma 1: Let $h[0], \dots, h[L-1]$ be drawn i.i.d. from a zero-mean distribution with variance $1/L$, i.e., $\mathbb{E}_{\mathbf{h}}\{h[l]\} = 0$, $l = 0, 1, \dots, L-1$, and $\mathbb{E}_{\mathbf{h}}\left\{\sum_{l=0}^{L-1}|h[l]|^2\right\} = 1$. Then

$$f(L) \triangleq \mathbb{E}_{\mathbf{h}}\left\{\sum_{m=1}^{L-1}|\gamma_m|^2\right\} = \frac{L(L-1)}{2L^2} \le \frac{1}{2} \tag{20}$$

for all $L \ge 1$, and $f(L)$ is a monotone increasing function of L and converges to 1/2 as L increases without bound.

Proof: Please see [4]. ■

Theorem 1: For any two-tap channel $\mathbf{h}$ with zero-mean and independent tap coefficients, $\eta_{\mathrm{EN,LMP}}(\boldsymbol{\gamma}(\mathbf{h})) \ge 1/2$, and thus $\overline{\eta}_{\mathrm{EN,LMP}} \ge 1/2$ for $L = 2$.

Proof: Note that $\gamma_0 = |h[0]|^2 + |h[1]|^2$ and $\gamma_1 = |h[0]h[1]^*|$

$$\begin{aligned} \eta_{\mathrm{EN,LMP}}(\boldsymbol{\gamma}(\mathbf{h})) &= \frac{1}{1 + \frac{2|\gamma_1|^2}{\gamma_0^2}} \\ &= \frac{1}{1 + \frac{2|h[0]|^2|h[1]|^2}{(|h[0]|^2+|h[1]|^2)^2}} \\ &\ge \frac{1}{2}. \end{aligned}$$

The last inequality holds since $(|h[0]|^2 + |h[1]|^2)^2 \ge 2|h[0]|^2|h[1]|^2$. ■

Theorem 2: Under the same conditions as in Lemma 1, for sufficiently large L, the average ARE is lower-bounded by 1/2, i.e.,

$$\overline{\eta}_{\mathrm{EN,LMP}} \ge LB(L) \ge \frac{1}{2} \tag{21}$$

and $LB(L)$ converges to 1/2 with rate of $O(1/L)$ as L increases unboundedly.

Proof: Since $\gamma_0 = \sum_{l=0}^{L-1}|h[l]|^2$ and $\mathbb{E}\{\gamma_0\} = 1$, we have by the strong law of large numbers (SLLN) for sufficiently large L

$$|\gamma_0 - 1| \le \epsilon$$

almost surely. Thus, we have for sufficiently large L

$$1 - \epsilon' \le (1-\epsilon)^2 \le \gamma_0^2 \le (1+\epsilon)^2 \le 1 + \epsilon' \tag{22}$$

almost surely for any $\epsilon' > 0$. Then we get (23), shown at the bottom of the next page. Here, (a) is by Jensen's inequality, (b) is by applying (22), (c) is by the definition of $f(L)$ in Lemma 1, and (d) is because ϵ' is arbitrary and $f(L) \le 1/2$ by Lemma 1. The second claim is by substituting $f(L) = (1/2)(1 - 1/L)$ from Lemma 1 in (23). ■

Theorems 1 and 2 provide a fundamental limit for the average loss of the energy detection compared with the LMP detection caused by ignoring the signal correlation in multipath fading channels for two tap channel and large L, i.e., very strong correlation, respectively. Since the average ARE is greater than 1/2, the number of samples for energy detection is no larger than twice of the LMP detection for the same performance in multipath fading channels even with large number of taps. Fig. 2 shows the average ARE of the energy detection to the LMP detection with respect to the number L of taps in multipath fading

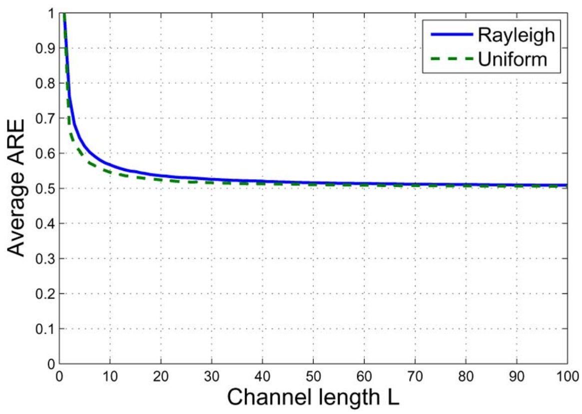

Fig. 2. $\overline{\eta}_{\mathrm{EN,LMP}}$ for L-tap FIR channel with equal tap power.

channels with Rayleigh distribution and circularly uniform distribution for channel coefficients. Although it is not proved for intermediate values of L, it is seen in the figure that the average ARE is larger than 1/2 for intermediate values too.

IV. An Algorithm Exploiting Signal Correlation

As shown in Fig. 2, the performance of energy detection can be improved especially for large L by exploiting the signal correlation. Here, we propose a two-step algorithm exploiting the signal correlation. Assuming that the channel does not vary for the sensing period, the secondary user first estimates the correlation $\hat{\mathbf{\Sigma}}_{\mathbf{y}}$ of the observation signal to yield a signal correlation estimate $\theta^2\hat{\mathbf{\Sigma}}_r = (\hat{\mathbf{\Sigma}}_{\mathbf{y}} - \sigma^2\mathbf{I})^+$, where $(\cdot)^+$ is a limiting operation to yield a positive semi-definite signal covariance matrix. Next, we apply the LMP test based on the estimated signal correlation $\hat{\Sigma}_{\mathbf{r}}$, i.e.,

$$\hat{T}_{LMP}(\mathbf{y}_n) = \frac{1}{n}\mathbf{y}_n^H\hat{\Sigma}_{\mathbf{r}}\mathbf{y}_n \underset{H_1}{\overset{H_0}{\lessgtr}} \tau_3. \tag{24}$$

Simulation was performed to evaluate the performance of the proposed algorithm under equal power Rayleigh fading FIR channels with $L = 7$, $SNR = -12$ dB and $P_F = 0.1$. Fig. 3 shows the miss detection probability for the energy detection, LMP detection and the proposed method for which a maximum likelihood estimation is used for covariance estimation. It is observed that the proposed algorithm improves the performance over the energy detection and yields almost the same performance as the LMP detection.

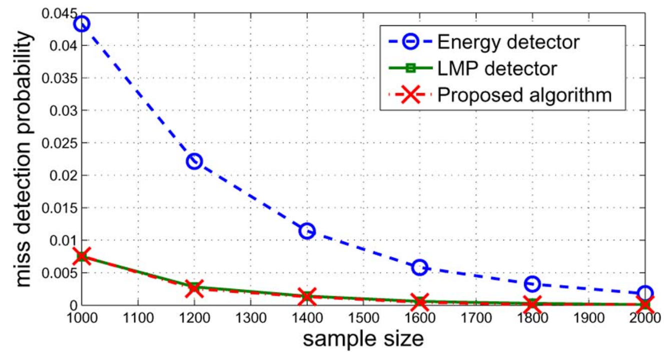

Fig. 3. Miss detection probabilities with respect to sample size.

V. Conclusion and Discussion

We have considered the energy detection of signals under multipath fading. We have investigated the loss of energy detection compared with the LMP detection caused by ignoring signal correlation using Pitman's ARE, and have shown that under the L-tap FIR channel with i.i.d. zero-mean tap coefficients with equal power, the average performance loss of energy detection is no greater than 50% in sample size for the same performance compared with the LMP detection. We have also proposed an algorithm that exploits the signal correlation, and have evaluated its performance numerically. The proposed algorithm yields almost the same performance as the LMP detection with the exact knowledge of signal correlation.

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$$\overline{\eta}_{\mathrm{EN,LMP}} = \mathbb{E}\left\{\frac{\gamma_0^2}{\gamma_0^2 + 2\sum_{m=1}^{L-1}|\gamma_m|^2}\right\} = \mathbb{E}\left\{\frac{1}{1+\frac{2\sum_{m=1}^{L-1}|\gamma_m|^2}{\gamma_0^2}}\right\} \overset{(a)}{\geq} \frac{1}{1+2\mathbb{E}\left\{\frac{\sum_{m=1}^{L-1}|\gamma_m|^2}{\gamma_0^2}\right\}} \overset{(b)}{\geq} \frac{1}{1+\frac{2\mathbb{E}\left\{\sum_{m=1}^{L-1}|\gamma_m|^2\right\}}{(1-\epsilon')}} \overset{(c)}{\geq} \frac{1}{1+\frac{2f(L)}{(1-\epsilon')}} \overset{(d)}{\geq} \frac{1}{2}. \tag{23}$$