Completion Time Minimization and Robust Power Control in Wireless Packet Networks

Chris T. K. Ng, Muriel Médard, and Asuman Ozdaglar
Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology, Cambridge, MA 02139
Email: {ngctk, medard, asuman}@mit.edu

Abstract

A wireless packet network is considered in which each user transmits a stream of packets to its destination. The transmit power of each user interferes with the transmission of all other users. A convex cost function of the completion times of the user packets are minimized by optimally allocating the users' transmission power subject to their respective power constraints. It is shown that, at all ranges of SINR, completion time minimization can be formulated as a convex optimization problem and hence can be efficiently solved. When channel knowledge is imperfect, robust power control is considered based on the channel fading distribution subject to outage probability constraints. The problem is shown to be convex when the fading distribution is log-concave in exponentiated channel power gains; e.g., when each user is under independent Rayleigh, Nakagami, or log-normal fading.

I. Introduction

Power control plays a crucial role in the operation of a wireless network, as it strives to provide maximum benefits to the users within the confines of available resources. In particular, in a wireless network, each user's transmit power interferes with the transmission of all other users; therefore, power allocation has a significant impact on the quality-of-service (QoS) experienced by the network users. The user benefits derived from a given power allocation assignment can be characterized by different performance metrics. Traditionally, a common performance metric is a utility function of the rates of the users.

However, maximizing a concave utility function over the feasible rate region in a wireless network is not necessarily a convex optimization problem [1]–[5]. In the high signal to interference-plus-noise (SINR) regime, rate utility maximization can be approximately formulated as a convex optimization problem [1], and hence it can be efficiently solved. For voice or video applications that need to maintain at least a moderate minimum rate, the high-SINR regime is often an appropriate assumption. On the other hand, in wireless sensor networks or delay-insensitive data applications, a user may wish to transmit at arbitrarily low rates (i.e., at low SINRs) depending on the channel conditions. At medium to low SINR, rate utility maximization are handled by iterative approximation methods [1], [2]. In this paper, we study a different network performance metric that is motivated by data applications. In particular, we consider the scenario where each user transmits a stream of packets to its destination, and we wish to minimize a convex cost function of the user packet completion times. We show that completion time minimization and the corresponding optimal power allocation can be formulated as a convex optimization problem at all ranges of SINR.

Moreover, we consider imperfect channel knowledge due to channel fading, and formulate the minimization as a stochastic programming problem. The channel gains are modeled as random variables: an

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outage event occurs when the SINR realization falls below the transmitter's SINR target. Robust power control is considered where each user is subject to an outage probability constraint. We show that for a wide class of commonly used channel fading distributions, e.g., Rayleigh, Nakagami, and log-normal, robust power control can be posed a convex optimization problem.

Optimal power control in wireless networks is studied in [1]; it shows that maximizing concave rate utility functions can be formulated as geometric programming (GP) problems in the high-SINR regime, which are convex and hence their solutions can be computed efficiently. In the medium- to low-SINR regime, [6] describes an iterative approximation method to maximize a concave rate utility function by solving a series of GPs, and [2] proposes an algorithm to maximize a linear function of the rates by approximating the feasible SINR region by a series of polyblocks. In [3], sufficient conditions are presented for the convexity of the feasible QoS region, with optional constraints on the allocation of user power. Utility maximization through joint optimization of adaptive modulation, rate allocation, and power control is investigated in [5]. In [7] power control is studied in frequency-selective Gaussian interference channels. Outage probabilities corresponding to different fading distributions for network users and interferers are derived in [8]. For interference-limited wireless networks, optimal power control is considered in [9] under Rayleigh fading subject to outage probability constraints.

The rest of the paper is organized as follows. Section II describes the channel model and the optimization framework. In Section III, minimizing a cost function of the completion times is posed as a convex optimization problem. Section IV explains how the completion time region and its corresponding rate region are related. Section V considers robust power control against imperfect channel knowledge subject to outage probability constraints. Conclusions are presented in Section VI.

II. System Model

A. Wireless Channels

Consider the scenario in which M users are communicating in a wireless packet network. Each user i consists of a transmitter i and a corresponding receiver i, where $i = 1, \ldots, M$. Transmitter i wishes to send a stream of equal-length packets to receiver i, where each packet has L_i bits. We assume the complex baseband additive white Gaussian noise (AWGN) channel model between the transmitters and receivers:

$$Y_i = \sum_{j=1}^{M} \sqrt{H_{ij}} X_j + Z_i, \tag{1}$$

where Y_i is the observed signal at receiver i, X_j is the signal sent by transmitter j, H_{ij} is the channel power gain from transmitter j to receiver i, and Z_i is independent Gaussian noise with power N_i. We consider coherent detection: thus H_{ij} may be assumed real as detection is unaffected by phase change in the received signal. In the subsequent sections, we consider different channel knowledge assumptions where H_{ij}'s may represent known constants or random variables. Suppose transmitter i has transmit power constraint $\bar{P}_i$. When transmitter i transmits at a power level of $P_i \leq \bar{P}_i$, in the capacity limit, the transmission rate R_i (bits per channel use) achieved by user i is given by:

$$R_i = \log(1 + S_i), \tag{2}$$

where $\log$ is base 2, and S_i is the signal to interference-plus-noise ratio (SINR) at receiver i. When the interference from the other transmitters is treated as noise, the SINR at receiver i is:

$$S_i = \frac{H_{ii} P_i}{N_i + \sum_{j \neq i} H_{ij} P_j}. \tag{3}$$

We assume each user has a sufficient number of packets backlogged at the transmitter so that it is always transmitting, and we assume L_i is sufficiently large to allow transmission close to the channel capacity

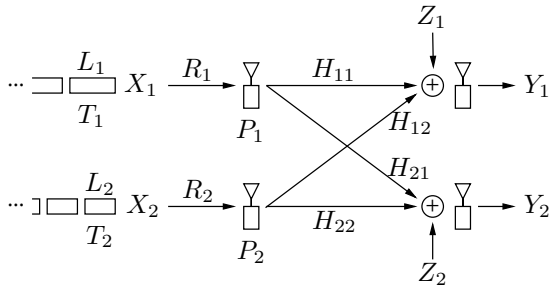

Fig. 1. An AWGN wireless packet network with $M = 2$ users.

limit. The completion time (number of channel uses) of the transmission of each of user i's packet is given by:

$$T_i = L_i/R_i. \tag{4}$$

For example, an AWGN wireless packet network with $M = 2$ users is illustrated in Fig. 1.

B. Completion Time Cost Function

We consider the problem of minimizing a convex cost function of the completion times $T_1, \ldots, T_M$ by optimally choosing the users' transmission power subject to the power constraints: $0 \le P_i \le \bar{P}_i$, $i = 1, \ldots, M$. Let $J(\mathbf{T})$ be the completion time cost function, where $\mathbf{T} \triangleq \left[T_1 \ldots T_M\right]^T$, and other vectors are denoted similarly in this paper. We assume $J(\mathbf{T})$ is jointly-convex in $T_1, \ldots, T_M$; the convexity penalizes overlong completion times. For example, the following completion time cost functions are convex [10]:

$$J_r(\mathbf{T}) = \mathbf{T}_{[1]} + \cdots + \mathbf{T}_{[r]} \tag{5}$$

$$J_p(\mathbf{T}) = \left((T_1)^p + \cdots + (T_M)^p\right)^{1/p}, \quad p \ge 1. \tag{6}$$

In (5), $\mathbf{T}_{[i]}$ denotes the ith largest component of $\mathbf{T}$. Thus the cost function $J_r(\mathbf{T})$ is the sum of the r longest completion times. As special cases, $r = 1$ represents the maximum completion time: $\max\{T_1, \ldots, T_M\}$, and $r = M$ represents the sum of the completion times: $\sum_{i=1}^{M} T_i$. In (6), the cost function $J_p(\mathbf{T})$ is the ℓ_p-norm of the user completion times.

III. Packet Completion Time Minimization

A. Perfect Channel Estimation

We first consider the scenario where the channel power gains H_{ij}'s can be accurately estimated and they are known by all users:

$$H_{ij} = G_{ij}, \quad i = 1, \ldots, M, \quad j = 1, \ldots, M, \tag{7}$$

where G_{ij}'s are known constants. In this case, the minimization of the completion time cost function $J(\mathbf{T})$ can be mathematically formulated as the following optimization problem:

$$\begin{aligned}
&\text{minimize} \quad J(\mathbf{T}) && (8)\\
&\text{over} \quad T_i, R_i, P_i\\
&\text{subject to}\\
&\quad T_i \ge L_i/R_i && (9)\\
&\quad R_i \le \log\left(1 + \frac{G_{ii}P_i}{N_i + \sum_{j\ne i} G_{ij}P_j}\right) && (10)\\
&\quad 0 \le P_i \le \bar{P}_i, && (11)
\end{aligned}$$

where $i = 1, \ldots, M$, and the problem data $G_{ij} \in \mathbb{R}_+$, $\bar{P}_i, N_i, L_i \in \mathbb{R}_{++}$ are given. The constraint (10) in the optimization problem is not convex [1]–[3], [5]. However, we show in the next section (8)–(11) can be transformed into a convex optimization problem, and hence its solution can be efficiently computed.

B. Convex Optimization

To formulate the the completion time minimization problem given in (8)–(11) as a convex optimization problem, we first rewrite the constraints (9), (10) as:

$$T_i \geq \frac{L_i}{\log(1+S_i)}, \quad i = 1, \ldots, M \tag{12}$$

$$S_i \leq \frac{G_{ii}P_i}{N_i + \sum_{j \neq i} G_{ij}P_j}, \quad i = 1, \ldots, M. \tag{13}$$

Next we apply the change of variables:

$$\tilde{S}_i \triangleq \ln S_i, \qquad \tilde{P}_i \triangleq \ln P_i, \qquad i = 1, \ldots, M, \tag{14}$$

where $\ln$ is the natural logarithm. The completion time minimization in (8)–(11) then becomes:

$$\begin{aligned}
&\text{minimize} \quad J(\mathbf{T}) && (15)\\
&\text{over} \quad T_i, \tilde{S}_i, \tilde{P}_i\\
&\text{subject to}\\
&\quad T_i \geq \frac{L_i}{\log\big(1+\exp(\tilde{S}_i)\big)} && (16)\\
&\quad \ln\left\{\exp(\tilde{S}_i - \tilde{P}_i - \ln G_{ii} + \ln N_i) + \sum_{j\neq i} \exp(\tilde{S}_i - \tilde{P}_i + \tilde{P}_j - \ln G_{ii} + \ln G_{ij})\right\} \leq 0 && (17)\\
&\quad \tilde{P}_i \leq \ln \bar{P}_i, && (18)
\end{aligned}$$

where $i = 1, \ldots, M$. Note that the constraint in (17) follows from rewriting (13) as:

$$S_i P_i^{-1} G_{ii}^{-1} N_i + \sum_{j \neq i} S_i P_i^{-1} P_j G_{ii}^{-1} G_{ij} \leq 1, \tag{19}$$

and taking logarithm on both sides after applying (14). The change of variables is similar to the transformation techniques in geometric programming (GP) problems [1]. In particular, the log-sum-exp function in constraint (17) is convex [10]. The convexity of (16) can be verified from its second-order conditions, as the right-hand side of (16) is twice-differentiable and its second derivative is positive:

$$\begin{aligned}
\frac{d^2}{dx^2}\big[\log(1+e^x)\big]^{-1} &= \frac{e^x \ln 2\big[2e^x - \ln(1+e^x)\big]}{(1+e^x)^2\big[\ln(1+e^x)\big]^3} && (20)\\
&> 0, && (21)
\end{aligned}$$

which follows from the inequality $y > \ln(1+y)$ for $y > 0$. Note that the transformation in (14) does impose a slight loss of generality as we assume $P_i \neq 0$. Nevertheless, the formulation in (15)–(18) is otherwise valid for all ranges of SINR, and its solution can be efficiently computed by standard numerical methods in convex optimization [10]. Note that we may consider additional linear or convex constraints on $\mathbf{T}$, and sum power constraints on subsets of $\mathbf{P}$: they can be readily incorporated in the optimization problem without violating its convexity.

C. Fading Channels and Power Adaptation

In this section we consider fading channels, i.e., the channel gains H_{ij}'s in (1) experience random variations. In particular, we assume the channel gains can be characterized by a set of $s \in \{1, \ldots, S\}$ discrete fading states:

$$\mathbf{H} = \begin{cases} \mathbf{G}^{(1)} & \text{with probability } p_1 \\ \vdots & \\ \mathbf{G}^{(S)} & \text{with probability } p_S \end{cases}, \qquad \sum_{s=1}^{S} p_s = 1, \tag{22}$$

where $\mathbf{H} \in \mathbb{R}_+^{M \times M} \triangleq \left[H_{ij}\right]$ is the channel power gain matrix, and $\mathbf{G} \in \mathbb{R}_+^{M \times M} \triangleq \left[G_{ij}\right]$ is defined similarly. For example, the discrete states may represent a finite set of quantized channel estimates. We consider slow fading where the duration of a fading state is long compared to the packet completion times. We assume the channel state s can be accurately estimated and it is known by all users.

We first consider the case where each user can adapt its transmission power level according to the fading state. Suppose user i transmits at power level $P_i^{(s)}$ in fading state s, subject to the average power constraints:

$$\mathrm{E}[P_i] \triangleq \sum_{s=1}^{S} p_s P_i^{(s)} \leq \bar{P}_i, \quad i = 1, \ldots, M. \tag{23}$$

To minimize a cost function of the expected completion times, the optimization problem can be formulated as:

$$\begin{aligned} &\text{minimize} \quad J(\mathrm{E}[T_1], \ldots, \mathrm{E}[T_M]) && (24)\\ &\text{over} \quad \mathrm{E}[T_i], T_i^{(s)}, S_i^{(s)}, P_i^{(s)} \\ &\text{subject to} \\ &\quad \mathrm{E}[T_i] = \sum_{s=1}^{S} p_s T_i^{(s)} && (25)\\ &\quad T_i^{(s)} \geq \frac{L_i}{\log\left(1 + S_i^{(s)}\right)} && (26)\\ &\quad S_i^{(s)} \leq \frac{G_{ii}^{(s)} P_i^{(s)}}{N_i + \sum_{j \neq i} G_{ij}^{(s)} P_j^{(s)}} && (27)\\ &\quad 0 \leq P_i^{(s)} && (28)\\ &\quad \sum_{s=1}^{S} p_s P_i^{(s)} \leq \bar{P}_i, && (29) \end{aligned}$$

where $s = 1, \ldots, S$; $i = 1, \ldots, M$. The optimization in (24)–(29) can then be transformed into a convex optimization problem by similar techniques as described in Section III-B. Note that to minimize the expected value of the cost function, it can be handled similarly by replacing the objective function in (24) by:

$$\mathrm{E}[J(\mathbf{T})] = \sum_{s=1}^{S} p_s J(T_1^{(s)}, \ldots, T_M^{(s)}), \tag{30}$$

where convexity is preserved in the nonnegative weighted sum of convex functions.

In the case where the user cannot adapt its transmission power level to the fading state (i.e., the transmitter is under a short-term power constraint), the optimization problem is similar to (24)–(29), but with the average power constraint in (29) replaced by separate power constraints for each fading state:

$$P_i^{(s)} \leq \bar{P}_i, \quad s = 1, \ldots, S, \quad i = 1, \ldots, M. \tag{31}$$

Note that under the short-term power constraints of (31), minimizing the expected cost function (30) decomposes into S independent optimization problems: i.e., each of $J(T_1^{(s)}, \ldots, T_M^{(s)})$, for $s = 1, \ldots, S$, can be minimized separately.

IV. Relations to Rate Region

In general, in a wireless network as defined in (1)–(4), minimizing a convex cost function $J(\mathbf{T})$ of the completion times $T_1, \ldots, T_M$ is not equivalent to maximizing a concave utility function $U(\mathbf{R})$ of the rates $R_1, \ldots, R_M$. In particular, maximizing $U(\mathbf{R})$ over the rate region is in general non-convex [3]: at high SINR it can be approximately formulated as a GP, and in the medium- to low-SINR regime there are iterative approximation methods [1], [2]. Suppose the cost function $J_+(\mathbf{T})$ is convex and nondecreasing in each argument T_i, then completion time minimization is a special case of rate utility maximization where the optimization problem can be formulated as convex. To see that minimizing $J_+(\mathbf{T})$ can be posed as a rate utility maximization problem, we define the corresponding rate utility function:

$$U_T(\mathbf{R}) \triangleq -J_+(L_1/R_1, \ldots, L_M/R_M). \tag{32}$$

Note that minimizing $J_+(\mathbf{T})$ is equivalent to maximizing $U_T(\mathbf{R})$, and the utility function $U_T(\mathbf{R})$ is concave in $\mathbf{R}$ as prescribed by the convexity composition rules [10].

Nevertheless, in the converse, some rate utility maximization problems can be formulated as minimizing convex functions of the completion times. Suppose we minimize a positively weighted sum of the completion times:

$$J_w(\mathbf{T}) = w_1 T_1 + \cdots + w_M T_M, \quad \mathbf{w} \succeq 0, \ \mathbf{1}^T \mathbf{w} = 1, \tag{33}$$

where $\succeq$ denotes component-wise inequality, then it is equivalent to maximizing:

$$U_d(\mathbf{R}) = -\frac{w_1'}{R_1} - \cdots - \frac{w_M'}{R_M}, \tag{34}$$

where $U_d(\mathbf{R})$ is the utility function that corresponds to minimum potential delay fairness [11], with $w_i' = w_i L_i$, $i = 1, \ldots, M$. In addition, minimizing $J_w(\mathbf{T})$ in (33) is also equivalent to maximizing the weighted harmonic mean of the rates:

$$U_h(\mathbf{R}) = \left(\frac{w_1'}{R_1} + \cdots + \frac{w_M'}{R_M} \right)^{-1}. \tag{35}$$

Note that by applying Jensen's inequality on the convex function $1/x$ for $x \in \mathbb{R}_{++}$, we have

$$\frac{1}{w_1' R_1 + \cdots + w_M' R_M} \leq \frac{w_1'}{R_1} + \cdots + \frac{w_M'}{R_M}, \tag{36}$$

which implies:

$$w_1' R_1 + \cdots + w_M' R_M \geq U_h(\mathbf{R}). \tag{37}$$

Hence maximizing $U_h(\mathbf{R})$ provides a lower bound to $\max w_1' R_1 + \cdots + w_M' R_M$, which represents a weighted throughput of the wireless network. In particular, the bound is tight when $R_1 = \cdots = R_M$ as equality is achieved in (36); therefore, maximizing the minimum rate in (38) below can be formulated as

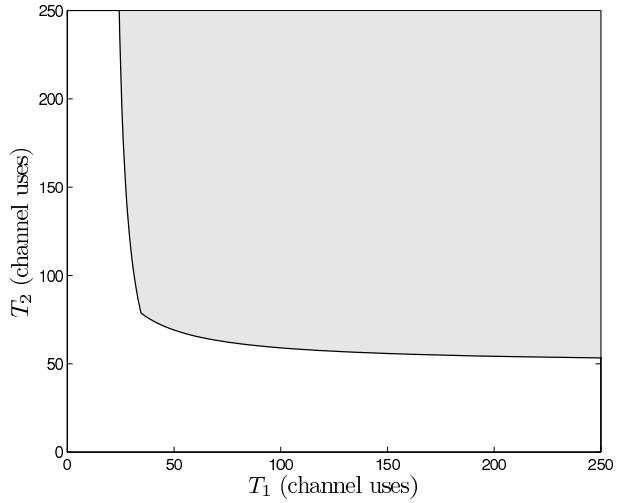

Fig. 2. Completion time region ($\bar{P}_1 = \bar{P}_2 = 0\,\mathrm{dB}$).

a minimization of the convex cost function $J_x(\mathbf{T})$ as given in (39), which corresponds to the maximum completion time:

$$U_n(\mathbf{R}) = \min\{R_1, \ldots, R_M\} \tag{38}$$

$$J_x(\mathbf{T}) = \max\{T_1, \ldots, T_M\}. \tag{39}$$

Moreover, the entire rate region achievable under (10)–(11) can be characterized in terms of the corresponding completion time region. Specifically, the completion time region as characterized in (16)–(18) is convex, and its boundary are given by the minimizer of $J_w(\mathbf{T})$ in (33) over all $\mathbf{w} \succeq 0$, $\mathbf{1}^T\mathbf{w} = 1$. In turn, from the monotonicity of (4), each minimal completion time vector $\mathbf{T}^\star = \arg\min J_w(\mathbf{T})$ corresponds to a maximal rate vector $\mathbf{R}^\star$ on the boundary on the rate region (10)–(11), with $R_i^\star = L_i/T_i^\star$, for $i = 1, \ldots, M$. As a numerical example, we consider the following 2-user AWGN wireless packet network:

$$\mathbf{G} = \begin{bmatrix} 0.42 & 0.89 \\ 0.63 & 0.15 \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \tag{40}$$

$$\bar{\mathbf{P}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \mathrm{dB}, \qquad \mathbf{N} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \mathrm{dB}, \tag{41}$$

with maximum completion time constraints: $T_i \leq 100L_i$. The completion time region and its corresponding rate region are shown in Fig. 2 and Fig. 3, respectively. Note that the power constraints as given in (41) belong to the low SINR regime, where the high-SINR GP approximation does not readily apply. The completion time region is convex in Fig. 2; however, note that its rate region counterpart is non-convex as can be observed in Fig. 3.

V. Robust Power Control

A. Outage Probability Constraints

In Section III, we assume that the channel power gains H_{ij}'s can be accurately estimated. However, in a fading environment where the channel estimates are updated not as fast as the channels vary, the transmitters may not know the H_{ij}'s perfectly. In this section, we consider the scenario where the channel gains are modeled as random variables:

$$\mathbf{H} = \mathbf{W} \triangleq \left[W_{ij}\right], \tag{42}$$

where the users know the joint probability distribution of W_{ij}'s but do not know their realization. As in Section III-C, we assume the duration of a fading state is long compared to the packet completion times.

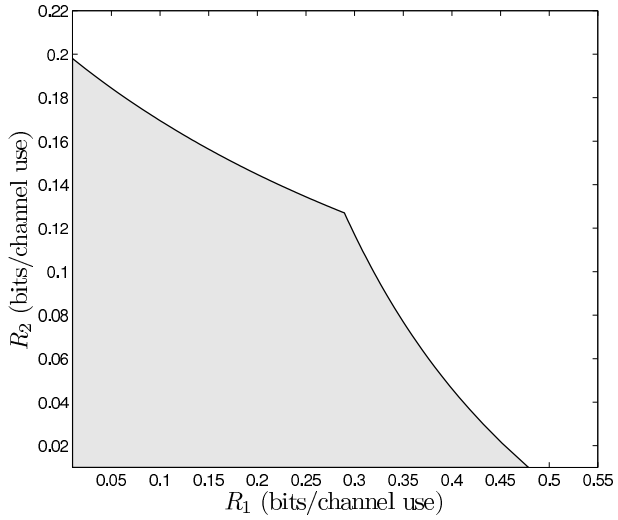

Fig. 3. Rate region ($\bar{P}_1 = \bar{P}_2 = 0\,\mathrm{dB}$).

Based on the channel distribution, each user i chooses a target SINR S_i. Should the realized channel SINR fall below the target S_i, the receiver cannot decode the transmitter's message, and it results in an *outage* event. To ensure the network operates with an acceptable level of reliability, we extend the completion time minimization framework in Section III to additionally consider constraints imposed on the permissible probability of outage. Specifically, we minimize the completion time cost function subject to a set of outage probability constraints: $q_i \in [0,1]$, $i = 1, \ldots, M$, where we stipulate that the probability of user i's transmission in outage not exceed q_i.

Incorporating the outage probability constraints q_i's, the minimization of the completion time cost function is described by the following stochastic programming [12] problem:

$$\begin{aligned}
&\text{minimize} \quad J(\mathbf{T}) && (43)\\
&\text{over} \quad T_i, S_i, P_i\\
&\text{subject to}\\
&\quad T_i \geq \frac{L_i}{\log(1+S_i)} && (44)\\
&\quad \Pr\left\{\frac{P_i W_{ii}}{N_i + \sum_{j\neq i} P_j W_{ij}} \leq S_i\right\} < q_i && (45)\\
&\quad 0 \leq P_i \leq \bar{P}_i, && (46)
\end{aligned}$$

where $i = 1, \ldots, M$. We show that the minimization in (43)–(46) can be posed as a convex optimization problem for a wide class of channel fading distributions commonly considered in wireless communications.

B. Robust Convex Optimization

In terms of the transformed variables in (14), we first define the *reliability function* as:

$$\begin{aligned}
\Phi_i(\tilde{S}_i, \tilde{\mathbf{P}}) &\triangleq \Pr\{\text{user } i \text{ not in outage}\} && (47)\\
&= \Pr\left\{\tilde{W}_{ii} > \ln\left\{\exp(\tilde{S}_i - \tilde{P}_i + \ln N_i) + \sum_{j\neq i} \exp(\tilde{S}_i - \tilde{P}_i + \tilde{P}_j + \tilde{W}_{ij})\right\}\right\}, && (48)
\end{aligned}$$

where (48) follows from rearranging (45) with the additional change of variables:

$$\tilde{W}_{ij} \triangleq \ln W_{ij}, \quad i = 1, \ldots, M, \quad j = 1, \ldots, M. \tag{49}$$

Next, we characterize the reliability function in terms of the channel distribution. Let $\mathbf{W}_i$ be an M-component non-negative random vector that corresponds to the ith row of the channel matrix $\mathbf{W}$:

$$\mathbf{W}_i \triangleq \begin{bmatrix} W_{i1} & \dots & W_{iM} \end{bmatrix}^T, \tag{50}$$

and $\mathbf{w}_i \in \mathbb{R}_+^M$ a realization of $\mathbf{W}_i$. Under (49), the transformed vectors $\tilde{\mathbf{W}}_i$, $\tilde{\mathbf{w}}_i$ are defined similarly. Let $f_{\mathbf{W}_i}(\mathbf{w}_i)$ denote the joint probability distribution function (pdf) of $\mathbf{W}_i$. Theorem 1 below describes the sufficient condition that establishs the log-concavity of $\Phi_i(\tilde{S}_i, \tilde{\mathbf{P}})$, under which (43)–(46) can be posed as the following convex optimization problem:

$$\begin{aligned} &\text{minimize} \quad J(\mathbf{T}) && (51)\\ &\text{over} \quad T_i, \tilde{S}_i, \tilde{P}_i \\ &\text{subject to} \\ &T_i \geq \frac{L_i}{\log\big(1+\exp(\tilde{S}_i)\big)} && (52)\\ &\ln \Phi_i(\tilde{S}_i, \tilde{\mathbf{P}}) \geq \ln(1-q_i) && (53)\\ &\tilde{P}_i \leq \ln \bar{P}_i, && (54) \end{aligned}$$

where $i = 1, \dots, M$. Let $\exp(\mathbf{w}_i)$ denote component-wise exponentiation of the vector $\mathbf{w}_i$.

Theorem 1: The reliability function $\Phi_i(\tilde{S}_i, \tilde{\mathbf{P}})$ is log-concave in $\tilde{S}_i, \tilde{\mathbf{P}}$ if $f_{\mathbf{W}_i}\big(\exp(\mathbf{w}_i)\big)$ is log-concave in $\mathbf{w}_i$.

The proof is given in the appendix. For example, the above condition is satisfied when each user experiences independent fading distributed as [13]: i) Rayleigh (richly scattered environments), ii) Nakagami (significant line-of-sight propagation), or iii) log-normal (shadowing due to signal attenuation). Therefore, in all these cases, completion time minimization subject to outage probability constraints can be formulated as convex optimization problem (51)–(54).

VI. Conclusions

In this paper we consider minimizing a convex function of the completion times of user packets by optimally allocating transmission power in a wireless network. We first focus on the scenario where the channel gains can be estimated accurately and are known by all users. We show that completion time minimization can be formulated as a convex optimization problem, and hence the corresponding optimal power allocation can be efficiently computed. The optimization formulation is valid for all ranges of SINR, which is especially pertinent for wireless sensor networks or delay-insensitive data applications where the users may transmit at moderate or low SINRs. Under fading channels with transmission power adaptation across fading states, an average power constraint can be incorporated into the optimization problem. We show that completion time minimization is a special case of rate utility maximization for which the optimization problem can be posed as convex. In particular, in a wireless network, although the feasible rate region is non-convex, the corresponding completion time region is shown to be convex. Finally, we consider robust power control under imperfect channel knowledge in fading channels. Completion times are minimized subject to outage probability constraints over the fading distribution, and we show that for a wide class of commonly used fading distributions, e.g., Rayleigh, Nakagami, and log-normal, robust power control can be posed as a convex optimization problem.

Appendix

Proof of Theorem 1

We first introduce, in terms of the transformed variables in (49), the notation of $\tilde{\mathbf{W}}_{-i} \in \mathbb{R}^{M-1}$ representing the interfering channel random vector:

$$\tilde{\mathbf{W}}_{-i} \triangleq \begin{bmatrix} \tilde{W}_{i1} & \dots & \tilde{W}_{i\,i-1} & \tilde{W}_{i\,i+1} & \dots & \tilde{W}_{iM} \end{bmatrix}^T, \tag{55}$$

and $\tilde{\mathbf{w}}_{-\mathbf{i}}$ is a realization of $\tilde{\mathbf{W}}_{-i}$. The pdf of $\tilde{\mathbf{W}}_{-i}$ is given by the marginal:

$$f_{\tilde{\mathbf{W}}_{-i}}(\tilde{\mathbf{w}}_{-i}) = \int_{\tilde{W}_{ii}} f_{\tilde{\mathbf{W}}_i}(\tilde{\mathbf{w}}_i)\, d\tilde{W}_{ii}. \tag{56}$$

Next, conditioning on $\tilde{\mathbf{W}}_{-i} = \tilde{\mathbf{w}}_{-i}$, the reliability function in (48) is given by:

$$\Phi_i(\tilde{S}_i, \tilde{\mathbf{P}}) = \int_{\tilde{\mathbf{W}}_{-i}} \phi_i(\tilde{S}_i, \tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-i})\, \mathbf{d}\tilde{\mathbf{w}}_{-i}. \tag{57}$$

For notational convenience, we write $\phi_i(\tilde{S}_i, \tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-i})$ as a composition of:

$$\phi_i(\tilde{S}_i, \tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-i}) \triangleq \bar{F}_{\tilde{W}_{ii}}\big(g_i(\tilde{S}_i, \tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-i})\big)\, f_{\tilde{\mathbf{W}}_{-i}}(\tilde{\mathbf{w}}_{-i}) \tag{58}$$

$$\bar{F}_{\tilde{W}_{ii}}(\tilde{w}_{ii}) \triangleq 1 - F_{\tilde{W}_{ii}}(\tilde{w}_{ii}) \tag{59}$$

$$g_i(\tilde{S}_i, \tilde{\mathbf{P}}, \tilde{\mathbf{w}}_{-i}) \triangleq \ln\Bigg\{\exp(\tilde{S}_i - \tilde{P}_i + \ln N_i) + \sum_{j\neq i} \exp(\tilde{S}_i - \tilde{P}_i + \tilde{P}_j + \tilde{w}_{ij})\Bigg\}, \tag{60}$$

where $F_{\tilde{W}_{ii}}(\tilde{w}_{ii})$ in (59) is the cumulative distribution function (cdf) of $\tilde{W}_{ii}$, and $\bar{F}_{\tilde{W}_{ii}}(\tilde{w}_{ii})$ is referred to as its complementary cdf, which is a nonincreasing function in $\tilde{w}_{ii}$.

Proof: In the construction of $\Phi_i(\tilde{S}_i, \tilde{\mathbf{P}})$ in (57), with the application of Lemma 1 below, log-concavity is preserved [10], [12] in the integration in (57), (56); multiplication in (58); complementary cdf in (59); and composition of a logarithmically concave, nonincreasing function with a convex function in (60). ■

Lemma 1: Under transformation (49), $f_{\tilde{\mathbf{W}}_i}(\tilde{\mathbf{w}}_i)$ is log-concave in $\tilde{\mathbf{w}}_i$.

Proof: Consider the logarithm of the pdf of transformed random vector $\tilde{\mathbf{W}}_i$:

$$\ln f_{\tilde{\mathbf{W}}_i}(\tilde{\mathbf{w}}_i) = \ln\Bigg\{\exp\Big(\sum_{j=1}^{M} \tilde{w}_{ij}\Big) f_{\mathbf{W}_i}\big(\exp(\tilde{\mathbf{w}}_i)\big)\Bigg\} \tag{61}$$

$$= \sum_{j=1}^{M} \tilde{w}_{ij} + \ln f_{\mathbf{W}_i}\big(\exp(\tilde{\mathbf{w}}_i)\big), \tag{62}$$

where log-concavity of $f_{\mathbf{W}_i}\big(\exp(\tilde{\mathbf{w}}_i)\big)$ in $\tilde{\mathbf{w}}_i$ follows from assumption from Theorem 1. ■

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