Bright spatial solitons in non-Kerr media: stationary beams and dynamical evolution

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Received January 30, 1997; revised manuscript received June 6, 1997

We give a comprehensive summary of the results on self-guided beams, bright spatial solitary waves, emphasizing the most recent advances. To be stable in a bulk medium, such beams should propagate in a non-Kerr medium. It is emphasized that all spatial solitons are generically the same, independent of the type of nonlinear medium, and such free-propagating beams can execute interesting dynamics, including spiraling around one another, self-tapering, periodically changing their shape, cross section, or polarization, and inelastically colliding to annihilate one another, fuse, or create a new beam. Most of these types of solitary-wave dynamics have recently been confirmed experimentally for self-guided beams propagating in different non-Kerr materials. Significantly, there exists no temporal analog (e.g., for pulse propagation in fibers) even for stationary spatial solitons of a bulk medium. [S0740-3224(97)03211-6]

1. INTRODUCTION

Recent years have shown increased interest in self-guided localized beams propagating in bulk nonlinear materials (see, e.g., Ref. 1 and references therein). These are free-standing beams that do not require waveguides. Such beams are commonly referred to by physicists as bright spatial solitons even though they do not adhere to the formal mathematical definition of solitons, which is solely the preserve of a one-dimensional beam of a cubic Kerr medium described in the paraxial approximation.

One of the important recent advances in this field was the development of the unifying conceptual approach[1] based on the notion that a self-guided beam induces a waveguide, permitting direct application of the physics and results of the theory of linear guided waves. The observation that a beam creates a waveguide and guides itself in it is old. For example, the notion of the soliton-induced waveguide[2] was clearly stated in some early papers on the self-focusing of light by Chiao *et al.*[3] and Akhmanov *et al.*,[4] and it has been rediscovered often since that time (see, e.g., Ref. 5 as a typical example). But it was not realized until recently that this concept alone could be used by means of a self-consistency principle to permit novel predictions and to yield new closed-form expressions for both stationary-beam propagation and for beam dynamical evolution and even for beam interactions (see the original papers[6–9,11–16] and also the review papers[1,17,18]). This idea has significantly extended our knowledge of self-guided beams in non-Kerr media, allowing a theoretical approach to be developed even in cases far from those described by integrable models. The most important fact is that many of the predicted phenomena have now been confirmed experimentally.

Our purpose in the present paper is to give a comprehensive summary of the main theoretical advances in the theory of self-guided beams, based on the linear perspective concept. Because a comprehensive review of dark spatial solitons is available elsewhere,[19] here we concentrate mostly on spatially localized beams, known as bright solitons, with fields monotonically decreasing from the beam center. After giving some background physics, we first emphasize that a stationary (scalar) bright soliton in an isotropic nonlinear medium must have a cross section that is circularly symmetric[20] and that the medium's nonlinearity must be in some sense saturating (noncubic or non-Kerr; e.g., see Ref. 6) if the beam is to be stable in a bulk material.[9] Second, our discussion is related mostly to the dynamics of beams in bulk materials. We particularly emphasize that there is no temporal analog of spatial bright solitons[17] for reasons elaborated below.

2. PHYSICAL BACKGROUND

Simple physics explains the existence of spatial solitons.[1,20] First we recall the physics of an optical waveguide (see, e.g., Ref. 21). Optical beams have an innate tendency to spread as they propagate in a homogeneous medium. However, beam diffraction can be compensated for by beam refraction if the refractive index is increased in the region of the beam. The resulting optical waveguide can provide an exact balance between diffraction and refraction if the medium is uniform in the direction of propagation.

A similar effect of diffraction suppression can be produced solely by nonlinearity. As has been well established in many experiments,[22] some materials can exhibit considerable optical nonlinearity when their properties are modified by light propagation. In particular, if a nonlinearity leads to a change of the refractive index of the medium in such a way that it generates an effective positive lens to the beam, the beam can become self-trapped and propagate unchanged (stationary propagation) without any external waveguiding structure.[3] These stationary self-guided beams are known as spatial optical solitons, which can exist with profiles of a certain form allowing local compensation of the beam diffraction by the nonlinearity-induced change in the material's refractive index.

Until recently optical soliton theory was based primarily on the nonlinear Schrödinger equation,[23] describing one-dimensional beams of a Kerr (cubic) nonlinear medium in the so-called paraxial approximation. This certainly is the appropriate model for temporal solitons propagating enormous distances along existing optical fibers. But the model is both inappropriate and unphysical for bright spatial beams propagating in bulk materials. Indeed, as was recognized long ago (see, e.g., Ref. 24), stationary solutions of the $(2 + 1)$-dimensional nonlinear Schrödinger equation are unstable and exhibit collapse (see, e.g., Refs. 25 and 26). Saturation has been suggested as a way to stabilize the self-focused beams (see, e.g., Ref. 27), the effect also known in some other fields (see, e.g., Ref. 28). Recent advances in this field[20] have been based on a very simple physics and the so-called qualitative approach, showing that stationary beams must have a circular cross section and that they must also propagate in a non-Kerr (particularly, saturating) medium if they are to be stable in a bulk medium. The first elegant closed-form expression for stable beams of circular symmetry, the so-called induced optical fiber, was presented in Ref. 6. Furthermore, theory has also shown that the behaviors of beams in non-Kerr materials are qualitatively similar and that, in general, they all obey a generalized nonlinear Schrödinger equation.[1] This has led to a number of predictions about spatial solitons and their dynamical evolution and interactions that are not possible from the one-dimensional nonlinear Schrödinger equation.[1,16,17] Finally, applications of spatial solitons involve millimeter lengths and not the kilometers of temporal solitons.

From this perspective we understand that there is no simple mapping between temporal and spatial solitons in bulk materials. Spatial solitons are a much richer and more complex phenomenon. Recent experiments, particularly those in the past year or so, have borne out many of the theoretical predictions mentioned above, especially those for non-Kerr materials.

In particular, it was recently demonstrated theoretically and experimentally that self-guided beams can be observed in materials with a strong photorefractive effect,[29–32] in vapors with strong saturation of the refractive index,[33,34] and also as a result of the phase-matched two- and three-wave parametric interactions in $\chi^{(2)}$ nonlinear crystals.[35–37] In all these cases propagation of self-guided waves is observed in non-Kerr materials that are described by models more general than the cubic nonlinear Schrödinger equation.

3. UNIFYING CONCEPTUAL APPROACH

That a stationary self-guided beam is a mode of the waveguide it induces is self-evident and has been known for more than 30 years, from the first prediction of the phenomenon.[2–4] Yet only recently has it been recognized that this fact provides the foundations for a self-consistent method for actually obtaining closed-form expressions for both stationary and nonstationary self-guided beams (spatial solitons) as well as their interactions (see overviews presented in Refs. 1, 12, and 18). This new theoretical approach—called the linear perspective—has led to many predictions that ultimately have been confirmed experimentally.

The success of the linear perspective concept is based on the fact that all spatial solitary waves are qualitatively the same. In particular, all stationary self-guided beams can be treated as the modes of a (linear) axially uniform waveguide. This waveguide is induced by the interaction of light with the nonlinear medium and is in general anisotropic, e.g., as for the two-wave solitary waves of a quadratic [or $\chi^{(2)}$] medium (see Ref. 1). This elementary concept allows us to borrow physics and exact soliton descriptions directly from the pages of waveguide theory even without any prior knowledge of solitons.[1,12,18] More generally, soliton dynamics can also be understood from this elementary approach, but the induced waveguides are then axially nonuniform.[1,16] All of this applies to different nonlinear media, including wave mixing in $\chi^{(2)}$ media (see Section 9 of Ref. 1) and self-trapping in photorefractive materials, as pointed out in Ref. 1. Conceptually speaking, nonlinear beams interact with matter to create their own waveguides. We emphasize that these waveguides are linear and that they can be of arbitrary shape and form. Beams then propagate along their own induced waveguide according to the familiar physics of linear optics. For example, in the simplest case a soliton is one mode of the waveguide it induces; more generally, it is any two modes of the induced waveguide, which explains the coexistence of different classes of solitons such as dark and bright. Vector solitons are the exceptional case in which the modes are degenerate.[1,12] Periodic (higher-order) solitons can be viewed as being created by the beating of two or several modes,[13] and so on. This significantly generalizes the soliton as originally envisaged[3] and provides the first physical explanation for mysterious phenomena such as radiation free collisions and periodic oscillations that result from scaling up a soliton.[23] The fact that every nonlinear problem has a linear equivalent provides a powerful conceptual tool, one that guides us in a physical manner to the fundamental equations and to their solutions, as well as providing us the insight necessary for predictions and for interpreting experiments.

Spatial optical solitons are solutions of Maxwell's equations. These equations are greatly simplified for solitons of any practical material when the weak-guidance approximation is assumed to be valid. We emphasize that this is solely because the maximum refractive index approximately equals the minimum refractive index, so that the components of the vector electric field $\mathbf{E}$ obeys

$$2ikn_0 \frac{\partial \mathbf{E}}{\partial z} + \nabla_\perp^2 \mathbf{E} + k^2(\mathbf{n}^2 \cdot \mathbf{E} - n_0^2 \mathbf{E}) = 0, \qquad (1)$$

where $\mathbf{E}$ is the vector that describes the slowly varying wave envelope, $\nabla_\perp^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$, n_0 is the refractive index at zero intensity, and $\mathbf{n} = \mathbf{n}(|\mathbf{E}|^2)$ is the tensor refractive index induced by a beam of finite amplitude. This result is easily derived directly from Maxwell's equations (see Section 8 of Ref. 1). Equation (1) can also be derived for wave mixing; then the components of the electric field are the harmonic envelopes.[1]

4. STATIONARY SPATIAL SOLITONS

A rich variety of stationary non-Kerr solitons exist in analogy with those of a Kerr medium. The fundamental physical concept that a soliton is (self-consistently) a mode of the effective linear waveguide it induces[1] allows us, in principle, to gain a deep insight into the properties of solitary waves, knowing the structure and the type of this waveguide. For non-Kerr bright solitons of any dimension many useful physical results, including existence and stability, can be extracted with the help of an even simpler, qualitative approach, the soliton sketch,[20] based on the fact that a stationary soliton is a balance between the beam diffraction and the nonlinearity-induced change in the refractive index.

Stationary scalar solitary waves have the form

$$\mathbf{E}(\mathbf{r}, z) = \mathbf{e}E(\mathbf{r}; \beta)\exp(i\beta z), \tag{2}$$

where $\mathbf{r} = (x, y)$, $\mathbf{e}$ is the unit polarization vector, and $E(\mathbf{r}; \beta)$ is a real envelope that depends on the propagation constant β. The parameter β defines both the shape and the intensity of the stationary beam. For the stationary beam $E(\mathbf{r}; \beta)$ does not involve the spatial distance z when Eq. (2) is reduced to a simple eigenvalue equation. Profiles of the stationary waves can be found as separatrix trajectories that start from and return to a critical point corresponding to vanishing boundary conditions, $E(\mathbf{r}; \beta) \to 0$ for $|\mathbf{r}| \to \infty$. From the mathematical point of view the regions of the existence of bright-solitary-wave solutions with decaying tails can be determined by analysis of the critical points. For scalar solitons the corresponding ordinary differential equations are integrable (a system with one degree of freedom), and the structure of the critical points does not allow the existence of complicated (e.g., multihump[12,38,39]) solutions. For vector solitons, e.g., two different orthogonal (degenerate) modes of the waveguide they induce (see Ref. 12 and also Ref. 1, Sections 3.1 and 3.2), the stationary waves are described by a system of coupled (and generally nonintegrable) ordinary differential equations that may display many exotic solutions (including multihump localized solutions, solutions with oscillating tails, etc.), which are all usually unstable. The existence of such solitons is a direct consequence of the complex critical points of the corresponding dynamical system describing the stationary localized modes.

Two-component solitons, or the more general cases of vector solitons, were first studied by Berkhoer and Zakharov,[40] and later Manakov[41] demonstrated the existence of exact analytical solutions for this kind of two-component solitary wave, the so-called Manakov solitons. Vector solitary waves were discussed in their application to temporal solitons in optical fibers,[38] and they were also demonstrated experimentally.[42,43] However, as was shown in Ref. 12, a vector spatial soliton is a very particular case of a more general dynamic soliton, which can be described as two different modes of its induced waveguide. Among other things, this leads to interesting predictions about the beam polarization dynamics.[1,12]

Apart from the vortex soliton,[9,10] no other spatial soliton is mathematically stable in a Kerr (cubic) nonlinearity.[9] However, simple physical reasoning shows that nonlocality or saturation imposes stability.[20] Furthermore, it can be shown[20] for scalar solitons that stationary beams must have a circularly symmetric cross section. Clearly there is no temporal analog of bright spatial solitons, i.e., beams that have circular cross sections and propagate in non-Kerr materials. Just as there are many types of circular (linear) waveguide profile, there exist correspondingly many types of induced waveguide, depending on the medium. We consider only monotonically decreasing spatial (bright) solitons, but dark solitons exist that are dimmer than their flat background, and so do brightlike dark solitons that are brighter than their flat tails.[44] A qualitative theory was recently developed[20] that enables us to determine the existence and stability of bright solitons simply by inspecting the refractive index versus intensity curve that characterizes any scalar nonlinear medium. This gives a direct physical meaning to abstract mathematical concepts such as bifurcation. The first analytical expression for a stable soliton of circular cross section was for an ideal saturating medium, and that led to the prediction of stable induced optical fibers for light-guiding light.[6] Subsequently, the logarithmic nonlinearity[16] was shown to have stationary solitons that are independent of intensity. Both of these theoretically predicted phenomena were observed experimentally.[30,33] It was also predicted that there are stationary vector solitons in which the two field components are orthogonal (a mode of an induced anisotropic waveguide) but travel at the same speed.[1,12] This is a contrived situation. More generally the two components travel at different speeds, causing a rotation of polarization as first predicted in Ref. 12. This can explain the possibility of having two parallel vector solitons,[14] which would not be possible with scalar self-localized beams.

Solitons in nonlocal media. Above, we have been considering solitons in a nonlinear medium with a purely local response. In that case the intensity at position x produces a change in the refractive index at the given point x only. Solitary waves can be also supported by nonlocal nonlinearities such as, for example, photorefractive materials. In such materials a light beam of radius ρ creates a circularly symmetrical refractive-index change whose spatial extent is ρ_m. The smaller ρ/ρ_m, the more highly nonlocal the process. The collisions, interactions and deformations of solitons in a highly nonlocal medium are solutions of the effective equation for a linear harmonic oscillation and hence have an elegant description.[45] A highly nonlocal medium is unique in that it can support (scalar) stationary beams of noncircular symmetry.

5. SOLITON STABILITY

Existence of stationary solutions in a non-Kerr medium does not guarantee the stability of self-guided beams, and therefore the stability becomes one of the most important issues for the analysis of self-trapping in a non-Kerr medium. Bright solitons of any dimension are known to be stable provided[25,26,46] $\mathrm{d}\beta/\mathrm{d}P > 0$, where $P = \int_V |\mathscr{E}(\mathbf{r};\ \beta)|^2 \mathrm{d}\mathbf{r}$ is the beam power and β is the soliton propagation constant. A simple mathematical relation tells us[20] that the condition for solitary-wave stability can be simplified to the form $\mathrm{d}I/\mathrm{d}P > 0$, where I is the maximum intensity of the beam. The stability criterion for guided waves becomes more complicated in the case of spatially inhomogeneous media, e.g., in waveguide structures (see, e.g., Refs. 47 and 48), but general results have now been obtained even for the case of arbitrary nonlinearity.[26]

This criterion of soliton stability is usually valid for the bright solitons that constitute a one-parameter family, i.e., those whose shape is defined solely by the beam propagation constant or the maximum intensity. In particular, it has also been shown that a similar criterion applies for two-wave solitons in $\chi^{(2)}$ materials.[49]

Linear stability analysis does not allow us to predict the subsequent evolution of unstable (bright and dark) solitons. This can be investigated with the help of the numerical beam propagation method. Recently an elegant analytical approach, valid near the point of the soliton stability change (stability threshold), was suggested and elaborated in detail[50,51] for solitary waves described by the generalized nonlinear Schrödinger equation. This allows us to analyze the nonlinear regime of the soliton instability when the beams diffract, collapse, or switch to a novel (stable) state with long-lived oscillations of their amplitude. These periodic or oscillating solitons[12] are generally possible in models of different dimensions, and they are due to the existence of soliton internal modes[51]; this phenomenon can also be understood as a beating between two (or more) modes of the induced waveguides.[13]

Many novel soliton states demonstrated recently for the case of two interacting fields are unstable; i.e., they cannot be realized experimentally. A general theory of stability for bright vector solitons, rotating or dynamic solitons,[12] is still an open problem. However, recently it was demonstrated that the threshold of instability for two-parameter solitons, analogous to the condition $\mathrm{d}\beta/\mathrm{d}P$ for the scalar solitons, is given by the following criterion:

$$\frac{\partial(\mathscr{F}_1,\ \mathscr{F}_2)}{\partial(\beta_1,\ \beta_2)} \equiv \frac{\partial \mathscr{F}_1}{\partial \beta_1}\frac{\partial \mathscr{F}_2}{\partial \beta_2} - \frac{\partial \mathscr{F}_1}{\partial \beta_2}\frac{\partial \mathscr{F}_2}{\partial \beta_1} = 0, \tag{3}$$

where $\mathscr{F}_j$ ($j = 1, 2$) are two powerlike (Manley–Rowe) invariants of the system describing coupled (e.g., bright–dark) solitons or vector solitons of two interacting polarizations and β_j are two independent parameters of the stationary localized (soliton) solution. This result seems to be valid for any dimension and for different types of vector (or coupled) soliton described by two independent parameters introduced by two nontrivial invariants of the model. For example, in the case of coupled bright–dark solitons, $\mathscr{F}_1$ is the power of the bright component P, and β_1 is the propagation constant of the bright component, whereas $\mathscr{F}_2$ is the total momentum M and β_2 is the soliton velocity V.[52] The same result holds for the stability of three-wave parametric solitons in a $\chi^{(2)}$ medium[53] and for the models in which Galilean invariance is absent, such as two-wave parametric solitons with the walk-off effect.[53] In this latter case the second parameter is the soliton velocity V, and the second invariant is the soliton momentum M.

6. SOLITON DYNAMICS

Since the Kerr solitons in (1 + 1) dimensions are described by the integrable cubic nonlinear Schrödinger equation, soliton interactions are known to be elastic. This is, however, not the case for solitons in three-dimensional space in non-Kerr materials, those that have a rich dynamics and exhibit many interesting features of their interaction, including spiraling, fusion, and generation of new solitons.

The challenging problem of theory is to describe the trajectory of beams that are not stationary and to describe them in (2 + 1) dimensions. No temporal analogue exists for such problems. This is clearly a difficult task, so it has in the past been necessary to restrict the treatment to merely delivering beam stability.

Soliton steering in three dimensions, including spiraling. One of the important physical concepts that allows us to treat beam propagation and interaction is the notion that a self-guided beam moving in a medium with a slowly varying refractive index can be described as an effective optical ray. The ray (or geometrical optics) method is well known (e.g., see Ref. 21) and is used to account for various phenomena of scattering and to account for wave propagation in acoustics and electrodynamics (see, e.g., Ref. 54). Applied to self-guided beams through the method of invariants,[7] the ray method allows us to describe the beam trajectories like rays in a graded index fiber. Indeed, in a medium with a slowly varying refractive index the quasi-stationary beam [Eq. (2)] can be presented in the form

$$\mathscr{E}(\mathbf{r},\ z) = \hat{E}(\mathbf{r} - \mathbf{r}_0;\ \beta)\exp[iS(\mathbf{r} - \mathbf{r}_0,\ z)], \tag{4}$$

assuming that the beam propagation constant, $\beta = \partial S/\partial z$, varies slow with z. Equations for the beam trajectory $\mathbf{r}_0(z)$ can be then derived from the relations for the invariants of the model [Eq. (1)], the power $P = \int |\mathscr{E}|^2 \mathrm{d}A$, and the Hamiltonian $H = \int \{|\nabla_\perp \mathscr{E}|^2 - k^2 F(I)\} \mathrm{d}A$, where $F(I) = \int_0^I (n^2 - n_0^2)\mathrm{d}I$. These equations are

$$\frac{\mathrm{d}\mathbf{r}_0}{\mathrm{d}z} = \mathbf{k}, \quad \frac{\mathrm{d}\mathbf{k}}{\mathrm{d}z} = -\frac{\partial W}{\partial \mathbf{r}_0}, \tag{5}$$

where, for a single beam, the effective potential W is defined as

$$W(\mathbf{r}_0,\ z) = \frac{\displaystyle\int (n^2 - n_0^2)|\mathscr{E}|^2 \mathrm{d}A}{n_0^2 \displaystyle\int |\mathscr{E}|^2 \mathrm{d}A}.$$

Similar equations hold when the change of the refractive index is produced by another beam, with a modification of the effective potential W.

This powerful method allows us to treat a variety of different problems involving beam propagation in a non-Kerr medium. Significantly, some theoretical predictions have now been confirmed experimentally: First, theory has predicted that stationary beams of circular cross section can spiral about each other when propagating in a saturating medium.[7,8] In other words, it is actually possible to control the direction of one beam with another beam in a bulk medium. This was recently observed experimentally.[55]

Furthermore, it was recently shown to be possible to treat the solitons as particles with mass and to directly calculate their interaction from the classical force laws that predate Maxwell's equations.[14]

Soliton collisions. A simple qualitative theory predicted[11] that colliding beams in any non-Kerr medium can annihilate each other, fuse, or give birth to any number of new solitons. This was also observed experimentally.[56,57]

Dynamics of a single beam: mighty morphing solitons. Most important, a simple exact solution,[16] which leads to the evolution of the beam intensity described by the expression

$$I = I_0 \, \frac{\rho_{x0}\rho_{y0}}{\rho_x \rho_y} \exp\left(-\frac{x^2}{{\rho_x}^2} - \frac{y^2}{{\rho_y}^2} \right),$$

allows us to demonstrate analytically how Gaussian (laserlike) beams evolve dynamically as they propagate in a medium with a logarithmic nonlinearity. In a nonstationary regime the beam characteristic radii, ρ_x and ρ_y, vary periodically, and in general at every fixed distance the beam cross section is elliptical. The elliptical cross section changes as the beam propagates. This was recently observed[58] experimentally, and it was also shown that the logarithmic nonlinearity is an excellent model for realistic saturation. It seems that Gaussian-like beams will behave similarly in a wide variety of non-Kerr materials.

7. LIGHT-GUIDING LIGHT

If bright light beams can induce a stable optical fiber,[6] then such beams can guide a beam of another frequency or polarization or steer[8] and direct[11] another bright beam in (2 + 1) dimensions. We put both phenomena under the category of light-guiding light for (three-dimensional) photonic devices. This is an area much promoted by both theoretical predictions[1,6,8,11,59] and experimental observation.[31,33,34,37,60] The past few years have witnessed enormous progress in this field.

8. CONCLUSIONS

This paper has presented a summary of the recent theoretical advances in three-dimensional self-guided beams (spatial optical solitons) propagating in non-Kerr media. While the variety of self-guided beams in non-Kerr materials is rich, they are all generically similar, independent of their origin, including solitary waves in photorefractive and $\chi^{(2)}$ materials. Such a notion permits the development of a unifying conceptual approach based on the physics of induced optical waveguides. Indeed, from the physical point of view all spatial solitary waves can be treated as modes (or combinations of different modes) of the waveguides they induce, and their dynamical properties are similar to those of optical rays. There is no temporal analog of spatial solitons propagating in bulk materials. Recent experiments (Refs. 30, 31, 33, 34, 37, 55, 57, 58) have confirmed a number of theoretical predictions, including: stable solitons of circular symmetric cross section[6] (induced optical fibers leading to light-guiding light), directing solitons in three dimensions, e.g., solitons that can spiral about one another,[7,8] inelastic collisions leading to fusion, annihilation, or the birth of new beams,[11] and, finally, periodic changes of beam shape and beam cross section when the beam is not stationary.[16]

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