Wireless Pers Commun
DOI 10.1007/s11277-011-0465-6

Proportional Fair Scheduling with Capacity Estimation for Wireless Multihop Networks

Jae-Han Jeon · Jong-Tae Lim

Abstract In this paper, we investigate the scheduling problem to achieve the proportional fairness among the flows in wireless multihop networks with time-varying channel capacity. Using the signal to interference noise ratio and the outage probability, we present an estimate of time-varying capacity. Then, we achieve the proportional fairness in terms of maximizing the network utility function with consideration of fast fading without measurement of channel state information. Finally, we show that the proposed scheme results in better performance compared to the existing schemes through the simulation results.

Keywords Scheduling · Proportional fairness · Optimization · Wireless multihop networks

1 Introduction

Network utility maximization (NUM) is firstly proposed by Kelly et al. [1] to allocate resources to the flows in the wired network. With a properly chosen utility of each flow, the network utility is maximized while the network throughput satisfies the capacity constraint. However, it is not simple to apply the NUM framework to the wireless network because the wireless channel capacity is time-varying. Thus, the NUM problem in the wireless multihop network is one of the representative researches in the last decade [2–4]. Initially, the maximum clique approach using the conflict graph [2] has been proposed for the wireless multihop network with the fixed channel capacity and extended to the case of time-varying capacity using the activation set [3]. Moreover, Neely et al. [4] have generalized the results of NUM with time-varying capacity by using the dynamic control algorithm. However, since the results of [3,4] assume the fixed channel state during a time slot and the perfect channel state information (CSI) measurement, it is hard to apply to the fast fading wireless channel environment and the performance is degraded. To overcome the degradation, the probabi-

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Published online: 06 December 2011

listic CSI estimation using the channel distribution has been proposed. Kar et al. [5] have proposed the scheduling policy in consideration of the time-varying channel between the CSI measurement and it can be applied to the results of [3,4]. However, since the interference is varying according to not only the channel distribution but also the activation set in the wireless multihop network, their performance is also degraded.

The capacity with outage [6,7] is useful in the fast fading channel where it is impossible to follow the instantaneous channel state in a time slot. The capacity with outage is defined in [6] as the maximum rate that can be transmitted over a channel with some probability that the transmission cannot be decoded in the receiver. Then, the average rate correctly received over many transmissions is the product of the data rate and the corresponding success probability, i.e., 1—the outage probability. Thus, if we choose the data rate properly, we obtain the maximum rate in the fast fading channel. The result of [7] has presented the outage probability of transmission in the Rayleigh fading environment. Note that it is hard to determine the optimal data rate for achieving the fixed outage probability in the wireless multihop network where the interference is varying according to the activation set. Thus, it needs a scheme to achieve the fixed outage probability of the transmission in the wireless multihop network.

In this paper, we propose a time-varying capacity estimation based on the relationship between the signal to interference noise ratio (SINR) and the capacity with outage [6] in the Rayleigh fading environment [7]. From the estimated capacity, we formulate the NUM problem in terms of maximizing the log function which achieves the proportional fairness [1] with consideration of fast fading without measurement of CSI. With the simulation results, we claim that the proposed scheme improves the performance of the wireless multihop network.

2 System Description

We consider a wireless multihop network described by the undirected graph, $G = (N, L)$, where N is the set of the network nodes and L is the set of the network links. The flow is defined as a path from the source node to the destination node, which is composed of the multiple links between nodes, and the set of flows is defined as S. We assume that the routing path of each flow is fixed. Also, we don't consider the power control of each node, i.e., each node transmits with an equal fixed power. We consider the time division multiple access (TDMA) as the medium access scheme so that the time is slotted. In order to reflect the fast fading wireless channel, we consider the case where the duration of a time slot, $[t, t+1)$, is larger than the coherence time for fast fading. Then, the channel state is varying during a time slot due to the fast fading and the transmission during a time slot experiences the various channel states. Finally, some notations are summarized in Table 1.

3 Proportional Fair Scheduling with Capacity Estimation

In order to resolve the medium access control in the wireless multihop network, we adopt the concept of the activation set as shown in [3]. The activation set is defined as a collection of links which can transmit simultaneously. We describe the activation set as a mathematical vector form as follows: Let K be the total number of the activation sets and $|L|$ be the total number of the links. Then, the k-th activation set is described as a indicator vector $\mathbf{I}_k = [I_{1k} \dots I_{lk} \dots I_{|L|k}]^T, l \in L, k = 1, \dots, K$ with

Table 1 Notations

Notation	Definition
γ_{lk}	SINR of link l in k-th activation set
$\gamma_{lk}^{th}(t)$	SINR threshold at time t
$\hat{\gamma}_{lk}^{th}(t)$	Average SINR threshold at time t
$\hat{\gamma}_{lk}^{ths}$	Stable equilibrium point of average SINR threshold
C_{lk}	Shannon's capacity of link l in k-th activation set
$\tilde{C}_{lk}(t)$	Estimated capacity at time t
$\hat{C}_{lk}(t)$	Average estimated capacity at time t
$\hat{C}_{lk}^{s}$	Stable equilibrium point of average estimated capacity
$\bar{e}_{lk}(C_{lk})$	Outage probability when transmitter sends data with C_{lk}
$e_{lk}(t)$	Outage indicator at time t
e_{lk}^{d}	Desired outage probability
$a_k(t)$	Scheduling indicator of k-th activation set at time t
$\bar{a}_k$	Average fraction of time for scheduling

$$I_{lk} = \begin{cases} 1, & \text{if link } l \text{ is activated in activation set } k \\ 0, & \text{if link } l \text{ is not activated in activation set } k \end{cases} \tag{1}$$

Note that the link capacity varies due to not only the fading but also the scheduled activation set. Thus, to accommodate the capacity variation, we introduce the SINR of link l in the k-th activation set, γ_{lk}, [7] as follows:

$$\gamma_{lk} = \frac{G_{ll} F_{ll} P_t}{\sum_{n \neq l,\ n \in L_a(k)} G_{nl} F_{nl} P_t + \sigma^2} \tag{2}$$

where G_{ij} is the fixed path gain from the transmitter of link i to the receiver of link j and F_{ij} is the associated fast fading component of the channel. Note that F_{ij} is assumed to be unit mean exponentially distributed with the Rayleigh fading channel [7], i.e., $E[F_{ij}] = 1$. P_t is the transmitter power and σ^2 is the noise power. $L_a(k)$ is the set of the activation links when the k-th activation set is scheduled, i.e., $L_a(k) = \{n \in L | I_{nk} = 1\}$. From the SINR, we obtain the Shannon's capacity [6] of link l in the k-th activation set, C_{lk}, as follows:

$$C_{lk} = W \log_2 \left[1 + \gamma_{lk}\right] \tag{3}$$

where W is the bandwidth of the network.

3.1 Capacity Estimation

Since the SINR is time-varying during a time slot due to fast fading, the capacity with outage [6,7] is useful. Note that the capacity with outage is defined in [6] as the maximum rate that can be transmitted over a channel with some probability that the transmission cannot be decoded in the receiver. Then, the average rate correctly received over many transmissions is the product of the data rate and the corresponding success probability, i.e., 1—the outage probability. According to the result of [7], when the transmitter sends the data with C_{lk}, the outage probability in the Rayleigh fading channel, $\bar{e}_{lk}(C_{lk})$, is described as follows:

$$\bar{e}_{lk}(C_{lk}) = 1 - \exp\left[-\frac{\sigma^2(2^{C_{lk}/W} - 1)}{G_{ll}P_t}\right] \times \prod_{n \neq l,\ n \in L_a(k)} \frac{1}{1 + \frac{(2^{C_{lk}/W}-1)G_{nl}P_t}{G_{ll}P_t}} \tag{4}$$

Note that the SINR threshold in [6,7] is described by $(2^{C_{lk}/W} - 1)$ from (3). The transmission during a time slot results in a successful transmission or a failed transmission according to the outage probability from (4). Thus, using the outage indicator of transmission and the desired outage probability, e^d_{lk}, the SINR threshold, γ^{th}_{lk}, is updated as follows:

$$\gamma^{th}_{lk}(t+1) = \gamma^{th}_{lk}(t) + \theta\left[e^d_{lk} - e_{lk}(t)\right] \tag{5}$$

where θ is a small positive control gain and $e_{lk}(t)$ is the outage indicator which indirectly reflects the CSI during a time slot described as

$$e_{lk}(t) = \begin{cases} 1, & \text{transmission is failed} \\ & \text{on link } l \text{ in activation set } k \text{ at time } t \\ 0, & \text{transmission is successful} \\ & \text{on link } l \text{ in activation set } k \text{ at time } t \end{cases} \tag{6}$$

Then, as it follows from (3), the resulting capacity is estimated by using the SINR threshold with the exponential-weighted moving average as follows:

$$\tilde{C}_{lk}(t+1) = (1-\theta)\tilde{C}_{lk}(t) + \theta\left\{W\log_2\left[1 + \gamma^{th}_{lk}(t)\right]\right\} \tag{7}$$

If the transmission fails, i.e, $e_{lk}(t) = 1$, the SINR threshold is thought to be higher than the optimal SINR threshold for the desired outage probability. Thus, the SINR threshold of (5) should be decreased and the estimated capacity is decreased in (7). Otherwise, we should increase the SINR threshold to achieve higher throughput. As shown in [8], the average SINR threshold, $\hat{\gamma}^{th}_{lk}(t)$, and the average estimated capacity, $\hat{C}_{lk}(t)$, are obtained as follows:

$$\begin{aligned} \hat{\gamma}^{th}_{lk}(t+1) &= \hat{\gamma}^{th}_{lk}(t) + \theta\left[e^d_{lk} - \bar{e}_{lk}(\hat{C}_{lk}(t))\right] \\ \hat{C}_{lk}(t+1) &= (1-\theta)\hat{C}_{lk}(t) + \theta\left\{W\log_2\left[1 + \hat{\gamma}^{th}_{lk}(t)\right]\right\} \end{aligned} \tag{8}$$

Note that the outage probability, $\bar{e}_{lk}(\hat{C}_{lk}(t))$, is the strictly increasing function of $\hat{C}_{lk}(t)$, so that we conclude that for any $\hat{C}_{lk}(t)$, there exists a unique $\bar{e}_{lk}(\hat{C}_{lk}(t))$ from (4). Thus, from the linearization method in [8], the stable equilibrium point of (8) is obtained by

$$\hat{\gamma}^{ths}_{lk} = 2^{\hat{C}^s_{lk}/W} - 1, \quad \hat{C}^s_{lk} = \bar{e}^{-1}_{lk}(e^d_{lk}) \tag{9}$$

Thus, we show that the estimated capacity is same as the capacity with the desired outage probability.

3.2 Optimization with Time-varying Capacity

If the k-th activation set is scheduled at time t, the scheduling indicator of the k-th activation set, $a_k(t) = 1$, otherwise $a_k(t) = 0$. Since only one activation set is scheduled at each time t, $\sum_{k=1}^{K} a_k(t) = 1$. Moreover, to accommodate the variation of link capacity with each activation set, we introduce the link rate with the activation set. Define y_{lk} as the link l's rate when the k-th activation set is scheduled and $A(l)$ be the index set of the activation set related to link

l, i.e., $A(l) = \{k | I_{lk} = 1 \text{ where } k \in [1, \dots, K]\}$. Then, $\sum_{k \in A(l)} y_{lk} = \sum_{s \in S(l)} x_s, \forall l \in L$ where $S(l)$ is the set of flows which traverse link l and x_s is the injection rate of flow $s \in S$. Thus, with consideration of both the capacity with outage and the activation set, the NUM problem to maximize the log function for the proportional fairness [1] is formulated as follows:

$$\begin{aligned} \max_{x_s \geq 0, y_{lk} \geq 0, \bar{a}_k \geq 0} \quad & \sum_{s \in S} \log(x_s) \\ \text{s.t.} \quad & y_{lk} \leq \bar{R}_{lk} \bar{a}_k, \ \forall k \in A(l), \ \forall l \in L \\ & \sum_{k \in A(l)} y_{lk} = \sum_{s \in S(l)} x_s, \ \sum_{k=1}^{K} \bar{a}_k = 1 \end{aligned} \tag{10}$$

where $\bar{R}_{lk} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \tilde{C}_{lk}(t) [1 - e_{lk}(t)]$ is the average achievable capacity with outage on link l in activation set k, and $\bar{a}_k = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} a_k(t)$ is the average fraction of time for the scheduling. Note that the inequality constraint of (10) means that the link rate of each activation set should not be larger than the estimated capacity of each activation set. To solve the optimization problem in (10), we define the partial Lagrangian, $V(x, y, \bar{a}; p)$, without equality constraints as follows:

$$\begin{aligned} V(x, y, \bar{a}; p) &= \sum_{s \in S} \log(x_s) - \sum_{l \in L} \sum_{k \in A(l)} p_{lk} \left(y_{lk} - \bar{R}_{lk} \bar{a}_k \right) \\ &= \sum_{s \in S} \log(x_s) - \sum_{l \in L} \sum_{k \in A(l)} p_{lk} y_{lk} \\ &\quad + \sum_{l \in L} \sum_{k \in A(l)} p_{lk} \bar{R}_{lk} \bar{a}_k \end{aligned} \tag{11}$$

where $p_{lk} \in p$ is the Lagrangian multiplier. From the Lagrangian in (11), the objective function of the dual problem is described by

$$D(p) = \max_{x_s \geq 0, y_{lk} \geq 0, \bar{a}_k \geq 0} V(x, y, \bar{a}; p) = D_c(p) + D_s(p) \tag{12}$$

where $D_c(p)$ is the rate control subproblem described as

$$\begin{aligned} \max_{x_s \geq 0, y_{lk} \geq 0} \quad & \sum_{s \in S} \log(x_s) - \sum_{l \in L} \sum_{k \in A(l)} p_{lk} y_{lk} \\ \text{s.t.} \quad & \sum_{k \in A(l)} y_{lk} = \sum_{s \in S(l)} x_s, \ \forall l \in L \end{aligned} \tag{13}$$

and $D_s(p)$ is the scheduling subproblem described as follows:

$$\begin{aligned} \max_{\bar{a}_k \geq 0} \quad & \sum_{l \in L} \sum_{k \in A(l)} p_{lk} \bar{R}_{lk} \bar{a}_k \\ \text{s.t.} \quad & \sum_{k=1}^{K} \bar{a}_k = 1 \end{aligned} \tag{14}$$

The dual problem is described as $\min_{p \geq 0} D(p)$. Since the objective function is strictly concave and the feasible set is a convex set, the optimum of the dual problem is then the optimum of the primal problem in (10).

As the term $\sum_{k\in A(l)} p_{lk} y_{lk}$ from (13) is described as the weighted sum of $\sum_{k\in A(l)} y_{lk}$ and y_{lk} should be non-negative, we select the minimum value of p_{lk} for the optimal solution. Thus, the optimal injection rate of flow s, x_s^*, is described as follows:

$$x_s^* = x_s(p) = \frac{1}{\sum_{l\in L(s)} \min\limits_{k\in A(l)} p_{lk}} \tag{15}$$

where $L(s)$ is the set of links where flow s traverses. Also, the optimal rate of link l in activation set k, y_{lk}^*, is obtained as follows:

$$y_{lk}^* = y_{lk}(p) = \begin{cases} \sum\limits_{s\in S(l)} x_s^*, & \text{if } k = k^* \\ 0, & \text{if } k \neq k^* \end{cases} \tag{16}$$

where $k^* = \arg\min\limits_{k\in A(l)} p_{lk}$ is the index of the optimal activation set. From (15), (16), we obtain the optimal injection rate of flow s and the rate of link l in activation set k. In contrast to the rate control subproblem from (13), the solution of the scheduling subproblem from (14) is obtained as follows:

$$\bar{a}_k^* = \bar{a}_k(p) = \begin{cases} 1, & \text{if } k = k^{**} \\ 0, & \text{if } k \neq k^{**} \end{cases} \tag{17}$$

where $k^{**} = \arg\max\limits_{k} \sum_{l\in L_a(k)} p_{lk} \bar{R}_{lk}$.

Now, we present an algorithm to solve the dual problem using the subgradient algorithm [3,9] because the dual problem $D(p)$ is not differentiable. For each Lagrangian multiplier, p_{lk}, we can obtain the subgradient of $D(p)$, i.e., $\frac{\partial D(p)}{\partial p_{lk}}$ from (12)–(14). Then, using the subgradient of $D(p)$, the Lagrangian multiplier, p_{lk}, is updated as follows:

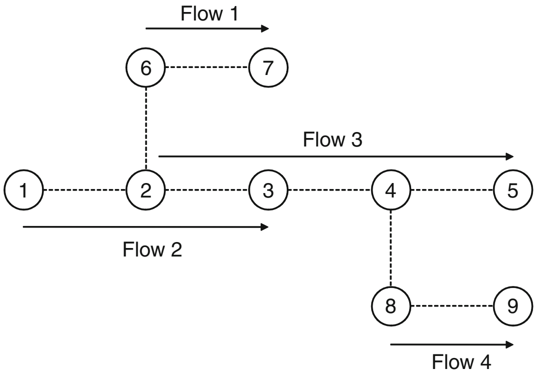
$$p_{lk}(t+1) = \left[p_{lk}(t) + \alpha \left\{y_{lk}(p(t)) - \bar{R}_{lk}\bar{a}_k(p(t))\right\}\right]^+ \tag{18}$$

where $[\cdot]^+ = \max(0, \cdot)$ and α is a small positive step size of the subgradient algorithm for stability [9]. Since $\bar{R}_{lk}$ from (17) and (18) is described as the time average of infinite samples, we implement $\bar{R}_{lk}$ as the average of finite previous samples such as $\frac{1}{t+1}\sum_{\tau=0}^{t} \tilde{C}_{lk}(\tau)\,[1 - e_{lk}(\tau)]$. Then, we achieve the proportional fairness from (15)–(18).

4 Simulation Results

In this section, we show the behavior of the proposed algorithm and compare with the previous results. In simulation, we consider a TDMA-based wireless multihop network. We set that the nodes in the wireless multihop network are stationary. Also we set that all links suffers the additive white Gaussian noise (AWGN). Thus, the Rayleigh fading is considered in the transmission. In the network, the fixed path gain, G_{ij}, is set to $\left(d_{ij}\right)^{-4}$ where d_{ij} is the distance between the transmitter of link i and the receiver of link j. The noise power, σ^2, is 10^{-9} mW. The bandwidth of the network, W, is fixed to 1 MHz. The random fast fading component, F_{ij}, is independently generated for all i and j at every 16 ms of the coherence time (with 2.4 GHz carrier frequency and 5 km/h velocity from [10]). The duration of a time slot is set to 5 times of the coherence time. The desired outage probability is set to 0.1 for all links in the network. The network utility is defined as $U_s(x_s) = \sum_{s\in S} \log(x_s)$ for the proportional fairness [1]. Finally, we use the MATLAB as a tool of the simulation to show the effectiveness of the proposed algorithm.

Fig. 1 Network topology



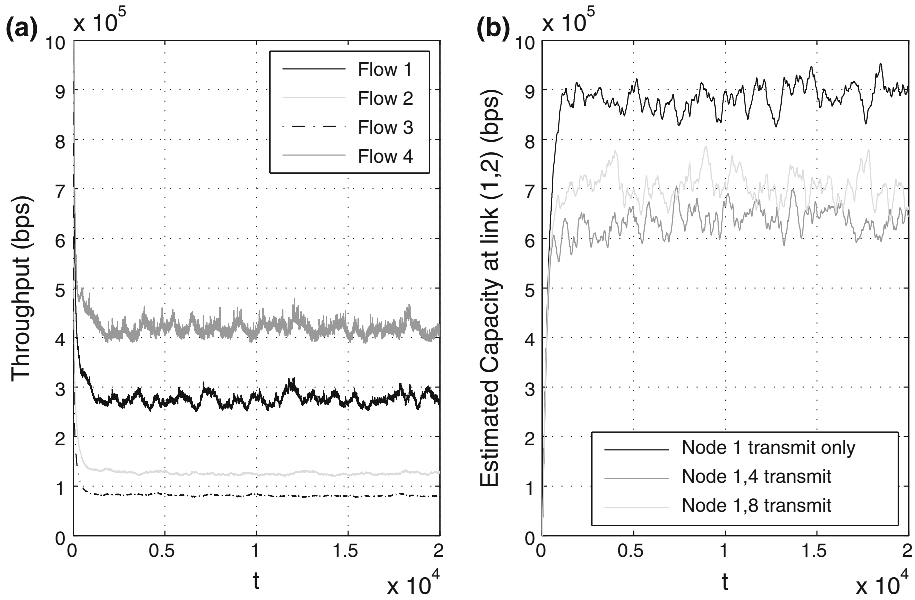


Fig. 2 Flow throughputs and estimated capacity at link (1, 2)

Firstly, we show the throughput and the estimated capacity of the proposed scheme with the network topology in Fig. 1. The control gain θ is set to 0.02 and the transmit power is 0.8 mW for all nodes in the network and the distance between adjacent nodes is 100 m. The left hand side graph of Fig. 2 shows the throughput performance of all flows in the networks. With the capacity estimation algorithm in Sect. 3, the trajectories of throughput converge. Moreover, the throughput of each flow is different according to the interference of the wireless multihop network. For example, flow 3 shows the lowest throughput because it suffers both the maximum interference and the maximum hop count while flow 4 achieves the highest throughput with both the minimum interference and the minimum hop count as shown in Fig. 1. The right hand side graph of Fig. 2 shows the convergence of the estimated capacity. We confirm that the estimated capacity is higher if the interference is lower in the activation set. Then, when only node 1 transmits, the estimated capacity of link (1, 2) is the highest.

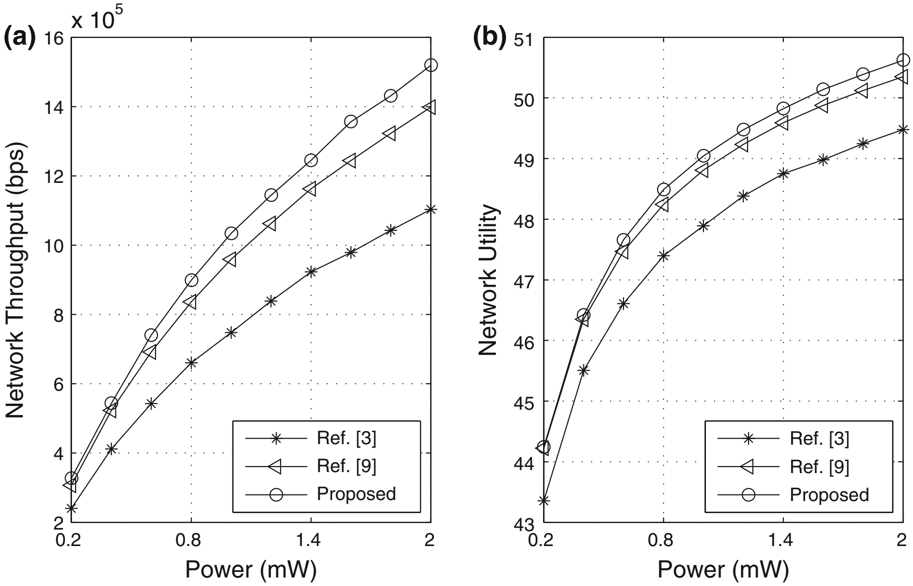


Fig. 3 Comparison of network throughput and utility

We compare the proposed scheme with the results of [3] and [5] in terms of the network throughput and utility by varying the transmit power in Fig. 3. Due to the outage between the CSI measurement, the performance of [3] is degraded. Although the result of [5] estimates the capacity between the CSI measurement, the performance is still degraded due to the outage from the statistical estimation which is highly dependent on the measured CSI at the beginning of the time slot. Since the proposed algorithm controls the outage every time slot, the throughput performance is about 30% better than the result of [3] and about 7% better than the result of [5].

5 Conclusion

We propose a new proportional fair scheduling scheme in the wireless multihop network. In order to consider the time-varying capacity, we propose the capacity estimation scheme using the outage probability in the Rayleigh fading environment. With the simulation results, we show that the proposed scheduling scheme performs better than the previous results.

Acknowledgments This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2011-0004861).

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