PERGAMON

Transportation Research Part A 33 (1999) 337–364

TRANSPORTATION
RESEARCH
PART A

Fuzzy logic systems for transportation engineering: the state of the art

Dušan Teodorović *

Faculty of Transport and Traffic Engineering, University of Belgrade, Vojvode Stepe 305, 11000 Belgrade, Yugoslavia

Received 19 August 1997; received in revised form 11 May 1998

Abstract

The paper presents a classification and analysis of the results achieved using fuzzy logic to model complex traffic and transportation processes. Fuzzy logic is shown to be a very promising mathematical approach to modeling traffic and transportation processes characterized by subjectivity, ambiguity, uncertainty and imprecision. The basic premises of fuzzy logic systems are presented as well as a detailed analysis of fuzzy logic systems developed to solve various traffic and transportation engineering problems. Emphasis is put on the importance of fuzzy logic systems as universal approximators in solving traffic and transportation problems. Possibilities are shown regarding the further application of fuzzy logic in this field. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Transportation modeling; Transportation planning; Real time operations and control of transportation networks; Fuzzy logic; Approximate reasoning; Soft computing

1. Introduction

In the past several decades a variety of deterministic and/or stochastic models have been developed to solve complex traffic and transportation engineering problems. These mathematical models use different formulae and equations to solve such problems. During their education and training, engineers are most often directed to the use of exclusively objective knowledge (formulae and equations). However, when solving real-life engineering problems, linguistic information is often encountered that is frequently hard to quantify using 'classical' mathematical techniques. This linguistic information represents subjective knowledge. Since we are unable to quantify some linguistic information, different assumptions are made in the models. Through the assumptions made by the analyst when forming the mathematical model, the linguistic information is very

* Tel.: +381-11-4890-580; fax: +381-11-466-294; e-mail: teodor@eunet.yu

PII: S0965-8564(98)00024-X

often ignored. On the other hand, a wide range of traffic and transportation engineering parameters are characterized by uncertainty, subjectivity, imprecision and ambiguity. Human operators, dispatchers, drivers and passengers use this subjective knowledge or linguistic information on a daily basis when making decisions. When we are behind the wheel, backing out of a parking place, we understand the passenger sitting next to us quite well when he says to 'wait a bit for this speeding car to pass'. We would certainly not be able to quickly process the information that 'the car is going 32.5 mph and is 52.7 meters away from our car'. In order to properly use such information we would have to write formulae, make calculations and reach a decision about when to finally leave the parking spot. We also understand quite well when told not to use a certain route since 'there is considerable congestion on it'. Most drivers did not attend a course on 'Traffic Flow Theory' and 'describing' congestion using numerical values related to traffic flow parameters would be of no use. These simple examples show that in certain situations we accept linguistic information much more easily than numerical information. In the same vein, we are perfectly capable of accepting approximate numerical values and making decisions based on them. In a great number of cases we use approximate numerical values exclusively. For example, when subjectively estimating travel time between two nodes in a network we use expressions of the type 'travel time is around thirty minutes'. Not a single driver will ever say that travel time between two nodes is '17 minutes and 26 seconds'. The assertion that travel time is 'approximately 20 minutes' results from a subjective estimation, i.e. our subjective feeling. This assertion is neither the result of some measurement, nor is it the execution of the random variable that represents travel time. If we were to record travel time between two nodes over a longer period of time, we would receive a series of different values, each one being an execution of the random variable—travel time. When we subjectively estimate travel time we do not have numerical information about probability density functions, rather our own estimate based on experience and intuition. It should be emphasized that the subjective estimation of different traffic parameters differs from dispatcher to dispatcher, driver to driver, and passenger to passenger.

When solving real-life traffic and transportation problems we should not use only objective knowledge (formulae and equations) or only subjective knowledge (linguistic information). We simply cannot and should not ignore the existence of linguistic information, i.e. subjective knowledge. Fuzzy logic is an extremely suitable concept with which to combine subjective knowledge and objective knowledge.

The primary goal of this review paper is to acquaint the reader with the basic elements of fuzzy set theory, fuzzy logic, fuzzy logic systems, the relationship between fuzzy logic and probability theory, applications of fuzzy logic to date in traffic and transportation engineering, and to indicate the directions for future research in this area.

The paper is organized as follows. The basic assumptions of fuzzy set theory are presented in Section 2. Section 2 also deals with the relation between fuzzy set theory and probability theory. The characteristics of fuzzy logic systems are presented in Section 3. Section 3 also concerns fuzzy logic systems for transportation engineering, including fuzzy transportation planning models, transportation investment projects, traffic control models, vehicle routing and scheduling, accident analysis and prevention, level of service considerations, air transportation and river transportation applications. Section 4 contains the final remarks and directions for future research.

2. Basic assumptions of fuzzy set theory

More than 30 years have passed since Zadeh's paper appeared (1965), introducing the concept of a fuzzy set. Ample scientific literature has been written on fuzzy sets and fuzzy logic, and a large number of commercial products based primarily on the principles of fuzzy logic has appeared on the market.

In the classic theory of sets, very precise bounds separate the elements that belong to a certain set from the elements outside the set. In other words, it is quite easy to determine whether an element belongs to a set or not. For example, if we denote by A the set of stoplight intersections in a city, we conclude that every intersection under observation belongs to set A if it has a stoplight. Element x's membership in set A is described in the classic theory of sets by the membership function $\mu_A(x)$, as follows:

$$\mu_A(x) = \begin{cases} 1, \text{if and only if x is a member of A} \\ 0, \text{if and only if x is not a member of A} \end{cases}$$

Many sets encountered in reality do not have precisely defined bounds that separate the elements within the set from those outside the set. Thus, it might be said that a certain stoplight has a 'long' waiting time. If we denote by **A** the set of 'long waiting time at a stoplight', the question logically arises as to the bounds of such a defined set. Does a waiting time of 25 s belong to this set? What about 15, 37 or 60 s? All these times, depending on the context and perceptions could belong to the set called 'long waiting time at a stoplight'. Other members of this set could equal 18, 32, 55 s, etc. On the other hand, we intuitively feel that a waiting time of 60 s belongs to the set called 'long waiting time at a stoplight' 'more' or 'stronger' than a waiting time of 18 seconds. In other words, there is more truth in the statement that a waiting time of 60 s is a 'long waiting time at a stoplight' than in the statement that a waiting time of 18 seconds is a 'long waiting time at a stoplight'. Within the context of this simple example we can fully appreciate Kosko's (1993) observation that 'everything is a matter of degree'. All waiting times at a stoplight can be treated as long. If we introduce a set called 'short waiting time at a stoplight', we note that, depending on the context and perceptions, different waiting times at a stoplight can also be treated as short. Finally, we can ask ourselves whether a waiting time of 15 s is long, short or perhaps medium. The answer is very simple. A waiting time of 15 s is long, short and medium, all at the same time. In other words, a waiting time of 15 s belongs to the sets 'long waiting time', 'medium waiting time' and 'short waiting time' with different grades of membership. The membership function for fuzzy sets can take any value from the closed interval [0, 1]. Fuzzy set **A** is defined as the set of ordered pairs $\mathbf{A} = \{x, \mu_A(x)\}$, where $\mu_A(x)$ is the grade of membership of element x in set **A**. The greater $\mu_A(x)$, the greater the truth of the statement that element x belongs to set **A**.

Up until the middle of this century, probability theory was considered the only theory capable of explaining uncertainty phenomena. During the past several decades, alternative theories appeared that also try to explain uncertainty phenomena, including the most important: fuzzy sets (Zadeh, 1965, 1996; Klir and Folger, 1988; Self, 1990; Zimmermann, 1991; Kosko, 1992b, 1994; Bezdek, 1993; Tyler, 1993; Bezdek and Pal, 1992), evidence theory (Shafer, 1976), imprecise probabilities (Kruse and Meyer, 1987; Waley, 1991), possibility theory (Dubois and Prade, 1988), rough sets (Pawlak, 1991)), and fuzzy measures (Wang and Klir, 1992)). A number of researchers from the field of probability theory have strongly opposed the new theories, fuzzy set theory in particular (Lindley, 1994). I firmly believe that both theories should exist side by side, that it is possible to combine them, and that both have a very important role in explaining complex transportation processes. Bezdek (1994) completes his brilliant paper with the statement that 'fuzzy memberships... represent similarities of objects to imprecisely defined properties', and that probability 'conveys information about relative frequencies'. When solving different traffic and transportation problems, Bayes's formula will still be used, and we will continue to use normal, exponential or some other probability density function to describe certain laws. But we must also be tolerant and accept the fact that some other phenomena are properly described using fuzzy membership functions.

3. Fuzzy logic systems for transportation engineering

Basic results linked to the development of fuzzy logic date from Zadeh (1973) and Mamdani and Assilian (1975). Introducing a concept he called 'Approximate Reasoning', Zadeh successfully showed that vague logical statements enable the formation of algorithms that can use vague data to derive vague inferences. Zadeh assumed his approach would be beneficial above all in the study of complex humanistic systems. Realizing that Zadeh's approach could be successfully applied to industrial plant controllers, Mamdani and Assilian (1975) applied this method to control a pilot-scale steam engine. They used fuzzy logic in order to express linguistic rules. Pioneer papers in the field of fuzzy controllers include Mamdani (1974), Kickert and van Nauta Lemke (1976), Ostergard (1976), and Tong (1976). Tong (1977) made a control engineering review of fuzzy systems. Regarding the application of fuzzy logic in engineering, the tutorial given by Mendel (1995) is of utmost importance, as is the recently published book by Ross (1995).

What is a fuzzy logic system? Mendel (1995) explains the concept of a fuzzy logic system-a (FLS) as follows: 'In general a FLS is a nonlinear mapping of an input data (feature) vector into a scalar output (the vector output case decomposes into a collection of independent multi-input/single-output systems)'. We would also note that a fuzzy logic system maps (most often crisp) inputs into crisp outputs.

The basic elements of every fuzzy logic system are rules, fuzzifier, inference engine and defuzzifier (Fig. 1).

Input data are most often crisp values. The task of the fuzzifier is to map crisp numbers into fuzzy sets (cases are also encountered where inputs are fuzzy variables described by fuzzy membership functions). Models based on fuzzy logic consist of 'If–Then' rules. A typical 'If–Then' rule would be:

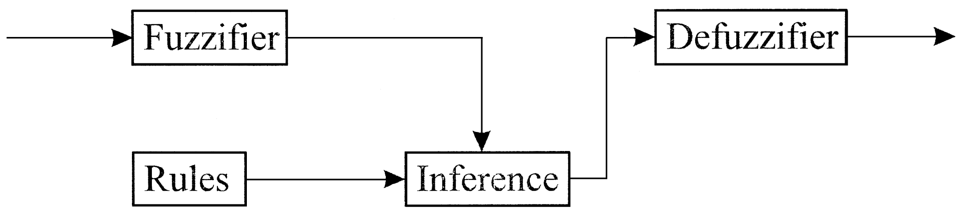

Fig. 1. Fuzzy logic system.

If the ratio between the flow intensity and capacity of an arterial road is SMALL
Then vehicle speed in the flow is BIG

The fact following 'If' is called a premise or hypothesis or antecedent. Based on this fact we can infer another fact that is called a conclusion or consequent (the fact following 'Then'). A set of a large number of rules of the type:

If premise
Then conclusion is called a fuzzy rule base.

We would note that in classical expert systems, rules are derived exclusively from human experts. In fuzzy rule-based systems, the rule base is formed with the assistance of human experts; recently, numerical data has been used as well as through a combination of numerical data-human experts. An interesting case appears when a combination of numerical information obtained from measurements and linguistic information obtained from human experts is used to form the fuzzy rule base. In this case, rules are extracted from numerical data in the first step. In the next step this fuzzy rule base can (but need not) be supplemented with the rules collected from human experts. A fuzzy rule base obtained from numerical data can be used to solve the same type of problem solved by artificial neural networks. The inference engine of the fuzzy logic maps fuzzy sets onto fuzzy sets. A large number of different inferential procedures are found in the literature. In most papers and practical engineering applications, minimum inference or product inference is used. During defuzzification, one value is chosen for the output variable. The literature also contains a large number of different defuzzification procedures. The final value chosen is most often either the value corresponding to the highest grade of membership or the coordinate of the center of gravity.

Drivers, passengers or dispatchers make decisions about route choice, mode of transportation, most suitable departure time or dispatching trucks. In each case the decision-maker is a human. The environment in which a human expert (human controller) makes his decisions is most often very complicated, making it extremely hard to formulate a suitable mathematical model. Thus, the development of fuzzy logic systems seems justified in such situations. Special importance should be given to the fact that Wang and Mendel (1992) showed that a fuzzy logic system (representing a nonlinear mapping) is capable of approximating any real continuous function to arbitrary accuracy.

Pappis and Mamdani (1977) published the first paper in which a practical traffic and transportation problem was solved using fuzzy logic. In the mid- and late-1980s, a group of Japanese authors made a significant contribution to fuzzy set theory applications in traffic and transportation. Nakatsuyama et al. (1983), Sugeno and Nishida (1985) and particularly Sasaki and Akiyama (1986, 1987, 1988) solved complex traffic and transportation problems indicating the great potential of using fuzzy set theory techniques. At the end of the 1980s and beginning of the 1990s, the fuzzy set theory in traffic and transportation became extensively used at American universities. The pioneering work of the research team of the University of Delaware deserves special attention, headed by Professor Shinya Kikuchi (Chakroborthy, 1990; Chakroborthy and Kikuchi, 1990; Perincherry, 1990; Perincherry and Kikuchi, 1990; Teodorović and Kikuchi, 1990, 1991; Kikuchi et al., 1991, 1993; Kikuchi, 1992). In the early and mid-1990s, interest in fuzzy logic applications in traffic and transportation increased in other world universities. Different traffic and transportation problems successfully solved using fuzzy set theory techniques were presented in the works of Chen et al. (1990), Tzeng and Teng (1993), Lotan and Koutsopoulos (1993a,b), Xu and Chan, 1993a,b), Teodorović and Babić (1993), Akiyama and Shao (1993), Chang and Shyu (1993), Chanas et al. (1993), Teodorović and Babic (1993), Akiyama amd Yamanishi (1993), Deb (1993), Nanda and Kikuchi (1993), Perkinson (1994), Hsiao et al. (1994), Vukadinović and Teodorović (1994), Teodorović et al. (1994), Teodorović (1994), Teodorović and Kalić (1995), Milosavljević et al. (1996), Teodorović and Pavković (1996) and Tzeng et al. (1996).

Let us try to explain why fuzzy logic systems were developed for transportation engineering. Great many of the problems in the field of transportation planning and traffic control are often ill defined, ambiguous and vague. As already mentioned, many traffic and transportation problems, phenomena and parameters are characterized by subjectivity. It is hard to disregard the fact that subjective judgment is present in problems dealing with the choice of route, mode of transportation and carrier, a driver's perceptions and reactions, an established level of service, defining safety standards, defining criteria to rank alternative transportation plans and projects, etc. It should also be emphasized that both deterministic and stochastic models that have been developed to solve a variety of complex traffic and transportation problems are characterized by mathematics based on binary logic. Without denying the importance of binary logic as the basis for the development of many scientific disciplines and technology leading to the prosperity of man's society, we must note that it cannot deal effectively with passengers', dispatchers' or drivers' feelings of uncertainty, vagueness and ambiguity. Since the fuzzy set theory recognizes the vague boundary that exists in some sets, different fuzzy set theory techniques need to be used in order to properly model traffic and transportation problems characterized by ambiguity, subjectivity and uncertainty.

3.1. Trip generation

Trip generation problem was solved using fuzzy logic byKalić and Teodorović (1997b). Fuzzy rule base was generated by learning from numerical examples. For this purpose, the procedure proposed by Wang and Mendel (1992a) was used. Firstly, the available set of data was divided into two subsets: the first was used for generation of the fuzzy rule base, and the other was intended to be a control data subset. After the fuzzy rule base was created, the obtained fuzzy system was tested on both subsets of data. The estimate of the number of trips for the subsets was also determined using artificial neural networks, as well as by multiple linear regression. The training of the neural network was done using simulated annealing technique. After the testing, the fuzzy logic approach proved to give the closest estimate of the actual number of trips generated in a given area.

One of the most important problems in the field of transportation planning is the problem of origin–destination estimation from link counts. In order to decrease the cost of passenger surveys, traffic count are undertaken on certain links of the transportation network. In the next phase an origin–destination matrix is estimated from the link counts. In the last two decades a greater number of models have been developed for origin–destination estimation from link counts. Xu and Chan (1993a,b) were the first to use the fuzzy set theory techniques when analyzing the problems arising from the poor quality of link count data. As known, traffic counts are often subject to errors. Xu and Chan (1993a,b) particularly pointed out the problem of traffic counts in countries such as China in which mixed urban traffic flow with heavy bicycle volume is common. In such situations the heavy bicycle volume also makes precise vehicular counts very difficult. Xu and Chan (1993a,b) estimated an origin–destination matrix with fuzzy weights. They based their method on that of Chan et al. (1986) which was developed for a non-fuzzy case.

3.2. Trip distribution

Trip distribution problem was also solved using fuzzy logic by Kalić and Teodorović (1996, 1997a). Like in the case of trip generation problem, Kalić and Teodorović (1996) generated fuzzy rule base by learning from numerical examples. Again, Wang and Mendel (1992a) procedure for generation of the fuzzy rule base was applied, and the obtained fuzzy system was tested. The aim of testing was to estimate the number of air passengers travelling between major industrial cities and given regions. In comparison to non-fuzzy methods, the fuzzy logic approach gave the best estimate of the actual number of observed air passengers. In subsequent research, Kalić and Teodorović (1997b) improved previous results by combining genetic algorithms with fuzzy logic: fuzzy rule base generated by learning from numerical examples was randomly changed a certain number of times, thus producing the pool of fuzzy rule bases, each of which represented one individual in the initial population. The value of the fitness function was taken to the mean square deviation between the actual and fuzzy-estimated number of trips. By using the common genetic operators (reproduction, crossover, and mutation), several generations of fuzzy rule bases were obtained. In this way, the final fuzzy rule base was created, based on which the best estimate of the actual number of trips was acquired.

3.3. Modal split

Teodorović and Kalić (1996) used fuzzy logic to solve the mode choice problem. The authors illustrated the possibilities of fuzzy logic in solving the mode choice problem using a hypothetical numerical example. The fuzzy rule base was generated using available numerical data on the differences between the travel times and travel costs of competitive modes. In other words, learning from examples generated the fuzzy rule base. The technique proposed by Wang and Mendel (1992a) was used to generate the fuzzy rule base by learning from examples. There was extremely good agreement between the results obtained using fuzzy logic and real results.

Quadrado and Quadrado (1996) used fuzzy logic to determine the accessibility of different transportation modes in the Lisbon Metropolitan Area. The authors first pointed out that all variables used in the 'classical' way of calculating accessibility are characterized by fuzziness. A fuzzy rule base was developed for each mode of transportation. Accessibility was shown as a percentage between 0 and 100%. One of the typical rules to calculate accessibility reads:

If the number of people involved in movements is VERY BIG
and the time used in those movements is VERY SMALL
Then the accessibility is HYPER BIG

The authors thus put an end to the tradition of treating accessibility as a deterministic concept and offered the possibility of a better representation of accessibility.

3.4. Route choice

In the past four decades, the route choice problem, along with the traffic assignment problem, has been considered by a large number of authors worldwide. In addition to the large number of prominent works devoted to the problem of traffic assignment, mention should also be made of the books by Florian (1976) and Sheffi (1985). Teodorović and Kikuchi (1990) were first to model the complex route choice problem using fuzzy logic. They used fuzzy inference techniques to study the binary route choice problem. Akiyama et al. (1993) also developed a model for route choice behavior based on the fuzzy reasoning approach. Lotan and Koutsopoulos (1993a,b) developed models for route choice behavior in the presence of information based on concepts from approximate reasoning and fuzzy control. The research of Lotan and Koutsopoulos (1993a,b) is particularly important within the context of ongoing research in Intelligent Vehicle Highway Systems (IVHS). Teodorović and Kalić (1995) developed an approximate reasoning algorithm to solve the route choice problem in air transportation. Akiyama and Tsuboi (1996) studied route choice behavior by multi-stage fuzzy reasoning.

The following discussion will present the results of Teodorović and Kikuchi's (1990) work. Let us consider the simplest case when the user is to choose one of two possible paths between the origin and destination of movement. Perceived travel time is very often 'fuzzy' and can be treated as a fuzzy set. In other words, when subjectively estimating travel time between two points, expressions are used such as 'it takes about 20 minutes from point C to point D', 'you'll get there in about half an hour', etc. Teodorović and Kikuchi (1990) assumed that users choose their paths based on a comparison of the characteristics of alternative paths. Despite the fact that travel time is a measurable parameter, a driver's notion of travel time when he/she makes the route choice is often fuzzy. Thus, driver's preference for a route is based on a comparison of two fuzzy numbers. As a result, his/her certainty of choosing a route is also not crisp. The Teodorović and Kikuchi (1990) model can represent the decision process under two layers of uncertainty: driver's judgment of time and driver's preference.

Let us denote by **TA** and **TB**, respectively, fuzzy sets representing estimated travel time (by one user) along path A and path B. When comparing travel times along paths A and B, the user might estimate, for example, that travel time along path A, **TA** is 'much shorter than **TB**', 'shorter than **TB**', 'equal to **TB**', 'longer than **TB**', and 'much longer than **TB**'. These fuzzy sets are denoted in the following manner: **MLTB**—'much less than **TB**', **LTB**—'less than **TB**', **GTB**—'greater than **TB**', **MGTB**—'much greater than **TB**'.

Let us assume that a user has a specific preference regarding the choice of each of the possible paths through the network. This preference can be 'stronger' or 'weaker'. Let us introduce into the discussion a preference index that can take values on the interval from 0 to 1. When the user has an absolute preference for a specific path we consider the preference index to be equal to 1. This preference index decreases along with a decrease in the strength of the preference. Fuzzy sets such as 'very strong', 'strong', 'medium' can express the strength of the user's preference, 'weak' and 'very weak'.Teodorović and Kikuchi (1990) proposed the following rules of approximate reasoning to establish the transport network user's preference strength:

If	**TA = MLTB**	then	**PA = VERY STRONG**
If	**TA = LTB**	then	**PA = STRONG**

If	**TA = TB**	then	**PA = MEDIUM**
If	**TA = GTB**	then	**PA = WEAK**
If	**TA = MGTB**	then	**PA = VERY WEAK**

where **PA** is the preference index associated with path A.

An approximate reasoning algorithm was used to establish the preference strength for every network user. Once the preference index has been determined for each network user, an algorithm must be developed that will determine the number of users along individual links. Such algorithms are known in the literature as network 'loading' algorithms. Teodorović and Kikuchi (1990) developed a network 'loading' algorithm in which the basic input data are the preference indexes of individual network users. Teodoroviæ and Kikuchi assumed that the perceived travel times are distributed around a previously measured most likely travel time. The 'loading' algorithm is based on the concept of hybrid numbers. [Kaufmann and Gupta (1985) introduced the concept of a hybrid number representing the 'combination' of a fuzzy number and a random variable].

Lotan and Koutsopoulos (1993a,b) also studied route choice behavior problem. They indicated the great importance of studying this problem, especially in the light of intensive research connected with the development of Intelligent Vehicle Highway Systems. When developing their approximate reasoning algorithm Lotan and Koutsopoulos used rules dealing with perceived travel times (or other measures of attractiveness) and rules dealing with real-time traffic information. Real-time traffic information on current traffic conditions was primarily used to indicate how default behavior changes. The attractiveness of different alternatives was measured on a scale ranging from -1 to 1. A value of -1 indicates a complete aversion towards taking the alternative, while a value of $+1$ corresponds to an assured choice of the alternative. Once the initial set of rules was established, in order to improve the model it was necessary to test, update and expand the rules.

Continuing the research of Lotan and Koutsopoulos (1993a,b), Vythoulkas and Koutsopoulos (1994) studied the modeling of discrete choice behavior using techniques and concepts from fuzzy set theory and neural networks. The authors used the neuro-fuzzy approach in order to calibrate the proposed model. They assumed that the membership functions of the fuzzy sets that appear in different rules are bell-shaped (gaussian functions). The authors presented the problem of rule generation as an integer programming optimization problem. Vythoulkas and Koutsopoulos (1994) used the fuzzy neural network procedure proposed by Lin and Lee (1991) to optimize the parameters of the membership functions, as well as to adjust the rule weights. The developed neural network was trained using an adaptation of the generalized delta rule. Two sets of data were used in order to test the proposed approach. The first group represented stated preference data collected by Lotan and Koutsopoulos (1993b) referring to the choice of one out of three alternative routes in Boston in the presence of information. The second group consisted of revealed preference data of the choice between rail and car for intercity travel. Data were obtained by a population survey in the city of Nijmegen in The Netherlands. The authors made a number of experiments with different combinations of input variables and finally concluded that the most important variables for mode choice decision are travel time, travel cost and rail access/egress time. Rule premises used differences between travel time by rail and travel time by car and differences between travel cost by rail and travel cost by car. The model was calibrated using an error function that was the sum of the squares of the deviations between actual choice and calculated preference.

Akiyama and Tsuboi (1996) used multi-stage fuzzy reasoning to describe the driver decision making process on road networks. Their paper considers the multi-route choice problem.Akiyama and Tsuboi (1996) first made a survey and collected data. Experiment participants were asked about the number of alternative routes they use going from their origin to their destination (the authors asked the respondents to cite at most three alternatives they may use). For each route considered they defined its characterizing factors (travel time, degree of congestion and risk of accidents). All the participants estimated the values of all the factors using their own perception and experience. Appropriate fuzzy numbers could then represent these values. The third survey question dealt with the values of the utilities of the nominated routes. The authors assumed the existence of two stages in the driver decision making process. The first approximate reasoning algorithm they proposed determined the utilities of the alternative routes. A typical rule in this approximate reasoning algorithm reads:

If	T is large	and CR is small	and RA is medium
Then	V is small		

where:

(a) T = perceived travel time
(b) CR = congestion of a route
(c) RA = risk of accident
(d) V = utility for the route.

The first input variable DF in the second approximate reasoning algorithm is the difference between the utilities associated with the shortest path and the second shortest path. The second input variable DS is the difference in the utilities between the second shortest path and the third shortest path. The third input variable N refers to the number of alternative routes. When $N = 2$, variable DS does not appear in the premises. A typical rule of the second approximate reasoning algorithm reads:

If	$N = 3$	and DF is medium	and DS is medium
Then	$FR(1)$ is large		
	$FR(2)$ is medium		
	$FR(3)$ is medium		

where:

(a) DF = difference of utilities between the shortest path and the second shortest path
(b) DS = difference of utilities between the second shortest path and the third shortest path
(c) $FR(i)$ = degree of frequency for route i.

Akiyama and Tsuboi (1996) also developed a neural network model for the second stage of estimation. The neurons in the input layer represent the number of alternative routes and the values of the utilities of individual alternative routes. The network output is route frequency.

Connecting weights are determined using the back propagation method. The two-stage model based on a combination of fuzzy logic (first stage) and neural network (second stage) produced somewhat better results than the two-stage model based on fuzzy logic in both stages.

3.5. *Traffic assignment*

Many papers devoted to the traffic assignment problem were published in world literature. There are two characteristics common to most of these papers: they are based on Wardrop's principle and they are travel time crisp. Akiyama et al. (1994) observed that 'in real world, however, the driver can only use fuzzy traffic information even if several types of information are available'. These authors succeeded in formulating the fuzzy user equilibrium with fuzzy travel time. They confirmed their assumption about drivers' perception of time as triangular fuzzy numbers by organizing the survey on the network comprised of Hanshin Expressway and urban streets in the Osaka area. In their paper, the authors introduced 'the descriptive method of route choice behavior to design the traffic assignment model'. They showed that 'the state of user equilibrium is also generated even if fuzziness of travel time exists'. Akiyama et al. (1994) developed Fuzzified Frank–Wolfe algorithm to solve the problem considered. This pioneer paper undoubtedly presents a significant study on the relationship between traffic information and drivers' behavior.

3.6. *Transportation investment projects*

Tzeng and Teng (1993) considered transportation investment project selection with fuzzy multi-objectives. The basic idea of the paper was to show the possibilities of using the fuzzy set theory in transportation investment planning.

Smith (1993) applied fuzzy set theory techniques in the evaluation of potential suburban railway station locations. His goal was to find the most convenient railway station locations among the set of potential station locations on one of three possible railway line extensions to Brisbane's suburban network. Smith (1993) considered the following criteria when evaluating potential station locations: (a) Magnets and Generators (regional business/shopping centers, secondary schools, hospitals, recreation facilities,...); (b) Population Trends (the growth or decline of the population in the station area); (c) Site Centrality ('based on an assessment of the proximity of each potential station location to the 'center', 'core' or 'focus' of the built-up area to be serviced by the station'); (d) Land Resumption Costs; (e) Headwork Costs (all infrastructure and services required for the station operation); (f) Population Catchment Area (a distance of 800 meters was used, since around 70% of railway commuters walk to the station); (g) People Traveling by Bus (number of bus passengers); (h) Topographical Considerations (contour lines on topographical maps, vegetated areas,..); (i) Bus Interchange (the possibility of bus/rail interchange). Each of the above criteria was based on the linguistic scale: 'Low', 'Medium' and 'High'. Smith (1993) used linear additive weighting to make his multicriteria evaluation.

3.7. *Traffic control at the intersection*

Fuzzy traffic control problems were the first problems from the field of traffic and transportation in which fuzzy logic was used. Pappis and Mamdani (1977) were first to use the principles of fuzzy logic to control the isolated signalized intersection of two one-way streets and developed a model based on linguistic control instruction. Pappis and Mamdani (1977) assumed a uniform distribution of vehicles arriving at the intersection. They also presupposed that the vehicle detectors were placed sufficiently upstream from the intersection, so that it is possible to inform the controller about vehicle arrivals at the intersection within the next 11 s. The authors assumed as well that the cycle was divided into two periods 'actually red' and 'actually green' and that vehicles left the line at the same intensity at which they joined it. Pappis and Mamdani developed an approximate reasoning algorithm to control traffic at the intersection. This algorithm consists of a large number of rules of the following type:

If	T is very short and A is greater than none and Q is any
Then	E is very short

where:

(a) T = Fuzzy variable that denotes the time that has elapsed since the last light change at the intersection
(b) A = Fuzzy variable denoting the number of vehicles from the priority direction that have passed through the green light during the considered time period
(c) Q = Fuzzy variable representing the number of vehicles in line on the one-way street that does not have priority
(d) E = Fuzzy variable 'extension' which has values identical to fuzzy variable T representing the extension given to the present state of the system.

Fuzzy variables T, A and Q are input variables whose values should determine the value of output variable E. Fuzzy variables A and Q could be assigned the value 'many vehicles', 'more than several vehicles', 'few vehicles', etc. Variables T and E are assigned values 'very short', 'short', 'medium', etc. Bearing in mind the present state of the system, every 10 s a different set of five control rules was applied with the purpose of determining the extension (in s). The fuzzy logic model compared to 'classical' approaches used previously to solve this problem achieved better results (from the viewpoint of average time loss per vehicle).

Chang and Shyu (1993) produced a fuzzy expert system to evaluate whether a traffic signal is required in an intersection. They used 8-h volume, 4-h volume, peak-hour volume, progressive movement, peak-hour delay, accident experience and traffic system, and school crossing as criteria to help the decision maker when deciding whether a traffic signal is required in an intersection. The approximate reasoning algorithm developed by Chang and Shyu (1993) consists of rules such as the following:

If	Intersection is located in the city
and	The number of lanes on major street is 1
and	Peak hour traffic volume on major street (both directions) is very large
and	The number of lanes on minor street is 1
and	Peak hour traffic volume on minor street is very large
Then	a signal is required.

The proposed approximate reasoning algorithm was illustrated on the town of Tainan, Taiwan.

Sayers and Bell (1996) solved the traffic responsive signal control problem using fuzzy logic. These authors were very successful in building fuzzy logic into a modular system executing traffic control. The developed model enables both second-by-second and cycle-by-cycle decisions about the apportionment of green time. From raw data for each signal group, Sayers and Bell (1996) used fuzzy logic to calculate both the number of seconds of green time that the signal group will require in the next cycle, and the 'weight (a value between 0 and 1) as indicator of the urgency with which the signal group requires green'.

3.8. Traffic control in the corridor

The model of Nakatsuyama et al. (1983) is based on the concept of the Fuzzy Logic Controller proposed by Pappis and Mamdani (1977). They coordinated two consecutive intersections along an arterial road. After the light changes (from red to green) at the first intersection at one time point t, a green light appears at the other intersection at another time point $t+\mathrm{n}$. Elapsed time n is the so-called offset whose value is determined using fuzzy control rules. Fuzzy control developed by Nakatsuyama et al. (1983) is based on traffic conditions. Nakatsuyama et al. (1983) compared a fuzzy logic controller to a standard vehicle-actuated controller for different values of traffic flow rates. When compared, they had considerably shorter average delay times using fuzzy logic than using a standard vehicle-actuated controller.

Sasaki and Akiyama (1986, 1987, 1988) developed a fuzzy traffic control system on an urban expressway. The authors showed that control of an urban expressway depends upon a skilled operator's judgment and decisions. The main goal of the Sasaki and Akiyama papers was to describe the operator's judgment process using fuzzy logic. A simple fuzzy reasoning model for on-ramp control was presented in the papers. The developed model was tested on the Osaka–Sakai route of the Hanshin expressway. Restricting the number of booths and closing the gates carried out traffic control on the urban expressway. The main comclusion of the papers is that the proposed fuzzy logic traffic controller system effectively describes the judgment process and takes the place of human operators.

Chen et al. (1990) studied a fuzzy controller for freeway ramp metering. The developed model was tested on the San Francisco–Oakland Bay Bridge. The fuzzy controller was based upon six linguistic variables (inputs) and three linguistic variables (outputs). Chen et al. (1990) showed that the fuzzy controller was very efficient in reducing efficiency losses due to incidents. The authors showed savings of 148 passenger-h for a typical short incident. For longer incidents there were savings of 328 passenger-h compared to the existing controller. The developed controller also significantly reduced the min-miles of congestion.

3.9. Network control

Chiu (1992) developed adaptive traffic signal control at the intersection using fuzzy logic. This approach provided for control of a small network of intersections. It is important to point out that the signal timing parameters (cycle time, phase split and offset) adjusted as functions of the local traffic condition and of the signal timing parameters at adjacent intersections. These parameters were adjusted independently. In other words, a fuzzy rule base was developed to make adjustment for each of these parameters. Chiu (1992) says that 'although independent adjustment of these parameters may result in one parameter change working against another, no conflict was evident in simulations under various traffic conditions', also remarking that 'since incremental adjustments are made at every phase change, a conflicting adjustment will most likely be absorbed by the numerous successive adjustments'. While making the cycle time adjustment, the goal was to maintain a good degree of saturation on the approach with highest saturation. The inputs to the rules for cycle time adjustment were the highest degree of saturation on any approach and the highest degree of saturation on its competing approaches. The output of the rule was 'the amount of adjustment to the current cycle time, expressed as a fraction of the current cycle time'. One of the typical rules in this fuzzy rule base is:

If the highest degree of saturation on any approach is HIGH
and the highest degree of saturation on its competing approaches is NOT HIGH
Then the cycle change is POSITIVE SMALL.

Phase split is adjusted so as to maintain equal degrees of saturation on competing approaches. While making the offset adjustment, the goal was to coordinate adjacent signals in order to minimize the number of stops in the direction with the highest possible value of the traffic flow. Among other numerical examples on which the model in question was tested, the author considered a small network of intersections formed by six streets. As a measure of quality of the developed model, the author used the average waiting time per vehicle per s spent in the network. During first 30 min of the simulation all intersection had fixed cycle times. From the 30th to 60th min of simulation three intersections functioned on developed fuzzy rules, while the other intersections had fixed cycle times. From the 60th to 90th min all the intersections worked on fuzzy rules. Insignificant improvements were accomplished during the 30 to 60-minute period, while significant betterment was achieved in the period from 60th to 90th min, when all the intersections were adaptive. It can be noted that the work of Chiu (1992) is a pioneer work in the area of adaptive traffic signal control on the network using fuzzy logic. The results obtained by Chiu (1992) can be of great importance for further research in this field.

3.10. Accident analysis and prevention

Fuzzy set theory techniques were also recently applied to the issues in the area of accident analysis and prevention. Akiyama and Shao (1993) investigated the problem of constructing traffic safety facilities on urban expressways. They pointed out that traffic safety projects consist of local countermeasures whose cost and effectiveness are evaluated in monetary terms. In the general case, a countermeasure is the construction of safety facilities at each heavy traffic point on the expressway. In the planning phase consideration must be given to construction costs and benefits from reducing the number of accidents. The basic problem studied by Akiyama and Shao (1993) was determining the kind of countermeasure to be feasibly carried out within the budget. The authors noted that safety costs cannot be defined deterministically, and the cost and benefit of alternatives cannot be measured without fuzziness. In other words, when evaluating certain alternatives, factors such as feeling of safety, driving comfort, etc., must be taken into account. The authors used incremental cost–benefit analysis with fuzzy constraints and dynamic programming with fuzzy constraints to solve this problem. The model developed was illustrated on the Hanshin Expressway in Japan.

Incident detection and classification and the identification of accident-prone locations also belong to the class of problems that can be solved by fuzzy set theory techniques. Sayed et al. (1995) used fuzzy pattern recognition to identify accident-prone locations. These authors pointed out that the identification of such locations is the first step towards highway safety improvement. Sayed et al. (1995) stressed that accidents can be caused by road environment-related factors, by driver-related factors, by vehicle-related factors or different combinations of these factors. For the purposes of this paper, the authors considered '7000 accident records randomly selected between the years 1989 and 1992'. Each accident was described with the following 14 variables: degree of curvature, road grade, speed limit, surface condition, weather condition, lighting condition, land use, accident time, accident location, accident type, severity, traffic control device, use of a restraint device, volume/capacity ratio and vehicle type. The authors used the fuzzy K-NN algorithm Keller et al. (1985) to determine the degree to which each accident was caused by driver-related, vehicle-related and road-related factors. The algorithm was tested on three sets of data, each containing 300 accidents. As the authors noted, using this algorithm enabled the very successful classification of accidents into a finite set of categories.

Busch et al. (1995) used fuzzy reasoning for traffic state estimation and incident detection on motorways. Input variables in fuzzy logic were speed density difference between neighboring cross-sections, trend factor for the last N intervals and traffic volume trend at the neighboring cross-sections. The output was probability of a traffic situation obtained in a three-level process. The authors made tests with both simulation studies and real data, attaining satisfactory results in regards to reliability of state estimation and incident detection, and reaction time.

Schretter and Hollatz (1996) used fuzzy logic to determine the required period of waiting after a traffic accident. The authors considered the case when nobody is present at the site of an accident (the driver causes an accident without involving any other driver). The authors indicated that the duration of the required waiting time (until someone arrives who will take part in establishing all relevant facts regarding the accident) is conditioned by the amount of damage, the expectation level of someone's arrival, the site of the accident, the time of the accident, and traffic density. Two fuzzy rule bases were developed. One of the typical rules in one of the rule bases reads:

If	the damage is VERY SEVERE and the expectation level of someone's arrival is POSITIVE
Then	the waiting period is VERY LONG

The results obtained based on the fuzzy expert system were compared to court decisions. Satisfactory agreement was obtained between these two groups of results. The developed system (prescribing the required waiting time after a traffic accident) can considerably facilitate the work of the courts and insurance companies involved in determining responsibility and damage appraisal.

3.11. Level of service

Chakroborthy and Kikuchi (1990) discussed the application of fuzzy set theory to the analysis of highway capacity and level of service. They showed the limitations of the current procedure to determine highway capacity and service level. Chakroborthy and Kikuchi pointed out that fuzzy numbers can be used to represent the values of input variables (ideal capacity, sight distance, volume of traffic, headway between cars) and output variables (adjustment factors, actual capacity, level of service criteria) which are involved in calculating capacity and service level. The authors showed that it is much better if the Level of Service Categories are defined as fuzzy sets. It was also shown how fuzzy composition could be used to represent parametric relationships. Highway and traffic characteristics are so complex that relationships between parameters can be described only in approximate terms. The authors pointed out the possibilities of using fuzzy relations and the max–min composition method when determining the relationship between parameters. The authors illustrated how to use fuzzy relations and the max–min composition method to determine the adjustment factor to ideal capacity when average lane width is around 10 feet instead of 12 feet (which is ideal). It was also shown that there are cases in which adjustment factors depend directly on human perceptions and decision-making processes. The authors considered the example of a driver's acceptance gap (the critical gap is the minimum gap in a traffic stream, which a driver wishing to cross will accept). Chakroborthy and Kikuchi (1990) showed that a driver's acceptance of a gap depends on his perception of the gap and the speed of the oncoming traffic. They concluded that a driver's decision to accept a gap is in fact based on an approximate reasoning process and so they developed an approximate reasoning algorithm to estimate the approximate critical gap which a driver would choose under given conditions.

Ndoh and Ashford (1994) developed a model to evaluate airport passenger services using fuzzy set theory techniques. In their words, 'The literature on transportation level of service evaluation indicates a strong impetus to move away from a strictly capacity/volume or time/space based measure to one that directly incorporates the perception of passengers'. Studying airport passenger service, Ndoh and Ashford (1994) pointed out the hierarchical structure of the service system and proposed that the service system level of service be decomposed into its component service attributes (information, waiting time at processing activities, convenience at processing activities in terms of physical efforts, availability of seats, etc.). Each component service attribute can be assigned a linguistic variable name (high, medium, etc.). Ndoh and Ashford (1994) defined the overall level of service **Z** as:

$$\mathbf{Z} = \sum_{i=1}^{n} [W_i] * [L_i] / \sum_{i=1}^{n} [W_i]$$

where:

(a) n = the number of component service attributes
(b) W_i = the fuzzy weight of component i (importance factor of component i)
(c) L_i = the fuzzy quality of service component i.

The obtained level of service **Z** must be 'translated' into a suitable linguistic term. Customary linguistic terms that must be previously defined include 'Good', 'Tolerable' and 'Bad'. Let us denote by **A** the known fuzzy set representing 'one of the natural language expressions used for rating level of service'. Ndoh and Ashford proposed to calculate the Euclidean distances between fuzzy set **Z** and individual fuzzy sets **A**, indicating that the best natural language is equivalent to the fuzzy set **A** corresponding to the shortest Euclidean distance from fuzzy set **Z**. The methodology they presented was illustrated on the example of evaluating the Processing Activity Subsystem for departures with the components: Check-in, Security and Passport Control. The methodology proposed by Ndoh and Ashford (1994) is certainly quite compatible with the way passengers perceive transport services ('in terms of imprecise and vague linguistic values').

Pattnaik and Ramesh Kumar (1996) developed methodology to define level of service of urban roads taking into account users' perceptions. The authors assigned appropriate linguistic terms to the categories of level of service A to F (which are defined by Highway Capacity Manual). Speed, volume and standard deviation served as the evaluating criteria for the level of service. Based on surveys they have done, and taking into consideration perceptions of the users, the authors determined membership functions for different fuzzy sets, and importance factors assigned to them. They also categorized surveyed people according to age, thus determining users' perceptions more appropriately.

3.12. Vehicle and crew routing, scheduling and dispatching problems

Vehicle and crew routing, scheduling and dispatching problems appear in various transportation activities. Hundreds of papers have been published in world literature during the past three decades treating different aspects of vehicle routing, scheduling and dispatching problems. Significant reviews were given by Larson and Odoni (1981), Bodin et al. (1983), Solomon and Desrosiers (1988) and Golden and Assad (1988). Combinatorial optimization methods or different heuristic algorithms with deterministic characteristics traditionally solve complex vehicle routing problems. In the past several years papers have appeared in which attempts were made to use fuzzy logic and other fuzzy set theory techniques to solve these very important vehicle and crew routing, scheduling and dispatching problems.

Perincherry and Kikuchi (1990) studied the transshipment problem in a fuzzy environment. The transshipment problem deals with the optimal allocation of goods and services between supply locations and demand locations with an added dimension involving intermediate points where goods can be stored to satisfy the demand at a later date. The input data are demands, supplies, transportation cost and the costs of inventory at the transshipment points. In most real-world problems, the data are given as estimates and are not likely to be precise. Perincherry and Kikuchi (1990) introduced the principles of fuzzy linear programming to solve the transshipment problem under the conditions of fuzziness in input data. They assumed fuzziness in the quantity of supplies available and the quantities demanded. The travel time between the locations and the costs are assumed to be available in the form of precise information. Perincherry and Kikuchi (1990) mentioned that the goal in a fuzzy transshipment problem is not always to minimize the cost, rather to schedule the shipments at a 'reasonable cost'. This 'reasonable cost' is represented by a fuzzy set. In order to illustrate the application of the model developed, a problem was presented involving the scheduling of empty freight cars by a large manufacturing concern having many plants and distribution centers.

In most vehicle routing models, it is assumed that travel time, transport costs and the distance between pairs of nodes in the network are constant values known in advance. On the other hand, the travel time between two nodes in a network involves an uncertainty due to traffic conditions, the type of driving, weather conditions, choice of streets, etc. Dispatchers-decision makers most often make a subjective estimate of travel time based on their experience and intuition, expressing the estimated travel time as 'short', 'long', 'about twenty minutes', etc. When studying the 'classical' vehicle routing problem, Teodorović and Kikuchi (1991) treated travel time and transportation costs between two nodes in a network as fuzzy numbers. They modified the Clarke and Wright (1964) algorithm when travel times in a network are treated as fuzzy numbers. Teodorović and Kikuchi (1991) expressed the level of uncertainty through parameter ß, where $ß=0$ corresponds to the highest possible uncertainty, while $ß=1$ corresponds to the situation in which the exact travel time through the network is known with complete certainty. After generating all travel times using the algorithm, a set of vehicle routes was obtained corresponding to a certain parameter ß. Different sets of vehicle routes can be generated for different values of parameter ß depending on the level of uncertainty used to estimate travel times in the network.

Kikuchi (1992) solved the problem of scheduling demand-responsive transportation vehicles using fuzzy arithmetic techniques. He developed 'a vehicle routing and scheduling method for the time-window based many-to-many, advanced reservation multi-vehicle dial-a-ride problem'. Kikuchi was the first to introduce fuzziness to the dial-a-ride problem, treating the desired time of vehicle arrival ('I would like to be picked up about 9 a.m.') and the travel time between nodes ('travel time is about 20 minutes') as fuzzy numbers. Kikuchi's approach is sequential (routes are designed one by one), has two stages (first the initial route is developed, and then insertions are made) and fuzzy (desired times of departures and travel times are treated as fuzzy numbers).

We often do not have precise data available regarding demand at certain nodes (the start-up operations of a distributing system are just beginning, the records of demand at some nodes are not up-to-date, etc.). The problem discussed by Teodorović and Pavković (1996) can be defined as follows: For known locations of the depot and nodes to be served, known vehicle capacity, and demand at the nodes which is only approximate (represented by triangular fuzzy numbers), design a set of vehicle routes that minimizes costs. The algorithm developed by Teodorović and Pavković (1996) to route vehicles when demand at the nodes is uncertain is based on the heuristic 'sweep' algorithm and fuzzy logic.

Transportation companies receive a great number of requests every day from clients wanting to send goods to different destinations. Each transportation request is characterized by a large number of attributes, including the most important: type of freight, amount of freight (weight and volume), loading and unloading sites, preferred time of loading and/or unloading and the distance the freight is to be transported. Transportation companies usually have fleets of vehicles consisting of several different vehicle types. In addition to the characteristics of the transportation request, when assigning a specific type of vehicle to a specific transportation request, the dispatcher must also bear in mind the total number of available vehicles, the available number of vehicles by vehicle type, the number of vehicles temporarily out of working order, and vehicles undergoing technical examination or preventive maintenance work. Milosavljević et al. (1996) considered the case when dispatching is carried out every day based on the principle 'today for tomorrow'. In other words, dispatchers have a set amount of time (1 day) to match available vehicles to transportation requests, which are to begin the following day. Milosavljević et al. (1996) developed a heuristic algorithm to solve the vehicle assignment problem that included several approximate reasoning algorithms. For every vehicle type, a corresponding approximate reasoning algorithm was developed to determine the dispatcher's preference strength in terms of meeting a specific transportation request with the type of vehicle in question. The approximate reasoning algorithms for each type of vehicle differed from each other in terms of the number of rules they contained and the shape of the membership functions of individual fuzzy sets. A typical rule found in these algorithms is:

If	suitability in terms of distance is HIGH and capacity utilization is HIGH
Then	preference is VERY HIGH

The quality of the solution is measured on the basis of the total number of realized ton-km. The results showed that using the developed model produced a greater number of ton-km than when the dispatching process was carried out by experienced dispatchers.

3.13. Air transportation

Larkin (1985) made the first application of fuzzy logic in the field of Air Traffic Control. He developed a model of an autopilot controller based on fuzzy logic. Most airports in the world are equipped with the Instrumental Landing System (ILS) that guides approaching aircraft to the runway. Larkin (1985) interviewed a large number of experienced pilots in order to develop a fuzzy rule base consisting of 125 rules. All pilots were given the following objective: 'to land the aircraft while maintaining an airspeed of 75 mph, rate of descent of 350 fpm and glide-slope angle of three degrees'. In order to derive fuzzy rules, questions were formulated and asked of the pilots. All questions had the following format:

> 'If the current rate of descent is around x fpm, the current airspeed is around y mph above/below the desired airspeed and the glide slope is about z degrees above/below the desired glide slope, by about how much would you change the engine speed and the elevator angle?'

Larkin's (1985) research indicated the exceptional possibilities provided by fuzzy logic in the field of Air Traffic Control.

In recent years there has been considerable congestion in some elements of the air traffic control system. The most frequent congestion occurs at airports. Meteorological conditions (visibility, wind velocity, etc.) have a direct influence on airport capacity. Since meteorological conditions cannot be precisely forecast over a longer period of time, the capacity of an airport can only be approximately determined over the long run. In the same vein, some airports frequently have time periods in which 'demand' is greater than the airport's capacity. In such conditions, the air traffic control service might not allow a certain number of aircraft to take off at their planned departure times. In this way, aircraft delay (caused by congestion) takes place on the ground instead of in the air. Airport congestion can appear during one time period (1 hour) or over a greater number of time intervals. Air traffic flow management consists of undertaking actions to minimize the negative consequences of congestion. In other words, there is a need to determine which aircraft will not be served during a considered time period and to set their new departure times.In the Teodorović and Babić (1993) air traffic flow management model based on fuzzy logic, the authors considered a starlike network with several departure airports and one landing airport. They developed heuristic algorithm to manage air traffic flows when landing airport capacity is decreased. It is based on the tactic in which a certain number of aircraft are 'sacrified', so that the number of aircraft with delays is minimized. Teodorović and Babić (1993) assumed that the decision-maker has certain preferences towards servicing certain aircraft. This preference must be dependent on aircraft size and delay. In other words, larger aircraft have higher priority to be served. Taking into account the applied tactic in which a certain number of aircraft are 'sacrified', lower priority was given to aircraft that could not be served during a longer period of time (these aircraft have longer delays). The decision-maker has a subjective feeling regarding aircraft size and delay times. The decision-maker feels that aircraft are 'big', 'medium-sized' or 'little'. The decision-maker might also estimate delay time to be 'big', 'medium' or 'little'. In other words, aircraft size and delay can be treated as fuzzy variables.The approximate reasoning algorithm developed by Teodorović and Babić (1993) to establish preference strength regarding aircraft to be served consists of nine rules of the following type:

If the aircraft is LITTLE and the delay is LITTLE
Then preference is MEDIUM.

The presented approximate reasoning algorithm enables the establishment of the preference index value, or the strength of the decision maker's preference regarding service to a specific aircraft in the considered time interval.

The route choice problem in air transportation is similar to the route choice problem in city or intercity road transportation. The differences in these problems are primarily found in the parameters that influence route choice. Teodorović and Kalić (1995) considered the simplest case when a passenger in air transportation must choose between two possible routes going from his origin to his destination. In their paper, they assume that passengers base their route choice on a comparison of travel time and flight frequency on the alternate routes. We will denote respectively by dT and dF the differences in travel time and flight frequency between one route and the other. We will also denote by P the percentage of passengers who use the first considered route. In order to determine the probability of the choice of the first route (the percentage of passengers who use the first considered route), Teodorović and Kalić (1995) developed an approximate reasoning algorithm composed of rules of the following type:

If δT = very big negative and δF = any
Then P = very very big.

The approximate reasoning algorithm developed by Teodorović and Kalić (1995) was tested using real data taken from the doctoral dissertation of Ghobrial (1983) (data on traffic in both directions between 13 pairs of cities in the Atlanta, Georgia region, USA). Extremely good agreement was obtained between the values calculated using the approximate reasoning algorithm and real values (one can easily calculate and note very low values of the average relative deviation and maximum relative deviation between the algorithm-obtained and real values.)

Teodorović et al. (1994) used fuzzy logic and fuzzy mathematical programming in solving the airline network design problem. Using Generalized Floyd's algorithm, the initial set of route candidates for air transportation was defined. The final set of the route candidates was determined by applying fuzzy logic. Annual frequencies of flying along the certain routes were established by fuzzy mathematical programming. This method was used to compensate for the incapacity to precisely calculate the yearly number of passengers travelling between certain O–D pairs, the incapacity to figure operational costs, and the incapability of analysists (decision makers) to very accurately define specific constraints inherent in the model.

3.14. River transportation

Studying the process of transporting bulk freight in river traffic, Vukadinović and Teodorović (1994) developed an approximate reasoning model to control the process of loading, transporting and unloading gravel. The problem of transporting bulk freight in river traffic will be presented using their example. Let the suction dredging machine be located downstream from the harbors where the gravel is to be transported. A tugboat pushes compositions of two, three or at most four barges. After loading gravel at the loading point, the tug pushes the barges upstream towards the harbors where the gravel is to be delivered. When the barges and gravel reach the unloading harbors, the dispatcher in charge of traffic decides on the number of barges that should be left at each harbor. After leaving a certain number of barges in the harbor, the tugboat continues to navigate upstream until the entire composition (all the barges it has pushed) has been left in harbors. Then the tugboat changes direction, heading downstream to pick up the empty barges along the way. The dispatcher in charge of river traffic decides on the number of empty barges that the tugboat should pick up at each harbor.A dispatcher manages the entire process of loading, transporting and unloading in conditions of uncertainty, since the goals, constraints and consequences of actions undertaken cannot be perceived with precision. Subjectively evaluating the situation, the dispatcher makes certain decisions with very little time at his disposition.

The approximate reasoning algorithms developed by Vukadinović and Teodorović (1994) were applied in the case of two unloading harbors—Pančevo and Belgrade (Yugoslavia). The main goal of this research was to show the possibilities of using approximate reasoning models to control the process of transporting bulk freight in river traffic. Vukadinović and Teodorović (1994) developed two approximate reasoning algorithms. The first calculates the number of barges that the tugboat should leave in Pancevo and the second the number of empty barges it should take in Belgrade.The algorithm that calculates the number of barges to be left in Pancevo consists of rules of the following type:

If	waiting time to unload in Pancevo is LARGE
and	waiting time to unload in Belgrade is LARGE or AVERAGE
Then	the number of barges left in Pancevo is AVERAGE.

The approximate reasoning algorithm to calculate the number of empty barges that are picked up in Belgrade consists of rules of the following type:

If	the number of empty barges in Belgrade is LARGE
and	the number of empty barges in Pancevo is ANY
Then	the number picked up in Belgrade is LARGE

Vukadinović and Teodorović (1994) tested these approximate reasoning algorithms on a large number of numerical examples. Waiting times to unload varied from 0–12 h. The number of empty barges was from 0–4. In addition to calculating the number of barges that were left or picked up for different input values, dispatchers were surveyed as well. Dispatchers were asked the number of barges they would leave, or the number of empty barges they would pick up within the framework of an offered scenario. In other words, the dispatchers were asked such questions as 'If you estimate that the waiting time to unload is 3 hours in Pancevo and 8 hours in Belgrade, how many barges would you leave in Pancevo?' A comparison between model output results and output results from the dispatcher survey indicates very good agreement between dispatcher decisions and the results of the approximate reasoning algorithms. Computer testing showed that the developed models are applicable in real time.

4. Validation of results obtained by applying fuzzy logic to modeling transportation phenomenon and directions for future work

Up to now, there has been only one research in which the number of trips generated in a given area was estimated using artificial neural networks, as well as by multiple linear regression and fuzzy logic. After the testing, the fuzzy logic approach proved to give the closest estimate of the actual number of trips generated in a given area. Certainly, the obtained results should be taken tentatively, and further research in this area appreciated and carefully examined.

In addition, there has been only two other similar research studies: one pertaining to the application of fuzzy logic to the trip distribution problem, and one regarding the modal split problem. In both cases, very small relative deviations between the values calculated using the approximate reasoning algorithms and the real values were obtained. However, these testings were done on hypothetical, almost school examples, so it would be necessary to verify the suggested approach by a real transport study of a given region.

In several papers, fuzzy logic was also applied to the problem of complex route choice modeling. Research in this area began by modeling a simple two-route choice problem, just to shortly afterwards extend to modeling a multi-choice problem. It has also been attempted to put the research in this area into the context of IVHS problems (in specific models, both real time traffic information and perceived travel times were used). It would be of great importance to develop fuzzy logic models, which would treat the relationship between the traffic information and drivers' behavior. In particular cases, fuzzy logic models gave considerably better results than those obtained from the Logit model. The major drawback of these models is that they were tested on hypothetical numerical examples. The exception to this is a study done in The Netherlands, which encompassed revealed preference data of the choice between rail and car for intercity travel. The model was calibrated using an error function that was the sum of the squares of the deviations between the actual choice and calculated preference. Very promising results were obtained which fact points out to the necessity of further applications of fuzzy logic to the problem of complex route choice modeling. In future research, using fuzzy logic to solve choice problems (route choice, modal split, choice of carrier, choice of the passenger class in the aircraft, etc.) will primarily need to entail the generation of appropriate fuzzy rules from available numerical data.

Traffic control is the oldest field to which fuzzy logic has been applied in traffic and transportation. In this area, the attempts have been made to use fuzzy logic to perform traffic control at the intersection, as well as corridor and network controls. The very first research in the area of traffic control as related to the isolated intersection showed that better results (from the viewpoint of average time loss per vehicle) could be achieved using fuzzy logic than by 'classical' approaches used previously to solve this problem. Existing research in the area of traffic control also revealed that the fuzzy logic traffic controller system could effectively describe the judgment process and take place of the human operators. The obtained results also point out that the fuzzy controller could be very efficient in reducing lost passenger hours, as well as minute miles of congestion in the case of traffic incidents. In the area of traffic control, it would be very significant to attempt to develop approximate reasoning algorithms related to parts of the network with a greater number of signalized intersections.

Current experience in trying to solve the problem of congestion in the Air Traffic Control system indicates that further application of fuzzy logic in this field could eventually lead to the successful resolution of very urgent problems.

Vehicle and crew routing and scheduling problems are another important field of research for fuzzy set theory applications. Fuzziness related to travel times between different pairs of nodes, fuzziness related to the moment required service should begin, as well as fuzziness related to quantities that need to be picked up and/or delivered to certain nodes require the development of appropriate models based on fuzzy set theory. Limited experience to date in this field has shown that fuzzy logic could prove to be a very good technique to solve various routing and scheduling problems in real time.

It can be noted that a considerable number of fuzzy logic approaches to solve diverse traffic and transportation problems have been developed. These theoretical attempts must be supported by more practical applications of different ideas from the fuzzy set theory. In some other fields of human activity a notably greater agreement between theoretical results and practical applications has been achieved. As already known, there exists a great many number of commercial products that operate on the principles of fuzzy logic (vacuum cleaners, washing machines, microwave ovens, cameras, etc.). On the other hand, there are still no practical fuzzy logic applications in the fields of traffic control or vehicle routing and scheduling. The assessment of the benefits of fuzzy logic will be more readily performed with the increase in number of successful practical applications of the fuzzy logic in traffic control and transportation planning. It is most definite, however, that the attempts of applying fuzzy logic to the fields of transportation engineering will present a great challenge for the theoreticians and practitioners in the period to come.

5. Conclusion

Many problems encountered when studying complex transportation systems are highly nonlinear. As already noted, a fuzzy logic system is a nonlinear system that maps a crisp input vector into a crisp scalar output. When solving a large number of different traffic and transportation problems, this is what we actually do: map a crisp input vector into a crisp scalar output.

Fuzzy logic could be used successfully to model situations in which people make decisions in an environment that is so complex that it is very hard to develop a mathematical model. Such situations for example often occur in the field of traffic and transportation when studying the work of dispatchers or modeling choice problems. Present experience shows that there is room for the development of different approximate reasoning algorithms when solving complex problems of this type.

Until a few years ago the 'trial and error' procedure was the customary method used to develop fuzzy logic systems, and so researchers designed fuzzy logic systems independently of numerical training data. Wang and Mendel, 1992a,b; Jang (1992) and Horikawa et al. (1992) proposed separately from each other to tune fuzzy logic system parameters using numerical training data. This idea was accepted by a great many researcher and in the past two or three years a large number of papers have appeared in which the fuzzy rule base and/or shape of the membership function are determined using numerical training data. Designing fuzzy logic systems using numerical training data is certainly one of the very important tasks in the area of fuzzy logic systems for transportation engineering as well.

In the past several years, significant theoretical results have been achieved in the field of fuzzy logic systems. Wang and Mendel (1992c) and Kosko (1992a) showed that fuzzy logic systems are universal approximators, which means that a fuzzy logic system 'can uniformly approximate any real continuous nonlinear function to an arbitrary degree of accuracy' (Mendel, 1995). This existence theorem shows us above all the possibilities offered by fuzzy logic, but does not indicate the manner in which to create fuzzy logic systems. In other words, the number of inputs, the number of fuzzy sets used to describe fuzzy variables, and the number of rules essentially influence the quality of the solution generated by a fuzzy logic system. It should be emphasized that feedforward neural networks and fuzzy logic systems are techniques that can be used to solve the same class of problems. Since fuzzy logic system parameters can be initialized using expert knowledge, while weights in feedforward neural networks are most often initialized randomly, fuzzy logic systems are tuned much more quickly than the tuning of feedforward neural networks. Fuzzy logic systems provide two other very important advantages. They can use existing linguistic knowledge very successfully, and they treat uncertainty in an appropriate manner.

The basic goal of this paper was to classify and analyze results in the application of fuzzy logic when modeling complex traffic and transportation processes. The results obtained show that fuzzy set theory and fuzzy logic present a promising mathematical approach to model complex traffic and transportation processes that are characterized by subjectivity, ambiguity, uncertainty and imprecision. As already noted, the benefits from the fuzzy logic will be more accurately assessed as the number of successful practical applications of the fuzzy logic in traffic control and transportation planning increases.

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