Energy Systems manuscript No.
(will be inserted by the editor)

Optimal Power Flow: A Bibliographic Survey II

Non-Deterministic and Hybrid Methods

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Received: date / Accepted: date

Abstract Over the past half-century, Optimal Power Flow (OPF) has become one of the most important and widely studied nonlinear optimization problems. In general, OPF seeks to optimize the operation of electric power generation, transmission, and distribution networks subject to system constraints and control limits. Within this framework, however, there is an extremely wide variety of OPF formulations and solution methods. Moreover, the nature of OPF continues to evolve due to modern electricity markets and renewable resource integration. In this two-part survey, we survey both the classical and recent OPF literature in order to provide a (see part I) for the state of the art in OPF formulation and solution methods. The survey contributes a comprehensive discussion of specific optimization techniques that have been applied to OPF, with an emphasis on the advantages, disadvantages, and computational characteristics of each. Part I of the survey provides an introduction and surveys the deterministic optimization methods that have been applied to OPF. Part II of the survey (this article) examines the recent trend towards stochastic, or non-deterministic, search techniques and hybrid methods for OPF.

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Keywords electric power systems · optimal power flow · non-deterministic optimization · stochastic search · hybrid methods · heuristics · global optimization · survey

1 INTRODUCTION

This article is part II of a two part survey of Optimal Power Flow (OPF). For a full introduction, we refer readers to part I of the survey, *cf.* Frank et al. (2012), which provides a general introduction to the OPF problem, describes the key requirements for OPF methods, and surveys deterministic optimization methods that have been applied to OPF. Part II of the survey (this article) examines the recent trend towards non-deterministic search techniques (also known as heuristic, stochastic, or random search methods) and hybrid methods for OPF and gives the survey conclusions. These methods have become popular because they have a theoretical advantage over the deterministic methods with respect to handling of nonconvexity, dynamics, and discrete variables Biskas et al. (2006); Qiu et al. (2009).

The remainder of this article is organized as follows: In Section 2 we review non-deterministic optimization methods that have been applied to OPF. In Section 3, we survey hybrid methods, that is, methods that consist of the combination of various established OPF techniques. In each section, we first describe the applied methodology and, second, we survey the relevant literature. While some paragraphs discuss the papers in chronological order, others highlight streams of research. Finally, we conclude both part I and part II of this survey in Section 4. Various abbreviations and acronyms used throughout the article are summarized in Appendix A.

2 NON-DETERMINISTIC OPTIMIZATION METHODS

In the past two decades, a number of non-determinisitc optimization methods have been developed and applied to global optimization problems to overcome the weak global search capabilities of many conventional deterministic optimization algorithms, *cf.* He et al. (2004); Alrashidi and El-Hawary (2009). Spall (2003) gives a general introduction to these heuristic, or random search, optimization methods. Many of these techniques have been applied to OPF problems, including Ant Colony Optimization (ACO), Artificial Neural Networks (ANN), Bacterial Foraging Algorithms (BFA), Chaos Optimization Algorithms (COA), various Evolutionary Algorithms (EAs), Particle Swarm Optimization (PSO), Simulated Annealing (SA), and Tabu Search (TS).

2.1 Ant Colony Optimization

Ant Colony Optimization (ACO), initially proposed by Colorni et al. (1991) and Dorigo (1992), is a class of probabilistic algorithms modeled after the pathing behavior of ants, *cf.* Dorigo and Stützle (2004); Dorigo et al. (2008). ACO is a parallel search over several constructive computational threads based on local problem data and a dynamic memory structure containing information on the quality of previously obtained results. ACO was inspired by the observation of ant colonies establishing shortest route paths between the colony and food sources. The technique is based on a probabilistic pheromone model, consisting of a set of model parameters called the pheromone trail parameters. The pheromone values are updated using previously generated solutions in such a way that the probability of generating high-quality solutions increases over time. Unlike some other stochastic algorithms, such as SA and GA, ACO can be run continuously and adapt to changes in real time, *cf.* Venayagamoorthy and Harley (2007).

Recently, ACO has been applied to a number of OPF problems with several objective functions; Lee and Vlachogiannis (2005) survey some of these. Teng and Liu (2003) successfully employed ACO approaches to deal with the optimum switch relocation problem, finding the ACO solutions more reliable than those produced by a GA. Swarup (2005) applied ACO to economic load dispatch and generator scheduling problems. Kalil et al. (2006) proposed ACO techniques for optimal reactive power dispatch in order to improve voltage stability conditions and reduce transmission losses while providing voltage profile monitoring. The authors indicate that ACO outperformed both EP and AIS in both solution quality and computation time.

ACO has been successfully applied to several combinatorial optimization problems within the OPF field. Vlachogiannis et al. (2005) formulated a reactive power control problem as a combinatorial optimization problem prior to applying ACO. Simon et al. (2006) confirmed that ACO is a suitable approach to the combinatorial unit commitment problem. Allaoua and Laoufi (2008, 2009) used ACO to minimize the total fuel cost of thermal generating units while also retaining an acceptable system performance level in terms of limits on generator real and reactive power outputs, bus voltages, shunt capacitors/reactors, transformer tap settings and power flow on transmission lines. Their method is notable because it saves computation time by decomposing the constraints into active and passive constraint sets. The active constraints are used to calculate the optimal solution set using ACO while the passive constraints are enforced by a Newton-Raphson power flow algorithm. Simulation results showed that their ACO method outperforms previously published EAs in computational speed as well as solution quality. Gasbaoui and Allaoua (2009) also used ACO to solve a combinatorial OPF problem with multiple objectives, including fuel cost minimization, voltage profile improvement, and voltage stability enhancement. The authors report that the ACO approach performed better than both classical techniques and GAs.

2.2 Artificial Neural Network

Artificial Neural Networks (ANNs) are computational tools based on the operation of biological neural networks. ANNs operate on the principle of parallel processing, analogous to the operation of the human brain. Consequently, ANNs are quite fast, especially when dealing with large volumes of data with unknown mathematical correlation. Apart from on-line processing and classification capabilities, the main advantage of ANNs is the capability of dealing with stochastic variations of the scheduled operating point given increasing data. The theory of ANNs has been discussed extensively in several textbooks, *cf.* Ripley (1996); Jurada (1997); Dreyfus (2005).

ANNs have been used in a broad range of applications in power systems operation and control; Haque and Kashtiban (2005) reviewed a number of these. Nguyen (1995, 1997) developed a general Neural Network (NN) architecture for OPF which can include different types of objective functions and constraints. The proposed approach adopts the Newton-Raphson method for its implementation on NNs. A principal feature of the NN OPF is the ability of individual NN modules to handle specific constraints. Paralleling these modules leads to high speed computation. The algorithm also exploits the sparsity of the matrices encountered in OPF problems. Hartati and El-Hawary (2001) proposed a NLP-based approach to an OPF problem where an ANN augmented the cost function by computing suitable penalty terms. The approach showed faster convergence in the active OPF problem than conventional methods.

Two papers report the use of ANN to compute optimal capacitor switching and control. Santoso and Tan (1990) propose an expert system using a two-stage ANN to execute real-time control of multi-tap capacitors installed on a distribution system for a nonconforming load profile such that the system losses are minimized. The authors claim that this method is suitable for on-line implementation of the capacitor control even for a very large distribution system because of the much reduced computation time compared with traditional optimization processes. Later, Das and Verma (2001) developed an ANN-based approach for computing optimal capacitor switching in a distribution system. They also report drastically reduced computation time in comparison to traditional approaches: on the order of 100 times faster, even for a realistic number of capacitors in the system.

2.3 Bacterial Foraging Algorithm

Inspired by the patterns exhibited by bacteria while foraging for food, Passino (2002) introduced the Bacterial Foraging Algorithm (BFA). In time-varying environments, natural selection tends to eliminate bacteria with poor foraging strategies, *cf.* Passino (2002) Liu and Passino (2002); Passino (2005). After many generations, poor foraging strategies are either eliminated or reshaped into good ones. BFA mimics these patterns to optimize a solution pool.

Although relatively new, BFA has attracted interest in the power systems community. Mishra (2007) and Tripathy and Mishra (2007) applied BFA to optimize the real power losses and voltage stability limits of a mesh power network. This was formulated as a multi-objective OPF problem with the unified power flow controller (UPFC) location, UPFC series injected voltage, and transformer tap positions as the controllable variables. The authors reported that the BFA was superior to interior point SLP techniques. Li et al. (2007b) developed a BFA with varying population for the OPF problem. The authors explored the the mechanisms of bacterial chemotaxis, quorum sensing, and proliferation for the first time.

One drawback of BFA is that it is not always able to effectively track the global optimal solution in dynamic environments, *cf.* Passino (2002). In order to address this shortcoming, Tang et al. (2006) presented an approach called Dynamic BFA (DBFA) to solve the OPF problem with dynamic loads. The variations of power loads and system topology were simulated as regular and irregular environmental changes. The results demonstrated the adaptability of DBFAs to various environmental changes and the authors reported DBFA as superior to both traditional BFAs and PSO methods. Tang et al. (2008) subsequently applied a DBFA to minimize the power system fuel cost with the OPF embedded in an environment with dynamically changing loads. Simulation results showed that in comparison with BFAs and PSO methods, DBFAs can adapt more rapidly to load changes and track the fuel cost global optimum more closely.

2.4 Chaos Optimization Algorithm

Chaos is a universal phenomenon, occurring naturally in many deterministic systems. Chaos exhibits diverse, complex, and sophisticated rules under apparent disorder. A system can make the transformation from a regular periodic system to a chaotic system simply by altering one of the controlling parameters, *cf.* Shengsong et al. (2003). Chaotic movement has the properties of ergodicity, intrinsic stochasticity, and regularity, and can therefore go through every state in a certain area according to its own rule without repetition. Chaos Optimization Algorithms (COAs) as introduced by Li and Jiang (1998) exploit these concepts, employing chaotic variables to search for an optimal solution. Being relatively new random search methods, COAs have already attracted great attention, *cf.* Yang et al. (2007b). COAs have several favorable characteristics that are particularly well suited for OPFs, including the ability to escape from local optima via chaotic motions, insensitivity to initial values, high search velocity, and gradual global convergence.

COA is gradually being applied to engineering practice, including OPF. Jiang et al. (1999) proposed a COA to solve the economic dispatch problem of a hydro power plant. Zhijiang et al. (2002) also applied a COA to economic dispatch and OPF, reporting that simulation results verified the precision of the COA solution. Xu et al. (2000) applied a mutative scale COA to the economic operation of power plants. However, the results showed that the method is time-consuming. Subsequently, Han and Lu (2008) used an improved mutative scale COA to solve an economic load dispatch problem. According to the authors, their algorithm is highly efficient and can be applied to many power system problems, such as economic operation, OPF, system identification and optimal control. Recently, COAs have also been combined with various other exact and heuristic optimization algorithms; see Section 3.

2.5 Evolutionary Algorithms

Evolutionary Algorithms (EAs) include a broad array of techniques based on the theory of biological evolution where a solution pool is maintained to mimic the evolution of individuals inside a population. By design, EAs are effective for problems which evolve over time and need to be solved repeatedly. Furthermore, EAs make no assumptions on the differentiability, convexity or smoothness of the functions present in the optimization model and are very well suited for parallel algorithms due to the presence of a solution pool. EAs are well suited for OPF problems where multiple objective functions are present or a set of solutions (rather than one solution) is desirable.

The label EA applies to a diverse set of algorithms, and some debate exists as to which of these algorithms are properly classified as EAs. We refer the interested reader to Yu and Gen (2010) for an in depth discussion of the various EAs, including similarities and differences. Here, we classify as EAs the following algorithms that have been applied to OPF: Artificial Immune Systems (AIS), Differential Evolution (DE), Evolutionary Programming (EP), and Genetic Algorithms (GAs).

2.5.1 Artificial Immune Systems

The first paper on Artificial Immune Systems (AIS) was published by Kephart (1994). The AIS methodology is based on three principles of biological immune systems: proliferation, mutation, and selection. Proliferation is the capability of generating new individuals, leading to a dynamic optimization process. Mutation is the ability to search through the solution space by altering the solutions in the pool. Selection is responsible for eliminating low-affinity cells, *i.e.*, poor solutions. Optimization algorithms based on AIS are called immune algorithms (IAs) and explore these three principles. Modern IAs are inspired by three different theories explaining adaptive immune system: clonal selection, negative selection and immune network algorithms.

AIS has seen some application to OPF. Liao (2006) applied an IA to the short-term unit commitment problem with linearized transmission constraints. The proposed algorithm differs from conventional AIS approaches in the use of variable (rather than fixed) crossover and mutation ratios, the use of a memory cell, and the use of an annealing immune operator. The authors claim that their algorithm does not fall into locally optimum solutions and can quickly and correctly find the full set of globally optimum solutions, although no formal proof is provided. The authors compared the IA solution with those obtained by dynamic programming, Lagrangian relaxation, GA, SA and TS methods, reporting that the IA returned better solutions with respect to the objective function values. Xiangzheng (2007) applied an IA to an ORPF control problem. The developed techniques allow the dispatcher to control the reactive power, to reduce the power loss, and to improve the power quality.

de Mello Honório et al. (2007) developed a modified AIS optimization methodology, combining AIS with a gradient vector in order to improve both computational effort and search robustness. The numerical information of the gradient leads to a more efficient hypermutation process, and, consequently, local optima are approached faster. Hugang et al. (2008) also developed a modified IA for OPF. The authors designed a multi-objective, adaptive IA to solve an ORPF problem incorporating static voltage stability. The IA has two additional parts compared to existing IAs. The first defines both partial affinity and global affinity to evaluate the antibody affinity to the multi-objective functions; the second uses adaptive crossover, mutation and clone rates for antibodies to maintain the antibodies' diversity. This improvement results in a dynamic balance between individual diversity and population convergence.

In addition to the relatively few AIS applications to OPF problems available in the literature, results in other engineering fields, *cf.* Castro and Zubben (2000);

Carpaneto et al. (2006), are promising and may encourage power engineers to further explore IAs.

2.5.2 Differential Evolution

Differential Evolution (DE) is a population-based, direct stochastic search algorithm originally proposed by Storn and Price (1995) for optimization problems over a continuous domain. It has since been extended to cope with MINLP optimization problems, *cf.* Lampinen and Zelinka (1999). DE combines simple arithmetic operators with the classical evolutionary operators of crossover, mutation and selection to evolve from a randomly generated starting population to a final solution. DE uses a greedy, rather than stochastic, approach to solve the problem. The differential mutation mechanism is the key element distinguishing DE from the other population-based techniques. To generate trial parameter vectors, DE adds the weighted difference between two population vectors to a third vector. No separate probability distribution is required, which makes DE completely self-organizing.

The one-to-one competition of offspring makes DE significantly faster in convergence than other EAs. Unfortunately, this faster convergence results in a higher probability of converging on a local, rather than global, optimum, *cf.* Coelho and Mariani (2007). In order to overcome this drawback and to avoid employing a large population, Chiou and Wang (1998) added two phases to DE: the accelerated phase and the migrating phase. Price et al. (2005) and Onwubolu and Davendra (2009) are good sources for more detailed discussion of DE methods and the edited volume by Chakraborty (2008) documents recent developments in DE.

DE has been applied to several engineering problems in different areas including OPF, *cf.* Liang et al. (2007b). One area of focus has been the application of DE to OPF problems with complex cost curves or unconventional generator characteristics. Coelho and Mariani (2007) implemented DE algorithms for solving economic dispatch problems with transmission line constraints and losses. The algorithms account for nonlinear generator features, such as ramp rate limits and prohibited operating zones. Vaisakh and Srinivas (2008) applied DE to OPF with both conventional and unconventional cost characteristics. Sayah and Zehar (2008) developed DE algorithms for solving OPF with non-smooth and non-convex generator fuel cost curves. The authors suggested effective modifications in the mutation rule, enhancing the convergence rate while improving the solution quality. The authors also showed the empirically that their modified DE algorithm outperforms the classical DE algorithms in global convergence speed and obtains similar results compared to EP and TS methods.

DE is also suited to OPF problems that include transient stability constraints or complex controls. Bakare et al. (2007) applied DE to the Nigerian power grid in order to optimize voltage profiles and system losses via control of system reactive power. The approach achieved a significant reduction of real power losses while simultaneously keeping the voltage profiles within the acceptable limits. Basu (2008) used a DE algorithm to minimize the generator fuel cost in optimal power flow control with flexible AC transmission systems (FACTS) devices, including thyristor-controlled series capacitors and phase shifters. Test results showed that the proposed DE approach can obtain better solutions requiring less CPU time than EP and GA approaches. Cai et al. (2008) developed a robust and efficient method for solving transient stability-constrained OPF problems based on DE.

Their method allows the incorporation of detailed dynamic models of the system by combining time-domain simulation and transient energy functions. The authors demonstrated the ability to use the algorithm for large-scale systems via a parallelized implementation.

Several authors have altered the DE algorithm to improve performance for OPF problems. Liang et al. (2007a) developed an enhanced DE for ORPF. In contrast to conventional DE, several sub-populations are maintained, updated and unified using a cooperative co-evolutionary architecture. The major challenge for this approach is selecting an effective problem decomposition; the authors suggest a voltage-VAR sensitivity-based power system decomposition. This approach avoids the use of a large population size to overcome premature convergence of conventional DE methods. Computational tests show that their enhanced DE approach yields better solutions than conventional DE methods while requiring similar time. Changa et al. (2007) also applied a modified DE towards optimal reactive power planning. In order to enhance the convergence speed of their algorithm, the authors implemented two main changes compared to conventional DE: a multi-direction search scheme and a carefully balanced search space reduction scheme. Their algorithm consistently found better solutions when compared to SA, GA, and conventional DE.

Abou El Ela et al. (2009, 2010) applied a DE-based approach for an OPF problem with soft constraints. The authors considered five different objective functions: minimization of fuel cost, improvement of the voltage profile, enhancement of system voltage stability (also during contingency condition), and a non-smooth piecewise quadratic cost function. (As each objective function is considered on its own, this is not a multi-objective approach.) The authors discussed the robustness for all five objective functions of their DE for small test instances. However, the different contributions of the two papers published in 2009 and 2010 is not clear.

Finally, DE has also been used in multi-objective OPF. Varadarajan and Swarup (2008) presented a DE approach to compute pareto-optimal solutions for OPF problem with multiple objectives. For the active power dispatch problem, total emissions and generation costs were the competing objectives considered (both quadratic functions), while power losses and voltage deviation were the objective functions considered for the reactive power dispatch problem. The authors discussed an empirical method to obtain a good population size.

2.5.3 Evolutionary Programming

Fogel (2006) invented Evolutionary Programming (EP) with the initial intent of using simulated evolution as a learning process to generate artificial intelligence. EP has developed into a computational optimization method which can avoid being trapped in local optima via the use of the mutation operator and selection scheme, *cf.* Ongsakul and Jirapong (2005).

Evolutionary computation techniques have found many applications in power systems, *cf.* Wu and Ma (1995); Abido (2004). Wong and Yuryevich (1999) applied EP to an OPF problem containing highly nonlinear generator input/output cost curves, reporting promising results. Ongsakul and Jirapong (2005) developed a multi-objective, EP-based approach for the optimal allocation of FACTS devices to maximize the total transfer capability of power transactions between source and sink areas in deregulated power systems. The authors used penalty functions to enforce constraints, including real and reactive power generation limits, voltage limits, line flow limits, and FACTS devices operation limits. Test results indicated that OPF with optimally placed FACTS devices by EP could enhance the total transfer capability value far more than OPF without FACTS devices. Aminudin et al. (2007) successfully applied EP to improve the load margin of a power system, considering operational cost and loss reductions.

Although EP applications in OPF have been well investigated in academic studies, few have been used in industrial power systems. This is because the number of computations required to solve practical OPF problems with EP is very large, limiting its usefulness over more efficient algorithms. Some work has been done to improve EPs for OPF applications. Tangpatiphan and Yokoyama (2009) developed an improved EP algorithm for OPF considering steady-state voltage stability. The proposed algorithm incorporates crossover techniques from real-coded GAs (that is, GAs having real-valued parameters, allowing for optimization in real-valued search spaces) to enhance the offspring generation process. Test results showed that the improved EP algorithm is capable of working with both convex and non-convex objective functions. Zhihuan et al. (2010) proposed three improved strength pareto EP algorithms for an optimal ORPF, incorporating problem-specific local search strategies to improve convergence characteristics.

2.5.4 Genetic Algorithm

Genetic Algorithms (GAs) are evolutionary search algorithms based on the mechanics of natural genetics, *cf.* Tang and Kwong (1999); Haupt (2004). In order to simultaneously explore the search space and increase the performance of generated solutions, GAs combine elements of directed and stochastic search with the exploitation of historical information from previous solution guesses, *cf.* Kamal et al. (2004). GAs generally avoid termination at local optima as the population of solutions is distributed throughout the search space and new solutions are produced from random processes. Eventual convergence to a global optimum can be proven, for instance, if the best solution is always maintained in the solution pool, *cf.* Rudolph (1994).

A GA starts with the generation of initial individuals representing the candidate solutions. For each individual, or offspring, the fitness (objective) function of the problem is evaluated. The individuals with the best fitness function values are selected for the next iteration. The offspring then undergo crossover and mutation operations to create a new solution population. The crossover is a transfer of information between individuals in the population and new offspring. This is the primary difference between GAs and EP: while GA stresses crossover functions, EP emphasizes mutation. GAs have received significant attention in the OPF field; Karthikeyan et al. (2009) survey some major contributions in this area.

Applications of GA to OPF first appeared in the late 1990's. Song et al. (1997) applied GAs to a combined environmental/economic dispatch problem, employing fuzzy logic to adjust crossover and mutation probabilities. Lai et al. (1997) presented GAs for OPF under both normal and contingency conditions; Zhang et al. (1998) applied an improved GA to ORPF. Numnonda and Annakkage (1999) introduced advanced crossover and mutation operators in a GA to solve an economic dispatch problem, although transmission cost are ignored. Bakirtzis et al. (2002)

extended the work of Lai et al. and Numnonda and Annakkage by including switchable shunt devices and transformer tap settings as discrete control variables. The enhanced GA used by Bakirtzis et al. also incorporated problem-specific operators to solve larger OPF problems. Their use of penalties in the fitness function lead to a discussion by Sood et al. (2003) on the dependence of the convergence of the enhanced GA on these penalties.

Bouktir et al. (2004) used a GA to minimize system fuel costs while maintaining secure limits on power outputs of generators, bus voltages, shunt capacitors/reactors and transformer tap settings. The authors decomposed the problem into active and reactive constraints, where the active constraints are those that directly affect the objective function. The GA handles the active constraints while the remaining constraints are maintained by conventional power flow.

Todorovski and Rajicic (2006) proposed an initialization procedure for voltage angles at generator buses that yields a good starting point for GA-based OPF. The goal of the procedure is to obtain a feasible or near-feasible starting point at the outset, avoiding the need to deal with constraint violations in the solution population. Test results confirmed performance improvements of the GA OPF procedure in computational time by benchmarking it to various other GAs for OPF present in the literature. Todorovski and Rajicic further show computational robustness of their approach by considering three different generator cost curves (quadratic, piece-wise quadratic, and quadratic with a sinusoidal component).

Recently, Mahdad et al. have contributed a significant amount of literature regarding the use of GAs in OPF problems, *cf.* Mahdad et al. (2008a,b, 2009b,a, 2010). The authors have focused on the development of efficient parallel GAs using decomposition techniques. The active and reactive power subproblems are solved separately by parallel flexible GAs. The goal of this approach is to improve the overall execution time of the algorithm. The authors include variables for modern power system controls, including FACTS devices and static VAR compensators. Mahdad et al. (2009a) and Mahdad et al. (2010) extensively benchmark their proposed algorithm against other GAs—including a fuzzy-GA—EP, DE, ACO, and the simulation packages MATPOWER and PSAT. The authors report that their proposed algorithm can obtain competitive solutions in reasonable computational time.

Other researchers have also examined the use of GAs for optimal placement and operation of FACTS devices in power systems. Both Leung and Chung (2000) and Banu and Devaraj (2008) discuss the application of GAs to optimally place FACTS devices in relation to security or contingency requirements. Lai and Sinha discuss the application of GAs to FACTS devices in their book chapter as well, *cf.* Lai and Sinha (2008).

Several recent papers have applied real-coded GA to OPF. Gaing and Chang (2006) presented a real-coded mixed-integer GA for solving non-convex OPF problems with transmission security and bus voltage constraints. Each individual in the proposed GA is represented as a mixture of continuous and discrete control variables. Later, Kumar and Renuga (2009) compared EP and real-coded GA as applied to reactive power planning problems. The authors concluded that in the case of optimization of a non-continuous and non-smooth function, real-coded GA outperforms EP and "always leads to the global optimum points of the multi-objective reactive power planning problem." However, no theoretical proof is provided to confirm this claim. Subbaraj and Rajnarayanan (2009) used self-adaptive real-coded GA to solve an optimal reactive power dispatch problem. The self-adaptation is introduced by applying a simulated binary crossover operator. The authors reported that their approach can handle all types of decision variables and produces near optimal solutions in less computation time than EP methods.

Kumari and Maheswarapu (2010) solved a multi-objective OPF by an enhanced GA. Their GA employs elitism via an external population. To enhance performance, a quadratic power flow routine, rather than the GA itself, solves the polar form of the power flow equations. Combinations of the three objectives—generation costs, system transmission losses, and a system voltage stability index—are considered simultaneously to obtain a multi-objective optimization problem. The authors benchmarked their algorithm against a PSO approach in which the objective function values are fuzzified in order to compute the pareto set. Results showed that the GA method computes superior solutions than the fuzzy PSO approach when multiple objectives are present.

Kumar and Mohan (2010) used GAs to solve the unit commitment problem with line flow constraints. The authors compared their results with those obtained using lambda iteration techniques for economic dispatch and concluded that their GA reduces the power losses.

2.6 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a population-based stochastic optimization technique introduced by Kennedy and Eberhart (1995). PSO is based on processes arising naturally in socially organized colonies such as flocks of birds and schools of fish. PSO exploits a population of individuals to explore promising regions within the search space. In the search procedure, each individual (particle) moves within the decision space over time and changes its position in accordance with its own best experience and the current best particle, *cf.* Poli et al. (2008). PSO is capable of evolving towards a global optimum with a random velocity by its memory mechanism, *cf.* Hajian-Hoseinabadi et al. (2008). Though similar to EAs, categorically PSO is a swarm algorithm (like ACO).

Compared with other stochastic optimization methods, PSO has comparable or superior convergence rates and stability for several difficult optimization problems, *cf.* Mo et al. (2007). However, as with many heuristic approaches, a primary drawback of traditional PSO is premature convergence when the parameters are not chosen correctly, especially while handling problems with many local optima, *cf.* Gaing and Liu (2007).

PSO has been widely applied to electric power system problems in general, *cf.* Yang et al. (2007a), and OPF problems specifically, *cf.* Yumbla et al. (2008). There is a significant amount of variety among the algorithms used. Zhao et al. (2004) presented a PSO method using a non-stationary, multi-stage assignment penalty function to convert a constrained optimization problem into an unconstrained one. However, the convergence of the algorithm is quite sensitive to the choice of penalty coefficients. Wang et al. (2005) demonstrated the feasibility of a modified PSO algorithm where each particle obtains information not only from itself and the best in the group but in addition also from other group members. The authors claim that this technique speeds up the convergence towards a global optimum.

Gaing (2005) presented an efficient mixed-integer PSO algorithm for constrained OPF.

Using PSO, Swapur (2006) solved an OPF problem with continuous and discrete variables considering both normal and contingency states. Swapur demonstrated the algorithm's speed (in terms of the number of power flow computations executed) as well as the superiority of the results in comparison to GAs. Kim et al. (2007) successfully implemented a parallel PSO algorithm; the parallel approach reduced computation time compared to sequential PSO algorithms. Hajian-Hoseinabadi et al. (2008) presented a PSO algorithm to minimize the total fuel cost while considering active and reactive power limits of generators, voltage profile of load buses, and transmission line flow limits. The key idea of their modified PSO method is to exploit the information contained in the worst experiences (of each individual and within the group). This is in contrast to conventional PSO algorithms, where the best experiences are used. Computation tests on small problem instances showed that their modified PSO outperforms the conventional PSO in terms of convergence to better solutions.

PSO has been applied to a number of OPF problem types, including ORPF, security problems, and multi-objective problems. Zhang and Liu (2004) reported the successful application of PSO to ORPF. Coath et al. (2004) applied PSO to solve a reactive power and voltage control problem incorporating wind farms. Zhao et al. (2005) applied an PSO method towards ORPF. In the authors use an algorithm similar to that of Wang et al. in that the social influence term in the velocity update formula for each particle includes the information of several particles, compared to just the best particle in the conventional PSO. The authors establish an adaptive parameter updating rule which allows a proof of global convergence of their improved PSO. Comparisons with GAs, EP and conventional PSO showed that their improved PSO requires less computation time to achieve better solutions.

Vlachogiannis and Lee (2006) proposed three PSO algorithms for reactive power and voltage control: enhanced general passive congregation, local passive congregation, and coordinated aggregation. The authors compared the proposed PSO algorithms with an IPM-based OPF algorithm, a conventional PSO algorithm, and an EA, demonstrating the performance of the proposed algorithms. Li et al. (2007a) proposed an adaptive PSO algorithm to solve an ORPF, introducing the concept of species into the population diversity measure. Simulation results showed a fast global convergence rate with robust computation. Pouya and Lesani (2009) implemented an angle-based PSO method for the optimal procurement of reactive power in an open electricity market. The reactive power management problem was formulated as an NLP problem with nonlinearity in both the objective and the constraints, where the voltage stability constraint was implemented as a soft constraint to guarantee the security of the system. The authors reported that their method has higher speed and accuracy than standard PSO techniques.

In the realm of system security, Yoshida et al. (2001) applied PSO to a reactive power and voltage control problem, considering voltage security assessment. The proposed method expands the original PSO methods to handle MINLP problems and determine an online VAR Control strategy with continuous and discrete control variables. This method was compared with reactive TS and enumeration methods on practical power systems with promising results. Gaing and Liu (2007) presented a multi-objective constriction PSO method with mutation mechanisms for solving a security-constrained OPF problem. The method incorporates both steady-state security and transient stability constraints. The four objectives were minimization of total generation cost, enhancement transmission security, reduction of transmission losses, and improvement of the bus voltage profile under normal and post-contingency states. The proposed method incorporates a new cognitive behavior of particles which allows more effective exploration of the search space. Onate and Ramirez (2007) developed a novel PSO technique with reconstruction operators to solve a security constrained OPF problem. Reconstruction operators guarantee that the search for the optimal solution occurs only in the feasible space, reducing computation time while improving the quality of the solution.

In addition to Gaing and Lui's work, Vlachogiannis and Lee (2005) and Abido (2008) both applied PSO to multi-objective OPF. Vlachogiannis and Lee successfully implemented parallel vector evaluated PSO, minimizing both the real power losses in the transmission lines and the voltage magnitudes at the load busses. Abido's formulation simultaneously optimized competing fuel cost and voltage stability objectives. Abido used a clustering algorithm to manage the size of the pareto optimal set.

2.7 Simulated Annealing

Simulated Annealing (SA) is a generic, probabilistic meta-heuristic for global optimization that was proposed by Kirpatrick et al. (1983). In each step of the SA algorithm, the current solution is replaced by a random nearby solution, chosen with a probability that depends on the difference between the corresponding function values and on a global temperature parameter that is gradually decreased as the process continues, *cf.* Ingber (1993). The dependency is such that the current solution changes almost randomly when the temperature is large but moves increasingly "downhill" (toward an improved objective function value) as the temperature goes to zero. The allowance for "uphill" moves saves the method from becoming stuck at local minima, a common problem in greedier methods. SA is guaranteed to converge asymptotically to a global optimal solution, *cf.* Aarts and Korst (1989). In addition, SA is relatively easy to implement and therefore suitable for a wide range of problems. For recent advances in SA, refer to the edited book by Chibante (2010).

Several authors have applied SA to OPF. Hsiao et al. (1993) used SA to solve a contingency-based optimal VAR sources planning problem while considering real and reactive power balance equations. In testing, the authors claim that the proposed algorithm is suitable for large-scale power systems. Wong and Fung (1993) developed a general SA-based economic dispatch algorithm which incorporates transmission losses through the use of a quadratic loss formula. Although the test results demonstrate that the algorithm is able to find a global or near global optimal solution, its computation time is high. Roa-Sepulveda and Pavez-Lazo (2001) also used SA techniques to solve an economic dispatch problem where the transmission constraints are modeled in polar form. However, the authors also reported long computation times. Pure SA approaches for OPF have been replaced by hybrid methods within the last decade; see Section 3.

2.8 Tabu Search

Tabu Search is an iterative improvement procedure introduced by Glover, *cf.* Glover (1989, 1990b,a). The search process is partly based on a hill-climbing method that discovers a solution by defining a neighborhood and then moving to the solution with the minimum cost function within the neighborhood. TS employs a tabu list that plays an important role as a memory function, storing a number of visited states along with a number of states that might be considered unwanted. The tabu list controls search directions so that the solution escapes from local minima and prevents cycling by using flexible memory structures. Faigle and Kern (1992) proved the global convergence of TS by exploiting similarities to SA.

By now, TS is an established optimization approach which has been applied to various power system optimization problems with impressive success, *cf.* Mori and Hayashi (1998). Together with other heuristic search algorithms, such as GA, TS was singled out as "extremely promising" for the future treatment of practical applications in the early 1990's, *cf.* Glover (1989); Bland and Dawson (1991). Twenty years later, this research is still ongoing, though other meta-heuristics have gained more attention recently.

Mori and Hayashi (1998) proposed a parallel TS-based method for voltage and reactive power control. The parallel scheme improves the solution quality by computing the neighborhood in a parallel way (by using two cores) with two different tabu lengths (one core per tabu length). The parallel TS method is efficient in comparison to conventional TS, SA and GA. Abido (2002) presented an efficient and reliable TS-based approach to set the optimal control variables of the general OPF problem, examining various objective functions and constraints. The proposed approach outperformed both EP and deterministic NLP techniques. Kulworawanichpong and Sujitjorn (2002) developed an efficient TS algorithm for solving the OPF problem accounting for both real and reactive power. To assess the usefulness and advantages of the technique, the authors compared the TS results with those obtained from SQP and EP optimization methods and concluded that TS outperforms the other techniques in terms of computation time.

Nualhong et al. (2004) applied a reactive TS algorithm towards the OPF problem. The power flow constraints are given in polar form. Emissions in terms of total pollutant tons are modeled as a function with quadratic and exponential terms. The total emissions are combined via a weighting together with fuel cost into one objective function to be minimized. Their reactive TS improves upon the search process of standard TS methods by implementing an adaptive modification of the tabu length. The authors reported that reactive TS can yield better solutions while significantly reducing the computational time compared with standard TS methods.

Altun and Yalcinoz (2008) studied the economic dispatch problem with a quadratic power transmission loss function in the power generation variables. Four so-called soft computing methods were discussed and benchmarked. Among the tested methods are TS and GA which computed good solutions for the small test instances. However, the authors did not incorporate the full set of OPF constraints. Muthuselvan and Somasundaram (2009) applied TS to an SCED problem. The formulation incorporated both base and contingency case power flow constraints. The authors reported that the algorithm was sufficiently efficient and reliable for application to utility-scale systems.

2.9 Summary of Non-Deterministic Optimization Methods

The non-deterministic, stochastic search methods discussed are all meta-heuristic approaches. Though each methodology has its own philosophy, the fundamental idea unifying all the discussed meta-heuristics is the systematic exploration of the search space using a heuristic improvement scheme.

Meta-heuristics are typically very versatile with respect to problem format. They can handle all the types of non-convexities present in the OPF problems and complicating constraints due to various problems in which OPFs are embedded. OPF problems may have many local optima, and most meta-heuristics are able to escape local optima and converge, at least in theory, to a global optimum. This is typically achieved by managing a solution pool and (indirectly) keeping track of past algorithm performance.

However, all the meta-heuristics discussed tend to be computationally intensive. As a result, the scalability of non-deterministic OPF methods often lags that of well-developed deterministic OPF methods, even for MINLP formulations, *cf.* Zhang and Tolbert (2005); Qiu et al. (2009); Xia and Elaiw (2010). For instance, Biskas et al. (2006) showed that dynamic PSO and enhanced GA were both slower and achieved inferior solutions to the use of relaxation methods to solve MINLP OPF formulations using commercially available NLP solvers. Furthermore, meta-heuristics possess several parameters which must be tuned to ensure good performance. In many cases, penalty functions for the constraints must also be selected and tuned. Some methods are more sensitive to the parameter and penalty choices than others, affecting their computation time as well as theoretical convergence properties. This makes comparisons between methods difficult, as poor parameter selection may make a certain method appear artificially slow in comparison to its peers.

The non-deterministic methods discussed in this section are summarized in Table 2.9. The second column reports the standard parameters for each method—fewer parameters is typically preferable. The actual number of parameters for a particular algorithm may change in variations of the method: some parameters may be eliminated, or additional parameters may be added. The third column indicates whether or not the meta-heuristic can theoretically compute a global optimal solution given appropriate parameter choices and algorithmic tuning. More precisely, a "Yes" in the third column means that the probability of finding a globally optimal solution approaches one when the algorithm is allowed to run infinitely long—this is also known as "convergence in value". If no theoretical results on the convergence are available, then we mean that there are no results for the methodology or slight variants of it; convergence can always be achieved by hybridization, *i.e.*, combination of techniques which have desirable convergence properties. The fourth table column provides a single reference which we consider as a good starting point for novices to this methodology. Finally, the last column provides a few remarks and OPF-specific suggestions.

Table 1 Comparison of non-deterministic techniques

Method	Standard Parameters	Opt.[a]	Ref.	Comments
ACO	pheromone evaporation and weighting	yes	Dorigo and Stützle (2004)	solid theoretical understanding; designed for discrete variables only—extensions exist to cope with continuous variables; combination with NLP methods for OPF suggested
AIS	clonal rate, mutation rate, matching number	–	de Castro and Timmis (2002)	EA; learning algorithm; promising for security applications in OPF and in hybrid methods for OPF
ANN	energy function weights	–	Dreyfus (2005)	learning-based data processing algorithm mainly used for classification and pattern recognition; training sets for optimization problems not required; can handle non-linearities and integrality; suitable for on-line OPF problems as high-speed implementations are possible
BFA	various parameters related to chemotaxis, swarming, reproduction, elimination, and dispersal	–	Passino (2005)	theory not well understood, yet; no analytic gradient information required as approximations are used (can be seen as a disadvantage for OPF problems as gradients are available); suitable for noisy objective functions within OPF
COA	chaos mapping, chaos disturbance	yes	Li and Jiang (1998)	successful for OPF particularly in hybrid methods to enhance convergence (compared to the meta-heuristic method(s) alone)
DE	differentiation constant, crossover constant, population size	yes	Feoktistov (2006)	newest among the EAs discussed; empirical studies show good convergence properties for OPF problems
EP	population size, number of generations	yes	Fogel (2006)	EA; conventionally very simple; well suited when uncertainty is present in OPF and in hybrid methods for OPF
GA	crossover probability, mutation probability, population size	yes	Reeves and Rowe (2003)	EA; reasonably well understood theory among the EA algorithms; suitable for large-scale OPF due to parallelism of GAs
PSO	swarm size, cognitive parameter, social parameter	yes	Clerc (2006)	theory reasonably well understood; reasonably robust to parameters for OPF problems; modified PSO algorithms demonstrate empirically good convergence properties for OPF problems
SA	initial temperature, annealing rate	yes	Laarhoven and Aarts (1987)	only suggested as a hybrid method for OPF due to slow convergence speed
TS	tabu list length	yes	Glover and Laguna (1997)	conceptually very simple; a good starting point for OPF algorithms

[a] yes Shown to converge to global optimal solution given sufficient time
– No theoretical results exist, to the best knowledge of the authors

In the literature, almost every meta-heuristic method is claimed as being robust with respect to parameter choices, being easy to implement, and having "good" convergence properties. However, in practice, the required parameter tuning takes considerable effort when solving real-world OPF problems. Furthermore, there is only very little known on the convergence rate of the meta-heuristics discussed. Moreover, authors often state that in comparative studies, their particular version of any given meta-heuristic performs better than other methods. This yields a situation in which every method is (situationally) regarded as the best. Therefore, the reader should exercise caution when evaluating the appropriateness of any given meta-heuristic to a given OPF formulation.

3 HYBRID METHODS

Hybrid methods combine several different optimization techniques into one algorithm. If done right, then the advantages of each method can be used to overcome the disadvantages of the others, leading to a very powerful algorithm. Typically, hybrid methods can achieve significant improvements (*e.g.* in computation time, convergence properties, solution quality, or parameter robustness) over each of the individual methods. Hybrid methods have gained popularity in the last decade for various OPF applications.

For a discussion of the individual deterministic methods referenced in this section, including definitions for the relevant acronyms, we refer the interested reader to part I of this survey, *cf.* Frank et al. (2012). Additionally, the acronyms used throughout this section are expanded in Appendix A.

3.1 Deterministic Methods Combined

SQP combined with Quasi-Newton: Lin et al. (2004) developed a hybrid method to solve an OPF with discrete control variables. The algorithm is based on ordinal optimization theory, which finds "good enough" solutions with "high probability." The basic idea of their algorithm is as follows: First, linearize all discrete variables in the OPF problem to obtain a continuous NLP which is then solved with SQP methods. Second, round any continuous variables which must be discrete to a set of allowable values. Ordinal theory helps to avoid an exponential number of choices for this rounding procedure. Third, fix the discrete variables and rank the solutions by solving approximated quadratic optimization problem via a Newton-type method. Fourth, for the best solutions obtained in step three, fix the discrete variables and solve the resulting continuous OPF problem to select the best solution. Extensions to an online version of the algorithm are also discussed. The authors benchmarked their method with a conventional approach (use the first step of the hybrid method and round the appropriate variables to their closest discrete values) and TS. Their hybrid method computes better solutions than the conventional method by consuming marginally more time and finds solutions comparable to the TS method much faster.

IPMs combined with Benders Decomposition: Borges and Alves (2007) solved a nonlinear security constrained OPF problem by using distributed processing for real-time operation. The authors considered active and reactive controls, initially based on a message passing interface which was then integrated into an energy management system. The idea of their hybrid algorithm as follows: Benders' cuts are iteratively generated to linearly approximate the N contingency configuration constraints while the nonlinear Benders' Master Problem (representing an OPF problem with additional linear constraints) is solved using IPMs. The authors give a parallel version of their algorithm as well.

IPMs combined with Lagrangian Relaxation and Newton's method: Lage et al. (2009) proposed a penalty/modified barrier method for ORPF to minimize the active power losses in transmission lines. First, the OPF problem is transformed into an equality constrained optimization problem by introducing slack variables. In order to ensure the non-negativity of the slack variables, smooth penalty functions are introduced using Lagrangian multipliers in combination with a barrier parameter. Newton's method is then used to solve the first order necessary conditions for the objective function (including the penalty function) to iteratively update the Lagrange multipliers. One important feature of this hybrid algorithm is that the optimal trajectory is allowed to pass through both the feasible and infeasible regions.

PC-IPMs combined with Newton's Method and Line Search: Han et al. (2009) introduced a so-called sequential feasible optimal method to solve OPF problems. In each iteration, their algorithm uses a two stage approach. In the first stage, a new point with improving objective function value (compared to the previous iteration's solution) is computed. In the second stage, the new point is slight changed to enforce feasibility. The first stage is based on the computation of a direction of decent for the objective function using PC-IPMs. Feasibility in stage two is ensured by solving various auxiliary problems using Newton's method and line search techniques. The hybrid algorithm converges to a KKT point of the OPF problem and maintains feasibility in each iteration.

3.2 Deterministic and Non-deterministic Methods Combined

Deterministic approaches tend to be computationally much quicker than non-deterministic approaches. However, deterministic methods are typically limited to providing locally optimal solutions—at best—and the quality of the solution obtained is sensitive to the starting point. To overcome the drawback of getting trapped on local optima, local search techniques can be combined with global search procedures provided by non-deterministic methods. Very often, the idea is that meta-heuristics deal with the discrete decision variables of the problem and local search techniques are employed to handle the remaining continuous NLP portion.

IPMs combined with meta-heuristics: Shengsong et al. (2002) proposed a hybrid optimization method consisting of two stages: a global search by COA and a local search by PC-PDIPM. Computational tests show that the local search speeds convergence when the COA is close to global optimal solutions. Furthermore, the hybrid method was able to compute the same optimum in all 300 tests performed. Therefore, robustness of global convergence is improved, compared to each single methodology alone. One year later, the same authors implemented a hybrid two-stage optimization algorithm for solving OPF problems with multimodal characteristics, *cf.* Shengsong et al. (2003). This time, COA was combined with

SLP. The resulting linearized subproblems in SLP are solved using PC-PDIPM. Again, the authors reported an improvement in the global convergence robustness of their hybrid method compared to COA or SLP alone.

Chuanwena and Bomp (2005) combined chaotic PSO with linear IPMs to solve an ORPF problem. Chaotic mapping is combined with PSO to enhance global search while the IPM is used as local solver. The proposed hybrid method was benchmarked against a conventional PSO method on two problems: shunt capacitor minimization and regular transformer optimization. Both computational tests showed that the hybrid method converges in fewer iterations and to better solutions compared to the conventional PSO alone.

Liu et al. (2006) combined immune GAs and IPMs to solve a dynamic ORPF problem. The continuous variables are handled by a nonlinear IPM while the discrete variables are solved by immune GAs. Computational times are not reported and no benchmarking against other methods was performed.

Raju et al. (2009) proposed an IPM in combination with evolutionary PSO to solve an OPF. The formulation incorporates FACTS devices, such as static synchronous series compensators. The objective function includes fuzzy logic composite criteria, combined fuzzy severity index, and the system real power loss. The case studies compared the results obtained with and without inclusion of FACTS devices.

LP and SQP combined with GA: Younes et al. (2007) developed a sequential hybrid method combining a GA with the LP and SQP OPF algorithms available in the MATPOWER software, *cf.* Zimmerman et al. (2011). The GA is used first to generate an approximate optimal solution, which is then fine-tuned using MATPOWER to obtain an exact local optimum. This sequential method combines the global search characteristics of GA with the rapid convergence to a local optimum from deterministic methods. Computational results show that high quality solutions are obtained compared to the GA or MATPOWER alone. To yield similar quality solutions, the GA alone requires more time than the hybrid algorithm.

Newton's method combined with PSO: Rashidi and El-Hawary (2007) presented a hybrid PSO method to solve an OPF with multimodal characteristics. The proposed algorithm makes use of PSO's global search capabilities to allocate the optimal control settings. The non-linear power flow equations with continuous variables are handled via a conventional Newton-Raphson power flow algorithm. The power flow ensures that the continuous variables remain in the feasible region, avoiding a penalty approach for constraint violations. Extensive tuning was done on the PSO parameters of the hybrid algorithm. The proposed algorithm was tested with three different objective functions to be minimized: system real power losses, fuel cost, and nitrogen oxides emissions of the generating units. The hybrid method computed better solutions than the MATPOWER software (SQP-based) for all tested instances (containing only continuous variables).

Newton's method combined with SA: Chen et al. (1997) considered an OPF problem with both continuous and discrete variables. Their hybrid algorithm treats the continuous variables via Newton's method while the discrete variables are handled via a SA-type algorithm. The typical temperature reduction rules of SA algorithms are replaced by so-called Mean Field Equations, enhancing the convergence of the algorithm compared to standard SA.

Direct search combined with EP: Gopalakrishnan et al. (2003) applied a hybrid EP to reactive power planning. The objective function to be minimized is the sum of the cost associated with network losses and the installation cost. The hybrid method uses EP to generate good solutions while a direct search method computes a local optimum quickly. As such, the global search ability of EP is combined with a fast local search algorithm as a fine-tuning procedure. Their hybrid method computed better solutions than EP alone in computational tests on small instances.

Direct search combined with BFA: Panigrahi and Pandi (2009) combined a BFA with the Nelder-Mead method (a direct search technique which evaluates vertices of simplices). Their hybrid method is largely based on BFA where the chemotaxis step is extended by the Nelder-Mead method to find a solution with lower fitness function value, where the fitness function consists of a quadratic generation cost function and penalty functions for constraint violations. The proposed method is used to solve a OPF problem whose solution is then used in a second optimization problem to reduce transmission line congestions. Computational benchmarking revealed that the hybrid method converges faster to a better solution compared to conventional BFA, GA, and PSO.

SLP combined with local heuristic search: Aoki et al. (1988) used SLP together with a heuristic mixed-integer programming optimization method to calculate the optimal placement of new capacitor banks while accounting for the discrete nature of capacitor installations. Their heuristic is based on a local search method of an optimal solution of the LP-relaxation together with an improvement procedure. The LP-relaxation is obtained by first linearizing the load flow equations at the current best solution (the main idea of SLP) and then relaxing the integrality requirements on the binary decision variables, which model the capacitor unit installation decisions.

Tangent vector technique combined with PSO: Esmin et al. (2005) and Esmin and Lambert-Torres (2006) presented a two-stage PSO approach for minimization of power loss. The first stage identifies a set of buses where the voltage instability is most likely to cause a voltage collapse. These buses are determined via a tangent vector which contains the information on the changes of the system variables with respect to changes in the parameters. The PSO technique is then used to optimize the shunt reactive power compensation needed for each bus.

3.3 Non-deterministic Methods Combined

The predominant idea of combining several meta-heuristics into one hybrid method is to overcome slow convergence and/or to improve the global convergence properties of the individual meta-heuristics. As such, the hybrid methods tend to be much more tailored to OPF problems than the conventional meta-heuristic methods.

GA combined with other meta-heuristics: Liu et al. (2000) combined GA, SA and TS techniques to solve ORPF problems. The authors presented three variations of their algorithm, but the main scheme is to use a GA-SA algorithm to compute starting solutions for TS, which then performs a global search. SA is combined to help GA escape local optima, while TS is used to overcome potential local convergence of SA for a low temperature parameter. Computational results show fast convergence of all three hybrid methods to high quality solutions.

Das and Patvardhan (2003) also applied a hybrid GA-SA method to OPF. Up to four objective functions are considered in a multi-objective optimization fashion: minimization of generation cost, emissions, and transmission losses, and maximization of a security index. SA is used to update the probability of selection of different objective functions to be improved, based on the average difference of objective function values obtained.

Nakawiro and Erlich (2009) proposed a speedup strategy for OPF that uses a dedicated ANN to perform the function of a power flow program. The ANN is combined with a GA, which performs the optimization. Tests show that their method significantly speeds up the solution process compared to GAs alone, while providing solutions of similar quality.

PSO combined with SA: Sadati et al. (2009) proposed a PSO-SA hybrid optimization technique for solving an under-voltage load shedding problem with detailed transmission modeling. SA is used to independently generate new solutions which are then included in the swarm for PSO if their fitness function is good enough. Computational comparisons with conventional PSO and SQP methods demonstrated the fast and consistent convergence of the hybrid algorithm.

DE combined with other meta-heuristics: Abbasy et al. (2007) solved an optimal reactive power dispatch problem in power markets by integrating multi-agent systems and DE. Computational tests showed that their hybrid algorithm converges faster to better solutions using less CPU time compared to GA, PSO and conventional DE.

Chen (2008) combined DE and SA methods to solve an ORPF problem. The greedy updating rule of DE is replaced by the probabilistic updating of SA. The idea is to exploit the global convergence property of SA with the fast convergence rate of DE methods. Test results indicate that the hybrid algorithm is superior to conventional DE methods in both speed and solution quality.

3.4 Fuzzy Logic Combined with OPF

Fuzzy logic is not an optimization algorithm but rather a mathematical approach for dealing with incomplete or imprecise information. Fuzzy set methods have been used to hybridize many existing OPF algorithms, primarily to improve algorithm performance when inputs are unknown or uncertain. Fuzzy logic emerged from fuzzy set theory, developed by Zadeh (1965, 1996). The fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function. A key element of the theory is that a single element may be a member of several sets to varying degrees.

Fuzzy logic combined with LP: Miranda and Saraiva (1992) presented a fuzzy model for power system operation where load and generation uncertainties are modeled as fuzzy numbers. System behavior under known (though uncertain) injections were dealt with by a DC fuzzy power flow model. Two years later, the authors developed an improved DC fuzzy OPF model for planning purposes, *cf.* Saraiva and Miranda (1994). In their multi-parametric programming model, information about system loads is expressed in a subjective way either by expert input or by integrating a degree of future uncertainty. Testing showed that the fuzzy set approach achieved a significant reduction in computation time compared with sampling based heuristics while maintaining the quality of the results.

Venkatesh et al. (1999) solved a nonlinear optimal reactive power planning problem by adopting a successive multi-objective fuzzy LP framework. Each of the objectives and constraints are expressed as a fuzzy set, where a satisfaction parameter is assigned.

More recently, Chayakulkheeree and Ongsakul (2007) presented a fuzzy OPF algorithm with nonlinear fuzzy network constraints and generator ramp rate limits. The authors decomposed the problem into a linearized total fuel cost fuzzy minimization subproblem and a linearized total real power loss fuzzy minimization subproblem, which were then solved by fuzzy LP.

Gomes et al. (2009) presented a hybrid DC-OPF approach based on fuzzy logic considering load and generation cost uncertainties. The developed algorithms use multi-parametric linear optimization techniques that allow a more accurate description of the possible behavior of the system under the form of membership functions.

Fuzzy OPF converted to crisp OPF: Guan et al. (1995) applied a fuzzy optimization technique to OPF by taking into account the uncertainty of the inequality constraints in a large power system. The OPF with fuzzy constraints was first formulated as a fuzzy optimization problem, then converted into a crisp optimization problem. The authors used an efficient SLP method, with modifications, to solve the crisp problem. Numerical results show that this method is promising for handling uncertain constraints in practical power systems.

Ramech and Li (1997) also proposed a fuzzy logic approach for OPF that employed a fuzzy formulation that is subsequently converted to a crisp optimization problem and solved using a standard OPF method. The authors addressed a contingency constrained OPF problem, formulated in a decomposed form that allows for post-contingency corrective rescheduling. They developed a systematic procedure for specifying the tolerance parameters that are needed to obtain fuzzy membership functions for these fuzzy goals.

Fuzzy logic combined with Benders' Decomposition: Hahn et al. (2008) also used a decomposition approach, applying Benders Decomposition to a multi-objective, contingency constrained OPF problem. The goal was to evaluate available system transfer capability. The approach included a systematic procedure to specify the tolerance parameters, thereby obtaining fuzzy membership functions for these fuzzy goals. As with the algorithm of Ramech and Li (1997), the results allow for post-contingency corrective rescheduling.

Fuzzy logic combined with meta-heuristics: Song et al. (1997) used fuzzy logic to adjust crossover and mutation probabilities for a GA. Zhang and Liu (2005) used fuzzy logic to dynamically update the parameters in a PSO algorithm. The authors demonstrated that their hybrid method is superior to conventional PSO in terms of improved real power losses, voltage control and voltage stability.

Prasanna and Somasundaram (2009) and Prasanna et al. (2009) presented two algorithms for solving a security constrained OPF problems; both algorithms incorporate fuzzy logic into the mutation process—the first algorithm for EP and the second for TS. The motivation of these two hybrid methods is to reduce computational time compared to each of the meta-heuristics alone, which was computationally verified on small problem instances.

4 CONCLUSIONS

The diversity and versatility of OPF formulations has made it impossible for any single optimization technique to solve all OPF problems efficiently enough for practical applications. Hence, algorithms tailored to each specific problem type have had to be developed, as evidenced by the myriad of methods discussed. All the presented techniques—both deterministic methods, *cf.* Frank et al. (2012), and metaheuristic methods—have significant strengths in certain areas and weaknesses in others.

Deterministic methods for OPF have proven themselves reliable for many types of OPF problems. Nevertheless, none of the deterministic methods surveyed can guarantee global optimality and most cannot easily handle discrete variables. The deterministic optimization methods also generally have trouble handling qualitative constraints, are sensitive to initial conditions for both global convergence and the quality of the obtained solution, and require continuity and differentiability of the objective function (which is not always available in practical OPF problems). Moreover, each of the deterministic methods has tradeoffs with respect to the others in terms of reliability, accuracy, and computational performance, *cf.* Frank et al. (2012).

As a counter to the shortcomings of the deterministic methods, non-deterministic methods have been extensively applied to various OPF problems. These methods have excellent global search characteristics, and some have been shown to approach global optimality given sufficient search time and proper selection of control parameters. However, the random search methods typically lack an efficient method for enforcing constraints and are very expensive computationally, yielding impractically long execution times for large problems. The computational burden associated with heuristic methods has limited their application in practical OPF software, despite their theoretical advantages.

The most promising recent developments in the OPF field have been hybrid methods. In many cases, hybrid methods have been shown to be more robust and converge more quickly to optimal solutions than their individual component methods operating alone. However, the latest developments in global optimization, *cf.* Floudas and Gounaris (2009), have not yet been fully applied to OPF problems. In the future, we believe that *polylithic* modeling and solution techniques have potential for solving practical OPF problems, *cf.* Kallrath (2009, 2011). Such techniques provide successively tighter upper and lower bounds on the global optimal solution, providing convergence to the global optimum. Although there has been great progress in the field of global optimization in recent decades, additional improvements in speed and reliability are required before such methods are able to solve all forms of practical OPFs.

A Abbreviations

The following summarizes the meanings of abbreviations and acronyms used throughout the paper:

AC	Alternating Current
ACO	Ant Colony Optimization
AIS	Artificial Immune Systems

ANN	Artificial Neural Network
BFA	Bacterial Foraging Algorithm
COA	Chaos Optimization Algorithm
DBFA	Dynamic Bacterial Foraging Algorithm
DC	Direct Current
DE	Differential Evolution
EA	Evolutionary Algorithm
EP	Evolutionary Programming
FACTS	Flexible AC Transmission Systems
GA	Genetic Algorithm
IA	Immune Algorithm
IPM	Interior Point Method
KKT	Karush-Kuhn-Tucker (conditions for optimality)
LP	Linear Programming
MINLP	Mixed Integer-Nonlinear Programming
NLP	Nonlinear Programming
NN	Neural Network
OPF	Optimal Power Flow
ORPF	Optimal Reactive Power Flow
PC	Predictor-Corrector
PDIPM	Primal-Dual Interior Point Method
PSO	Particle Swarm Optimization
SA	Simulated Annealing
SCED	Security-Constrained Economic Dispatch
SLP	Sequential Linear Programming
SQP	Sequential Quadratic Programming
TS	Tabu Search
UPFC	Unified Power Flow Controller
VAR	Volt-Ampere Reactive

References

Aarts E, Korst J (1989) Simulated Annealing and Boltzmann Machines. John Wiley & Sons Ltd

Abbasy A, Tabatabaii I, Hosseini S (2007) Optimal Reactive Power Dispatch in Electricity Markets Using A Multiagent-Based Differential Evolution Algorithm. In: Power Engineering, Energy and Electrical Drives, 2007. POWERENG 2007. International Conference on, pp 249–254

Abido M (2002) Optimal power flow using tabu search algorithm. Electric Power Components and Systems 30:469–483

Abido M (2004) Multiobjective Optimal Power Flow Using Strength Pareto Evolutionary Algorithm. In: 39th International Universities Power Engineering Conference, vol 1, pp 457–461, uPEC 2004

Abido M (2008) Multiobjective Particle Swarm Optimization for Optimal Power Flow Problem. In: Power System Conference, 2008. MEPCON 2008. 12th International Middle-East

Abou El Ela A, Abido M, Spea S (2009) Optimal power flow using differential evolution algorithm. Electrical Engineering 91:69–78

Abou El Ela A, Abido M, Spea S (2010) Optimal power flow using differential evolution algorithm. Electric Power Systems Research 80(7):878–885, DOI 10.1016/j.epsr.2009.12.018

Allaoua B, Laoufi A (2008) Collective Intelligence for Optimal Power Flow Solution Using Ant Colony Optimization. Leonardo Electronic Journal of Practices and Technologies 13:88–105

Allaoua B, Laoufi A (2009) Optimal Power Flow Solution Using Ant Manners for Electrical Network. Advances in Electrical and Computer Engineering 9:34–40

Alrashidi M, El-Hawary M (2009) Applications of Computational Intelligence Techniques for Solving the Revived Optimal Power Flow Problem. Electric Power Systems Research 79:694–702

Altun H, Yalcinoz T (2008) Implementing soft computing techniques to solve economic dispatch problem in power systems. Expert Systems with Applications 35:1668–1678
Aminudin N, Rahman T, Musirin I (2007) Optimal Power Flow for Load Margin Improvement using Evolutionary Programming. In: The 5th Student Conference on Research and Development. SCOReD 2007, Malaysia
Aoki K, Fan M, Nishikori A (1988) Optimal VAR Planning By Approximation Method For Recursive Mixed-Integer Linear Programming. IEEE Transactions on Power Apparatus and Systems 3(4):1741–1747
Bakare G, Krost G, Venayagamoorthy G, Aliyu U (2007) Differential Evolution Approach for Reactive Power Optimization of Nigerian Grid System. In: IEEE Power Engineering Society General Meeting, pp 1–6
Bakirtzis A, Biskas P, Zoumas C, Petridis V (2002) Optimal Power Flow by Enhanced Genetic Algorithm. IEEE Transactions on Power Systems 17(2):229–236
Banu R, Devaraj D (2008) Genetic Algorithm Approach for Optimal Power Flow with FACTS devices. In: 4th International IEEE Conference Intelligent Systems, vol 23, pp 11–16
Basu M (2008) Optimal Power Flow with FACTS Devices Using Differential Evolution. International Journal of Electrical Power & Energy Systems 30(2):150–156
Biskas P, Ziogos N, Tellidou A, Zoumas C, Bakirtzis A, Petridis V (2006) Comparison of two metaheuristics with mathematical programming methods for the solution of OPF. IEE Proceedings – Generation, Transmission & Distribution 153(1):16–24, DOI 10.1049/ip-gtd:20050047
Bland J, Dawson G (1991) Tabu Search and Design Optimization. Computer-Aided Design 23(3):195–201
Borges C, Alves J (2007) Power System Real Time Operation based on Security Constrained Optimal Power Flow and Distributed Processing. In: IEEE Power Tech, Lausanne, pp 960–965
Bouktir T, Slimani L, Belkacemi M (2004) A Genetic Algorithm for Solving the Optimal Power Flow Problem. Leonardo Journal of Sciences 4:44–58
Cai H, Chung C, Wong K (2008) Application of Differential Evolution Algorithm for Transient Stability Constrained Optimal Power Flow. IEEE Transactions on Power Systems 23(2):719–728
Carpaneto E, Cavallero C, Freschi F, Repetto M (2006) ICARIS 2006. LNCS, Springer, Heidelberg, chap Immune Procedure for Optimal Scheduling of Complex Energy Systems, pp 309–320
de Castro L, Timmis J (2002) Artificial Immune Systems: A New Computational Intelligence Approach. Springer
Castro L, Zubben F (2000) Artificial Immune Systems: Part I - Basic Theory and Applications. Tech. rep., FEEC/UNICAMP, Campinas, Brazil
Chakraborty U (ed) (2008) Advances in Differential Evolution. Springer
Changa CF, Wong JJ, Chiou JP, Sua CT (2007) Robust Searching Hybrid Differential Evolution Method for Optimal Reactive Power Planning in Large-Scale Distribution Systems. Electric Power Systems Research 77:430–437
Chayakulkheeree K, Ongsakul W (2007) Optimal Power Flow Considering Non-Linear Fuzzy Network and Generator Ramprate Constrained. The International Energy Journal 8(2):131–138
Chen G (2008) Differential Evolution based Optimal Reactive Power Flow with Simulated Annealing Updating Method. In: International Symposium on Computational Intelligence and Design
Chen L, Suzuki H, Katou K (1997) Mean Field Theory for Optimal Power Flow. IEEE Transactions on Power Systems 12(4):1481–1486
Chibante R (ed) (2010) Simulated Annealing Theory with Applications. Sciyo, Croatia
Chiou J, Wang F (1998) A hybrid method of differential evolution with application to optimal control problems of a bioprocess system. In: Proceeding 1998 IEEE on evolutionary computation conference, vol 1, pp 627–632
Chuanwena J, Bomp E (2005) A hybrid method of chaotic particle swarm optimization and linear interior for reactive power optimisation. Mathematics and Computers in Simulation 68:57–65
Clerc M (2006) Particle Swarm Optimization. Wiley-ISTE
Coath G, Al-Dabbagh M, Halgamuge SK (2004) Particle Swarm Optimisation for Reactive Power and Voltage Control with Grid-Integrated Wind Farms. In: IEEE Power Engineering

Society General Meeting, vol 1, pp 303–308
Coelho L, Mariani V (2007) Improved Differential Evolution Algorithms for Handling Economic Dispatch Optimization with Generator Constraints. Energy Conversion and Management 48:1631–1639
Colorni A, Dorigo M, Maniezzo V (1991) Distributed Optimization by Ant Colonies. In: Actes de la première conférence européenne sur la vie artificielle, Elsevier, pp 134–142
Das B, Verma P (2001) Artificial neural network-based optimal capacitor switching in a distribution system. Electric Power Systems Research 60:55–62
Das D, Patvardhan C (2003) Useful multi-objective hybrid evolutionary approach to optimal power flow. In: IEE Proceedings - Generation, Transmission and Distribution, vol 150, pp 275–282
Dorigo M (1992) Optimization, Learning and Natural Algorithms. PhD thesis, Politecnico di Milano, Italy
Dorigo M, Stützle T (2004) Ant Colony Optimization (Bradford Books). The MIT Press
Dorigo M, Birattari M, Blum C, Clerc M, Sttzle T, Winfield A (eds) (2008) Ant Colony Optimization and Swarm Intelligence: 6th International Conference, ANTS 2008, Brussels, Belgium, September 22-24, 2008, Proceedings, Springer
Dreyfus G (2005) Neural networks: methodology and applications. Springer
Esmin A, Lambert-Torres G (2006) Loss Power Minimization Using Particle Swarm Optimization. In: International Joint Conference on Neural Networks. IJCNN '06., pp 1988–1992
Esmin A, Lambert-Torres G, de Souza AZ (2005) A Hybrid Particle Swarm Optimization Applied to Loss Power Minimization. IEEE Transactions on Power Systems 20(2):859–866
Faigle U, Kern W (1992) Some Convergence Results for Probabilistic Tabu Search. ORSA Journal on Computing 4(1):32–37
Feoktistov V (2006) Differential Evolution: In Search of Solutions (Springer Optimization and Its Applications), vol 5, 1st edn. Springer
Floudas C, Gounaris C (2009) A review of recent advances in global optimization. Journal of Global Optimization 45:3–38
Fogel D (ed) (2006) Evolutionary computation: toward a new philosophy of machine intelligence, third edition edn. John Wiley & Sons, Inc.
Frank S, Steponavice I, Rebennack S (2012) Optimal Power Flow: A Bibliographic Survey I, Formulations and Deterministic Methods. Energy Systems tba:tba
Gaing ZL (2005) Constrained optimal power flow by mixed-integer particle swarm optimization. In: Proceedings of 2005 IEEE Power Engineering Society General Meeting, vol 1, pp 243–250
Gaing ZL, Chang RF (2006) Security-Constrained Optimal Power Flow by Mixed-Integer Genetic Algorithm with Arithmetic Operators. In: IEEE Power Engineering Society General Meeting
Gaing ZL, Liu XH (2007) New Constriction Particle Swarm Optimization for Security-Constrained Optimal Power Flow Solution. In: Intelligent Systems Applications to Power Systems, 2007. ISAP 2007. International Conference on, pp 1–6
Gasbaoui B, Allaoua B (2009) Ant Colony Optimization Applied on Combinatorial Problem for Optimal Power Flow Solution. Leonardo Journal of Sciences 14:1–17
Glover F (1989) Tabu Search-Part I. ORSA Journal on Computing 1:190–206
Glover F (1990a) Tabu Search: A Tutorial. Interfaces 20(4):74–94
Glover F (1990b) Tabu Search-Part II. ORSA Journal on Computing 1(2):4–32
Glover F, Laguna M (1997) Tabu Search. Kluwer
Gomes B, Saraiva J, Neves L (2009) Impact of Load and Generation Price Uncertainties in Spot Prices. In: IEEE Bucharest Power Tech Conference, Bucharest, Romania
Gopalakrishnan V, Thirunavukkarasu P, Prasanna R (2003) Reactive Power Planning using Hybrid Evolutionary Programming Method. In: IEE Proceedings - Generation, Transmission and Distribution, vol 150, pp 275–282
Guan X, Liu W, Papalexopoulos A (1995) Application of a fuzzy set method in an optimal power flow. Electric Power Systems Research 34:11–18
Hahn T, Kim MK, Hur D, Park JK, Yoon Y (2008) Evaluation of available transfer capability using fuzzy multi-objective contingency-constrained optimal power flow. Electric Power Systems Research 78:873–882
Hajian-Hoseinabadi H, Hosseini S, Hajian M (2008) Optimal Power Flow Solution by a Modified Particle Swarm Optimization Algorithm. In: 43rd International Universities Power Engineering Conference. UPEC 2008, pp 1–4

Han F, Lu QS (2008) An improved chaos optimization algorithm and its application in the economic load dispatch problem. International Journal of Computer Mathematics 85(6):969–982

Han Z, Jiang Q, Cao Y (2009) Sequential feasible optimal power flow in power systems. Science in China Series E: Technological Sciences 52(2):429–435

Haque M, Kashtiban A (2005) Application of Neural Networks in Power Systems; A Review. World Academy of Science, Engineering and Technology 6:53–57

Hartati R, El-Hawary M (2001) Optimal Active Power Flow Solutions Using a Modified Hopfield Neural Network. In: Canadian Conference on Electrical and Computer Engineering, vol 1, pp 189–194

Haupt R (2004) Practical genetic algorithms. Wiley-IEEE

He S, Wen JY, Prempaint E, Wu Q, Fitch J, Mann S (2004) An improved particle swarm optimization for optimal power flow. In: Proceedings of 2004 International Conference on Power System Technology, vol 2, pp 1633–1637

Hsiao YT, Liu CC, Chiang H, Chen YL (1993) A New Approach for Optimal VAR Sources Planning in Large Scale Electric Power Systems. IEEE Transactions on Power Systems 8(3):988–996

Hugang X, Haozhong C, Haiyu L (2008) Optimal reactive power flow incorporating static voltage stability based on multi-objective adaptive immune algorithm. Energy Conversion and Management 49:1175–1181

Ingber AL (1993) Simulated annealing: Practice versus theory. Mathematical and Computer Modelling 18(11):29–57

Jiang C, Quan X, Zhang Y (1999) A chaotic optimization method for economical operation of hydro power plants. Journal of Huangzhong University of Science and Technology 27:39–40

Jurada J (1997) Introduction to Artificial Neural Systems. Jaico Publishing House

Kalil M, Musirin I, Othman M (2006) Ant Colony Based Optimization Technique for Voltage Stability Control. In: Proceedings of the 6th WSEAS International Conference on Power Systems

Kallrath J (2009) Combined Strategic Design and Operative Planning in the Process Industry. Computers and Chemical Engineering 33:1983–1993

Kallrath J (2011) Polylithic Modeling and Solution Approaches Using Algebraic Modeling Systems. Optimization Letters in print:tba

Kamal M, Rahman T, Musirin I (2004) Application of Improved Genetic Algorithms for Loss Minimisation in Power System. In: Proceedings National Power and Energy Conference, pp 258–262

Karthikeyan S, KPalanisamyl, Varghese L, Raglend I, Kothari D (2009) Comparison of Intelligent Techniques to Solve Economic Load Dispatch Problem with Line Flow Constraints. In: IEEE International Advance Computing Conference (IACC 2009)

Kennedy J, Eberhart R (1995) Particle Swarm Optimization. In: Proceedings of IEEE International Conference on Neural Networks IV, pp 1942–1948

Kephart J (1994) A biologically inspired immune system for computers. In: Proceedings of Artificial Life IV: The Fourth International Workshop on the Synthesis and Simulation of Living Systems, MIT Press, pp 130–139

Kim JY, Jeong HM, Lee HS, Park JH (2007) PC Cluster based Parallel PSO Algorithm for Optimal Power Flow. In: Intelligent Systems Applications to Power Systems, 2007. ISAP 2007. International Conference on, pp 1–6

Kirpatrick S, Gelatt C, Vecchi M (1983) Optimization by simulated annealing. Science 220:671–680

Kulworawanichpong T, Sujitjorn S (2002) Optimal Power Flow Using Tabu Search. IEEE Power Engineering Review 6:37–40

Kumar S, Renuga P (2009) Reactive Power Planning using Real GA Comparison with Evolutionary Programming. International Journal of Recent Trends in Engineering 1(3):124–148

Kumar V, Mohan M (2010) Solution to security constrained unit commitment problem using genetic algorithm. International Journal of Electrical Power & Energy Systems 32:117–125

Kumari M, Maheswarapu S (2010) Enhanced Genetic Algorithm based computation technique for multi-objective Optimal Power Flow solution. International Journal of Electrical Power & Energy Systems 32:736–742

Laarhoven P, Aarts E (1987) Simulated annealing: theory and applications, Mathematics and its applications, vol 37. Springer

Lage G, de Sousa V, da Costa G (2009) Power Flow Solution Using the Penalty/Modified Barrier Method. In: IEEE Bucharest Power Tech Conference, Bucharest, Romania
Lai L, Sinha N (2008) Modern heuristic optimization techniques: theory and applications to power systems, Wiley, chap Genetic algorithms for solving optimal power flow problems, pp 471–508
Lai L, Ma J, Yokoyama R, Zhao M (1997) Improved genetic algorithms for optimal power flow under both normal and contingent operation states. International Journal of Electrical Power & Energy Systems 19(5):287–292
Lampinen J, Zelinka I (1999) Mixed Integer-Discrete-Continuous Optimization by Differential Evolution. In: 5th International Mendel Conference on Soft Computing, Brno, Czech Republic, pp 77–81
Lee K, Vlachogiannis J (2005) Optimization of Power Systems based on Ant Colony System Algorithms: An Overview. In: Intelligent Systems Application to Power Systems, 2005. Proceedings of the 13th International Conference on, pp 22–35
Leung H, Chung T (2000) Optimal Power Flow with a Versatile FACTS Controller by Genetic Algorithm Approach. In: International Conference on Advances in Power System Control, Operation and Management, vol 1, pp 178–183
Li B, Jiang W (1998) Optimizing complex function by chaos search. Cybernetics and Systems 29(4):409–419
Li D, Gao L, Lu S, Ma J, Li Y (2007a) Adaptive Particle Swarm Optimization Algorithm for Power System Reactive Power Optimization. In: American Control Conference. ACC '07, pp 4733–4737
Li M, Tang W, Tang W, Wu Q, Saunders J (2007b) Applications of Evolutinary Computing, Springer Berlin Heidelberg, chap Bacterial Foraging Algorithm with Varying Population for Optimal Power Flow, pp 32–41. Lectures Notes in Computer Science
Liang C, Chung C, Wong K, Duan X (2007a) Parallel Optimal Reactive Power Flow Based on Cooperative Co-Evolutionary Differential Evolution and Power System Decomposition. IEEE Transactions on Power Systems 22(1):249–257
Liang C, Chung C, Wong K, Duan X, Tse C (2007b) Study of Differential Evolution for Optimal Reactive Power Flow. IET Generation, Transmission & Distribution 1(2):253–260
Liao GC (2006) Application of an immune algorithm to the shortterm unit commitment problem in power system operation. IEE Proceedings-Generation, Transmission and Distribution 153(3):309–320
Lin SY, Ho YH, Lin CH (2004) An Ordinal Optimization Theory-Based Algorithm for Solving the Optimal Power Flow Problem with Discrete Control Variables. IEEE Transactions on Power Systems 19(1):276–286
Liu F, Chung C, Wong K, Yan W, Xu G (2006) Hybrid Immune Genetic Method for Dynamic Reactive Power Optimization. In: International Conference on Power System Technology. PowerCon 2006, pp 1–6
Liu Y, Passino K (2002) Biomimicry of Social Foraging Bacteria for Distributed Optimization: Models, Principles, and Emergent Behaviors. Journal of Optimization Theory and Applications 115(3):603–628
Liu Y, Ma L, Zhang J (2000) GA/SA/TS Hybrid Algorithms for Reactive Power Optimization. In: Power Engineering Society Summer Meeting, 2000. IEEE, vol 1, pp 245–249
Mahdad B, Bouktir T, Srairi K (2008a) Optimal power flow of the algerian network using genetic algorithm/fuzzy rules. In: IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, pp 1–8
Mahdad B, Srairi K, Bouktir T, Benbouzid M (2008b) Optimal Power Flow for Large-Scale Power System with Shunt FACTS using Efficient Parallel GA. In: 34th Annual Conference of IEEE Industrial Electronics, 2008. IECON 2008
Mahdad B, Srairi K, Bouktir T (2009a) Dynamic Strategy Based Parallel GA Coordinated with FACTS devices to Enhance the Power System Security. In: Power & Energy Society General Meeting, 2009. PES '09. IEEE, pp 1–8
Mahdad B, Srairi K, Bouktir T (2009b) Optimal Power Flow with Environmental Constraints of the Algerian Network using Decomposed Parallel GA. In: IEEE Bucharest Power Tech Conference, Bucharest, Romania
Mahdad B, Bouktir T, Srairi K, Benbouzid ME (2010) Dynamic strategy based fast decomposed GA coordinated with FACTS devices to enhance the optimal power flow. Energy Conversion and Management 51:1370–1380

de Mello Honório L, da Silva AML, Barbosa DA (2007) A Gradient-Based Artificial Immune System Applied to Optimal Power Flow Problems. In: Artificial Immune Systems: 6th International Conference, ICARIS 2007, Santos, Brazil, August 2007 Procedings, L.N. De Castro and F.J. Von Zuben and H. Knidel

Miranda V, Saraiva J (1992) Fuzzy Modelling of Power System Optimal Loal Flow. IEEE Transactions on Power Systems 7(2):843–849

Mishra S (2007) Bacteria foraging based solution to optimize both real power loss and voltage stability limit. In: IEEE Power Engineering Society General Meeting

Mo N, Zou Z, Chan K, Pong T (2007) Transient Stability Constrained Optimal Power Flow Using Particle Swarm Optimisation. IET Generation, Transmission & Distribution 1(3):476–483

Mori H, Hayashi T (1998) New Parallel Tabu Search for Voltage and Reactive Power Control in Power Systems. In: IEEE International Symposium on Circuit and Systems – ISACAS'98, pp 431–434

Muthuselvan N, Somasundaram P (2009) Application of Tabu Search Algorithm to Security Constrained Economic Dispatch. Journal of Theoretical and Applied Information Technology 5:602–608

Nakawiro W, Erlich I (2009) A Combined GA-ANN Strategy for Solving Optimal Power Flow with Voltage Security Constraint. In: Asia-Pacific Power and Energy Engineering Conference, 2009. APPEEC

Nguyen T (1995) Neural network load-flow. In: IEE Proceedings-Generation, Transmission & Distribution, vol 142, pp 51–58

Nguyen T (1997) Neural Network Optimal-Power-Flow. In: Proceedings of the 4th International Conference on Advances in Power System Control, Operation and Management, Hong Kong, pp 266–271

Nualhong D, Chusanapiputt S, Phomvuttisarn S, Jantarang S (2004) Reactive tabu search for optimal power flow under constrained emission dispatch. In: IEEE Region 10 Conference 2004. TENCON, vol 3, pp 327–330

Numnonda T, Annakkage U (1999) Optimal power dispatch in multinode electricity market using genetic algorithm. Electric Power Systems Research 49:211–220

Onate P, Ramirez J (2007) Optimal Power Flow Solution with Security Constraints by a Modified PSO. In: IEEE Power Engineering Society General Meeting, pp 1–6

Ongsakul W, Jirapong P (2005) Optimal Allocation of FACTS Devices to Enhance Total Transfer Capability Using Evolutionary Programming. In: IEEE International Symposium

Onwubolu G, Davendra D (eds) (2009) Differential Evolution: A Handbook for Global Permutation-Based Combinatorial Optimization (Studies in Computational Intelligence). Springer

Panigrahi B, Pandi V (2009) Congestion management using adaptive bacterial foraging algorithm. Energy Conversion and Management 50:1202–1209

Passino K (2002) Biomimicry of bacterial foraging for distributed optimization and control. IEEE Control Systems Magazine 22(3):52–67

Passino K (2005) Biomimicry for Optimization, Control, and Automation. Springer Verlag

Poli R, Kennedy J, Blackwell T, Freitas A (eds) (2008) Particle Swarms: The Second Decade. Hindawi Publishing Corp US SR

Pouya K, Lesani H (2009) An Angle-Based PSO Approach For Reactive Power Management Problem. In: Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES, pp 1–6

Prasanna T, Somasundaram P (2009) OPF with FACTS devices in interconnected power systems using fuzzy stochastic algorithms. International Journal of Power and Energy Conversion 1:279–299

Prasanna T, Muthuselvan N, PSomasundaram (2009) Security Constrained OPF by Fuzzy Stochastic Algorithms in Interconnected Power Systems. Journal of Electrical Systems 5(1):P7

Price K, Storn R, Lampinen J (2005) Differential evolution: a practical approach to global optimization. Birkhäuser

Qiu Z, Deconinck G, Belmans R (2009) A literature survey of Optimal Power Flow problems in the electricity market context. In: IEEE/PES Power Systems Conference and Exposition. PSCE '09., Seattle, WA, pp 1–6

Raju C, Vaisakh K, Raju S (2009) An IPM-EPSO Based Hybrid Method for Security Enhancement Using SSSC. International Journal of Recent Trends in Engineering 2(5):208–212

Ramech V, Li X (1997) A Fuzzy Multiobjective Approach to Contingency Constrained OPF. IEEE Transactions on Power Systems 12(3):1348–1354
Rashidi MA, El-Hawary M (2007) Hybrid Particle Swarm Optimization Approach for Solving the Discrete OPF Problem Considering the Valve Loading Effects. IEEE Transactions on Power Systems 22(4):2030–2038
Reeves C, Rowe J (2003) Genetic algorithms: principles and perspectives: a guide to GA theory, 2nd edn. Springer
Ripley B (1996) Pattern Recognition and Neural Networks. Cambridge University Press
Roa-Sepulveda C, Pavez-Lazo B (2001) A Solution to the Optimal Power Flow Using Simulated Annealing. In: IEEE Porto Power Tech Conference, Porto, Portugal
Rudolph G (1994) Convergence Analysis of Canonical Genetic Algorithms. IEEE Transactions on Neural Networks 5:96–101
Sadati N, Amraee T, Ranjbar A (2009) A global Particle Swarm-Based-Simulated Annealing Optimization technique for under-voltage load shedding problem. Applied Soft Computing 9:652–657
Santoso N, Tan O (1990) Neural-Net Based Real-Time Control of Capacitors Installed on Distribution Systems. IEEE Transactions on Power Delivery 5(1):266–272
Saraiva J, Miranda V (1994) Evaluation of the performance of a fuzzy optimal power flow algorithm. In: Proceedings of 7th Mediterranean Electrotechnical Conference, vol 3, pp 897–900
Sayah S, Zehar K (2008) Modified Differential Evolution Algorithm for Optimal Power Flow with Non-smooth Cost Functions. Energy Conversion and Management 49:3036–3042
Shengsong L, Min W, Zhijian H (2002) A hybrid algorithm for optimal power flow using the chaos optimization and the linear interior point algorithm. In: Proceedings International Conference on Power System Technology, vol 2, pp 793–797
Shengsong L, Min W, Zhijian H (2003) Hybrid algorithm of chaos optimisation and SLP for optimal power flow problems with multimodal characteristic. In: IEE Proceedings - Generation, Transmission and Distribution, vol 150, pp 543–547
Simon S, Padhy N, Anand R (2006) An Ant Colony System Approach for Unit Commitment Problem. International Journal of Electrical Power & Energy Systems 28:315–323
Song Y, Wang G, Wang P, Johns A (1997) Environmental/econoimic dispatch using fuzzy logic controlled genetic algioriothms. IEE Proceedings-Generation, Transmission and Distribution 144(4):377–382
Sood Y, Padhy N, Gupta H (2003) Discussion of Optimal Power Flow by Enhanced Genetic Algorithm. IEEE Transactions on Power Systems 18(3):1219
Spall J (2003) Introduction to Stochastic Search and Optimization. Wiley-Interscience
Storn R, Price K (1995) Differential Evolution - a Simple and Efficient Adaptive Scheme for Global Optimization over Continuous Spaces. Tech. Rep. TR-95-012, ICSI
Subbaraj P, Rajnarayanan P (2009) Optimal reactive power dispatch using self-adaptive real coded genetic algorithm. Electric Power Systems Research 79:374–381
Swapur K (2006) Swarm intelligence approach to the solution of optimal power flow. Indian Institute of Science 86:439–455
Swarup K (2005) Ant Colony Optimization for Economic Generator Scheduling and Load Dispatch. In: Proceedings of the 6th WSEAS International Conference on Evolutionary Computing, Portugal, pp 167–175
Tang K, Kwong S (1999) Genetic algorithms: concepts and designs, 2nd edn. Springer
Tang W, Li M, He S, Wu Q, Saunders J (2006) Optimal power flow with dynamic loads using bacterial foraging algorithm. In: International Conference on Power System Technology
Tang W, Li M, Wu Q, Saunders J (2008) Bacterial Foraging Algorithm for Optimal Power Flow in Dynamic Environments. IEEE Transactions On Circuits And Systems – I: Regular Papers 55(8):2433–2443
Tangpatiphan K, Yokoyama A (2009) Optimal Power Flow with Steady-State Voltage Stability Consideration Using Improved Evolutionary Programming. In: 2009 IEEE Bucharest Power Tech Conference, Bucharest, Romania
Teng JH, Liu YH (2003) A novel ACS-based optimum switch relocation method. IEEE Transactions on Power Systems 18(1):113–120
Todorovski M, Rajicic D (2006) An Initialization Procedure in Solving Optimal Power Flow by Genetic Algorithm. IEEE Transactions on Power Systems 21(2):480–487
Tripathy M, Mishra S (2007) Bacteria foraging-based solution to optimize both real power loss and voltage stability limit. IEEE Transactions on Power Systems 22(1):240–248

Vaisakh K, Srinivas LR (2008) Differential Evolution based OPF with Conventional and Non-Conventional Cost Characteristics. In: Joint International Conference on Power System Technology and IEEE Power India Conference POWERCON, pp 1–9

Varadarajan M, Swarup K (2008) Solving multi-objective optimal power flow using differential evolution. IET Generation, Transmission & Distribution 2(5):720–730

Venayagamoorthy G, Harley R (2007) Swarm Intelligence for Transmission System Control. In: IEEE Power Engineering Society General Meeting, pp 1–4

Venkatesh B, Sadasivam G, Khan M (1999) Optimal reactive power planning against voltage collapse using the successive multiobjective fuzzy LP technique. In: IEE Proceedings on Generation, Transmission and Distribution, vol 146, pp 343–348

Vlachogiannis J, Lee K (2005) Reactive Power Control Based On Particle Swarm Multi-Objective Optimization. In: Proceedings of the 13th International Conference on Intelligent Systems Application to Power Systems, vol 1, pp 303–308

Vlachogiannis J, Lee K (2006) A Comparative Study on Particle Swarm Optimization for Optimal Steady-State Performance of Power Systems. IEEE Transactions on Power Systems 21(4):1718–1728

Vlachogiannis J, Hatziargyriou N, Lee K (2005) Ant Colony System-Based Algorithm for Constrained Load Flow Problem. IEEE Transactions on Power Systems 20(3):1241–1249

Wang CR, Yuan HJ, Huang ZQ, Zhang JW, Sun CJ (2005) A modified particle swarm optimization algorithm and its application in optimal power flow problem. In: Proceedings of 2005 International Conference on Machine Learning and Cybernetics, vol 5, pp 2885–2889

Wong K, Fung C (1993) Simulated annealing based economic dispatch algorithm. In: IEE Proceedings, vol 140, pp 509–515

Wong K, Yuryevich J (1999) Simulated Evolution and Learning, Springer Berlin / Heidelberg, chap Optimal Power Flow Method Using Evolutionary Programming, pp 405–412

Wu Q, Ma J (1995) Power System Optimal Reactive Power Dispatch Using Evolutionary Programming. IEEE Transactions on Power Systems 10(3):1243–1249

Xia X, Elaiw A (2010) Optimal dynamic economic dispatch of generation: A review. Electric Power Systems Research 80(8):975–986

Xiangzheng X (2007) Research on Reactive Power Optimizing Control Based on Immune Algorithms. In: The Eighth International Conference on Electronic Measurement and Instruments, pp 3–898 – 3–901

Xu H, Zhu Y, Zhang T (2000) Application of mutative scale chaos optimization algorithm in power plant units economic dispatch. Journal of Harbin Institute of Technology 32:55–58

Yang B, Chen Y, Zhao Z (2007a) Survey on Applications of Particle Swarm Optimization in Electric Power Systems. In: IEEE International Conference on Control and Automation ICCA, pp 481–486

Yang D, Li G, Cheng G (2007b) On the Efficiency of Chaos Optimization Algorithms for Global Optimization. Chaos, Solitons and Fractals 34:1366–1375

Yoshida H, Kawata K, Fukuyama Y, Takayama S, Nakanishi Y (2001) A Particle Swarm Optimization for Reactive Power and Voltage Control Considering Voltage Security Assessment. IEEE Transactions on Power Systems 15(4):1232–1239

Younes M, Rahli M, Abdelhakem-Koridak L (2007) Optimal Power Flow Based on Hybrid Genetic Algorithm. Journal of Information Science And Engineering 23:1801–1816

Yu X, Gen M (2010) Introduction to Evolutionary Algorithms. Springer

Yumbla P, Ramirez J, Coello C (2008) Optimal Power Flow Subject to Security Constraints Solved With a Particle Swarm Optimizer. IEEE Transactions on Power Systems 23(1):33–40

Zadeh L (1965) Fuzzy sets. Information and Control 8(3):338–353

Zadeh L (1996) Fuzzy Sets, Fuzzy Logic, Fuzzy Systems, Advances in Fuzzy Systems-Applications and Theory, vol 6. World Scientific Press

Zhang H, Zhang L, Meng F (1998) Reactive Power Optimization Based on Genetic Algorithm. In: International Conference on Power System Technology, vol 2, pp 1448–1453

Zhang W, Liu Y (2004) Reactive Power Optimization Based on PSO in a Practical Power System. In: IEEE Power Engineering Society General Meeting, vol 1, pp 239–243

Zhang W, Liu Y (2005) Fuzzy Logic Controlled Particle Swarm for Reactive Power Optimization Considering Voltage Stability. In: The 7th International Power Engineering Conference.IPEC 2005

Zhang W, Tolbert L (2005) Survey of Reactive Power Planning Methods. In: IEEE Power Engineering Society General Meeting, vol 2, pp 1430–1440

Zhao B, Guo C, Cao Y (2004) Improved Particle Swam Optimization Algorithm for OPF Problems. In: IEEE PES Power Systems Conference and Exposition, vol 1, pp 233–238
Zhao B, Guo C, Cao Y (2005) An Improved Particle Swarm Optimization Algorithm for Optimal Reactive Power Dispatch. In: IEEE Power Engineering Society General Meeting, vol 1, pp 272–279
Zhihuan L, Yinhong L, Xianzhong D (2010) Improved Strength Pareto Evolutionary Algorithm with Local Search Strategies for Optimal reactive Power Flow. Information Technology Journal 9:749–757
Zhijiang Y, Zhijian H, Chuanwen J (2002) Economic Dispatch and Optimal Power Flow Based on Chaotic Optimization. In: Proceedings of International Conference on Power System Technology, 2002. PowerCon, vol 4, pp 2313–2317
Zimmerman RD, Murillo-Sánchez CE, Thomas RJ (2011) MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education. IEEE Transactions on Power Systems 26(1):12–19, DOI 10.1109/TPWRS.2010.2051168