Robust Distributed Energy Management for Microgrids with Renewables

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***Abstract*—Due to the low communication overhead and robustness to failures, distributed energy management is of paramount importance in smart grids, especially in microgrids, which feature distributed generation (DG). Distributed economic dispatch for a microgrid with renewable penetration and demand-side management operating in the grid-connected mode is considered in this paper. To address the challenge of intrinsically stochastic availability of renewable energy sources (RES), a novel power scheduling approach involving the actual renewable energy as well as the energy traded with the main grid is introduced, effecting the supply-demand balance. Its optimality amounts to minimizing the microgrid net cost, which includes conventional DG cost as well as worst-case transaction cost stemming from the uncertainty in RES. Leveraging the dual decomposition, the optimization problem formulated is solved in a distributed fashion. Numerical results are reported to corroborate the effectiveness of the novel approach.**

I. Introduction

Microgrids are power systems comprising distributed energy resources (DERs) and electricity end-users, possibly with controllable elastic loads, all deployed across a limited geographical area [1]. Depending on their origin, DERs can come either from distributed generation (DG) or from distributed storage (DS). DG refers to small-scale power generators which can use fossil fuels, such as diesel generators and renewable energy sources (RES), as in wind or photovoltaic generation. DS paradigms include batteries, flywheels, and pumped storage. Specifically, DG brings power closer to the point it is consumed, thereby incurring fewer thermal losses and bypassing limitations imposed by a congested transmission network. Moreover, the increasing tendency towards high penetration of RES stems from their environmental-friendly and price-competitive advantage over conventional generation. Typical microgrid loads include critical non-dispatchable types and also elastic controllable ones.

Microgrids can also entail distribution networks with residential or commercial end-users, in rural or urban areas. A typical configuration is depicted in Fig. 1; see also [1]. The microgrid energy manager (MGEM) coordinates the DERs and the controllable loads. Each of the DERs and loads has a local controller (LC), which coordinates with the MGEM the scheduling of resources through the communications infrastructure. The microgrid can operate in two modes: either

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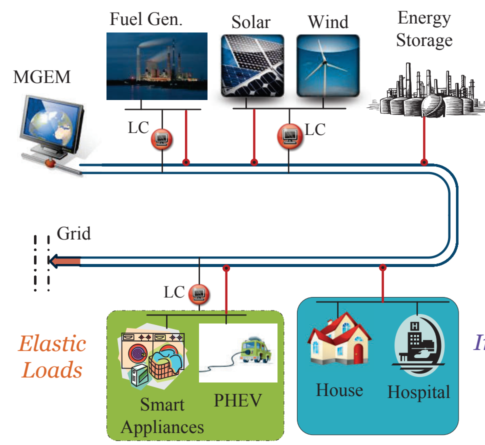

Fig. 1. Destributed control and computation architecture of a microgrid system. The microgrid energy manager (MGEM) coordinates the local controllers (LCs) of DERs and dispatchable loads.

connected or disconnected from the grid, the latter referred to as island mode.

In this context, the present paper deals with optimal energy management for both supply and demand of a microgrid incorporating renewable energy. The microgrid is connected to the main grid, while energy can be sold to or purchased from the main grid. Decentralized algorithms are developed, which are robust to the uncertainty of the available RES.

In addition to being computationally efficient, distributed power scheduling through the communication infrastructure connecting many DERs must also be resilient to communication outages or attacks. Furthermore, DER scheduling in microgrids must account for the random and nondispatchable nature of the RES.

Without incorporating the RES, energy management optimization problems including economic dispatch (ED), unit commitment (UC), and demand-side management (DSM) are outlined in e.g., [2]. Mixed integer programming problems are formulated for microgrid scheduling and DSM in [3], [4]. Based on deterministic RES models (e.g., those relating wind power with wind speed), ED problems are investigated in [5] and [6]. In all aforementioned works however, robust formulations accounting for the RES randomness are not pursued. Lyapunov stochastic optimization has been applied to maximize the long-term profit of a RES facility in [7]. Stochas-

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tic programming is also used to cope with the variability of RES. Single-period chance-constrained ED problems for RES have been studied in [8], yielding probabilistic guarantees that the load will be served. Direct coupling of uncertain renewable energy supply with deferrable demand is advocated in [9] using stochastic dynamic programming. Without DSM, robust scheduling problems with penalty-based costs for uncertain supply have been investigated in [10]. Recent works investigate energy scheduling with DSM and RES using only centralized algorithms [11], [12]. Game-theoretic approaches for microgrid distribution networks using cooperative and noncooperative games are introduced in [13]. Distributed algorithms are developed in [14], but they only coordinate DERs to supply a given load without considering the stochastic nature of RES.

This paper considers energy management of DERs and dispatchable loads with the aim of minimizing the microgrid net cost. The objective consists of costs for conventional DG, utility of elastic loads, and a worst-case transaction cost. The transaction cost stems from the ability of the microgrid to sell excess renewable energy to the main grid, or to import energy in case of shortage. A *robust* formulation accounting for the worst-case amount of harvested RES is developed. A novel model is introduced in order to maintain the supply-demand balance arising from the intermittent RES. Moreover, a transaction-price-based condition is established to ensure convexity of the overall problem. The separable structure of the resultant constrained problem is leveraged to develop a low-overhead *distributed* algorithm based on dual decomposition. The distributed implementation relies upon message exchanges between the MGEM and LCs.

The rest of the paper is organized as follows. Section II formulates the robust energy management problem, and Section III develops the distributed algorithm. Numerical results are reported in Section IV, while conclusions and research outlook directions are provided in Section V.

II. Robust Energy Management Formulation

Consider a microgrid comprising M conventional (fossil fuel) generators, N controllable (dispatchable) loads, and one RES facility (see also Fig. 1). The scheduling horizon is $\mathcal{T} := \{1, 2, \ldots, T\}$ (e.g., one day ahead). Let $P_{G_m}^t$ be the power produced by the mth conventional generator, and $P_{D_n}^t$ the power consumed by the nth dispatchable load at slot t, where $m \in \mathcal{M} := \{1, \ldots, M\}$, $n \in \mathcal{N} := \{1, \ldots, N\}$, and $t \in \mathcal{T}$. The *committed energy* from the RES delivered to the microgrid at slot t, which possibly includes the energy traded with the main grid in case of RES shortage, is denoted by P_R^t. The ensuing subsection details the RES uncertainty model as well as the transaction mechanism between the microgrid and the main grid. Subsection II-B formulates the microgrid energy management problem, which boils down to optimally scheduling the variables $P_{G_m}^t$, $P_{D_n}^t$, and P_R^t for all $t \in \mathcal{T}$.

A. Worst-case Transaction Cost

Let W^t denote the *actual renewable energy* harvested at time slot t. To capture the intrinsically stochastic and time-varying availability of RES, it is postulated that $\{W^t\}_{t=1}^T$ is unknown, but lies in a polyhedral uncertainty set

$$\mathcal{W} := \Bigg\{ \{W^t\} | \underline{W}^t \le W^t \le \overline{W}^t, \\ W_i^{\min} \le \sum_{t \in \mathcal{T}_i} W^t \le W_i^{\max}, \mathcal{T} = \bigcup_{i=1}^{I} \mathcal{T}_i \Bigg\}$$

where $\underline{W}^t$ ($\overline{W}^t$) denotes the lower (upper) bound on W^t. Moreover, considering RES aggregation over multiple periods, the time horizon $\mathcal{T}$ can be partitioned into consecutive but non-overlapping "sub-horizons" $\mathcal{T}_i$, $i = 1, 2, \ldots, I$, with the total wind power over the ith sub-horizon assumed bounded by $W_i^{\min}$ and $W_i^{\max}$; see also [12]. This RES uncertainty model is quite general and can take into account different geographical and meteorological factors. The only information it requires is these deterministic lower and upper bounds, namely $\underline{W}^t, \overline{W}^t, W_i^{\min}, W_i^{\max}, \forall\, i$, which can be determined via inference schemes based on historical data [15].

Supposing the microgrid operates in a grid-connected mode, a transaction mechanism between the microgrid and the main grid is postulated, whereby the microgrid can buy/sell energy from/to the spot market. Specifically, the shortage between the actual renewable energy produced and the one scheduled per slot t is given by $[P_R^t - W^t]^+$, while the surplus renewable energy is $[P_R^t - W^t]^-$, where $[a]^+ := \max\{a, 0\}$, $[a]^- := \max\{-a, 0\}$. The amount of shortage energy $[P_R^t - W^t]^+$ is bought with known purchase price α^t, while the surplus energy $[P_R^t - W^t]^-$ is sold to the main grid with known selling price β^t. The worst-case net transaction cost is thus given by

$$G(\{P_R^t\}) := \max_{\{W^t\} \in \mathcal{W}} \left\{ \sum_{t=1}^{T} \left(\alpha^t \left[P_R^t - W^t\right]^+ - \beta^t \left[P_R^t - W^t\right]^- \right) \right\}$$

where $\{P_R^t\}$ collects P_R^t for $t = 1, 2, \ldots, T$.

B. Microgrid Net Cost Minimization

The cost of the mth conventional generator is given by a strictly increasing and convex function $C_m^t(P_{G_m}^t)$. Typically, the chosen $C_m^t(P_{G_m}^t)$ is either piecewise linear or smooth quadratic. Moreover, the utility function of the nth dispatchable load, $U_n^t(P_{D_n}^t)$, is selected to be strictly increasing and concave. Similar to the generation cost, $U_n^t(P_{D_n}^t)$ is chosen either piecewise linear or smooth quadratic. Apart from dispatchable loads, there is also a fixed load demand from e.g., critical loads, denoted by L^t.

The energy management problem amounts to minimizing the microgrid social net cost; that is, the cost of conventional generation (ED) as well as the worst-case transaction cost (due to the volatility of RES) minus the load utility:

$$(P1) \quad \min_{\{P_{G_m}^t, P_{D_n}^t, P_R^t\}} \left\{ \sum_{t=1}^{T} \left(\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{n=1}^{N} U_n^t(P_{D_n}^t) \right) \\ + G(\{P_R^t\}) \right\} \qquad \text{(1a)}$$

subject to:

$$P_{G_m}^{\min} \le P_{G_m}^t \le P_{G_m}^{\max},\ \forall\, m \in \mathcal{M},\ \forall\, t \in \mathcal{T} \quad (1b)$$

$$P_{G_m}^t - P_{G_m}^{t-1} \le R_{m,\text{up}},\ \forall\, m \in \mathcal{M},\ \forall\, t \in \mathcal{T} \quad (1c)$$

$$P_{G_m}^{t-1} - P_{G_m}^t \le R_{m,\text{down}},\ \forall\, m \in \mathcal{M},\ \forall\, t \in \mathcal{T} \quad (1d)$$

$$\sum_{m=1}^{M} (P_{G_m}^{\max} - P_{G_m}^t) \ge S^t,\ \forall\, t \in \mathcal{T} \quad (1e)$$

$$P_{D_n}^{\min} \le P_{D_n}^t \le P_{D_n}^{\max},\ \forall\, n \in \mathcal{N},\ \forall\, t \in \mathcal{T} \quad (1f)$$

$$P_R^{\min} \le P_R^t \le P_R^{\max},\ \forall\, t \in \mathcal{T} \quad (1g)$$

$$\sum_{m=1}^{M} P_{G_m}^t + P_R^t = \sum_{n=1}^{N} P_{D_n}^t + L^t,\ \forall\, t \in \mathcal{T}. \quad (1h)$$

Constraints (1b)-(1e) stand for the minimum/maximum power output, ramping up/down limits, and spinning reserves, respectively, which capture the typical physical requirements of a power generation system. Constraints (1f) and (1g) correspond to the minimum/maximum power of the flexible load demand and committed energy. Constraint (1h) is the power supply-demand *balance equation* ensuring the total demand is satisfied by the power generation at any time.

Note that constraints (1b)-(1h) are linear, while $C_m^t(\cdot)$ and $-U_n^t(\cdot)$ are convex (possibly non-differentiable or non-strictly convex) functions. Consequently, the convexity of $(P1)$ depends on that of $G(\{P_R^t\})$, which is established in the following proposition.

Proposition 1. *If the selling price β^t does not exceed the purchase price α^t for any $t \in \mathcal{T}$, then the worst-case transaction cost $G(\{P_R^t\})$ is convex in $\{P_R^t\}$.*

Proof: Using that $[a]^+ + [a]^- = |a|$, and $[a]^+ - [a]^- = a$, $G(\{P_R^t\})$ can be re-written as

$$G(\{P_R^t\}) = \max_{\{W^t\} \in \mathcal{W}} \left\{ \sum_{t=1}^{T} \left(\delta^t |P_R^t - W^t| + \gamma^t (P_R^t - W^t) \right) \right\}$$

with $\delta^t := (\alpha^t - \beta^t)/2$, and $\gamma^t := (\alpha^t + \beta^t)/2$. Since the absolute value function is convex, and the operations of nonnegative weighted summation and pointwise maximum (over an infinite set) preserve convexity [16, Sec. 3.2], the claim follows readily. ■

An immediate corollary of Proposition 1 is that the energy management problem $(P1)$ is convex if $\beta^t \le \alpha^t$ for all t. The next section focuses on this case, and designs an efficient decentralized solver for $(P1)$.

III. Distributed Algorithm

Problem $(P1)$ is clearly separable, meaning that its cost and constraints are sums of terms, with each term dependent on different variables, namely $\{P_{G_m}^t\}$, $\{P_{D_n}^t\}$, and $\{P_R^t\}$. Note further that since the primal problem is convex, strong duality holds; hence, Lagrangian relaxation and the dual decomposition approach are applicable to yield a decentralized algorithm, as explained next. Coordinated by dual variables, the dual approach decomposes the original problem into several separate subproblems that can be solved parallelly by the LCs.

A. Dual Decomposition

Constraints (1e) and (1h) couple variables across different generators, loads, and the RES. Let $\{\mu^t\}$ and $\{\lambda^t\}$ denote Lagrange multipliers associated with (1e) and (1h), respectively. Keeping the remaining constraints implicit, the partial Lagrangian is given by

$$\begin{aligned}
&\mathcal{L}(\{P_{G_m}^t, P_{D_n}^t, P_R^t, \mu^t, \lambda^t\}) \\
&= \sum_{t=1}^{T} \left(\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{n=1}^{N} U_n^t(P_{D_n}^t) \right) + G(\{P_R^t\}) \\
&+ \sum_{t=1}^{T} \left\{ \mu^t \left(S^t - \sum_{m=1}^{M} (P_{G_m}^{\max} - P_{G_m}^t) \right) \right. \\
&\left. - \lambda^t \left(\sum_{m=1}^{M} P_{G_m}^t + P_R^t - \sum_{n=1}^{N} P_{D_n}^t - L^t \right) \right\}. \quad (2)
\end{aligned}$$

Then, the dual function can be written as

$$\mathcal{D}(\{\mu^t\},\{\lambda^t\}) = \min_{\substack{\{P_{G_m}^t, P_{D_n}^t, P_R^t\} \\ \text{s.t. } (1b)-(1d),(1f),(1g)}} \mathcal{L}(\{P_{G_m}^t, P_{D_n}^t, P_R^t, \mu^t, \lambda^t\})$$

and the dual problem is given by

$$\max \quad \mathcal{D}(\{\mu^t\}, \{\lambda^t\}) \quad (3a)$$

$$\text{s.t.} \quad \mu^t \ge 0, \lambda^t \in \mathbb{R},\ \forall\, t \in \mathcal{T}. \quad (3b)$$

The subgradient method will be employed to obtain optimal multipliers and power schedules. The iterative process is described next, followed by its distributed implementation.

1) Subgradient Iterations: The subgradient method amounts to running the recursions [17, Sec. 6.3]

$$\mu^t(k+1) = [\mu^t(k) + a g_{\mu^t}(k)]^+ \quad (4a)$$

$$\lambda^t(k+1) = \lambda^t(k) + a g_{\lambda^t}(k) \quad (4b)$$

where k is the iteration index; $a > 0$ is a constant stepsize; while g_{μ^t} and g_{λ^t} denote the subgradients of the dual function with respect to $\mu^t(k)$ and $\lambda^t(k)$, respectively. These subgradients can be expressed in the following simple forms

$$g_{\mu^t}(k) = S^t - \sum_{m=1}^{M} (P_{G_m}^{\max} - P_{G_m}^t(k)) \quad (5a)$$

$$g_{\lambda^t}(k) = L^t + \sum_{n=1}^{N} P_{D_n}^t(k) - \sum_{m=1}^{M} P_{G_m}^t(k) - P_R^t(k) \quad (5b)$$

where

$$P_{G_m}^t(k) \in \arg\min_{\substack{\{P_{G_m}^t\} \\ \text{s.t. } (1b)-(1d)}} \left\{ \sum_{t=1}^{T} [C_m^t(P_{G_m}^t) + (\mu^t(k) - \lambda^t(k)) P_{G_m}^t] \right\} \quad (6)$$

$$P_{D_n}^t(k) \in \arg\min_{\{P_{D_n}^{\min} \le P_{D_n}^t \le P_{D_n}^{\max}\}} \left\{ \sum_{t=1}^{T} [\lambda^t(k) P_{D_n}^t - U_n^t(P_{D_n}^t)] \right\} \quad (7)$$

$$P_R^t(k) \in \arg\min_{\{P_R^{\min} \le P_R^t \le P_R^{\max}\}} \left\{ G(\{P_R^t\}) - \sum_{t=1}^{T} \lambda^t(k) P_R^t \right\}. \quad (8)$$

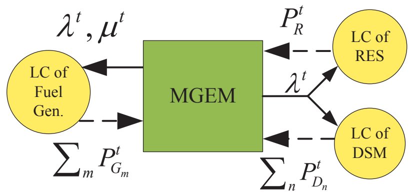

Fig. 2. Decomposition and message exchange.

Iterations are initialized with arbitrary $\lambda^t(0) \in \mathbb{R}$ and $\mu^t(0) \geq 0$. The iterates are guaranteed to converge to a neighborhood of the optimal multipliers [17, Sec. 6.3]. The size of the neighborhood is proportional to the stepsize, and can therefore be controlled by the stepsize.

The optimal power schedules are given by running averages of the iterates $P_{G_m}^t(k)$, $P_{D_n}^t(k)$, and $P_R^t(k)$ as

$$\bar{P}_{G_m}^t(k) = \frac{1}{k}\sum_{j=0}^{k-1} P_{G_m}^t(j) \tag{9}$$

and likewise for $\bar{P}_{D_n}^t(k)$ and $\bar{P}_R^t(k)$. The running averages are also guaranteed to converge to a neighborhood of the optimal solution [18].

2) Distributed Implementation: The form of the subgradient iterations easily lends itself to a distributed implementation utilizing the control and communication capabilities of a typical microgrid.

Specifically, the MGEM maintains and updates the Lagrange multipliers via (4). The LCs of conventional generation, dispatchable loads, and RES solve subproblems (6), (7), and (8), respectively. These subproblems can be solved if the MGEM sends the current multiplier iterates $\mu^t(k)$ and $\lambda^t(k)$ to the LCs. The LCs send back to the MGEM the quantities $\sum_{m=1}^M P_{G_m}^t(k)$, $\sum_{n=1}^N P_{D_n}^t(k)$, and $P_R^t(k)$, which are in turn used to form the subgradients according to (5). This interactive process of message passing is illustrated in Fig. 2.

B. Solving the LC Subproblems

This subsection shows how to solve each subproblem (6)-(8). Specifically, $C_m^t(\cdot)$ and $-U_n^t(\cdot)$ are chosen either convex piece-wise linear or smooth convex quadratic. Correspondingly, the first two subproblems (6) and (7) are essentially linear programs (LPs) or quadratic programs (QPs), which are easy to solve efficiently.

However, subproblem (8) is a convex nondifferentiable problem, which can be challenging to solve. Nondifferentiability comes from the absolute value operator, and also from the maximization with respect to $\{W^t\}$ in the definition of $G(\{P_R^t\})$. The bundle method is a state-of-the-art technique for convex nondifferentiable optimization problems [19], [17, Ch. 6], and is employed to solve (8).

Let $\tilde{G}(\{P_R^t\}) := \left\{G(\{P_R^t\}) - \sum_{t=1}^T \lambda^t P_R^t\right\}$ denote the cost in (8). By the generalization of Danskin's Theorem [17, Sec. 6.3], the subgradient of $\tilde{G}(\{P_R^t\})$ with respect to P_R^t, which is needed for the bundle method, can be obtained as

$$\partial\tilde{G}(\{P_R^t\}) = \begin{cases} \alpha^t - \lambda^t, & \text{if } P_R^t \geq W_*^t \\ \beta^t - \lambda^t, & \text{if } P_R^t < W_*^t \end{cases} \tag{10}$$

where for given $\{P_R^t\}$ it holds that

$$\{W_*^t\} \in \underset{\{W^t\}_{t=1}^T \in \mathcal{W}}{\arg\max} \left\{\sum_{t=1}^T \left(\delta^t|P_R^t - W^t| + \gamma^t(P_R^t - W^t)\right)\right\}. \tag{11}$$

Remark 1. *(Complexity of solving* (11)*).* In order to obtain $\{W_*^t\}$, the convex nondifferentiable function (11) should be maximized over $\mathcal{W}$. This is generally an NP-hard convex maximization problem, meaning the global optimal solution $\{W_*^t\}$ can not be obtained in polynomial time. However, for the specific problem here, the global solution is attained on the boundary of the feasible set [20, Ch. 3]. Therefore, the objective can be evaluated at all vertices of $\mathcal{W}$ to obtain the global solution. This straightforward approach still incurs exponential complexity. But if the cardinality of each sub-horizon $\mathcal{T}_i$ is not very large (e.g., when 24 hours are partitioned into 4 sub-horizons each comprising 6 time slots), then the complexity is affordable. Most importantly, the vertices of $\mathcal{W}$ need only be listed once, before the optimization.

The bundle method is described next. For notational brevity, let $\mathbf{p} := [P_R^1, \ldots, P_R^T]$, $\mathbf{p}^{\min} := P_R^{\min} \cdot \mathbf{1}$, $\mathbf{p}^{\max} := P_R^{\max} \cdot \mathbf{1}$, where $\mathbf{1}$ is the all-ones vector. A sequence $\{\mathbf{p}_\ell\}$ will be generated with guaranteed convergence to the optimal solution set [19], [17, Ch. 6]. The iterate $\mathbf{p}_{\ell+1}$ is obtained by minimizing the polyhedral approximation of $\tilde{G}(\mathbf{p})$ and a quadratic proximal regularization as follows

$$\mathbf{p}_{\ell+1} := \underset{\mathbf{p}^{\min} \preceq \mathbf{p} \preceq \mathbf{p}^{\max}}{\arg\min} \left\{\hat{G}_\ell(\mathbf{p}) + \frac{\rho_\ell}{2}\|\mathbf{p} - \mathbf{y}_\ell\|^2\right\} \tag{12}$$

where $\hat{G}_\ell(\mathbf{p}) := \max\{\tilde{G}(\mathbf{p}_0) + \mathbf{g}_0'(\mathbf{p} - \mathbf{p}_0), \ldots, \tilde{G}(\mathbf{p}_\ell) + \mathbf{g}_\ell'(\mathbf{p} - \mathbf{p}_\ell)\}$; $\mathbf{g}_\ell$ is the subgradient of $\tilde{G}(\mathbf{p})$ evaluated at the point $\mathbf{p} = \mathbf{p}_\ell$, which is calculated according to (10); $\mathbf{a}'$ denotes the transpose of $\mathbf{a}$; proximity weight ρ_ℓ is to control stability of the iterates; and the proximal center $\mathbf{y}_\ell$ is updated according to a query for descent

$$\mathbf{y}_{\ell+1} = \begin{cases} \mathbf{p}_{\ell+1}, & \text{if } \tilde{G}(\mathbf{y}_\ell) - \tilde{G}(\mathbf{p}_{\ell+1}) \geq \theta\eta_\ell \\ \mathbf{y}_\ell, & \text{otherwise} \end{cases} \tag{13}$$

where $\eta_\ell = \tilde{G}(\mathbf{y}_\ell) - \left(\hat{G}_\ell(\mathbf{p}_{\ell+1}) + \frac{\rho_\ell}{2}\|\mathbf{p}_{\ell+1} - \mathbf{y}_\ell\|^2\right)$, $\theta \in (0,1)$. By introducing an auxiliary variable, (12) can be re-written as a QP, which is efficiently solvable.

IV. Numerical Tests

In this section, preliminary numerical tests are carried out to verify the performance of the novel design for a microgrid consisting of $M = 2$ conventional generators, $N = 2$ dispatchable loads, and one RES scheduled over $T = 8$ hours. The generation costs and utilities of the elastic loads are set to be time-invariant as $C_1(P_{G_1}) = 0.4P_{G_1}^2 + 40P_{G_1}$, $C_2(P_{G_2}) = 0.3P_{G_2}^2 + 30P_{G_2}$, $U_1(P_{D_1}) = -0.6P_{D_1}^2 + 60P_{D_1}$, and $U_2(P_{D_2}) = -0.5P_{D_2}^2 + 55P_{D_2}$ (with units ¢). The

TABLE I
LOWER/UPPER LIMITS OF VARIABLES AND PARAMETERS

	P_{G_1}	P_{G_2}	P_{D_1}	P_{D_2}	P_R	W
Min (kWh)	10	8	5	8	0	52
Max (kWh)	50	45	35	40	35	139

TABLE II
DEMAND OF FIXED LOADS AND LIMITS OF ACTUAL RES

Slot	1	2	3	4	5	6	7	8
L^t (kWh)	25	30	45	58	70	67	50	42
$\underline{W}^t$ (kWh)	1	3	2.5	6	5	3.5	4	2
$\overline{W}^t$ (kWh)	15	18	20	30	25	19	27	20

simulation parameters are listed in Tables I–III (similar to [3]), while $I = 1$ is considered for the uncertainty set $\mathcal{W}$.

The optimal power schedules for the microgrid are depicted in Figs. 3 and 4 for the two selling price profiles in Table III, where $P_G^t := P_{G_1}^t + P_{G_2}^t$, $P_D^t = P_{D_1}^t + P_{D_2}^t$, and P_R^t, denote the total conventional power generation, total elastic demand, and the committed RES, respectively, which are the optimal solutions of $(P1)$. The meaning of W_{worst}^t (the worst-case RES at slot t) will be explained shortly.

A common observation for Figs. 3 and 4 is that the total conventional power generation P_G^t varies with the same trend across t as the fixed load demand L^t. Moreover, the optimal P_D^t and P_R^t in the first two time slots are larger than all other slots. This is because the demand for fixed loads is low during the first two slots, allowing the elastic loads power consumption to become larger in order to increase the utility. As a result, more committed energy P_R^t must be delivered to the microgrid to keep the power balance. The increased P_R^t for the first two slots is consistent with the corresponding lower purchase price. Moreover, Fig. 3 reveals that the committed energy P_R^t from slot 3 up to and including slot 6 is low. This is justifiable because the purchase price α^t as well as the selling price β^t are high during this interval, making it more economical to sell the renewable energy to the main grid, than to allocate it to the microgrid. Finally, the combination of reduced committed energy P_R^t with an increased load demand L^t between slots 3 and 6 causes a reduction of the dispatchable loads consumption P_D^t in these slots. Corresponding statements can be made for the low selling price case of Fig. 4.

Furthermore, by comparing Fig. 3 with Fig. 4, it is interesting to illustrate the worst-case transaction mechanism as follows. Consider first the schedule for slots 3 to 6. Since selling prices of Case 2 are much smaller than the ones of Case 1, the weaker motivation to sell makes the P_R^t's in Fig. 4 larger than the ones in Fig. 3. Meanwhile, by solving (11) using the optimal $\{P_R^t\}$, the optimal W_{worst}^t becomes smaller than the corresponding P_R^t in Case 2. This means that the microgrid net cost increases due to purchase transactions. In Case 1 on the other hand, the P_R^t's are already smaller than the lower bounds of W^t as shown in Fig. 3, which amounts to selling instead of purchasing energy. Moreover, the relative change of W_{worst}^t

TABLE III
PURCHASE AND SELLING PRICES FROM SPOT MARKET. THE UNITS OF α^t AND β^t ARE ¢/KWH.

Slot	1	2	3	4	5	6	7	8
α^t	15	20	57	65	80	77	50	45
Case 1: β^t	13.5	18	51.3	58.5	72	69.3	45	40.5
Case 2: β^t	1.5	2	5.7	6.5	8	7.7	5	4.5

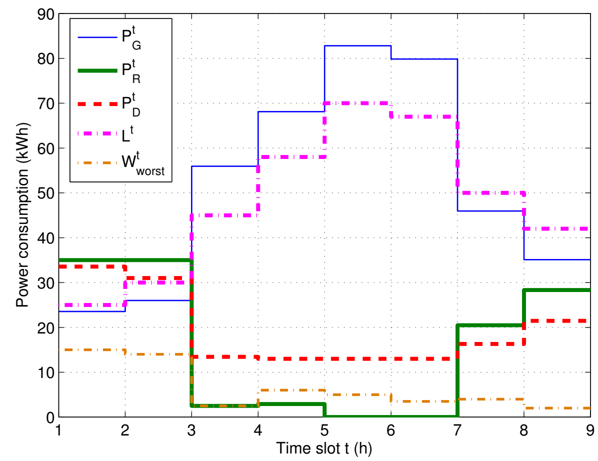

Fig. 3. Optimal power schedule (Case 1: high selling prices).

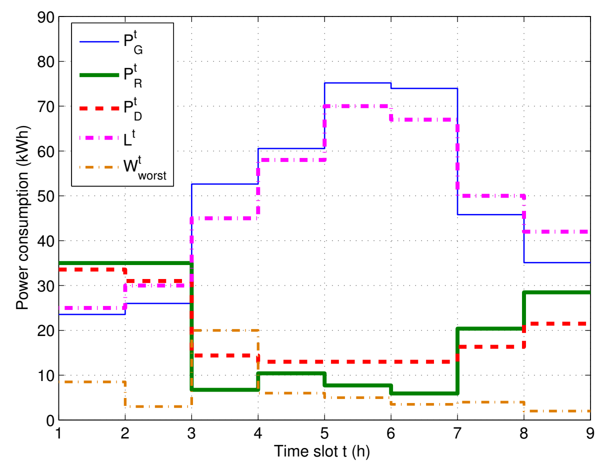

Fig. 4. Optimal power schedule (Case 2: low selling prices).

from Fig. 3 to Fig. 4 from slot 1 to 3 can also be explained by the effect of transaction prices and the uncertainty model. Specifically, the desire to increase the transaction cost, by the definition of function $G(\{P_R^t\})$, can be realized by smaller worst-case RES, i.e., with reduced W_{worst}^1 and W_{worst}^2 from Case 1 to Case 2. In order to keep the $\{W_{\text{worst}}^t\}$ feasible for $\mathcal{W}$ (i.e., $\sum_{t \in T} W^t \geq W^{\min}$ must be satisfied), the reduction of W_{worst}^1 and W_{worst}^2 from Case 1 to Case 2 causes an increase in W_{worst}^3. This makes the selling transaction happen in slot 3 in Case 2. However, the resultant revenue can not compensate for the extra cost due to the increased amount of purchased

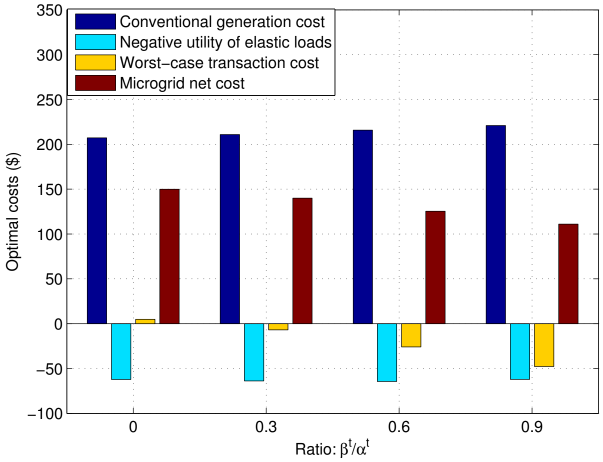

Fig. 5. Optimal costs.

energy in slots 1 and 2, which is determined by solving (11).

Fig. 5 shows the effect of different transaction prices on the microgrid net cost, where three times of the purchase price α^t in III is used. It can be clearly seen that the net cost decreases with the increase of the selling-to-purchase-price ratio β^t/α^t. When this ratio increases, the microgrid has a higher margin for revenue from the transaction mechanism. Obviously, if more renewable energy is sold rather than used within the microgrid, then the cost due to conventional generation may increase in order to supply the microgrid loads. Therefore, as depicted in Fig. 5, the microgrid net cost can be reduced as long as the obtained profit from the transaction is larger than the extra conventional generation cost.

V. Conclusions and Future Work

A distributed energy management approach was developed in this paper tailored to microgrids with high penetration of renewable energy sources. By introducing the notion of energy traded with the main grid, a novel model was introduced to deal with the challenging constraint of the supply-demand balance raised by the intermittent nature of renewable energy sources. Not only the conventional generation costs and utility of the adjustable loads are considered, but also the worst-case transaction cost is included in the objective. To schedule power in a distributed fashion, the dual decomposition method was utilized to decompose the original problem into smaller subproblems solved by the LCs of conventional generators, dispatchable loads, and the RES.

A number of interesting research directions open up towards extending the model and approach proposed in this paper. Specifically, distributed storage is a key feature that should be considered in future works. In addition, optimal power flow and the unit commitment problems are worth re-investigating with the envisaged growth of RES usage in microgrids.

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