Passivity Indices and Passivation of Systems with Application to Systems with Input/Output Delay

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***Abstract*— A passivation method to passivate a given system by using an input-output transformation matrix is introduced. This matrix generalizes the commonly used methods of series, feedback and feedforward interconnections to passivate a system. Through an appropriate design of this matrix, positive passivity levels can be guaranteed for the system. Further, this transformation matrix allows the use of a non-passive system as a controller to guarantee the passivity and stability of a feedback configuration.**

I. Introduction

Passivity and dissipativity characterize the energy consumption of a dynamical system. Passivity is preserved under parallel and feedback interconnections. Moreover, passivity implies stability under mild assumptions [1], [2]. Passivity-based controllers have been used to robustly control passive linear or nonlinear plants in many applications [3], [4]. Many physical systems, however, are not inherently passive. To apply the passivity based techniques to such non-passive systems, we need to passivate these systems through series, feedback and feedforward interconnections [5], [6]. Application of any of these methods requires some assumptions on the system. For instance, feedback alone cannot passivate systems that are non-minimum phase or have relative degree larger than one [6], [7].

In this paper, we introduce a passivation method for any finite-gain stable linear or nonlinear systems using an input-output transformation matrix. As shown in Fig. 1, consider a finite-gain stable system G. We show how the system can be passivated using a transformation matrix M, that can be realized through a combination of the commonly used series, feedback and feedforward passivation techniques. Through an appropriate design of this matrix, positive passivity levels can be guaranteed for the new system Σ_0. Compared to passivation through feedback as in [6], [7], our passivation method can be applied to systems that are non-minimum phase or have relative degree larger than one. In this sense, our method is a generalization of the series or feedback or feedforward interconnections to passivate a system. The system Σ_0 can also be used as a controller to stabilize and passivate another plant.

As an application, we focus on the case when system G is a stable linear system with an input/output delay. Such systems represent an important class of practical systems [8]. In some applications such as process control, the input/output delay is called transport delay [9]. Linear models for human operators are also of this form [10], [11], [12]. Passivity of linear systems with delayed state has been studied in e.g. [13], [14], [15]; however, these works use relaxed passivity definitions, where the passivity indices of a passive system are allowed to be negative. We show that such systems are non-passive by using the original definitions for passivity and passivity indices in [6], [16]. Linear systems with time delay may be passive under certain conditions, provided that the time delay appears in the denominator of the transfer function for the system [17], [18]. However, for systems with input/output delay, time delay appears in the numerator of the transfer functions and such systems cannot be passive. Our passivation method in Fig. 1 can be used to passivate such systems. Since this system can be a linear model of a human operator; this application shows that our proposed matrix may be useful to design an interface for human operators of a non-passive system in order to passivate it.

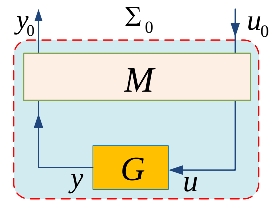

Fig. 1. The framework considered in the paper. G is a finite gain stable system. By designing an appropriate input-output transformation matrix M, the system Σ_0 is made passive with given passivity indices.

The main contributions of this paper are summarized as follows. (i) We introduce a passivation method that can be used to passivate any finite-gain stable linear or nonlinear systems. (ii) By using our passivation method, not only the passivity but also the desired passivity levels of the passivated system can be guaranteed. The passivated system can also be used as a controller to stabilize and passivate another non-passive plant. (iii) We show that linear systems with input/output delay are non-passive and our passivation method can be used to passivate such systems.

The rest of the paper is organized as follows. In Section II, we review some fundamental results in passivity and dissipativity theory. In Section III, we show that stable linear systems with input/output delay have negative passivity indices and such systems cannot be passivated by feedback alone. The main results are presented in Section IV. Specifically, the passivation method in terms of a transformation matrix is derived in Section IV-A. The use of the passivated system

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as a controller for a plant is discussed in Section IV-B. A numerical example is given in Section V to illustrate the results. Section VI concludes the paper.

Notation: The signal space under consideration is either the standard $\mathcal{L}_2$ space or the extended $\mathcal{L}_2$ space. The exact space will be clear from the context. We use $\mathbf{H}:\mathbf{u}\to\mathbf{y}$ (or simply H) to denote a dynamical system with input u and output y. We use the notations $u(t)$ and u for a signal interchangeably. We use $G(s)$ to denote the transfer function for a SISO linear system. The n-dimensional identity matrix is denoted by $I_{n\times n}$ or simply I by omitting the dimensions if clear from the context.

II. Background

Consider a dynamical system given by an operator $\mathbf{H}:\mathbf{u}\to\mathbf{y}$, where $\mathbf{u}\in\mathcal{U}$ denotes the input and $\mathbf{y}\in\mathcal{Y}$ denotes the corresponding output, and a real-valued function $w(u,y)$ defined on $\mathcal{U}\times\mathcal{Y}$, called *supply rate* [19]. We assume that $\int_{t_0}^{t_1}|w(u,y)|\mathrm{d}t<\infty$, for any t_0, t_1 and any input $u\in\mathcal{U}$.

Definition 1: An operator $\mathbf{H}:\mathbf{u}\to\mathbf{y}$ is said to be *dissipative* with respect to supply rate $w(u,y)$, if

$$\int_{t_0}^{t_1} w(u,y)dt \geq 0, \tag{1}$$

for all $t_1\geq t_0$, and all $u\in\mathcal{U}$. □

In particular, we can define passivity and $\mathcal{L}_2$ stability when the supply rate is in particular forms.

Definition 2: Suppose the system $\mathbf{H}:\mathbf{u}\to\mathbf{y}$ is dissipative. It is said to be

- *passive* if $w(u,y)=u^Ty$;
- *input feedforward passive* (IFP) if there exists a constant ν so that $w(u,y)=u^Ty-\nu u^Tu$; we call such a ν an IFP level, denoted as IFP(ν);
- *output feedback passive* (OFP) if there exists a constant ρ so that $w(u,y)=u^Ty-\rho y^Ty$; we call such a ρ an OFP level, denoted as OFP(ρ);
- *input-feedforward-output-feedback passive* (IF-OFP) if there exist constants δ and ϵ so that $w(u,y)=u^Ty-\delta y^Ty-\epsilon u^Tu$; we call such δ and ϵ passivity levels, denoted as IF-OFP(ϵ,δ);
- *finite-gain $\mathcal{L}_2$ stable* if there exists a constant $\gamma\neq 0$ so that $w(u,y)=\gamma^2u^Tu-y^Ty$, denoted as FGS(γ).

Further, if $\nu>0$, then the system is said to be *input strictly passive* (ISP); if $\rho>0$, then the system is said to be *output strictly passive* (OSP). Similarly, if $\delta>0$ and $\epsilon>0$, then the system is said to be *very strictly passive* (VSP). The largest IFP level ν is called the *IFP index* and the largest OFP level ρ is called the *OFP index*, respectively. □

Remark 1: If either one of two passivity indices is positive, we say that the system has an 'excess of passivity'; similarly, if either one of the two passivity indices is negative, we say that the system has a 'shortage of passivity'. □

In Definition 1 and 2, system $\mathbf{H}:\mathbf{u}\to\mathbf{y}$ can be either linear or nonlinear. If H is linear, then the IFP index can be defined in the following manner.

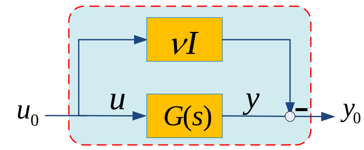

Fig. 2. Input feedforward passivity.

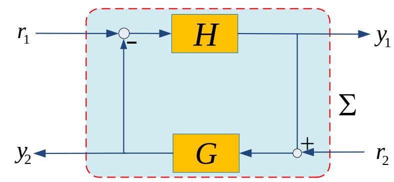

Fig. 3. Negative feedback interconnection of system H and system G.

Definition 3: The IFP index for a *stable*[1] linear system $G(s)$ is defined as

$$\nu(G(s)) \triangleq \frac{1}{2}\min_{w\in\mathbb{R}}\underline{\lambda}(G(jw)+G^*(jw)), \tag{2}$$

where $\underline{\lambda}$ denotes the minimum eigenvalue and G^* denotes the conjugate transpose of G. □

In Definition 3, the transfer function $G(s)$ may be rational or irrational. If $\nu\geq 0$, then the system $G(s)$ is called *passive* or *positive real*. If $\nu<0$, then the system $G(s)$ is not passive and ν can be interpreted as the minimum feedforward gain required for a stable non-passive linear system to become passive [6], [16], as shown in Fig. 2. When $G(s)$ is a single-input-single-output (SISO) system, we can test the passivity of $G(s)$ using its Nyquist plot. If the Nyquist plot of $G(s)$ is in the closed right-hand half of the complex plane, then the system is passive; otherwise, the system is not passive. For the special case when $G(s)$ is rational, if it is passive, then it must satisfy all of the following conditions [6], [21]: (i) stable; (ii) minimum phase; (iii) relative degree 0 or 1.

Remark 2: For a stable linear system $\mathbf{G}:\mathbf{u}\to\mathbf{y}$, the IFP index is given by the largest IFP level so that $\int_0^T u^Ty-\nu u^Tu\ \mathrm{d}t\geq 0$ for any $T\geq 0$ and any $u\in\mathcal{U}$. Further, if G has IFP($\tilde{\nu}$), then $\tilde{\nu}\leq\nu$. We show that the IFP index defined in Definition 3 is exactly the largest IFP level of G (see e.g. [22]). Therefore, the two definitions of IFP index of stable linear system are equivalent to each other. □

Consider the feedback configuration as shown in Fig. 3, the passivity and stability of a complex system Σ can be guaranteed from those of systems H and G which are much easier to analyze in practice.

Theorem 1 ([21]): Consider the feedback interconnection of two systems H and G in Fig. 3.

1) If system H and G are passive, then system Σ is passive.
2) If system H and G are output strictly passive (OSP), then system Σ is OSP;
3) If system H is IF-OFP(ϵ_1,δ_1) and system G is IF-OFP(ϵ_2,δ_2), where $\epsilon_1+\delta_2>0$, $\epsilon_2+\delta_1>0$, then

[1] A function $G(s)$ is called *stable* if it is analytic in the closed right half plane of the complex plane, see e.g. [20].

system Σ is finite gain stable (FGS). □

In Theorem 1, the passivity of both systems G and H is required to guarantee that system Σ is passive. However, it is not necessary for both system G and H to be passive to guarantee the stability of system Σ. For instance, if $\epsilon_1 < 0$ and $\delta_1 < 0$ (i.e. system H is not passive), we require that the system G has passivity levels $\epsilon_2 > -\delta_1 > 0$ and $\delta_2 > -\epsilon_1 > 0$ to compensate for the shortage of passivity of the system H.

Remark 3: In Fig. 3, we can view system H as a controller and system G as a plant. It can be seen that system Σ is passive only if both the plant and the controller are passive, which may be difficult to achieve in practice. □

If $r_2 = 0$, the feedback system is given by the mapping $\mathbf{r_1} \to \mathbf{y_1}$. We have the following less conservative result.

Theorem 2 ([23]): Consider the feedback interconnection of two systems H and G in Fig. 3. Assume that $r_2 = 0$.

1) If system H has OFP(ρ) and system G has IFP(ν) where $\rho + \nu > 0$, then the system $\mathbf{r_1} \to \mathbf{y_1}$ has OFP($\rho+\nu$). Further, the system $\mathbf{r_1} \to \mathbf{y_1}$ is finite-gain stable (FGS) with gain $\gamma \le \frac{1}{\rho+\nu}$.
2) If system H has IFP($\nu > 0$) and system G has OFP(ρ) where $\nu + \rho > 0$, then the system $\mathbf{r_1} \to \mathbf{y_1}$ has IFP($\min\{\nu, \rho + \nu\}$).

In Theorem 2, the system $\mathbf{r_1} \to \mathbf{y_1}$ is guaranteed to be OSP or ISP (stronger than just being passive). If system H has OFP $\rho < 0$, then the shortage of OFP in system H can be compensated by an excess of IFP of system G with $\nu > -\rho > 0$ so that the feedback system $\mathbf{r_1} \to \mathbf{y_1}$ is guaranteed to be OSP and FGS. Similarly, if system G has OFP $\rho < 0$, then the shortage of OFP in system G can be compensated by an excess of IFP of system H with $\nu > -\rho > 0$ so that the feedback system $\mathbf{r_1} \to \mathbf{y_1}$ is guaranteed to be ISP.

III. Linear Systems with Input/Output Delay

We are particularly interested in linear systems of the form $G(s) = G_0(s)\mathbf{e}^{-\tau s}$, where $G_0(s)$ denotes a SISO, stable, proper, rational transfer function and $\tau > 0$ denotes the input/output delay (also called transport delay) of the system. We show that: (i) such systems have negative passivity indices, i.e. such systems are not passive; (ii) state or output feedback cannot passivate such systems. These results motivate our main results in Section IV which can be applied to a large class of non-passive systems.

Remark 4: Linear systems in the form of $G(s) = G_0(s)\mathbf{e}^{-\tau s}$ can represent an important class of systems. In particular, common linear models for chemical processes and human operators, are of the form $G(s) = G_0(s)\mathbf{e}^{-\tau s}$, where τ denotes the time delay, see e.g. [6], [11], [12]. □

Proposition 1: Consider a linear system $G(s) = G_0(s)\mathbf{e}^{-\tau s}$, where $G_0(s)$ is SISO, stable, proper and rational and $\tau > 0$. Then, $G(s)$ has IFP index $\nu < 0$.

Proof: Let $G_0(jw) = A(w) + jB(w)$. When $w = 0$, we have $B(0) = 0$ and $A(0)$ denotes the gain of the system $G_0(s)$ at $w = 0$. We have

$$\begin{aligned}\nu_F(w) &\triangleq \frac{1}{2}(G(jw) + G^*(jw)) \\ &= A(w)\cos(\tau w) + B(w)\sin(\tau w) \\ &= \sqrt{A^2(w) + B^2(w)}\cos(\alpha(w)),\end{aligned}$$

where $\alpha(w) \triangleq \tau w - \beta(w)$ and $\beta(w) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is given by $\tan\beta(w) = \frac{B(w)}{A(w)}$. For $\alpha(w)$, when $w \to \infty$, the term τw dominates, and $\cos(\alpha(w))$ can take either positive or negative values. Thus, there exists a range of w, $w \in [w_1, w_2]$, so that $\nu_F(w) < 0$ for all $w \in [w_1, w_2]$. Therefore, $\nu = \min_{w\in\mathbb{R}} \nu_F(w) < 0$. ■

Proposition 1 shows that the IFP index of system $G(s)$ is negative, i.e. the system is non-passive. To obtain the exact value of the IFP index, we need to know the transfer function $G_0(s)$ and the exact value of the time delay τ. To obtain an approximate value of the IFP index of system $G(s)$, we can use the Padé approximation to approximate the pure time delay $\mathbf{e}^{-\tau s}$ by a rational transfer function. Denote $\mathbf{P}_i(s)$ as the i-th order Padé approximation of $\mathbf{e}^{-\tau s}$. Then, $G(s) = G_0(s)\mathbf{e}^{-\tau s}$ can be approximated by a rational transfer function $\tilde{G}(s) = G_0(s)\mathbf{P}_i(s)$. For example, a second order Padé approximation of $\mathbf{e}^{-\tau s}$ is given by

$$\mathbf{P}_2(s) \triangleq \frac{1 - \frac{\tau}{2}s + \frac{\tau^2}{12}s^2}{1 + \frac{\tau}{2}s + \frac{\tau^2}{12}s^2}.$$

One may obtain higher-order Padé approximations to better approximate $\mathbf{e}^{-\tau s}$ over a wider range of frequencies; however, the algebraic complexity for analysis and synthesis of the system will be increased.

In general, feedback alone cannot passivate systems of the form $G(s) = G_0(s)\mathbf{e}^{-\tau s}$. Given a strictly proper rational transfer function, it is feedback passive (i.e. can be passivated through feedback) if and only if it is minimum phase and it has relative degree one [6], [7]. For an irrational transfer function given by $G(s) = G_0(s)\mathbf{e}^{-\tau s}$, however, it is not obvious whether similar conditions can be found to check whether the system $G(s)$ can be feedback passive. One possible solution is to use the Padé approximation. For instance, by using second order approximation, the system can be approximated by $\tilde{G}(s) \triangleq G_0(s)\mathbf{P}_2(s)$. Clearly, $\tilde{G}(s)$ cannot be passivated by feedback alone since it has at least one zero in the right half complex plane. To passivate such systems, we may need feedforward or series passivation.

IV. Main Results

In this section, we consider a passivation method to alter the passivity levels of a given system to desired levels. When the passivated system is interconnected as a controller with another system that acts as a plant, the passivity and stability of the interconnected system can be guaranteed by appropriately designing the passivity levels of the passivated system.

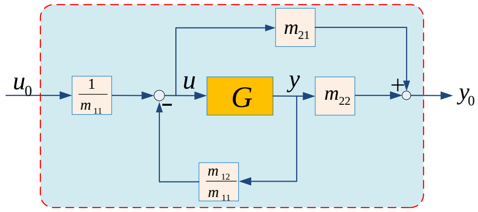

Fig. 4. Transformation M given by series, feedforward and feedback.

A. Passivation Method

Many methods are known for passivation of non-passive systems, such as series, feedback, feedforward or a combination of such schemes [5], [24], [25]. These passivation mechanisms require the system to satisfy certain properties, such as constraints on the relative degree, stability or minimum-phase property of the system. Consider system G and a general input-output transformation matrix M as shown in Fig. 1. This matrix M includes as special cases the commonly used passivation methods that use series, feedback and feedforward interconnections.

As shown in Fig. 1, $\begin{bmatrix} u_0 \\ y_0 \end{bmatrix} = M \begin{bmatrix} u \\ y \end{bmatrix}$, where the matrix M is constrained to be invertible and is defined as

$$M \triangleq \begin{bmatrix} m_{11}I & m_{12}I \\ m_{21}I & m_{22}I \end{bmatrix}.$$

For instance, consider the feedforward passivation in Fig. 2, where $u_0 = u$ and $y_0 = y - \nu u$ so that M is given by

$$\begin{bmatrix} I & 0 \\ -\nu I & I \end{bmatrix}.$$

Similarly, feedback and series passivation can also be represented by a transformation matrix M. In general, if $m_{11} \neq 0$, then M is a combination of series, feedback and feedforward passivation methods, see Fig. 4. By appropriate choices of the elements in M, we can obtain the desired passivity levels (ρ_0, ν_0) of the system $\Sigma_0 :\ \mathbf{u}_0 \to \mathbf{y}_0$. In the present paper, we focus on passivation using constant gains. It is possible to use e.g. transfer functions to replace the constants m_{ij} in M, where $i, j \in \{1, 2\}$.

The following result shows that the passivity levels of the system $\Sigma_0 :\ \mathbf{u}_0 \to \mathbf{y}_0$ depend on the gain γ of system G and m_{ij}, where $i, j \in \{1, 2\}$.

Theorem 3: Consider a system G which is finite gain stable with gain γ and a passivation matrix M as shown in Fig. 1. Then the system $\Sigma_0 : \mathbf{u}_0 \to \mathbf{y}_0$ is

1) passive, if M is chosen such that

$$m_{11} = m_{21}, \quad m_{22} = -m_{12}, \quad m_{11} \geq m_{22}\gamma > 0. \tag{3}$$

2) OSP with OFP level $\rho_0 = \frac{1}{2}\left(\frac{m_{11}}{m_{21}} + \frac{m_{12}}{m_{22}}\right) > 0$, if M is chosen such that

$$m_{21} \geq m_{22}\gamma > 0, \quad m_{11}m_{22} > m_{12}m_{21} > 0. \tag{4}$$

3) ISP with IFP level $\nu_0 = \frac{1}{2}(\frac{m_{21}}{m_{11}} + \frac{m_{22}}{m_{12}}) > 0$, if M is chosen such that

$$m_{11} \geq m_{12}\gamma > 0, \quad m_{12}m_{21} > m_{11}m_{22} > 0. \tag{5}$$

4) VSP with passivity levels $\delta_0 = \frac{1}{2}\frac{m_{11}}{m_{21}} > 0$ and $\epsilon_0 = \frac{a}{2}\frac{m_{21}}{m_{11}} > 0$, if M is chosen such that

$$m_{11} > 0, \quad m_{12} = 0, \quad m_{21} \geq \frac{m_{22}\gamma}{\sqrt{1-a}} > 0, \tag{6}$$

where $0 < a < 1$ is an arbitrary real number.

Proof: We provide the proof for the first item. The complete proofs are presented in [22]. For notational convenience, we denote $\langle u, y\rangle_T = \int_0^T u^T y \ \mathrm{d}t$. Since $u_0 = m_{11}u + m_{12}y$ and $y_0 = m_{21}u + m_{22}y$, it can be easily shown that

$$\begin{aligned} \langle u_0, y_0\rangle_T \ &= m_{11}m_{21}\langle u, u\rangle_T + m_{12}m_{22}\langle y, y\rangle_T \\ &+ \ (m_{11}m_{22} + m_{12}m_{21})\langle u, y\rangle_T, \end{aligned} \tag{7}$$

Since system G is finite gain stable with gain γ, we have

$$\langle y, y\rangle_T \leq \gamma^2 \langle u, u\rangle_T. \tag{8}$$

If M is chosen such that (3) is satisfied, then according to (7) and (8), we have

$$\begin{aligned} \langle u_0, y_0\rangle_T \ &= \ m_{11}^2\langle u, u\rangle_T - m_{22}^2\langle y, y\rangle_T \\ &\geq \ (m_{11}^2 - m_{22}^2\gamma^2)\langle u, u\rangle_T \\ &\geq \ 0. \end{aligned}$$

Therefore, the system $\mathbf{u}_0 \to \mathbf{y}_0$ is passive. ■

Remark 5: For the VSP case, we have $\delta_0\epsilon_0 = \frac{a}{4} < \frac{1}{4}$, which satisfies the necessary condition for the passivity levels given by $\epsilon_0\delta_0 \leq \frac{1}{4}$ [26]. The parameter a also provides us some freedom to design the passivity levels. For instance, if the parameter $a > 0$ is taken to be large (small), then we have a large (small) IFP level ϵ_0. □

In Theorem 3, the system G can be either linear or nonlinear provided that G is finite gain stable. In particular, system G can be the class of linear systems with input/output delay as we described in Section III. The choice of the transformation matrix M is not unique for guaranteeing the passivity or certain passivity levels of the system Σ_0.

B. Interconnected Systems

Consider the feedback interconnection of two systems Σ_0 and H as shown in Fig.5, where G is stable and given, r_1 denotes the reference input to the controller Σ_0 and r_2 denotes the disturbance input to the plant H. Our objective is to design the matrix M so that the interconnected system can be guaranteed to be passive or stable. We denote by Σ the interconnected system with mapping $[\mathbf{r}_1^T, \mathbf{r}_2^T]^T \to [\mathbf{y}_0^T, \mathbf{y}_2^T]^T$ in Fig.5. Assume that system G is finite gain stable with gain γ. Four cases are possible based on the passivity properties of the G and H. The following results can be obtained according to Theorem 1 and Theorem 3.

1) when both H and G are passive: We can choose $M = I$ (identity matrix). According to Theorem 1, system Σ is guaranteed to be passive.

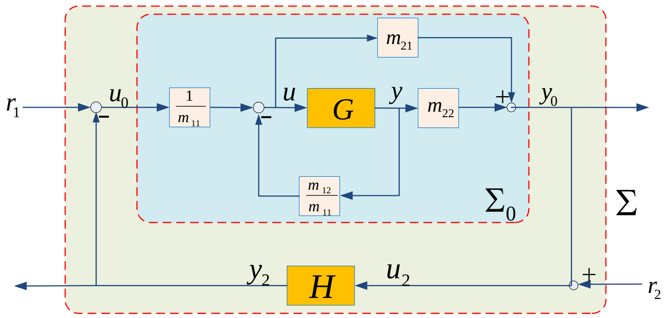

Fig. 5. Feedback Interconnection of Two Systems with Passivation M.

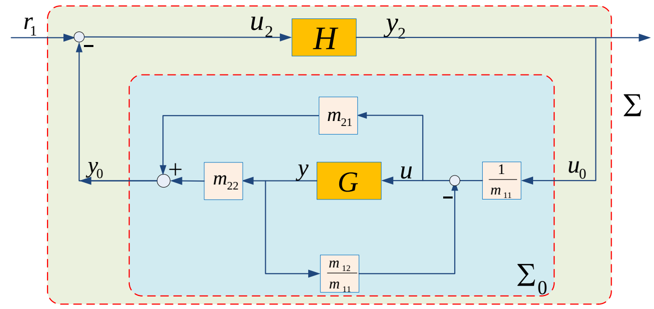

Fig. 6. Feedback Interconnection of Two Systems with only one input.

2) when H is passive, but G is not passive: We can choose M according to Theorem 3 so that G can be passivated through the transformation matrix M. For instance, if M is chosen such that (3) is satisfied, then system Σ_0 is passive and according to Theorem 1, system Σ is guaranteed to be passive. Furthermore, if M is chosen such that (6) is satisfied, then system Σ_0 is very strictly passive and according to Theorem 1, system Σ is passive and also finite gain stable.
3) when H is not passive, but G is passive: Assume that system H is IF-OFP($\delta < 0, \epsilon < 0$). We can choose M according to Theorem 3 so that the system Σ_0 have strictly positive passivity levels to compensate for the shortage of passivity in H. Specifically, we let

$$-\frac{a}{2\delta}m_{21} > \; m_{11} > \; -2m_{21}\epsilon,$$

and (6) be satisfied. Thus, the passivity levels of system Σ_0, denoted by (δ_0, ϵ_0), satisfy $\delta_0 + \epsilon > 0$ and $\epsilon_0 + \delta > 0$. Then, according to Theorem 1, system Σ is finite gain stable.
4) when both H and G are not passive: Assume that system H is IF-OFP($\delta < 0, \epsilon < 0$). Similar to the previous case, we can choose M so that Σ_0 is very strictly passive and system Σ is finite gain stable.

Consider the feedback configuration in Fig. 6 with only one control input r_1. We may guarantee the passivity of system Σ that maps r_1 to y_2 according to Theorem 2 and Theorem 3 even if the system H is non-passive.

Theorem 4: Consider the feedback configuration in Fig. 6, where r_1 can be seen as the disturbance to the plant H and G can be seen as a pre-designed stable controller. Assume that system H has OFP level $\rho < 0$. The system $\Sigma : \mathbf{r_1} \to \mathbf{y_2}$ is output strictly passive and finite gain stable,

1) if G has IFP level $\nu > -\rho > 0$ and $M = I$ (identity matrix). Furthermore, the gain of the system Σ is no larger than the value $\frac{1}{\rho+\nu}$;
2) if G is not passive and M is chosen such that

$$\nu_0 = \frac{1}{2}(\frac{m_{21}}{m_{11}} + \frac{m_{22}}{m_{12}}) > -\rho,$$

and (5) is satisfied. Furthermore, the gain of the system Σ is no larger than the value $\frac{1}{\rho+\nu_0}$. □

Remark 6: Note that in disturbance attenuation problems, we need the gain of the system $\mathbf{r_1} \to \mathbf{y_2}$ to be small. This can be done by setting $\nu_0 > 0$ to be large, which implies that the performance of disturbance attenuation would be improved by appropriate choice of the matrix M. □

System H and system G can be either linear or nonlinear in the above discussion. In particular, we assume that system G has a form of $G_0(s)\mathbf{e}^{-\tau s}$ as we described in Section III. For instance, system G could represent a human operator as a stable controller for the non-passive plant H.

Corollary 1: Consider the feedback configuration in Fig. 6, where r_1 can be seen as the disturbance to the plant H and $G = G_0(s)\mathbf{e}^{-\tau s}$, where $G_0(s)$ is stable, proper and rational and $\tau > 0$ denotes a constant delay. Assume that system H has OFP level $\rho < 0$. If M is chosen such that

$$\nu_0 = \frac{1}{2}(\frac{m_{21}}{m_{11}} + \frac{m_{22}}{m_{12}}) > -\rho,$$

and (5) is satisfied, then the system $\Sigma : \mathbf{r_1} \to \mathbf{y_2}$ is output strictly passive independent of the time delay τ. □

Remark 7: The above discussion can be seen as a generalization of Theorem 1 and Theorem 2, where the plant and the controller have to satisfy certain constraints, e.g. one of them has to be more than passive if the other one is less than passive. If such constraints cannot be satisfied, then we can design a transformation matrix M so that the passivity of the feedback system can still be guaranteed. □

V. Numerical Example

To illustrate the results, we provide a numerical example. Consider the feedback configuration in Fig. 5. The plant $H = \frac{s+1}{s-1}$ is unstable with OFP index given by $\rho = -1 < 0$. Thus, the plant H is non-passive. The controller $G = \frac{1}{0.2s+1}\mathbf{e}^{-0.1s}$ is finite-gain stable with gain $\gamma = 1$. The IFP index for G is given by $\nu = -0.293 < 0$ and thus the controller G is also non-passive. According to Theorem 1, if $M = I$ (i.e if $m_{11} = m_{22} = 1$, $m_{12} = m_{21} = 0$), then the closed-loop system Σ may be unstable. To see this, we build a Simulink model in Matlab. Let the input r_1 be a step with magnitude one as the reference input to the controller G and let r_2 be white noise with power 0.02 as the disturbance to the plant H. If $m_{11} = m_{22} = 1$, $m_{12} = m_{21} = 0$, then the output of the plant is plotted in Fig. 7. It can be seen from Fig. 7 that the closed-loop system goes unstable (of course, it is non-passive) if we set M to be the identity matrix.

To guarantee the stability of the closed-loop system, we can select M so that the system after passivation (i.e. system

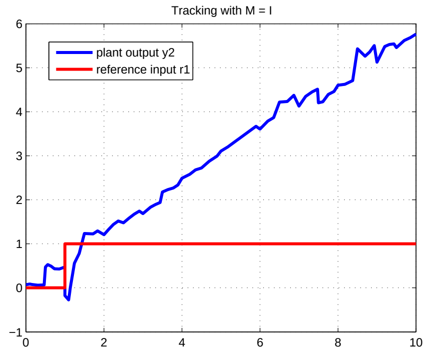

Fig. 7. The output of plant H if M is set to be the identity matrix.

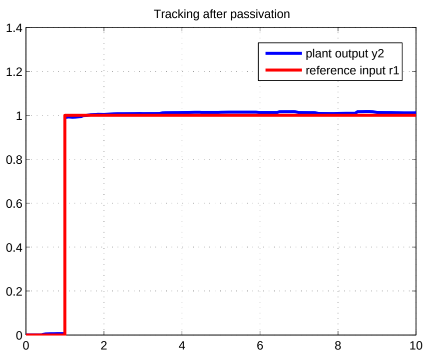

Fig. 8. The output of plant H if we select M according to Theorem 3.

Σ_0) is input strictly passive. According to Theorem 3, we let $m_{11} = 0.5$, $m_{12} = 0.4$, $m_{21} = 50$ and $m_{22} = 20$ so that (5) is satisfied and the IFP level for system Σ_0 is greater than 1. The output of plant H is plotted in Fig. 8. It can be seen that the closed-loop system is stable and the trajectory of the plant output y_2 is very close to the reference input r_1.

VI. Conclusion and Future Works

In this paper, we introduce a passivation method to passivate a given system by using an input-output transformation matrix. This matrix generalizes the commonly used methods of series, feedback and feedforward interconnections to passivate a system. Through an appropriate design of this matrix, positive passivity levels can be guaranteed for the system. Further, this transformation matrix allows the use of a non-passive system as a controller to guarantee the passivity and stability of a feedback configuration. In the future, we will investigate the following extensions: (i) when the transformation matrix has to optimize certain performance metrics in addition to satisfying the passivity requirements; (ii) when the time delay is time-varying instead of being a constant; (iii) when the transformation matrix contains transfer functions instead of constant gains.

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